2013 American Control Conference (ACC)
Washington, DC, USA, June 17-19, 2013

Formation Control of a Team of Single-integrator Agents with Measurement Error

Farzad Salehisadaghiani, Mohammad Mehdi Asadi, and Amir G. Aghdam

Abstract— **This paper investigates the formation control problem for a team of single-integrator agents subject to distance measurement error. Collision, obstacle and boundary avoidance are important features of the proposed strategy. It is assumed that upper bounds exist on the magnitude of the measurement error and its derivative w.r.t. the measured distance. A decentralized navigation function is then proposed to move the agents toward a desired final configuration which is defined based on the pairwise distances of the connected agents and the characteristics of the distance measurement error. Conditions on the magnitude of the measurement error and its derivative w.r.t. the measured distance are derived under which a new formation configuration can be achieved anywhere in the space due to the measurement error. This error-dependent formation can be determined exactly if the error model is available. If such a model is not available, the maximum discrepancy in the final distances can be obtained in terms of the maximum measurement error. Moreover, the control law designed based on the navigation function ensures collision, obstacle and boundary avoidance in the workspace. The efficacy of the proposed control strategy is demonstrated by simulation.**

I. INTRODUCTION

Control of multi-agent networks has received increasing attention in recent years due to its important real-world applications in mobile sensor networks, air traffic control and automated highway systems, to name only a few [1], [2], [3], [4], [5]. In this type of problem, it is desired to design a distributed control law to achieve a cooperative objective such as consensus, containment and formation [6], [7], [8], [9], [10]. For instance, in the consensus problem the objective is to drive all agents to a single point in the state space. In the containment problem, it is desired that the followers converge to the convex hull of the leaders. In the formation problem, on the other hand, the agents are to converge to a desired configuration in the workspace, which is defined in terms of the relative position of the agents.

Navigation functions are known to be effective tools in the design of cooperative control schemes for multi-agent systems [11], [12], [13], [14]. A class of triangulated graphs for algebraic representation of rigid formations is introduced in [15] to specify a mission cost for a group of vehicles. In [16] the formation behaviors are integrated with other navigational behaviors to enable a multi-agent network to reach the navigational goals. In [17], a novel decentralized control scheme is designed and implemented to achieve dynamic formation control and collision avoidance for a group of non-holonomic agents. The corresponding approach is Lyapunov-based, and guarantees the collision avoidance. In [18], formation control of unmanned aerial vehicles (UAVs) flying in an obstacle-laden environment is investigated. When static obstacles pop up during the operation, the UAVs are required to steer around them and also avoid collisions between each other. Three types of collision have been defined in [19] that should be avoided: 1) collision among the agents which are flying within the communication range of each other; 2) collision of an agent with a fixed obstacle; 3) collision of an agent and the boundary of the workspace.

Any cooperative objective such as the ones addressed in the previous paragraphs relies on some measurements (e.g. distance, speed, etc.) which are often assumed to be exact to simplify the analysis. However, it is known that in practice all measured quantities are subject to error, which can deteriorate the overall performance of the network significantly. This paper tackles the formation control problem in the presence of measurement error. The objective is to design a controller under which the agents converge to any desired configuration (or a sufficiently close neighborhood of it) in the presence of measurement error, while they maintain a minimum distance from each other and from any fixed obstacle in the workspace (including the boundaries). To this end, a navigation-based distributed control law is designed which uses the information about the magnitude of the measurement error and its derivative w.r.t. the measured distance. It is shown that under some conditions the formation configuration in the presence of measurement error can be determined.

The remainder of this paper is organized as follows. In Section II, some notations and definitions are provided and the problem statement is also given. Distributed navigation functions are introduced in Section III. In Section IV the construction of the goal function is discussed. Three functions are proposed in Section V which are subsequently used in the design of the controller with collision/obstacle/boundary avoidance property. It is shown in Section VI that the proposed functions meet the requirements of navigation functions. Simulation results are presented in Section VII to demonstrate the effectiveness of the proposed control strategy. Finally, concluding remarks are drawn in Section VIII.

The authors are with the Department of Electrical and Computer Engineering, Concordia University, 1455 De Maisonneuve Blvd. W., Montréal, Québec, H3G 1M8, Canada {fa_saleh, m_asadim, aghdam}@ece.concordia.ca

This work has been supported by the Natural Sciences and Engineering Research Council of Canada (NSERC) under Grant STPGP-364892-08.

II. Problem Formulation

Notation 1: A symmetric positive semidefinite matrix A is represented as $A \succeq 0$. Similarly, $A \succ 0$, $A \preceq 0$ and $A \prec 0$ denote a matrix A that is positive definite, negative semidefinite and negative definite, respectively.

Consider a set of N single-integrator agents represented by:

$$\dot{q}_i(t) = u_i(t) \tag{1}$$

where $q_i(t)$ and $u_i(t)$ denote the position and velocity of agent i at time t, respectively. Denote with $G = (V, E)$ the information flow graph, with $V = \{1, \dots, N\}$ the nodes (agents), and with $E \subset V \times V$ the edges (communication links among the agents). It is assumed that the information flow graph G is connected and undirected. Define a bounded workspace F with radius R_F, and assume the agents are point masses inside this workspace. Assume also that there are S fixed point obstacles $p_1, \dots, p_S$ in the workspace. The set of neighbors of agent i, denoted by $N_i(G)$, is a set consisting of any vertex in G which is connected to vertex i by an edge, i.e. $N_i := \{j \neq i | (i, j) \in E\}$. Moreover, the degree of the set of neighbors N_i is denoted by $d_i(G)$, i.e. $d_i(G) := |N_i|$. Assume that each agent can only communicate with its neighbors (which implies limited communication among the agents). Assume also that the agents' exchange information through the sensors mounted on them. In a physical system, measurements are always subject to error. In the design of a reliable control system, it is important to take measurement error into consideration in such a way that its effect in the closed-loop performance is minimized. Three main types of measurement error are described below.

Bias Error: When the real quantity is identical to zero but the measured value is not, the sensor is said to have *bias error* (also called *offset error*). The expected value of a measurement, made by a sensor subject to bias error can differ significantly from the actual mean value of the measured quantity. This type of error can be eliminated by proper calibration.

Drift Error: If the measured value gradually changes even when the quantity is fixed, the corresponding discrepancy is called *drift error*. The drift error is often caused by gradual changes in the environmental conditions such as temperature.

Random Error: Sometimes the measured quantity consists of rapidly changing signals of small magnitude. This is usually caused by sensor noise, whose effect on the system output can be significantly attenuated by using a proper filter (such as *Kalman filter*). Statistical techniques can be used to analyze the effect of noise in the system.

Let the error function for distance measurement between any two agents i and j be denoted by $\varepsilon_{ij}(\|q_i - q_j\|)$, where $\|\cdot\|$ denotes the Euclidean norm. Here ε_{ij} is assumed to be a C^2 positive scalar function of distance, $\|q_i - q_j\|$ [20]. The distance measurement error is assumed to be always less than or equal to the actual distance between the two agents, i.e.:

$$\varepsilon_{ij}(\|q_i - q_j\|) \leq \|q_i - q_j\| \tag{2}$$

Furthermore, the derivative of the error function is assumed to be bounded by 1, as follows:

$$\varepsilon'_{ij} = \frac{d\varepsilon_{ij}(\|q_i - q_j\|)}{d\|q_i - q_j\|} < 1 \tag{3}$$

For simplicity of notation, the argument of the function $\varepsilon_{ij}(\cdot)$ is omitted when no confusion can arise.

Definition 1: The communication region of agent i is a circle of radius R_i (communication radius) centered at the agent, and any agent inside this region is considered as a neighbor of agent i. The set of all neighbors of agent i is given by:

$$N_i := \{j \neq i \mid \|q_i - q_j\| - \varepsilon_{ij} \leq R_i\} \tag{4}$$

Definition 2: The *watch zone* of agent i is defined as a circle of radius d_i centered at the agent, where $d_i < R_i$. Any agent, fixed obstacle or workspace boundary inside the watch zone is considered as an obstacle. The set of obstacles of agent i are defined as those agents which are critically close to it, i.e.:

$$O_i := \{j \neq i \,|\, \|q_i - q_j\| - \varepsilon_{ij} \leq d_i\} \tag{5}$$

For simplicity, all agents are assumed to have the same communication radius, $R_i = R$, and the same watch zone radius $d_i = d$, for all $i \in \{1, \dots, N\}$. Suppose that L is the exact distance between the two agents i and j, and that due to the sensor error the measured distance is $L - \varepsilon_{ij}(\|q_i - q_j\|)$. Thus, if the distance $L - \varepsilon_{ij} < R_i$ is measured by the sensor of agent i, normally implying that the agent j is in the communication region of the agent i, but probably the real distance can be as great as $L \geq R_i$, resulting in losing the desired configuration.

III. Decentralized Navigation Functions

Navigation functions (NF) are real-valued maps realized through cost functions $\varphi(q)$, whose negated gradient field is attractive towards the desired pairwise destination and repulsive with respect to the obstacles (fixed point obstacles, potential agents inside the obstacle region, and also the boundary of the workspace) [21]. The main objective of a multi-agent control system is to reach a desired configuration in the workspace in terms of the relative position of the connected agents in the presence of distance measurement error. The control action for agent i is:

$$u_i = -\alpha_i \nabla \varphi(q_i) \tag{6}$$

where α_i is a positive scaling factor. The function $\varphi(q_i)$ is defined as:

$$\varphi(q_i) = \frac{\gamma(q_i)}{(\gamma(q_i)^k + \beta(q_i))^{1/k}} \tag{7}$$

where $\gamma(q_i)$ is the goal function and $\beta(q_i)$ is the obstacle function, which will be introduced later, and k is a tuning parameter ($k \geq 1$). It will be shown later that (7) meets all the requirements of a navigation function.

IV. Construction of the Goal Function

The goal function $\gamma(q_i) : F \to R^{\geq 0}$ has a unique minimum, which occurs when agent i is at the desired position w.r.t. its neighbors. It is defined as the summation of pairwise goal functions γ_{ij}, for all agents distinct from i:

$$\gamma(q_i) = \sum_{j=1, j\neq i}^{N} \gamma_{ij}(q_i, q_j). \tag{8}$$

The special structure of the goal function given above guarantees that the objective is satisfied if and only if the distances of agent i from all other agents approach the desired final distances. Note that the function $\gamma_{ij}(q_i, q_j)$ depends on the measured distances and the desired final distances w_{ij}'s for the error-free case as follows:

$$\gamma_{ij} = \begin{cases} \frac{1}{w_{ij}^2}(\|q_i - q_j\| - \varepsilon_{ij} - w_{ij})^2 & \|q_i - q_j\| - \varepsilon_{ij} \leq w_{ij} \\ \frac{1}{1+e^{a(\|q_i-q_j\|-\varepsilon_{ij}-(\frac{R+w_{ij}}{2}))}} & \|q_i - q_j\| - \varepsilon_{ij} > w_{ij} \end{cases} \tag{9}$$

where w_{ij} is assumed to be greater than d for all $i, j \in \{1, \ldots, N\}$. Note also that γ_{ij} is chosen as a convex function at a sufficiently small vicinity of the the point $\|q_i - q_j\| = \varepsilon_{ij} + w_{ij}$, because $\varphi(q_i)$ is to be minimum if agent i is positioned desirably w.r.t. its neighbors. To this end, a quadratic function is chosen for the first interval $\|q_i - q_j\| - \varepsilon_{ij} \leq w_{ij}$. Moreover, γ_{ij} takes its maximum value 1 at $\|q_i - q_j\| - \varepsilon_{ij} = 0$, i.e., when collision occurs. For the second interval, a sigmoid function is considered with the tuning parameter a. This coefficient is chosen in such a way that γ_{ij} equals zero as $\|q_i - q_j\| - \varepsilon_{ij}$ approaches w_{ij}, and equals one for $\|q_i - q_j\| - \varepsilon_{ij} > R$. It is important to note that γ_{ij} needs to be twice differentiable, so that the smooth functions are used in the construction of $\gamma(q_i)$. Note also that w_{ij} is the desired mutual distance between agents i and j in the absence of measurement error. However, in spite of the distance measurement error, agents will not arrive at the previous desired distances and in fact the desired configuration is being changed. To find the desired formation, the following equation needs to be solved for $\|q_i - q_j\|$:

$$\|q_i - q_j\| - \varepsilon_{ij}(\|q_i - q_j\|) - w_{ij} = 0 \tag{10}$$

If it is assumed that ε_{ij} is bounded by a known constant $\bar{\varepsilon}$, i.e. $\varepsilon_{ij} < \bar{\varepsilon}$. Thus, the new desired distance $\tilde{w}_{ij}$ between agents i and j in the presence of measurement error would be bounded by $w_{ij} + \bar{\varepsilon}$, i.e. $\|q_i - q_j\| = \tilde{w}_{ij}$, $\tilde{w}_{ij} < w_{ij} + \bar{\varepsilon}$, for $i, j \in \{1, \ldots, N\}$.

V. Construction of the Watch Zone Function

As noted earlier, the main objective of this work is to design a formation control scheme with the collision avoidance feature. Three types of collision are considered:

- Collision between two agents, represented by *collision avoidance function* $\beta_1(q_i, q_j)$ for agents i and j.
- Collision between an agent and a fixed obstacle, represented by *obstacle avoidance function* $\beta_2(q_i, p_k)$ for agent i and obstacle k.
- Collision of an agent and the boundary of the workspace, represented by *boundary avoidance function* $\beta_3(q_i)$ for agent i.

The watch zone of agent i is chosen as the product of the functions defined above, for all agents $j \in O_i$ and obstacles $k \in \{1, \ldots, S\}$, i.e.:

$$\beta(q_i) = \beta_3(q_i) \prod_{j\in O_i} \beta_1(q_i, q_j) \prod_{k\in\{1,\ldots,S\}} \beta_2(q_i, p_k) \tag{11}$$

With this choice of watch zone, it is guaranteed that if any type of collision (introduced above) occurs, the objective is not satisfied. The obstacle functions design procedure is described in the sequel.

Ideally, the obstacle function $\beta_1(q_i, q_j)$ associated with agents i and j should be equal to zero if agent i collides with agent j, and should be equal to one if the relative distance between the two agents is greater than d:

$$\beta_1(q_i, q_j) = \frac{1}{1 + e^{b(\|q_i - q_j\| - \varepsilon_{ij} - \frac{d}{2})}} \tag{12}$$

where b is a tuning factor which is chosen in such a way that β_1 approaches to the specific values. More precisely, β_1 reaches its unique minimum when the two agents collide, and its maximum when the two agents are outside of each other's watch zone. It is also important to note that β_1 belongs to C^2.

Assume that there are S fixed obstacles $\{p_1, \ldots, p_S\}$ in the workspace. For any agent q_i and fixed obstacle p_k define the following function:

$$\beta_2(q_i, p_k) = \frac{1}{1 + e^{c(\|q_i - p_k\| - \varepsilon_{ik} - \frac{d}{2})}} \tag{13}$$

If agent i is sufficiently far from the obstacles, then β_2 approaches one, which implies that it will not play any role in the product of the obstacle functions. If, on the other hand, a collision occurs, then the corresponding obstacle function becomes zero, which in turn makes the product of the obstacle functions zero.

As for the boundary of the workspace, the controller which is proposed later, treats it as infinitely many obstacles at radius R_F, which is the radius of the workspace. Now, similar to the obstacle function defined for the agent, consider a circle of radius $R_F - d > w_{ij}$ for the boundary. The region between this circle and the workspace boundary is defined as the *boundary avoidance margin*.

$$\beta_3(q_i) = \frac{1}{1 + e^{u(\|q_i\| - \varepsilon_i - (R_F - \frac{d}{2}))}} \tag{14}$$

The function $\beta_3(\cdot)$ has the property that it is equal to 1 as long as the corresponding agent is outside the boundary avoidance margin, and converges to 0 as the agent approaches the above region.

VI. Navigation Function Analysis

In this section, it is desired to show that $\varphi(q_i)$ is a navigation function. Navigation functions are used as control tools to direct the agents to their desired locations in the formation, where they are positioned properly w.r.t. their neighbors, in the presence of distance measurement error. However, the connectivity preservation of an agent and its neighbors is not guaranteed under such a navigation-function-based control strategy. As a remedy to this problem, it is assumed that a certain level of connection will always be held in the global system (this is in fact, a realistic assumption in practice, and simulations also verify that). The objective now is to investigate how the navigation function drives each agent to its desired location under the above assumption. To this end, let the navigation function be formally defined first.

Definition 3: Let $F \subset R^{2N}$ be a compact connected analytic manifold and denote its boundary with ϑF. A map $\varphi : F \to [0,1]$ is a navigation function if [21]:

1) It is analytic on F.
2) It has a unique minimum at $q_d \in \text{int}(F)$ (i.e. it is *Polar* on F).
3) Its Hessian at all critical points (zero gradient vector field) is full-rank (i.e., it is *Morse* on F).
4) $\lim_{q \to \vartheta F} \varphi(q) = 1$ (i.e., it is admissible on F).

Given $\xi > 0$, define $\beta^{nc}_{i,l}(\xi) = \{q_i : 0 < \beta_l(q_i, \cdot) < \xi, l \in \{1,2,3\}\}$. Partition the workspace to four regions of interest as follows, similar to the one in [21]:

1) The desired destination $F_d(q_i) = \{q_i : \|q_i - q_j\| - \varepsilon_{ij} = w_{ij}, \forall j \in N_i\}$;
2) the workspace boundary ϑF;
3) the set representing near collision regions $F_0(\xi) = \bigcup_{l \in \{1,2,3\}} \beta^{nc}_{i,l}(\xi) - F_d(q_i)$, and
4) the set representing the region sufficiently far from the watch zone $F_1(\xi) = F - (F_d(q_i) \cup \vartheta F \cup F_0(\xi))$.

To verify that $\varphi(\cdot)$ is a navigation function, it suffices to show that when the agents are located at the desired distances w.r.t. their neighbors, it constitutes an equilibrium point which is a non-degenerate local minimum. Furthermore, $\varphi(q)$ has no other critical points of the above form in the other subsets.

Lemma 1: The function $\varphi(\cdot)$ has a non-degenerate minimum at the desired formation.

Proof. To find the critical points of $\varphi(q_i)$, one can write the gradient as the following:

$$\nabla \varphi(q_i) = \frac{k\beta \nabla \gamma - \gamma \nabla \beta}{k(\gamma^k + \beta)^{\frac{1}{k}+1}}. \tag{15}$$

The critical points of $\varphi(q_i)$ are derived from the relation below:

$$\nabla \varphi(q_i) = 0 \Leftrightarrow k\beta(q_i)\nabla\gamma(q_i) = \gamma(q_i)\nabla\beta(q_i). \tag{16}$$

To show that this equilibrium is non-degenerate, it is required to prove that the Hessian is positive definite. One can use (16) to come up with the Hessian as the following:

$$\nabla^2 \varphi = \frac{1}{k(\gamma^k + \beta)^{\frac{1}{k}+1}} [(1 - \frac{1}{k})[\frac{\gamma}{\beta} \nabla\beta \nabla\beta^T] + k\beta \nabla^2 \gamma - \gamma \nabla^2 \beta] \tag{17}$$

Since $k \geq 1$, $\frac{\gamma}{\beta} > 0$ and $\nabla\beta\nabla\beta^T \succeq 0$, hence $(1 - \frac{1}{k})[\frac{\gamma}{\beta}\nabla\beta\nabla\beta^T] \succeq 0$. It is now required to show that the only remaining term, i.e. $k\beta\nabla^2\gamma - \gamma\nabla^2\beta$, is positive definite at the desired formation. To this end, it is important to note that $\nabla^2\beta = 0$ at the desired equilibrium. This results from the fact that, $\beta = 1$ as long as $w_{ij} > d$ and $w_{ij} < R_F - d$. Thus, the gradient and the Hessian are both equal to zero. Therefore, it suffices to show that $\nabla^2\gamma \succ 0$. This is straightforward and omitted due to space restrictions. ■

Lemma 2: All the critical points of $\varphi(\cdot)$ are the interior points of the workspace.

Proof. Assume that agent i (which is one of the critical points of $\varphi(q_i)$) is located on the boundary and that collision occurs between agents i, j. Then $\beta(q_i) = 0$, and thus from (15):

$$\nabla \varphi(q_i) = -\frac{\nabla\beta}{k\gamma^k} \tag{18}$$

Since agent i is on the boundary and at least one collision has occurred, it is concluded that:

$$\nabla\beta(q_i) \neq 0 \Rightarrow \nabla\varphi(q_i) \neq 0 \tag{19}$$

which contradicts the initial assumption in which q_i is a critical point. This completes the proof. ■

Lemma 3: For every $\xi > 0$, there exists a positive integer $N(\xi)$ such that if $k > N(\xi)$, then none of the critical points of $\varphi(q_i)$ are in $F_1(\xi)$.

Proof. Let $q \in F_1(\xi)$ be a critical point. Then:

$$k\beta\|\nabla\gamma\| = \gamma\|\nabla\beta\| \tag{20}$$

Hence, a sufficient condition for q not to be a critical point is:

$$\frac{\gamma\|\nabla\beta\|}{\beta\|\nabla\gamma\|} < k, \text{ for all } q \in F_1(\xi) \tag{21}$$

Note that we are analyzing the critical points which are away of the obstacles, $\beta_l(q_i, \cdot) > \xi$ for all $l \in \{1,2,3\}$. The proof follows immediately by choosing:

$$N(\xi) = \frac{1}{\xi} \frac{\max(\gamma)}{\min(\|\nabla\gamma\|)} (\sum_{j \in O_i} \max(\|\nabla\beta_1(q_i, q_j)\|) + \sum_{k \in \{1,\ldots,S\}} \max(\|\nabla\beta_2(q_i, p_k)\|) + \max(\|\nabla\beta_3(q_i)\|).$$

■

Lemma 4: There exists $\xi_0 > 0$ such that $\varphi(q_i)$ has no local minimum in $F_0(\xi)$, as long as $\xi < \xi_0$.

Proof. If $q \in F_0(\xi)$ is a critical point of φ_i, then $q \in \beta^{nc}_{i,l}(\xi)$ for some i. This implies that q is very close to some obstacles. It is desired now to show that $\nabla^2\varphi(q_i)$ has at least one negative eigenvalue. Define $\beta = \beta_\lambda \bar{\beta}_\lambda$ where β_λ is one of the collision functions appears in $\beta(q_i)$ and $\bar{\beta}_\lambda$ is product of all the collision functions except β_λ. One can choose a proper sigmoid function for β_λ. For instance, by using a function of the form $\beta_1(q_i, q_j)$ (the collision avoidance function), one can come up with the Hessian at the critical point as follows:

$$\nabla^2\varphi(q) = \frac{1}{k(\gamma^k + \beta)^{\frac{1}{k}+1}} (k\beta\nabla^2\gamma + (1 - \frac{1}{k})\frac{\gamma}{\beta} [\beta_\lambda^2 \nabla\bar{\beta}_\lambda \nabla\bar{\beta}_\lambda^T + 2\beta_\lambda\bar{\beta}_\lambda(\nabla\bar{\beta}_\lambda\nabla\beta_\lambda^T) + \bar{\beta}_\lambda^2 \nabla\beta_\lambda\nabla\beta_\lambda^T] - \gamma[\beta_\lambda\nabla^2\bar{\beta}_\lambda + 2 (\nabla\bar{\beta}_\lambda^T\nabla\beta_\lambda) + \bar{\beta}_\lambda\nabla^2\beta_\lambda]) \tag{22}$$

Choose a test vector of unit magnitude similar to the one in [21] which is orthogonal to $\nabla\beta_\lambda$ at a critical point q_c, i.e.:

$$\hat{v} = \frac{\nabla\beta_\lambda(q_c)^\perp}{\|\nabla\beta_\lambda(q_c)^\perp\|}$$

It is straightforward to verify that the following quadratic equation holds:

$$k(\gamma^k+\beta)^{1+\frac{1}{k}}\hat{v}^T\nabla\varphi^2\hat{v} = k\beta\hat{v}^T\nabla^2\gamma\hat{v} - \gamma\bar{\beta}_\lambda\hat{v}^T\nabla\beta_\lambda^2\hat{v} + (1-\frac{1}{k})\frac{\gamma}{\beta}\beta_\lambda^2\hat{v}^T\nabla\bar{\beta}_\lambda\nabla\bar{\beta}_\lambda^T\hat{v} - \gamma\beta_\lambda\hat{v}^T\nabla^2\bar{\beta}_\lambda\hat{v} \quad (23)$$

Now, take the inner-product of $\nabla\gamma$ and both sides of the equation $k\beta\nabla\gamma = \gamma\nabla\beta$ to obtain:

$$4k\beta\frac{(1-\varepsilon_{ij}^{'})^2}{w_{ij}^2} = \bar{\beta}_\lambda\nabla\beta_\lambda.\nabla\gamma + \beta_\lambda\nabla\bar{\beta}_\lambda.\nabla\gamma \quad (24)$$

Grouping the terms which are proportional to β_λ yields that two terms will appear in the RHS of the resultant equation. The first term is proportional to β_λ and can be made arbitrarily small by a proper choice of ξ. However, since this term can be positive, the second term which is proportional to $\bar{\beta}_\lambda$ should be strictly negative. Let the latter term be denoted by $Pr_{\bar{\beta}_\lambda}$, and rewritten as:

$$Pr_{\bar{\beta}_\lambda} = G_3\nabla\beta_\lambda.\nabla\gamma - B_3\gamma \quad (25)$$

where $G_3 := \frac{w_{ij}^2}{4(1-\varepsilon_{ij}^{'})^2}(G_1 + G_2\hat{v}^T(q_i-q_j)(q_i-q_j)^T\hat{v})$ and $B_3 := (B_1 + B_2\hat{v}^T(q_i-q_j)(q_i-q_j)^T\hat{v})$, and G_1, G_2, B_1 and B_2 are the functions of $\|q_i-q_j\|$. Note that G_3, B_3 and γ are strictly positive for any $q \in F_0(\xi)$ (which implies no collision has occurred). Thus, to complete the proof, it is straightforward to show that $\nabla\beta_\lambda.\nabla\gamma < 0$. Some algebraic calculations imply that for the following inequality $\max(Pr_{\bar{\beta}_\lambda})$ is strictly negative:

$$\xi < \frac{1}{1+e^{b(\frac{B_3}{G_3}\frac{\gamma}{A}-\frac{d}{2})}} \quad (26)$$

The proof follows now by choosing $\xi_0 = \frac{1}{1+e^{b(\frac{B_3}{G_3}\frac{\gamma}{A}-\frac{d}{2})}}$. ■

Proposition 1: At any point in time, $\varphi(q_i)$ is a navigation function if the parameter k has a value greater than a finite lower bound, and the distance measurement error (ε) satisfies the following conditions:

$$i)\ \varepsilon_{ij}(\|q_i-q_j\|) \le \|q_i-q_j\|,$$

$$ii)\ \varepsilon_{ij}^{'} = \frac{d\varepsilon_{ij}(\|q_i-q_j\|)}{d\|q_i-q_j\|} < 1.$$

Proof. The proof follows from Lemmas 1-4 so that $\varphi(q_i)$ satisfies all the conditions stated in Definition 3. ■

VII. Simulation Results

Example 1: Consider 3 single-integrator agents moving in a two-dimensional plane under the control law (6) and a point obstacle fixed in the origin of the workspace. The radius R_F of the workspace is assumed to be 60m. Let the obstacle avoidance region and communication be circles of radius $d = 5$m and $R = 40$m, respectively. Let also the control action parameters be $\alpha_i = 1$ for $i \in \{1,\dots,N\}$ and $k = 1$. Assume that the desired configuration is an equilateral triangle with edges of length $w_{ij} = 15$m. Assume also that the measurement error function has the following form:

$$\varepsilon_{ij} = \mu(1-e^{-\frac{\|q_i-q_j\|}{h}}) \quad (27)$$

It is straightforward to verify that the above function satisfies the conditions of proposition 1 for any $\mu < h$. Let h and μ be equal to 30m and 2m, respectively.

For the error function (27) and the given values for the parameters, $\|q_i-q_j\| = 15.82$m. Let the initial positions of the agents be marked by asterisks and the final positions by diamonds. Let also the obstacle avoidance region and the boundary avoidance margin be represented by dashed lines. Fig. 1 depicts the planar motion of the agents in this case. The agents are initially connected, with the pairwise distances chosen close to R (in order to test the goal function near the boundary of the communication region). As shown in Fig. 2, the pairwise distances approach the desired values (15.82m) as time increases.

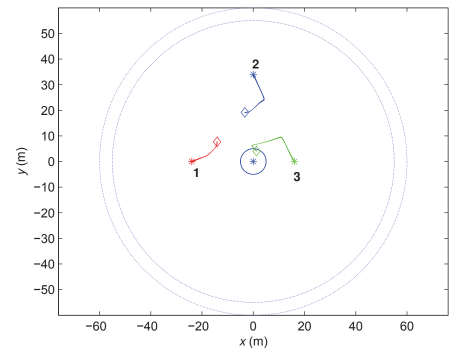

Fig. 1. Planar motion of the agents for the Example 1.

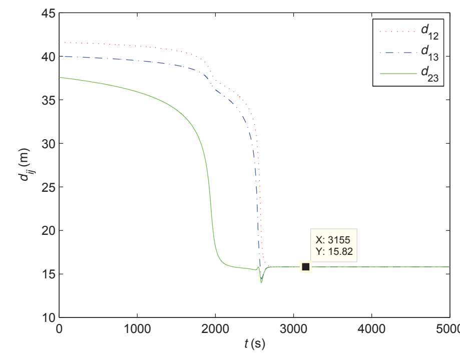

Fig. 2. Mutual distances among the agents for the Example 1.

Fig. 3 shows the planar motion of the agents for the case when agent 2 starts from inside the obstacle avoidance region

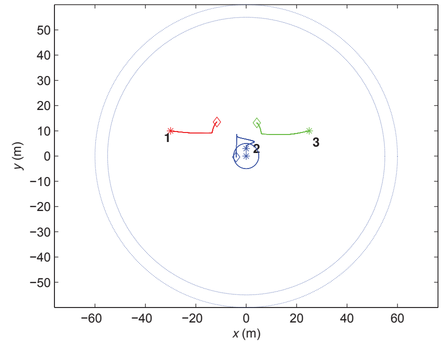

Fig. 3. Planar motion of the agents in Example 2.

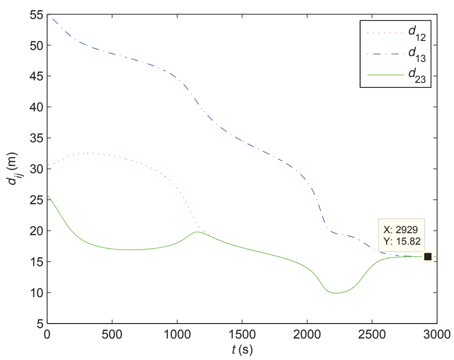

Fig. 4. Pairwise distances of the agents in Example 2.

(but no collision initially). As shown in Fig. 3, there is no collision in the trajectory of the agents (and in particular agent 2) as they move to their desired configuration. Fig. 4 demonstrates the final pairwise distances which are in accordance with the desired configuration.

VIII. Conclusions

A distributed navigation function-based controller is proposed to drive a group of single-integrator agents to a desired configuration. It is assumed that the distance measurements are subject to error, and that the agents should avoid collision and obstacles in the workspace as well as the boundaries of the workspace. The final formation is expressed in terms of the desired distances among the connected agents and the distance measurement error. The formation can be reached anywhere in the space and with any orientation, provided some sufficient conditions on the magnitude of the distance measurement error and its derivative w.r.t. the measured distance, hold. The navigation functions used to design the controller ensure collision avoidance between the agents as well as the obstacle and the workspace-boundary avoidance. As a suggestion for future work, the proposed approach can be used analogously to develop a formation control strategy for a team of double-integrator agents.

References

[1] D. V. Dimarogonas and K. J. Kyriakopoulos, "Connectedness preserving distributed swarm aggregation for multiple kinematic robots," *IEEE Transactions on Robotics*, vol. 24, no. 5, pp. 1213–1223, 2008.

[2] A. Ryan, M. Zennaro, A. Howell, R. Sengupta, and J. K. Hedrick, "An overview of emerging results in cooperative UAV control," in *Proceedings of the 43rd IEEE Conference on Decision and Control*, 2004, pp. 602–607.

[3] R. Horowitz and P. Varaiya, "Control design of an automated highway system," *Proceedings of the IEEE*, vol. 88, no. 7, pp. 913–925, 2000.

[4] C. Tomlin, G. J. Pappas, and S. Sastry, "Conflict resolution for air traffic management: A study in multiagent hybrid systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 509–521, 1998.

[5] N. E. Leonard, D. A. Paley, F. Lekien, R. Sepulchre, D. M. Fratantoni, and R. E. Davis, "Collective motion, sensor networks, and ocean sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48–74, 2007.

[6] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988–1001, 2003.

[7] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, 2006.

[8] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Transactions on Automatic Control*, vol. 52, no. 5, pp. 863–868, 2007.

[9] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, 2004.

[10] J. Cortes, S. Martinez, and F. Bullo, "Robust rendezvous for mobile autonomous agents via proximity graphs in arbitrary dimensions," *IEEE Transactions on Automatic Control*, vol. 51, no. 8, pp. 1289–1298, 2006.

[11] R. Fierro, A. K. Das, R. V. Kumar, and J. P. Ostrowski, "Hybrid control of formations of robots," in *Proceedings of IEEE International Conference on Robotics and Automation*, 2001, pp. 157–162.

[12] V. Gupta, B. Hassibi, and R. Murray, "Stability analysis of stochastically varying formations of dynamic agents," in *Proceedings of 42th IEEE Conference Decision and Control*, 2003, pp. 504–509.

[13] P. Ogren, M. Egerstedt, and X. Hu, "A control Lyapunov function approach to multiagent coordination," *IEEE Transactions on Robotics and Automation*, vol. 18, no. 5, pp. 847–851, October 2002.

[14] H. Yamaguchi and J. W. Burdick, "Asymptotic stabilization of multiple nonholonomic mobile robots forming group formations," in *Proceedings of IEEE International Conference on Robotics and Automation*, 1998, pp. 3573–3580.

[15] R. Olfati-Saber, W. B. Dunbar, and R. M. Murray, "Cooperative control of multi-vehicle systems using cost graphs and optimization," in *Proceedings of American Control Conference*, June 2003, pp. 2217–2222.

[16] T. Balch and R. Arkin, "Behavior-based formation control for multirobot teams," *IEEE Transactions on Robotics and Automation*, vol. 14, no. 6, pp. 926–939, 1999.

[17] S. Mastellone, D. M. Stipanovic, C. R. Graunke, K. A. Intlekofer, and M. W. Spong, "Formation control and collision avoidance for multiagent non-holonomic systems: Theory and experiments," *International Journal of Robotics Research*, vol. 27, no. 1, pp. 107–126, January 2008.

[18] X. Wang, V. Yadav, and S. N. Balakrishnan, "Cooperative UAV formation flying with obstacle/collision avoidance," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 4, pp. 672–679, July 2007.

[19] M. C. D. Gennaro and A. Jadbabaie, "Formation control for a cooperative multi-agent system using decentralized navigation functions," in *Proceedings of American Control Conference*, June 2006, pp. 1346–1351.

[20] F. Salehisadaghiani, A. Ajorlou, and A. G. Aghdam, "Distributed connectivity preservation of a team of single integrator agents subject to measurement error," in *Proceedings of IEEE Multi-Conference on Systems and Control*, September 2011, pp. 626–631.

[21] D. E. Koditschek and E. Rimon, "Robot navigation functions on manifolds with boundary," *Advances in Applied Mathematics*, vol. 11, pp. 412–442, 1990.