Efficient Finite-Difference Time-Domain Modeling of Driven Periodic Structures

Dongying Li* and Costas D. Sarris
Edward St. Rogers Sr. Department of Electrical and Computer Engineering
University of Toronto, Toronto, Ontario, Canada
dongyingli@waves.utoronto.ca, cds@waves.utoronto.ca

Abstract - The paper proposes an efficient method to analyze the response of periodic structures to non-periodic excitations in the time domain, employing the Finite-Difference Time-Domain (FDTD) method. To that end, the array-scanning method, which has been previously associated with frequency-domain, integral formulations of periodic structure problems, is translated into the context of FDTD. Hence, the application of periodic FDTD techniques, based on the simulation of a single unit cell, to problems involving non-periodic source excitations is enabled. While a validation case study involves a microstrip line on an electromagnetic band-gap substrate, the utilization of this approach to metamaterial structure modeling is envisaged.

Introduction

The recent interest in negative refractive-index (NRI) metamaterials stems from Veselago’s early work predicting media exhibiting simultaneously negative permittivity and negative permeability [1]. Metamaterial structures, such as split-ring resonator and strip-wires [2] and planar-loaded transmission line grids [3], are artificial dielectrics, made of periodically arranged inclusions. Thus, the design of NRI metamaterials is closely associated with the formulation of periodic structure modeling techniques.

Dispersion analysis of periodic structures can be performed by applying the Floquet periodic boundary conditions (PBCs). Since the latter are naturally cast in the frequency-domain, their translation into the time-domain and incorporation in the Finite-Difference Time-Domain (FDTD) method is performed through techniques such as the split-field and the sine-cosine [4]. However, driven periodic structures, excited by non-periodic sources, cannot be modeled by any of these techniques alone. Despite the high practical interest of this problem, previous FDTD studies have been limited to simulating driven truncated periodic structures, adding unit cells until convergence is achieved. Evidently, the efficiency of this approach largely depends on the nature of the problem at hand, and may be excessively cumbersome from a computational point of view.

This problem has been previously considered through formulations based on the time-domain Green’s function of truncated periodic arrays [5], while [6] proposed the coupling of integral equation methods with the array scanning method [7] in order to study the interaction of a microstrip line with a periodic substrate. As the interest in applying FDTD to these problems, especially in the area of metamaterials, continuously grows, this paper presents a possible, FDTD-based alternative.

In particular, the implementation of the array-scanning method of [7] within the context of FDTD is proposed, in order to incorporate non-periodic excitations into the sine-cosine method. For each Floquet wave vector, fields generated by an array of sources in a periodic structure are determined via the sine-cosine method
and Fourier-transformed to the frequency domain. Then, the array-scanning method is performed to isolate the response of the structure to a single source.

**Formulation**

In this section, the proposed methodology is explained. Consider the computational domain of Fig. 1, depicting a periodic structure along the z-direction. The application of PBCs implies the periodic extension of both the structure and the source to infinity (Fig. 2). This periodic problem can be readily solved in FDTD, yet is different from the problem of interest, namely the response of this periodic structure to the source of Fig. 1 only (that is without its periodic counterparts that appear in Fig. 2). The PBCs are implemented via the sine-cosine method [4] in two separate grids, with sine and cosine time-dependence respectively. These conditions are:

\[ \vec{E}(x + p) = \vec{E}(x)e^{-jk_x p} \]  

where \( p \) is the periodicity in the \( x \) direction and \( k_x \) is the \( x \) component of the Floquet’s wave vector.

By applying the sine-cosine method, field components within the unit cell of the structure of Figure 2 (excited by an infinite array of sources) are obtained. The sources have a progressive phase shift of \( \phi = k_x p \), implicitly specified by the PBCs. Further post-processing is performed in the frequency domain to find the fields within the structure under the center (zero phase shift) source, by the array scanning method of [7]:

\[ \vec{E}_N(\omega) = \frac{1}{N} \sum_{p=0}^{N-1} \vec{E}[\omega, \phi_x(p)], \text{ with } \phi_x(p) = p \frac{2\pi}{N} \]  

Here, \( \vec{E}_N(\omega) \) is the frequency-domain electric field sampled within the unit cell at a frequency \( \omega \), obtained by Fourier transform of the field time-series, determined by the FDTD simulation. The summation of (2) effectively cancels out all excitations within \( N \) cells, except the zero phase one. In the implementation of the method, \( N \) is chosen to be large enough for the result to converge.

**Numerical Results**

The method is validated by the numerical example of a microstrip line printed on a three-layer periodic substrate (Fig. 3), that was presented in [6]. The thickness of each substrate is 0.635 mm, while the relative permittivity of the three layers is 9.8, 3.2 and 9.8, respectively. The center layer includes periodic rectangular
blocks of 6.5×6.5 mm, filled with air. The spacing between the neighboring blocks is 14 mm in both directions. A microstrip line with a width of 3 mm is placed on the substrate and is aligned with the implanted air-blocks. In [6], a band-gap centered at 5.36 GHz was found for this structure.

For the FDTD analysis, a unit cell of this structure including a microstrip line section is analyzed. The FDTD cell size is 0.5×0.5×0.318 mm. The thickness of each layer of the substrates is modeled by two FDTD cells. The computational domain is 28×28×30 cells. The microstrip line is excited by a Gabor pulse from 2 GHz to 10 GHz. The number of the time steps in the simulation is 8192.

When both the x and y directions are terminated into PBCs, the Brillouin diagram of the structure can be determined. Fig. 4 shows this diagram, as determined by the sine-cosine method, along with results from Ansoft’s HFSS. The agreement of the two sets of data validates the periodic analysis technique, which the proposed array scanning method is based upon.

Furthermore, a second simulation, where PBCs are applied in the x-direction only is carried out. Along the y-axis, the microstrip is terminated into Mur’s first order absorbing boundary conditions. The same Gabor source is used and 16 points are sampled within the irreducible Brillouin zone of the structure, now being considered as a one-dimensional periodic one. Sampling and Fourier transforming incident and transmitted vertical electric fields at the input and output ports of the microstrip and using those to apply (2) at the N=16 points considered, provides for the calculation of the transmission coefficient S_{21}. The result of this process is depicted in Fig. 5, where the band-gap expected at 5.2 GHz is also apparent.

For comparison purposes, results produced by an FDTD simulation of a truncated structure with the size of 73×98 mm. 2×5 of air blocks implanted in the x and y direction are appended. Despite the difference in the structures simulated in the two cases, the reasonable agreement of their transmission coefficient results is expected, because of the strong confinement of microstrip fields.

It is necessary to note also that although the array-scanning method cancels out the effect of the periodic source excitations, the boundary conditions of the periodic microstrip lines still exist in the adjacent cells. This is a side-effect of the fact that the FDTD formulation does not employ surface currents on the microstrip as effective sources, although it could, but voltage sources at the input port. Hence, a minor (due to the aforementioned confinement), yet undesired coupling effect is caused. A rigorous solution to this problem is currently under development. Fundamentally, the FDTD analysis of the unit cell can provide all the information needed for the application of the array scanning technique.

**Conclusion**

The problem of applying FDTD to the analysis of driven periodic structures has been considered. A solution to this problem can be offered by the combination of the FDTD with the array scanning method. Hence, the possibilities stemming from fast periodic FDTD simulations (based on the discretization of a unit cell) are extended beyond the dispersion analysis to the determination of Green’s
functions of periodic structures. These possibilities evidently encompass a wide range of interesting applications in the area of metamaterials.

![Image 3. Single microstrip line on three layers of substrates implanted with periodic air blocks.]

Fig. 3. Single microstrip line on three layers of substrates implanted with periodic air blocks.

![Image 4. The Brillouin diagram of the microstrip line on one cell of periodic substrates.]

Fig. 4. The Brillouin diagram of the microstrip line on one cell of periodic substrates.

![Image 5. $S_{21}$ of the single microstrip line on periodic substrates using array scanning and $S_{21}$ of the corresponding truncated structure.]

Fig. 5. $S_{21}$ of the single microstrip line on periodic substrates using array scanning and $S_{21}$ of the corresponding truncated structure.

Acknowledgement

This work has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC), through a Strategic Grant.

REFERENCES