2011 IEEE International Symposium on Computer-Aided Control System Design (CACSD)
Part of 2011 IEEE Multi-Conference on Systems and Control
Denver, CO, USA. September 28-30, 2011

Distributed Connectivity Preservation of a Team of Single Integrator Agents Subject to Measurement Error

Farzad Salehisadaghiani, Amir Ajorlou, and Amir G. Aghdam

Abstract— **This paper deals with the connectivity preservation of multi-agent systems with state-dependent error in distance measurement. It is assumed that upper bounds on the measurement error and also its rate of change as a function of distance are available. A general class of distributed control strategies is then proposed for the distance-dependent connectivity preservation of the agents in the network. It is shown that if two neighboring agents are initially located at a distance closer than the required connectivity range, they are guaranteed to remain in the connectivity range at all times. The effectiveness of the proposed control strategies in consensus and containment problems is demonstrated by simulation.**

I. INTRODUCTION

There has been increasing interest in the control of multi-agent systems in the past decade due to their wide range of applications in real-world systems. Such applications include, for example, mobile sensor networks, air traffic control, and automated highway systems, to name only a few [1], [2], [3], [4], [5]. In this type of problem, it is desired to achieve a global objective such as consensus and containment by developing distributed control laws [6], [7], [8], [9], [10], [11]. For instance, in the consensus problem it is aimed that all agents converge to a single point in the state space. In the containment problem, on the other hand, it is desired that the followers converge to the convex hull of the leaders.

One of the common goals of any multi-agent control problem is the connectivity of the network. A pair of agents is said to be connected (via a communication link) if they are located in a sufficiently small distance from each other. The distributed connectivity preservation problem has been thoroughly investigated in prior literature [12], [13], [14], [15], [1], [16], [17], [18], [19], [20], [21]. Unbounded potential functions are often used in these papers, where an unbounded potential field is generated between any two agents which tend to move away from the connectivity range. However, such approaches may not be as effective in practice because actuators cannot handle infinite control signal. To remedy this shortcoming, a general class of bounded distributed potential functions with the connectivity preserving property is proposed in [22]. The idea behind this technique is to design the potential functions in such a way that when an edge is about to lose connectivity, the gradient of the potential function lies in the direction of the edge, aiming to reduce its size. The work [22] proposes an effective alternative to conventional unbounded potential functions. However, this work and all of the papers cited above assume that the distance measurement (which is required in any connectivity control law) has no error. It is known that all measurements are subject to error in practice, and this can negatively affect the control performance.

This paper investigates the problem of connectivity preservation for a team of single integrator agents subject to distance measurement error. It is assumed that upper bounds exist on the magnitude of the measurement error and its rate of change. These upper bounds are defined to ensure that the control signal will not be saturated. A general class of distributed control strategies is then proposed for the agents which has the connectivity preservation property. It is shown that if two neighboring agents are located within a more conservative distance from each other (compared to the case of the error-free measurements), they will remain in the connectivity range at all times. While the results of this work are presented for a static information flow graph, they can be easily extended to the more general case of dynamic edge addition.

The remainder of this paper is organized as follows. In Section II, some notations and definitions are given and also the problem statement is provided. The connectivity preserving control design is elaborated in Section III. Simulation results are presented in Section IV to demonstrate the effectiveness of the proposed control strategy. Finally, concluding remarks are drawn in Section V.

II. PROBLEM FORMULATION

The following definitions are borrowed from [22], and will prove convenient in presenting the main results.

Definition 1: [22] For a real or vector valued function $f(t)$, the index of $f(t)$ at time t, denoted by $\rho(f(t))$, is defined to be the smallest natural number n for which $f^{(n)}(t) \neq 0$, where $f^{(n)}(t)$ is the n-th derivative of $f(t)$ with respect to time.

Definition 2: The function f is said to be of class C^k if the derivatives $f^{(1)}, \dots, f^{(k)}$ exist and are continuous. A function of class C^∞ is referred to as a smooth function.

Definition 3: [22] *Multinomial coefficients* are defined as:

$$\binom{k}{r_1, r_2, \dots, r_\mu} := \frac{k!}{r_1! r_2! \dots r_\mu!}$$

where $r_1, r_2, \dots, r_\mu$ are nonnegative integers, and $k = r_1 + r_2 + \dots + r_\mu$. In the special case when $\mu = 2$, these coefficients are called *bionomial coefficients*, and are given by

The authors are with the Department of Electrical and Computer Engineering, Concordia University, 1455 De Maisonneuve Blvd. W., Montréal, Québec, H3G 1M8, Canada {fa_saleh, a_ajor, aghdam}@ece.concordia.ca

This work has been supported in part by the Natural Sciences and Engineering Research Council of Canada (NSERC) under Grant STPGP-364892-08, and in part by Motion Metrics International Corp.

$\binom{k}{r_1, r_2} = \binom{k}{r_1} = \binom{k}{r_2}$.

Notation 1: For any given function $h(x,y)$, the derivative $\frac{\partial h}{\partial y}h(x,y)_{|y=0}$ is represented by $\frac{\partial h}{\partial y}h(x,0)$ (and similarly, $\frac{\partial h}{\partial x}h(0,y) = \frac{\partial h}{\partial x}h(x,y)_{|x=0}$). Notice that while this may be considered standard notation, it is emphasized here for the sake of clarity, and to avoid possible confusion.

Consider a set of n single integrator agents in a plane with the control law of the form:

$$\dot{q}_i(t) = u_i = -\frac{\partial h_i}{\partial q_i} \tag{1}$$

where $q_i(t)$ denotes the position of agent i in the plane at time t, and h_i's are distributed potential functions. Denote with $G = (V,E)$ the information flow graph, with $V = \{1,\dots,n\}$ the vertices, and with $E \subset V \times V$ the edges. It is assumed that the information flow graph G is connected and undirected.

Definition 4: The set of neighbors of agent i, denoted by $N_i(G)$, is a set consisting of any vertex in G which is connected to vertex i by an edge, i.e., $N_i := \{j|(i,j) \in E\}$. Moreover, the degree of the set of neighbors N_i is denoted by $d_i(G)$.

Assume that each agent can only use the relative position of its neighbors in its local control law. Let the error function for distance measurement between any two agents i and j be denoted by $\varepsilon_{ij}(\|q_i - q_j\|)$, which is a smooth positive scalar function of distance ($\|\cdot\|$ denotes the Euclidean norm) and it occurs when agent i is sensing the position of agent j. The error function is assumed to be bounded with a known bound m, as follows:

$$|\,\varepsilon_{ij}(\|q_i - q_j\|)\,| \le m$$

Furthermore, the rate of change of the error function is assumed to be bounded by 1, i.e.:

$$\left|\frac{\partial \varepsilon_{ij}}{\partial \|q_i - q_j\|}\right| < 1$$

Assume that L is a real distance between the two agents i and j, but the distance $L - \varepsilon_{ij}(\|q_i - q_j\|)$ is measured instead. In the error-free connectivity preservation problem (i.e., perfect measurements) two agents i and j are said to be in the connectivity range if $\|q_i - q_j\| \le d$, where d is a pre-specified positive real number referred to as the *critical distance* [22]. However, in a practical setting, a more conservative condition needs to be adopted in order to ensure that connectivity is preserved in the presence of measurement error. More precisely, if the distance d is measured by a sensor, normally implying that the corresponding agent is in the connectivity range, the real distance can be as great as $d+m$, resulting in loss of connectivity. So, the critical distance in this case is adjusted to $d - m + \varepsilon$, where $m < d$ (note that the distance $d - m + \varepsilon$ is always less than d). The objective is to design a class of distributed potential function which preserve connectivity in this case. More precisely, it is desired to derive conditions under which if $\|q_i(0) - q_j(0)\| \le d - m$ for all $(i,j) \in E$, then $\|q_i(t) - q_j(t)\| \le d$ for all $(i,j) \in E$ and all $t \ge 0$.

III. Connectivity Preserving Controller Design

For every agent i, define:

$$\sigma_i(t) := \frac{1}{2}\sum_{j\in N_i(G)} (\|q_i(t) - q_j(t)\| - \varepsilon_{ij})^2 \tag{2}$$

$$\pi_i(t) := \frac{1}{2}\prod_{j\in N_i(G)} (d-m)^2 - (\|q_i(t) - q_j(t)\| - \varepsilon_{ij})^2 \tag{3}$$

$$\pi_{ij}(t) := \prod_{\substack{k\in N_i(G)\\ k\neq j}} (d-m)^2 - (\|q_i(t) - q_k(t)\| - \varepsilon_{ik})^2 \tag{4}$$

Consider a set of distributed smooth potential functions of the form $h_i(\sigma_i, \pi_i)$ with the following properties:

$$\frac{\partial h_i}{\partial \sigma_i}(\sigma_i, 0) = 0,\ \frac{\partial h_i}{\partial \pi_i}(\sigma_i, 0) < 0, \qquad \text{for } \sigma_i \in \mathbb{R}^+ \tag{5}$$

These are the same potential function used in [22] for designing connectivity preserving control laws in the ideal (error-free) case. The aim of this section is to show that using this type of potential function and under some conditions, the control law (1) is connectivity preserving. Using the formula $\frac{\partial h_i}{\partial q_i} = \frac{\partial h_i}{\partial \sigma_i}\frac{\partial \sigma_i}{\partial q_i} + \frac{\partial h_i}{\partial \pi_i}\frac{\partial \pi_i}{\partial q_i}$, one can rewrite the control law (1) as:

$$\begin{aligned}\dot{q}_i \quad = \quad & -\sum_{j\in N_i(G)} (((q_i - q_j) + \frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2}) - \frac{\partial}{\partial q_i}(\varepsilon_{ij}\|q_i - q_j\|)) \\ & (\frac{\partial h_i}{\partial \sigma_i} - \frac{\partial h_i}{\partial \pi_i}\pi_{ij}))\end{aligned} \tag{6}$$

Let $t_0 = \inf\{t|\exists(i,j) \in E : \|q_i - q_j\| > d - m + \varepsilon_{ij}\}$. Clearly, $\|q_i(t) - q_j(t)\| \le d - m + \varepsilon_{ij}$, for all $(i,j) \in E$ and $t \le t_0$. Construct a graph $G_d = (V_d, E_d)$ as the union of those edges $(i,j) \in E$ for which $\|q_i(t_0) - q_j(t_0)\| = d - m + \varepsilon_{ij}$. Define $s_{ij}(t) = (\|q_i(t) - q_j(t)\| - \varepsilon_{ij})^2$, for all $(i,j) \in E_d$. To prove the above claim, it suffices to show that there is a neighborhood of t_0 in which every s_{ij} is either decreasing or fixed.

The following lemmas will be used in the proof of the main results.

Lemma 1: Consider a real or vector valued function f of $x(t)$ in C^k. If $x^{(r)}(t_0) = 0$ for all $r \in \{1,2,\dots,k\}$, then:

$$f^{(r)}(x(t_0)) = 0, \forall r \in \{1,2,\dots,k\}$$

Proof. The proof is straightforward and is omitted due to space limitations. ■

Lemma 2: Suppose that $q_i^{(r)}(t) = q_j^{(r)}(t) = 0$, for all $r \in \{1,\dots,k-1\}$ and some t. Then:

$$\begin{aligned}(\varepsilon_{ij}^2 - 2\varepsilon_{ij}\|q_i - q_j\|)^{(k)} = {} & (2\frac{\varepsilon_{ij}}{\|q_i - q_j\|}\frac{\partial \varepsilon_{ij}}{\partial \|q_i - q_j\|} \\ & -2\frac{\partial \varepsilon_{ij}}{\partial \|q_i - q_j\|} - 2\frac{\varepsilon_{ij}}{\|q_i - q_j\|})(q_i(t) - q_j(t))^T \\ & (q_i^{(k)}(t) - q_j^{(k)}(t))\end{aligned} \tag{7}$$

Proof. It is required to take the derivative of the error terms iteratively, in order to find the k-th derivative. This can be carried out as follows:

$$\frac{d}{dt}(\varepsilon_{ij}^2-2\varepsilon_{ij}\|q_i-q_j\|)=\dot{\varepsilon}_{ij}(\varepsilon_{ij}-2\|q_i-q_j\|)+\varepsilon_{ij}$$
$$\frac{d}{dt}(\varepsilon_{ij}-2\|q_i-q_j\|)=(2\frac{\varepsilon_{ij}}{\|q_i-q_j\|}\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}-2$$
$$\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}-2\frac{\varepsilon_{ij}}{\|q_i-q_j\|})(q_i(t)-q_j(t))^T(\dot{q}_i(t)-\dot{q}_j(t))$$

Since $q_i^{(r)}(t)=q_j^{(r)}(t)=0$ for all $r\in\{1,\ldots,k-1\}$, hence:

$$(\varepsilon_{ij}^2-2\varepsilon_{ij}\|q_i-q_j\|)^{(k)}=(2\frac{\varepsilon_{ij}}{\|q_i-q_j\|}\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}$$
$$-2\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}-2\frac{\varepsilon_{ij}}{\|q_i-q_j\|})(q_i(t)-q_j(t))^T$$
$$(q_i^{(k)}(t)-q_j^{(k)}(t))$$

■

Lemma 3: Consider agents i and j in G_d, where j is the neighbor for which $\|q_i-q_j\|=d-m+\varepsilon_{ij}$. Suppose that $q_i^{(r)}(t)=q_j^{(r)}(t)=0$ for all $r\in\{1,\ldots,k-1\}$ and some t. Then:

$$s_{ij}^{(k)}(t)=A(q_i(t)-q_j(t))^T(q_i^{(k)}(t)-q_j^{(k)}(t)) \tag{8}$$

where $A>0$.

Proof. The proof follows on noting that $\left|\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}\right|<1$ and $|\varepsilon_{ij}|<m<d$, and is omitted here due to space constraints. ■

Lemma 4: Consider agents i and j in G_d, where j is the neighbor for which $\|q_i-q_j\|=d-m+\varepsilon_{ij}$. Then:

$$\|\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}(\varepsilon_{ij}\|q_i-q_j\|)\|<\|q_i-q_j\| \tag{9}$$

Proof. The left side of 9 can be expressed as:

$$\|\varepsilon_{ij}\frac{\partial\varepsilon_{ij}}{\partial q_i}-\frac{\partial\varepsilon_{ij}}{\partial q_i}\|q_i-q_j\|-\varepsilon_{ij}\frac{\partial}{\partial q_i}\|q_i-q_j\|\|=$$
$$\|\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}\frac{(q_i-q_j)^T}{\|q_i-q_j\|}(\varepsilon_{ij}-\|q_i-q_j\|)-$$
$$\varepsilon_{ij}\frac{(q_i-q_j)^T}{\|q_i-q_j\|}\|=\|\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}(m-d)-\varepsilon_{ij}\| \tag{10}$$

Now, using the inequality $m<d$, one arrives at:

$$\|\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}(m-d)-\varepsilon_{ij}\|<$$
$$\|\frac{\partial\varepsilon_{ij}}{\partial\|q_i-q_j\|}\|(d-m)+\varepsilon_{ij}<d-m+\varepsilon_{ij}$$

This completes the proof. ■

Lemma 5: Consider agents i and j in G_d, where j is the neighbor for which $\|q_i-q_j\|=d-m+\varepsilon_{ij}$. Then:

$$(q_i-q_j)^T((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}(\varepsilon_{ij}\|q_i-q_j\|))>0 \tag{11}$$

Proof. The proof follows directly from Lemma 4, and is omitted due to space limitations. ■

Lemma 6: Consider an agent i in G_d, and assume that $\eta=\min_{j\in N_i(G)}\{\rho(\pi_{ij})\}$. Assume also that $d_i(G_d)\geq 2$; then the following statements hold:

i) $\pi_{ij}^{(r)}=0$, for $0\leq r\leq\eta-1$, and $j\in N_i(G)$.
ii) $\pi_i^{(r)}=0$, for $0\leq r\leq\eta-1$.
iii) $(\frac{\partial h_i}{\partial\sigma_i})^{(r)}=0$, for $0\leq r\leq\eta-1$.
iv) $\rho(q_i)\geq\eta+1$.

Proof. The proof is similar to that of Lemma 3 in [22]. ■

Remark 1: Since the error function satisfies the conditions of Lemma 1, therefore $\rho(\varepsilon_{ij})\geq\rho(q_i)$ and $\rho(\varepsilon_{ij})\geq\rho(q_j)$.

Remark 2: In the case when $d_i(G_d)=1$, it is straightforward to show that $\dot{q}_i=((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}(\varepsilon\|q_i-q_j\|))\frac{\partial h_i}{\partial\pi_i}\pi_{ij}$, where agent j is the neighbor for which $\|q_i-q_j\|=d-m+\varepsilon_{ij}$.

Remark 3: If $\rho(\pi_{ij})$ is not the same for all $j\in N_i(G_d)$, then part (ii) of Lemma 6 also holds for $r=\eta$. Consequently, part (iii) also holds for $r=\eta$.

Lemma 7: For any agent i in G_d, let ν be one of the (possibly multiple) neighbors of i in G_d for which $\rho(q_\nu)=\max_{j\in N_i(G_d)}\{\rho(q_j)\}$. Then:

$$\rho(q_i)\geq 1+\sum_{\substack{j\in N_i(G_d)\\ j\neq\nu}}\rho(q_j) \tag{12}$$

Proof. The proof is similar to that of Lemma 4 in [22], on noting that:

$$\pi_{ij}^{(k)} = \sum_{\substack{r_1+\ldots+r_\mu=k\\ r_1,\ldots,r_\mu\geq 0}}\binom{k}{r_1,\ldots,r_\mu}\prod_{s=1}^{\mu}((d-m)^2-(\|q_i-q_{i_s}\|-\varepsilon_{ij})^2)^{(r_s)} \tag{13}$$

■

Lemma 8: Let $\rho_l(q_i)$ be the lower bound for $\rho(q_i)$ given in Lemma 7, i.e.:

$$\rho_l(q_i)=1+\sum_{\substack{j\in N_i(G_d)\\ j\neq\nu}}\rho(q_j) \tag{14}$$

where $\rho(q_\nu)=\max_{j\in N_i(G_d)}\{\rho(q_j)\}$. If ν is unique, then:

i) $\pi_{i\nu}^{(\rho_l(q_i)-1)}=\tilde{\pi}_{i\nu}\prod_{\substack{j\in N_i(G_d)\\ j\neq\nu}}(q_i-q_j)^Tq_j^{(\rho(q_j))}$, where $\tilde{\pi}_{i\nu}>0$.
ii) $q_i^{(\rho_l(q_i))}=\frac{\partial h_i}{\partial\pi_i}\tilde{\pi}_{i\nu}((q_i-q_\nu)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{i\nu}^2}{2})-\frac{\partial}{\partial q_i}(\varepsilon_{i\nu}\|q_i-q_\nu\|))\prod_{\substack{j\in N_i(G_d)\\ j\neq\nu}}(q_i-q_j)^Tq_j^{(\rho(q_j))}$.

Proof. The proof is omitted due to the space restrictions. ■

Lemma 9: Consider a real or vector-valued function f for which $f^{\rho(f(t))}(t)<0$, for some t. Then f is monotonically decreasing in the interval $[t,t+\varepsilon]$, for some $\varepsilon>0$.

Proof. The proof is similar to that of Lemma 1 in [22]. ■

Lemma 10: Define the subgraph $\tilde{G}_d$ of G_d as the union of those edges $e=(i,j)\in E_d$ for which $\min(\rho(q_i),\rho(q_j))<\infty$, and denote its set of edges with $\tilde{E}_d$. Then, for any $(i,j)\in\tilde{E}_d$, $\rho(s_{ij})=\min\{\rho(q_i),\rho(q_j)\}$ and $s_{ij}^{(\rho(s_{ij}))}<0$.

Proof. One can prove this lemma by induction on $\min(\rho(q_i),\rho(q_j))$. Start with $\min(\rho(q_i),\rho(q_j))=1$, and without loss of generality assume that $\rho(q_i)=1$. If $\rho(q_j)>1$, then $\dot{q}_j=0$, and hence from Lemma 3 and Remark 2:

$$\begin{aligned}\dot{s}_{ij} &= A(q_i-q_j)^T(\dot{q}_i-\dot{q}_j)\\ &= A(q_i-q_j)^T\frac{\partial h_i}{\partial \pi_i}\pi_{ij}((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\\ &\quad \frac{\partial}{\partial q_i}(\varepsilon_{ij}\|q_i-q_j\|))\end{aligned}\tag{15}$$

According to Lemma 5 and on noting that $A>0$, one can conclude that $\dot{s}_{ij}<0$. Also, if $\rho(q_j)=\rho(q_i)=1$, then:

$$\begin{aligned}\dot{s}_{ij} &= A(q_i-q_j)^T(((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}\\ &\quad (\varepsilon_{ij}\|q_i-q_j\|))\frac{\partial h_i}{\partial \pi_i}\pi_{ij}-((q_j-q_i)+\frac{\partial}{\partial q_j}(\frac{\varepsilon_{ji}^2}{2})-\\ &\quad \frac{\partial}{\partial q_j}(\varepsilon_{ji}\|q_j-q_i\|))\frac{\partial h_j}{\partial \pi_j}\pi_{ji})\\ &= A(q_i-q_j)^T\frac{\partial h_i}{\partial \pi_i}\pi_{ij}((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}\\ &\quad (\varepsilon_{ij}\|q_i-q_j\|))+A(q_j-q_i)^T\frac{\partial h_j}{\partial \pi_j}\pi_{ji}((q_j-q_i)+\\ &\quad \frac{\partial}{\partial q_j}(\frac{\varepsilon_{ji}^2}{2})-\frac{\partial}{\partial q_j}(\varepsilon_{ji}\|q_j-q_i\|))\end{aligned}\tag{16}$$

which yields $\dot{s}_{ij}<0$. Now, suppose that the lemma holds for $\min(\rho(q_i),\rho(q_j))<k$. To prove the lemma for $\min(\rho(q_i),\rho(q_j))=k$, assume without loss of generality that $\rho(q_i)=k$. Since $\rho(q_i)\leq\rho(q_j)$, using Lemma 7 one can easily show that $\max_{\omega\in N_i(G_d)}\{\rho(q_\omega)\}$ is unique, and in fact equals to q_j. As another consequence of Lemma 7, $\rho(q_\omega)<\rho(q_i)$ for $\omega\in N_i(G_d)$, $\omega\neq j$. Therefore, $\min(\rho(q_i),\rho(q_\omega))=\rho(q_\omega)<k$, and hence $\rho(s_{i\omega})=\rho(q_\omega)$ and $s_{i\omega}^{(\rho(s_{i\omega}))}<0$. This along with Lemmas 3 and 8 results in:

$$\begin{aligned}q_i^{(\rho_l(q_i))} &= \frac{\partial h_i}{\partial \pi_i}\tilde{\pi}_{ij}((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}\\ &\quad (\varepsilon_{ij}\|q_i-q_j\|))\prod_{\substack{\omega\in N_i(G_d)\\ \omega\neq j}}(q_i-q_\omega)^T q_\omega^{(\rho(q_\omega))}\\ &= \frac{\partial h_i}{\partial \pi_i}\tilde{\pi}_{ij}((q_i-q_j)+\frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}\\ &\quad (\varepsilon_{ij}\|q_i-q_j\|))\prod_{\substack{\omega\in N_i(G_d)\\ \omega\neq j}}-\frac{1}{A}s_{i\omega}^{(\rho(s_{i\omega}))}\end{aligned}\tag{17}$$

Thus:

$$\begin{aligned}(q_i-q_j)^T q_i^{(\rho_l(q_i))} &= (q_i-q_j)^T\frac{\partial h_i}{\partial \pi_i}\tilde{\pi}_{ij}((q_i-q_j)+\\ &\quad \frac{\partial}{\partial q_i}(\frac{\varepsilon_{ij}^2}{2})-\frac{\partial}{\partial q_i}(\varepsilon_{ij}\|q_i-q_j\|))\\ &\quad \prod_{\substack{\omega\in N_i(G_d)\\ \omega\neq j}}-\frac{1}{A}s_{i\omega}^{(\rho(s_{i\omega}))}<0\end{aligned}\tag{18}$$

from which one can conclude that $\rho(q_i)=\rho_l(q_i)$. On the other hand:

$$\begin{aligned}s_{ij}^{(\rho(q_i))} &= A(q_i-q_j)^T(q_i^{(\rho(q_i))}-q_j^{(\rho(q_i))})\\ &= A(q_i-q_j)^T q_i^{(\rho(q_i))}+A(q_j-q_i)^T q_j^{(\rho(q_i))}\end{aligned}\tag{19}$$

If $\rho(q_j)>\rho(q_i)$, then the second term in the right side of 19 vanishes, and it follows from (18) that $s_{ij}^{(\rho(q_i))}<0$. If $\rho(q_j)=\rho(q_i)$, the same inequality as (18) holds for $\rho(q_j)$. Therefore, both terms in (19) are less than zero, and hence $s_{ij}^{(\rho(q_i))}<0$. ■

Lemma 11: Consider the partition $E_d=E_\infty\cup\tilde{E}_d$. Then, for every $i\in V_\infty$,

i) $d_i(G_\infty)\geq 2$.
ii) $\dot{q}_i(t)=0$, for $t\geq t_0$.

Proof. The proof is similar to that of Lemma 7 in [22]. ■

Theorem 1: Under the conditions given by (5), the control law (1) is connectivity preserving.

Proof. To prove the theorem, it suffices to show that there is a neighborhood of t_0 in which for every $(i,j)\in E_d$, s_{ij} is either decreasing or fixed. It follows from Lemmas 9 and 10 that s_{ij} is decreasing in a neighborhood of t_0 for any $(i,j)\in\tilde{E}_d$. Also, from Lemma 11, s_{ij} is fixed for any $t\geq t_0$ and $(i,j)\in E_\infty$. The proof is completed on noting that $E_d=E_\infty\cup\tilde{E}_d$. ■

Corollary 1: For the case where the information flow graph G is a tree, the connectivity preservation is strict, meaning that if $\|q_i(0)-q_j(0)\|\leq d-m$ for all $(i,j)\in E$, then $\|q_i(t)-q_j(t)\|<d$, for all $(i,j)\in E$ and all $t>0$.

Proof. The proof is similar to that of Corollary 1 in [22]. ■

IV. Simulation Results

Example 1: Consider 4 single-integrator agents moving in a two-dimensional space with the control law given by (1) and the information flow graph G_1 depicted in Fig. 1. Let the potential function h_i $(i=1,2,\ldots,4)$ be chosen as:

$$h_i(\sigma_i,\pi_i)=\frac{\sigma_i}{\sigma_i+\pi_i+\pi_i{}^2}\tag{20}$$

It can be shown that the above function satisfies the conditions in (5), and hence the control law obtained by using this function is connectivity-preserving. Assume also that the error function has the following form:

$$\varepsilon_{ij}=m(1-e^{-\|q_i-q_j\|})\tag{21}$$

One can verify that the maximum value of the above function is m, and that function has the maximum rate of change of this function with respect to the relative distance is less than 1. Hence, this function satisfies all of the required conditions discussed earlier.

The control law (6) along with (5) implies that the velocity of each agent is directed towards a point inside the convex hull of its neighbors. This results in the contraction of the convex hull of the entire team, which in turn leads to the convergence of the agents to a single point.

Let d and m be equal to 1 and 0.1, respectively. The planar motion of the agents for the initial positions marked

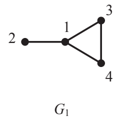

Fig. 1. The information flow graph for the multi-agent system of Example 1.

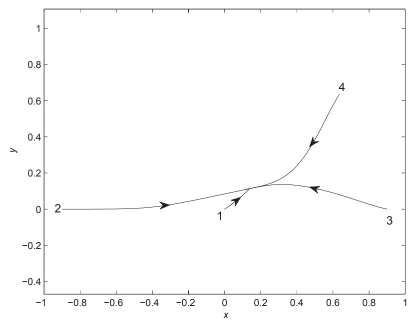

Fig. 2. The agents' planar motion in Example 1.

by the indices of the agents is shown in Fig. 2. Denote the relative distance between agent i and its neighbor j with d_{ij}, i.e., $d_{ij} = \|q_i - q_j\|$. The relative distances d_{12}, d_{13}, and d_{34} are depicted in Fig. 3, which confirms the convergence to consensus. Furthermore, the norm of the control inputs u_1, u_2 and u_3 are drawn in Fig. 4, which shows they are bounded at all times, as expected.

Example 2: Consider now a team of 3 static leaders and 3 followers with the information flow graph G_2 depicted in Fig. 5. The followers are desired to converge to the triangle of the leaders while preserving the connectivity of the information flow graph. Consider the following potential function:

$$h_i(\sigma_i, \pi_i) = -\pi_i \tag{22}$$

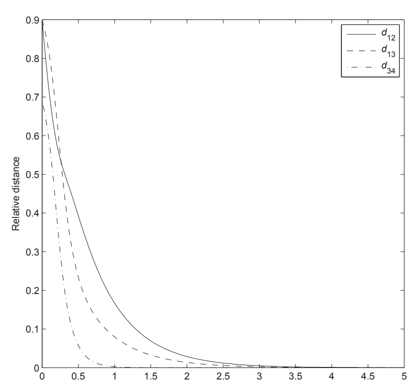

Fig. 3. The relative distances d_{12}, d_{13} and d_{34} in Example 1.

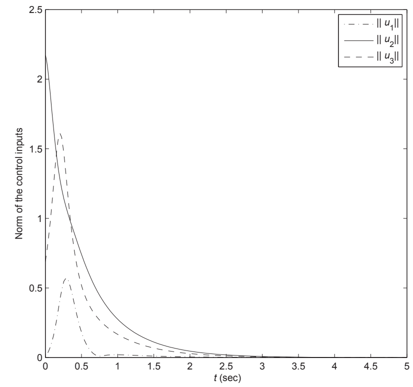

Fig. 4. The norm of the control inputs u_1, u_2 and u_3 in Example 1.

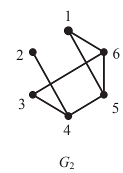

Fig. 5. The information flow graph for the multi-agent system of Example 2.

It can be easily verified that the function given above satisfies the conditions in (5), which means that the corresponding control law is connectivity preserving. Again assume that the error function has the same form as Example 1.

Assume that the error function has the same form as in Example 1. Let also d and m be equal to 1 and 0.1, respectively, and the initial position of each agent be marked by its label, as shown in Fig. 6. This figure shows the motion of the agents in the two dimensional plane. The resultant relative distances are sketched in Fig. 7, which confirm that the connectivity is preserved in the presence of the measurement error given above. The corresponding control inputs $\|u_4\|$, $\|u_5\|$ and $\|u_6\|$ are depicted in Fig. 8. This figure shows that the control inputs are bounded, although some of the agents are initially about to lose connectivity.

V. CONCLUSIONS

This work extends the connectivity preserving bounded control design technique for single integrator agents to the case when the distance measurements are subject to error. Sufficient conditions are presented for a class of distributed potential functions, which guarantee the connectivity preservation of the resultant control laws. It is assumed that the measurement error and its rate of change are bounded, with known bounds. Unlike existing methods, the potential function given in this work can be designed in such a way that the resultant bounded control inputs are connectivity preserving, even in the presence of the measurement error. The efficacy of the proposed control strategy is demonstrated by simulation for two different choices of connectivity preserving potential functions.

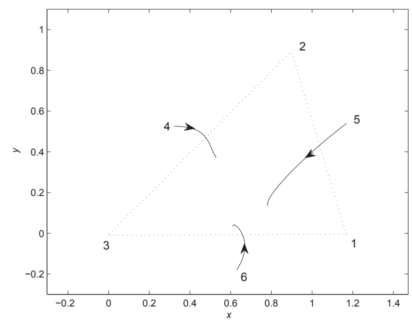

Fig. 6. The agents' planar motion in Example 2.

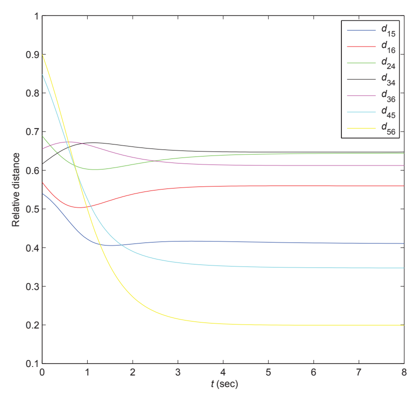

Fig. 7. The relative distances d_{15}, d_{16}, d_{24}, d_{34}, d_{36}, d_{45} and d_{56} in Example 2.

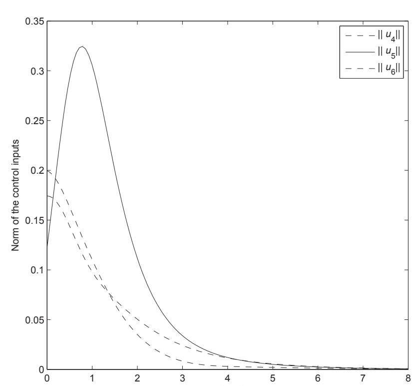

Fig. 8. The norm of the control inputs u_4, u_5 and u_6 in Example 2.

REFERENCES

[1] D. V. Dimarogonas and K. J. Kyriakopoulos, "Connectedness preserving distributed swarm aggregation for multiple kinematic robots," *IEEE Transactions on Robotics*, vol. 24, no. 5, pp. 1213–1223, 2008.

[2] A. Ryan, M. Zennaro, A. Howell, R. Sengupta, and J. K. Hedrick, "An overview of emerging results in cooperative UAV control," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, 2004, pp. 602–607.

[3] R. Horowitz and P. Varaiya, "Control design of an automated highway system," *Proceedings of the IEEE*, vol. 88, no. 7, pp. 913–925, 2000.

[4] C. Tomlin, G. J. Pappas, and S. Sastry, "Conflict resolution for air traffic management: A study in multiagent hybrid systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 509–521, 1998.

[5] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48–74, 2007.

[6] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.

[7] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[8] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[9] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[10] J. Cortes, S. Martnez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Transactions on Automatic Control*, vol. 51, no. 8, pp. 1289–1298, 2006.

[11] G. Lafferriere, A. Williams, J. Caughman, and J. J. P. Veerman, "Decentralized control of vehicle formations," *Systems and Control Letters*, vol. 54, no. 9, pp. 899–910, 2005.

[12] D. P. Spanos and R. M. Murray, "Robust connectivity of networked vehicles," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, 2004, pp. 2893–2898.

[13] Y. Kim and M. Mesbahi, "On maximizing the second smallest eigenvalue of a state-dependent graph laplacian," in *Proceedings of American Control Conference*, 2005, pp. 99–103.

[14] M. C. D. Gennaro and A. Jadbabaie, "Decentralized control of connectivity for multi-agent systems," in *Proceedings of the 45th IEEE Conference on Decision and Control*, 2006, pp. 3628–3633.

[15] M. M. Zavlanos and G. J. Pappas, "Distributed connectivity control of mobile networks," in *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007, pp. 3591–3596.

[16] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 916–922, 2007.

[17] M. M. Zavlanos, A. Jadbabaie, and G. J. Pappas, "Flocking while preserving network connectivity," in *Proceedings of the 46th IEEE Conference on Decision and Control*, 2007, pp. 2919–2924.

[18] M. Ji and M. Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE Transactions on Robotics*, vol. 23, no. 4, pp. 693–703, 2007.

[19] ——, "Distributed formation control while preserving connectedness," in *Proceedings of the 45th IEEE Conference on Decision and Control*, 2006, pp. 5962–5967.

[20] ——, "Connectedness preserving distributed coordination control over dynamic graphs," in *Proceedings of American Control Conference*, 2005, pp. 93–98.

[21] D. V. Dimarogonas and K. H. Johansson, "Decentralized connectivity maintenance in mobile networks with bounded inputs," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2008, pp. 1507–1512.

[22] A. Ajorlou, A. Momeni, and A. G. Aghdam, "A class of bounded distributed control strategies for connectivity preservation in multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 12, pp. 2828–2833, 2010.