COMPUTER DESIGN OF SUPER-ORTHOGONAL SPACE-TIME TRELLIS CODES

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Abstract

Super-orthogonal space-time trellis codes (SOSTTC) designed by hand can significantly improve the performance of space-time trellis codes. This paper introduces a new representation of SOSTTCs based on a generator matrix that allows a systematic and exhaustive search of all possible codes. This will verify that some of the known codes are optimal, and provides a means to easily implement encoders and decoders with a large number of states without relying on a graphic representation.

Keywords: *space-time coding, MIMO (Multiple-Input Multiple Output), diversity*

I. INTRODUCTION

Space-time trellis coding [1] provides a diversity gain and a coding gain to wireless communications systems employing multiple transmit antennas, thereby improving the error performance and the data rate of these systems. In [2], Jafarkhani and Seshadri introduced a new structure called Super-orthogonal space-time trellis codes (SOSTTCs) which can yield an additional coding gain of more than 2 dB while providing the highest possible rate. By concatenating a space-time block coding scheme with an outer trellis code, the diversity gain of the space-time block code is maintained and a coding gain is realized. Since the trellis coding gain is achieved through redundancy, the signal set of the inner code must be expanded to maintain full transmission rate. Note that it is not the signal constellation that is expanded but the set of orthogonal matrices, i.e., the number of available orthogonal matrices is increased. In order to accomplish this, Jafarkhani and Seshadri proposed a parameterized class of space-time block codes.

Beginning with the original space-time block code of Alamouti [3], a class of orthogonal designs or transmission matrices for two transmit antennas was created as follows:

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$$C(x_1, x_2, \theta) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix} \qquad (1)$$

where x_1 and x_2 are selected by input bits. The first row corresponds to the symbols transmitted in time slot 1, the second row, to the symbols in time slot 2. The first column corresponds to the symbols transmitted by antenna 1, the second column to the symbols of antenna 2. By varying θ, multiple orthogonal block codes can be constructed, and a super-orthogonal code is formed from the union of these codes. For M-PSK signal constellations, the signals x_1 and x_2 can be represented by $e^{\frac{j2\pi l}{M}}$, $l = 0,1,\cdots,M-1$ and θ can take on the values $\theta = 2\pi l'/M$, where $l' = 0,1,\cdots,M-1$, without expanding the signal set. To maximize the coding gain, the matrix sets are partitioned in a manner similar to Ungerboeck's method [4], but using the determinant criteria from [1] rather than Euclidian distance. Note that using the trace of the difference matrix would produce the same partitioning.

In addition to Jafarkhani and Seshadri's work [2], Siwamogsatham and Fitz [5], [6], and Ionescu [7] have also independently developed methods to expand the orthogonal matrix set. Siwamogsatham and Fitz's method applies a similar unitary transformation to the original orthogonal design to produce additional sets and Ionescu uses cosets which are equivalent to the rotations of [2]. For the QPSK signal constellation, the A, J, B and K sets of Siwamogsatham and Fitz correspond (respectively) to the θ = 0, $\pi/2$, π, and $3\pi/2$ rotations in [2] and the two cosets in [7], [8] correspond to the θ = 0 and π rotations.

The trellises presented in [2], [5]-[6] are all hand-designed to maximize the determinant of the difference matrix, while those in [7] also attempt to maximize a criterion based on the trace of the difference matrix. Rules for the code design are given. These rules are similar to Ungerboeck's rules [4] but use the determinant instead of the

Euclidean distance. While the hand-designed codes perform well, it is not known if they are optimal. Also, hand-designing codes for a large number of states becomes tedious. Furthermore, these codes are described using a graphic representation of the trellis, where all possible outputs for each state are listed [2], [5]-[8]. Hence, implementing the encoder or decoder requires listing the outputs of all the states, which can become quite cumbersome for a large number of states.

This paper presents a method for performing a computer search of codes. Using a compact matrix notation, similar to that in [9] a computer search is performed to find trellises that maximize both the determinant and trace criteria. Simulations are then performed to compare the performance of these new codes to that of the known ones. The matrix representation allows a more compact representation of the code, making the implementation of the encoder and decoder easier.

The organization of the paper is as follows. Section II provides the representation of SOSTTCs using a generator matrix. Section III discusses the details of the decoding and Section IV provides simulation results. Section V includes some concluding remarks.

II. CODE SEARCH

Maximizing the determinant and trace produced by a SOSTTC has proven effective in the design of optimal codes with a small number of states. However, as the number of states in the code increases, the complexity involved in the design of these codes increases proportionally and it becomes more difficult to produce optimal codes. In order to facilitate this, we show in this Section that the code can be represented by a simple generator matrix, allowing for a systematic and exhaustive computer search of all possibilities. This matrix representation is similar to that used for the search of space-time trellis-coded modulation as discussed in [9].

Fig. 1 shows a simple example of the graphical representation of an 8-state QPSK SOSTTC for a rate of 2 bits/s/Hz; this trellis has two parallel branches. Using the set partitioning for QPSK in [2], S_{000} means that x_1, x_2 in the transmission matrix (1) are equal to 0, 0 and 2, 2, respectively for the first and the second parallel branch. It can be seen that as the number of states gets larger, describing the code graphically becomes increasingly cumbersome.

In general, the generator matrix **G** of a SOSTTC for n_T transmit antennas is of the form r rows by n_T columns, where r is determined by the sum of the number of input bits m at each trellis level and the number of bits s needed to represent each state. Note that in this example each trellis level corresponds to a transmission matrix (1) and hence, a trellis level corresponds to two time slots. s is given by

$$s = \log_2 N$$

where N is the number of states. For an M-PSK constellation with the highest possible rate, m is given by

$$m = 2\log_2 M$$

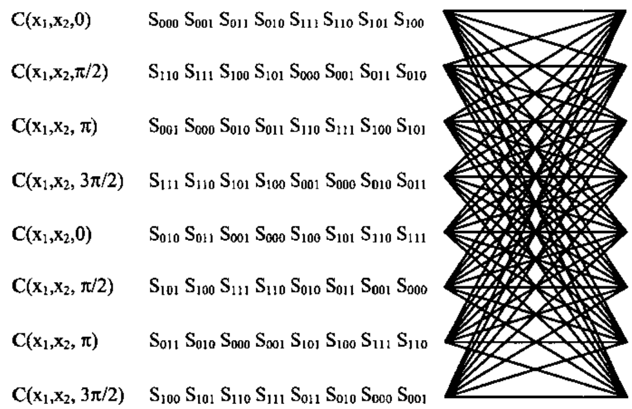

Fig. 1. 8-state SOSTTC with 2 parallel branches and a rate of 2 bits/s/Hz using QPSK.

As an example, an 8-state code requires three bits to represent each state. For QPSK with a rate of 2 bits/s/Hz and 2 transmit antennas, four bits are input at each trellis level. Hence, the generator matrix has 7 rows and 2 columns, as shown below

$$\mathbf{G} = \begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \\ a_5 & a_6 \\ a_7 & a_8 \\ a_9 & a_{10} \\ a_{11} & a_{12} \\ a_{13} & a_{14} \end{pmatrix} \quad (2)$$

where $a_i = 0$, 1, 2 or 3, and $i = 1, 2, \ldots, 14$. For an M-PSK modulation, $a_i = 0, 1, \ldots, M-1$.

The information sequence to be transmitted is $\mathbf{u} = (u_1, \ldots u_n)$, where $u_i = 0$ or 1. The number of information bits influencing the transmission matrix (1) of a given trellis level is equal to $(m+s)$, the number of rows of **G**. Let these bits be represented by a vector $\mathbf{u}_l$, where l is the trellis level. $\mathbf{u}_l$ can also be considered as a shift-register. When **G** is multiplied (modulo 4) by $\mathbf{u}_l$, we obtain two symbols x_1 and x_2 which are then mapped to a transmission matrix using the mapping scheme described by (1). The rotation θ depends on the current state and is determined in advance. As an example, if two rotations are used, all transitions originating from an odd state are assigned $\theta = 0$, and those from an even state, $\theta = \pi$. Hence, all possible outputs or transmission matrices of the SOSTTC can be obtained from $\mathbf{u}_l\,\mathbf{G}$. There is no need to list all possible outputs from each state, making the implementation of the encoder and decoder much easier. The generator matrix for the code in Fig. 1 is

$$G^T = \begin{pmatrix} 1 & 1 & 2 & 2 & 3 & 2 & 3 \\ 2 & 3 & 0 & 2 & 3 & 0 & 2 \end{pmatrix}$$

In this example of an 8-state code with two parallel branches, the bits of $\mathbf{u}_l$ can be considered as follows

$$\mathbf{u}_l = \begin{bmatrix} \overbrace{u_7 \quad u_6 \quad u_5}^{NextState} & \overbrace{u_4}^{ParallelBranch} & \overbrace{u_3 \quad u_2 \quad u_1}^{CurrentState} \end{bmatrix} \qquad (3)$$

Upon moving to the next transition the current state and parallel branch bits are discarded, the next state bits become the current state and four new input bits are shifted into the $\mathbf{u}_l$ register. The description of the bits of $\mathbf{u}_l$ given in (3) is not valid when the number of parallel branches or states changes and a lookup table must be used. For example, if an 8-state SOSTTC with 4 parallel branches is desired as shown in Fig. 2, a vector $\mathbf{u}_l$ with the same number of bits can be used but a lookup table is required to determine the three bits representing the next state from the current state and the input bits. At the next trellis level, these three bits become the current state bits of $\mathbf{u}_l$ and four new input bits are shifted into $\mathbf{u}_l$. The difference between this code and the code of Fig. 1 is that here the current state bits at the next trellis level are not simply the bits $u_7u_6u_5$ from the previous level but have been obtained from the lookup table. Fig. 2 shows the SOSTTC with 4 parallel branches and 2 rotations obtained using

$$G^T = \begin{pmatrix} 3 & 3 & 2 & 0 & 3 & 3 & 1 \\ 1 & 2 & 0 & 2 & 0 & 3 & 3 \end{pmatrix}$$

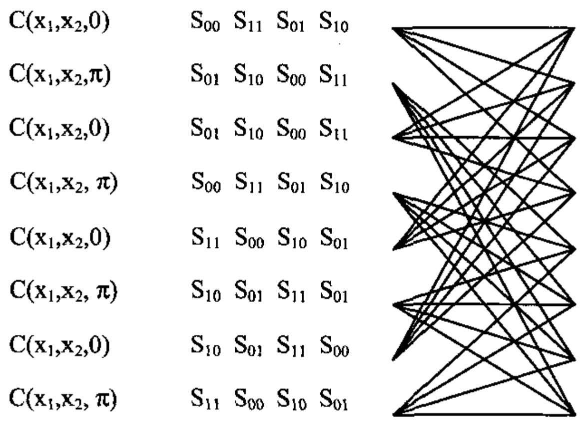

Fig. 2. 8-state SOSTTC with 4 parallel branches and a rate of 2 bits/s/Hz using QPSK.

By combining the approach of matrix multiplication with a lookup table, every possible 8-state trellis shape and permutation can be generated and tested. The lookup table is rather compact compared to what would be required to describe the whole trellis and can be determined in advance.

This idea can easily be extended to generate a code with any number of states or antennas for any signal constellation by increasing r and adding bits to the shift register accordingly. Hence, the matrix representation presented earlier is an efficient and compact way to describe a SOSTTC. Furthermore, it allows a computer design of a SOSTTC with a large number of states by performing an exhaustive or random search of all possible generator matrices $\mathbf{G}$ and selecting the matrix $\mathbf{G}$ which yields the best characteristics.

An exhaustive search among all possible 4^{14} or 268,435,456 matrices $\mathbf{G}$ for 8-state SOSTTCs has yielded thousands of codes with good characteristics (largest determinant and trace). There has been no SOSTTC which presented the largest determinant but not the largest trace. Many of these codes are equivalent, i.e., they generate the same outputs. The duplicate ones have been determined using a computer search and eliminated. The remaining codes have been simulated at a signal-to-noise ratio of 14 dB since the error probabilities of space-time codes usually do not crossover. In fact since all of the codes provide full diversity, the SNR-BER curve is a line at high SNRs (in a logarithmic scale) and therefore one point in the SNR-BER plane is good enough for comparison. The best new codes are presented in Table 1. It can be seen from Table 1 that the search has yielded codes which exhibit several improvements in both the minimum determinant and minimum trace compared to the codes in [5], [7]. The number of transitions of the paths that diverge from a state and remerge to that state is indicated in Table 1. The performance of the new codes will be compared to that of some known codes from [5], [7] in Section IV.

III. DECODING

Maximum-likelihood decoding is employed at the receiver end to reconstruct the transmitted signal. The decision metric was derived using the same method as in [10], and using Jafarkhani and Seshadri's [2] transmission matrix given by (1).

Let r_t^j be the signal received by antenna j at time t and let $\alpha_{i,j}$ be the path gain from transmit antenna i to receive antenna j. The signal r_t^j is given by [10]

$$r_t^j = \sum_{i=1}^{n_T} \alpha_{i,j} c_t^i + \eta_t^j$$

where c_t^i are the signals transmitted from the n_T antennas at time slot t and η_t^j are the independent noise samples.

With two transmit and m receive antennas the maximum-likelihood detection amounts to finding the values of s_1 and s_2 which minimize the decision metric

$$\sum_{k=1}^{m}\left(\left|r_1^k-\alpha_{1,k}s_1e^{j\theta}-\alpha_{2,k}s_2\right|^2+\left|r_2^k+\alpha_{1,k}s_2^*e^{j\theta}-\alpha_{2,k}s_1^*\right|^2\right)$$

which, upon expanding and deleting terms that are independent of the codewords, produces

$$\begin{aligned}-\sum_{k=1}^{m}\Big[&r_1^k\alpha_{1,k}^*s_1^*e^{-j\theta}+r_1^k\alpha_{2,k}^*s_2^*+\left(r_1^k\right)^*\alpha_{1,k}s_1e^{j\theta}+\left(r_1^k\right)^*\alpha_{2,k}s_2\\&-r_2^k\alpha_{1,k}^*s_2e^{-j\theta}+r_2^k\alpha_{2,k}^*s_1-\left(r_2^k\right)^*\alpha_{1,k}s_2^*e^{j\theta}+\left(r_2^k\right)^*\alpha_{2,k}s_1^*\Big]\\&+\left(\left|s_1\right|^2+\left|s_2\right|^2\right)\sum_{k=1}^{m}\sum_{i=1}^{2}\left|\alpha_{i,k}\right|^2\end{aligned}$$

This expression decomposes into two parts, the first of which is only a function of s_1

$$-\sum_{k=1}^{m}\left[r_1^k\alpha_{1,k}^*s_1^*e^{-j\theta}+\left(r_1^k\right)^*\alpha_{1,k}s_1e^{j\theta}+r_2^k\alpha_{2,k}^*s_1+\left(r_2^k\right)^*\alpha_{2,k}s_1^*\right]$$

$$+\left|s_1\right|^2\sum_{k=1}^{m}\sum_{i=1}^{2}\left|\alpha_{i,k}\right|^2$$

the second of which is only a function of s_2

$$-\sum_{k=1}^{m}\left[r_1^k\alpha_{2,k}^*s_2^*+\left(r_1^k\right)^*\alpha_{2,k}s_2-r_2^k\alpha_{1,k}^*s_2e^{-j\theta}-\left(r_2^k\right)^*\alpha_{1,k}s_2^*e^{j\theta}\right]$$

$$+\left|s_2\right|^2\sum_{k=1}^{m}\sum_{i=1}^{2}\left|\alpha_{i,k}\right|^2$$

Recognizing that the second and fourth terms of the previous expressions are the complex conjugates of the first and third terms respectively, the metrics reduce to

$$-\sum_{k=1}^{m}\left[2\Re\left(r_1^k\alpha_{1,k}^*s_1^*e^{-j\theta}+r_2^k\alpha_{2,k}^*s_1\right)\right]+\left|s_1\right|^2\sum_{k=1}^{m}\sum_{i=1}^{2}\left|\alpha_{i,k}\right|^2$$

for s_1, and

$$-\sum_{k=1}^{m}\left[2\Re\left(r_1^k\alpha_{2,k}^*s_2^*-r_2^k\alpha_{1,k}^*s_2e^{-j\theta}\right)\right]+\left|s_2\right|^2\sum_{k=1}^{m}\sum_{i=1}^{2}\left|\alpha_{i,k}\right|^2$$

for s_2.

Due to the orthogonality of the transmission matrix, the decoder can minimize the metrics for s_1 and s_2 separately, eliminating the need to compare all possible pairs of codewords.

For trellises with multiple parallel branches, the decoding is performed in two steps. At each decoding interval, the decoder first scans through the parallel branches and picks the branch with the smallest branch metric to represent the set. The other parallel branches are pruned off resulting in a trellis with only one branch per transition. The Viterbi decoding algorithm is then applied to this reduced trellis to find the path with the minimum accumulated metric. The initial pruning step greatly reduces the number of calculations required at this step, as the accumulated metric is not computed for the pruned off branches.

IV. SIMULATIONS RESULTS

Computer simulations using the techniques described in the previous section have been performed to determine the performance of our codes and select the best ones. Each frame consists of 130 transmissions out of each transmit antenna.

Computer simulation of the 8-state codes with one receive antenna yielded little or no improvement on already published codes [5], [7] as shown in Fig. 3. This would indicate that the known codes are optimal for all 8-state QPSK trellis shapes. This may be due to the fact that the known codes and the new codes with 4 and 8 parallel branches have the same determinant for parallel branches. To improve performance it would be necessary to search codes with a greater number of states. Preliminary results for 32-state codes are promising. The performance of the known 16-state codes is almost similar to that of the 8-state codes as reported in [5], [7]. Our new 16-state codes did not offer any improvement either.

V. CONCLUSION

A matrix representation of super-orthogonal space-time trellis codes has been presented in this paper. This representation allows a more compact description of SOSTTCs which does not require the listing of all possible outputs from each state and therefore, simplifies the implementation of encoders and decoders for codes with a large number of states. Furthermore, this representation can be used for a computer search of good SOSTTCs. An exhaustive search would yield the optimum code but may require an excessive amount of time if the number of states is large. An exhaustive search of 8-state codes has provided codes with an error performance similar to that of some known codes, which means that these codes designed by hand and the rules of thumb used to design them were excellent. However, codes with a larger number of states may benefit from a computer design.

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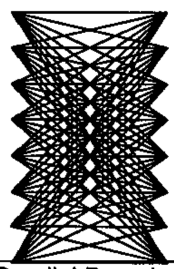
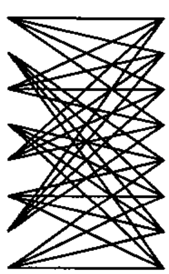
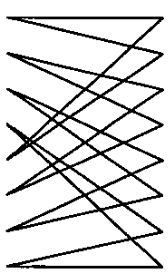
Table 1: Characteristics of 8 State QPSK Codes

TRELLIS SHAPE	DESIGNER	ROTATIONS	2 TRANSITIONS		3 TRANSITIONS		4 TRANSITIONS	
(2 Parallel Branches)			Min Det	Min Trace	Min Det	Min Trace	Min Det	Min Trace
	Code 1	4	28	12	28	12		
	$G^T = \begin{pmatrix} 1 & 1 & 0 & 2 & 3 & 2 & 1 \\ 1 & 3 & 3 & 2 & 1 & 2 & 1 \end{pmatrix}$							
	Code 2	2	36	12				
	$G^T = \begin{pmatrix} 2 & 1 & 0 & 2 & 1 & 0 & 1 \\ 3 & 1 & 2 & 2 & 0 & 1 & 1 \end{pmatrix}$							
(4 Parallel Branches)								
	IMYL [8]	2	36	12				
	Code 1	4	52	16	32	16		
	$G^T = \begin{pmatrix} 1 & 1 & 2 & 0 & 0 & 1 & 0 \\ 1 & 2 & 2 & 2 & 3 & 0 & 3 \end{pmatrix}$							
	Code 2	2	64	16				
	$G^T = \begin{pmatrix} 1 & 0 & 0 & 2 & 1 & 1 & 2 \\ 1 & 1 & 2 & 2 & 0 & 3 & 2 \end{pmatrix}$							
(8 Parallel Branches)								
	SF [5]	4	-	-	-	-		
	Code 1	4	-	-	-	-	80	20
	$G^T = \begin{pmatrix} 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}$							

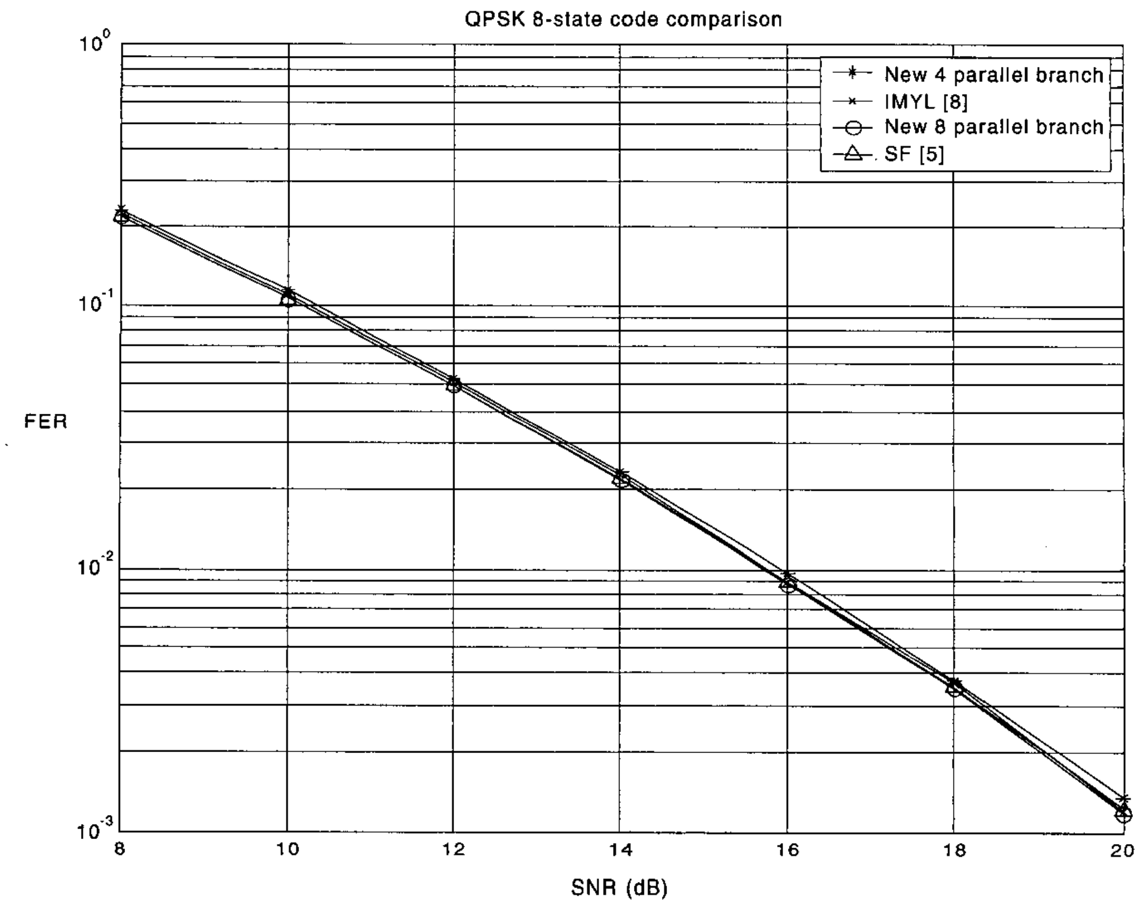


Fig. 3. Comparison of the frame error rates of 8-state codes.