A Theory of Grand Innovation Prizes

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Abstract

The past decade has witnessed a resurgence in innovation awards, in particular of Grand Innovation Prizes (GIPs) which are rewards to innovators developing technologies reaching performance goals and requiring breakthrough solutions. GIPs typically do not preclude the winner also obtaining patent rights. This is in stark contrast with mainstream economics of innovation theories where prizes and patents are substitute ways to generate revenue and encourage innovation. Building on the management of innovation literature which stresses the difficulty to specify ex-ante all the technical features of the winning technologies, we develop a model in which innovative effort is multi-dimensional and only a subset of innovation tasks can be measured and contracted upon. We show that in this environment patent rights and cash rewards are complements, and that GIPs are often preferable to patent races or prizes requiring technologies to be placed in the public domain. Moreover, our model uncovers a tendency for patent races to encourage speed of discovery over quality of innovation, which can be corrected by GIPs. We explore robustness to endogenous entry, costly public funds, and incomplete information by GIP organizers on the surplus created by the technology.

1 Introduction

Economists have long recognized the crucial role played by innovation in economic growth and - at least since Arrow (1962) – have considered channels that might cause under-investment in innovation relative to the socially optimal level. How to avoid such under-investment and how to provide greater innovation incentives is a central question in the economics of innovation literature. The patent system is the most important institution developed by policy-makers to spur innovation. Patents provide temporary monopoly rights as an incentive to innovation. However, it is well understood that patents come at a cost, since monopoly leads to inefficiencies. The natural alternative, prizes that come through money rather than monopoly rights, are viewed as difficult to implement in a way that generates appropriate incentives; proper incentives require that the prize be awarded only to genuine innovations, and in proportion to their usefulness. Prizes have nonetheless been growing as a way of rewarding innovation, both publically and privately. In the United States, former President Obama's Strategy for American Innovation strongly encouraged the use of innovation prizes and the America Competes Reauthorization Act of 2011 provided all federal agencies with power to offer innovation prizes (Williams, 2012).

This paper studies Grand Innovation Prizes (GIPs), which are a prominent class of innovation inducement prizes. GIPs are defined as large monetary rewards for innovators reaching a pre-determined set of performance targets (Kay, 2011; Murray et al, 2012). Critically, a GIP does not preclude the winner from also obtaining patent rights. This is different from the usual view of prizes and patents in the innovation literature where the two are viewed as alternative ways to generate revenue to encourage innovation, and the only reason patents are preferable to prizes is because of information or contracting frictions that prevent the prize from being sufficiently tied to success to be an effective incentive.

Our model of Grand Innovation Prizes provides an explanation for this coexistence of prizes and patents. Our setting departs from traditional models of innovation contest by assuming that the characteristics of the technology target can only be partially specified by the GIP organizer. Performance targets can be described but a full description of a solution is unavailable. We model this as the innovation having two dimensions. On one dimension, the performance goals for the prize to be awarded can be well specified and verified ex post. On another dimension, they cannot.

The use of prizes is not limited to the public sector, and their use in the private sector can be informative. Our multi-dimensional approach is consistent with existing descriptions of GIPs in the private sector. Often not much more is known than that a successful product requires significant research effort and breakthrough ideas. Since technical specifications are difficult to describe, GIPs differ from smaller-scale competitions as software development contests requiring more limited resources and for which the solution can be typically described in great detail (Boudreu et al., 2011).[1]

A number of case studies analyzed in the literature support the idea that specifying ex-ante the technical features for the winning technologies is the most challenging aspect of GIPs. For example, Murray et al (2012) describe the Progressive Insurance Automotive X Prize for the development of "*viable, super fuel-efficient vehicles that give people more car choices and make a difference in their lives.*" The broad objective of the prize was the development of vehicles able to revolutionize the automobile industry through a new generation of fuel efficient cars. Translating such broad aim into precise requirements for the contest is very hard. While technical metrics are required to set a target for

[1]A 2009 McKinsey report estimates that the total funds available from large prizes have more than tripled over the last decade to surpass $375 million with a large number of philanthropists entering the business of rewarding innovators (McKinsey, 2009). For example, Qualcomm and Nokia have offered multi-million dollar prizes for the development of affordable devices that can recognize and measure personal health information. Similarly, the Gates Foundation has offered an innovation award to immunize children in the poorest parts of the world. Such trend is likely to have been stimulated by the success of the 1996 $10 million Ansari Prize offered by the X PRIZE Foundation for a private space vehicle to launch a reusable manned spacecraft into space twice within two weeks (Kay, 2011; Murray et al, 2012).

the prize, the future impact of the technology may be related to characteristics that are difficult to specify ex-ante with unambiguous criteria. For example, the Progressive Insurance Automotive X Prize required vehicles to meet an efficiency standard of 100MPGe with CO2 emissions equivalent to <200g/mi. However, statements from the X-Prize Foundation indicate that the overall impact of the technology on consumers and follow-on innovation was unlikely to depend on fuel efficiency alone. Other dimensions related to the manufacturing process and consumer desirability would also be very important component of the welfare generated by the innovation.

The allocation of intellectual property rights in GIPs appears particularly in contrast with the microeconomic literature on innovation prizes (Wright, 1983; Scotchmer, 2004). The economics of innovation literature has treated prizes and patents as substitutes, and stressed that a prominent feature of prizes is the removal of the monopoly dead-weight loss generated by patents. In the choices of the private sector, things appear very different: patents and prizes are viewed by GIP organizers as complements and GIP rules tend to allow participants to keep the IP rights on their technologies. Murray et al (2012) report the following quote from an XPrize organizer: "*We have a standard for any XPrize that we have no interest in taking IP from teams with the exception of media rights to tell the story of the competition. It is not in our best interest to claim IP.....we need to allow teams to pursue their business in whatever way makes sense to them.*"

We argue that the informational motivation for the private use of GIPs might also be a rationale for their use by public policy makers. There may be additional reasons why it may be useful to allocate patent right by GIP rather than simply a patent. For instance, if rents from a patent are low, but either consumer surplus or externalities from the innovation are high, additional subsidy is valuable. Papers in the prize literature, such as Kremer (1998) or Galasso et al. (2016) emphasize replacing monopoly rights with prize revenue in such cases. When patent rights are not excessively costly and benefits are high, the policy maker might want to use both.

We begin our analysis with a simple model in which there is only one innovator. The planner wants to maximize welfare by rewarding successful innovation and has full information on the marginal benefit and marginal cost of innovative effort. A standard result in the innovation literature is that in the presence of full information prizes dominate patents generating larger welfare and innovation incentives. Because research effort in our model is multi-dimensional and only one dimension is contractible, this result does not hold in our environment.

We compare three different reward structures. The first one is a patent regime in which the innovator is granted a patent which allows him to extract market profits from the innovation. The second one is a prize regime in which the innovator obtains a cash reward if the technology meets a target specified by the prize organizer and the innovation is placed in the public domain. The third regime, that we label Grand Innovation Prize, is a hybrid system in which the innovator obtains a prize if the performance target is met and retains patent rights over the technology which allows him to extract additional revenue from consumers and licensee.

Neither the patent nor the prize regime can generate the first-best level of innovation in our model. In the patent regime under-investment arises because the patentee can appropriate only a fraction of the surplus generated. Prizes can correct the underinvestment by linking the reward to a performance target that maximizes social welfare. Nonetheless, because only a subset of innovative activities can be measured and contracted upon, the inventor has an incentive to disregard the non-measurable dimensions and to invest only on measurable activities. The hybrid GIP system generates larger innovation effort by attacking both of these under-investments. The under-investment in measurable activities that arises with the patent system is reduced thanks to the award that the innovator obtains if the effort target is met. The under-investment in unmeasurable activities that arises with prizes is reduced because the innovator obtains a fraction of the welfare generated, which depends on the entire set of innovative efforts. This result provides an explanation on the joint use of patents and prizes that is observed in Grand Innovation Prizes. Intuitively, the partial surplus appropriability typical of a patent regime combined with the non-measurability of certain aspects of innovative effort generates complementarity between the two instruments. We show that this complementarity implies that, under very general conditions, GIPs generate more welfare than the other two regimes.

We extend the analysis in several directions. First, we consider the case where research efforts along the two dimensions are complements or substitutes. We show that if either strong substitutability or strong complementarity is present, then a simple prize may perform better when compared to a GIP than in the baseline case of no interaction between the two inputs. If the two inputs are strong substitutes, then the under-investment in the unverifiable effort dimension induced by prizes is less problematic than in the baseline case because it is sufficient to induce a high effort in the other dimension for the innovation to succeed overall. If the two inputs are strong complements, then again it is sufficient to induce effort in the observable dimension because complementarity between the two effort levels will provide a built-in incentive for the innovator to exert effort along the non-observable dimension as well.

We consider further extensions to show that our results are robust to features that are standard in the innovation literature. In particular, we show that introducing costly public funds does not change our results qualitatively, and neither does incomplete information about the value of the innovation. Introducing competing innovators (either with a fixed number of innovators or with free entry) does not change the performance of simple prizes and GIPs, so our main comparison results are unchanged. However, patents perform worse with competing innovators than with a single innovator because each innovator may have an incentive to rush to the patent office, and file patents that provide very little improvements over existing products.

The rest of the paper is structured as follows. In Section 2, we provide further literature review. Section 3 sets up the baseline model and provides our main results. Section 4 revisits the baseline model to analyze the case of competing innovators. Section 5 considers the role of incomplete information, and costly public funds. Section 6 discusses the policy implications of our findings. Section 7 concludes. The proofs of all the results are relegated to the appendix.

2 Related Literature

This paper is connected to various strands of the literature on the economics of innovation and management of technology.

The first is the literature comparing patents and prizes as alternative mechanisms. Wright (1983) compares prizes, patents and research contracts as mechanisms to encourage innovation. He shows that any of these three policy tools can be optimal depending on the strength of three effects: research duplication, deadweight loss due to monopoly pricing and asymmetric information between research authority and innovating agent. Shavell and Van Ypersele (2001) provide a comparison of prizes and patents as mechanisms to incentivize innovation in a static framework. They show that neither system is superior and that a mechanism under which innovators can choose between prizes and patents is typically superior to a patent system. Weyl and Tirole (2012) study the optimal reward structure in the presence of multidimensional heterogeneity and non-manipulable market outcomes. In a static framework, they show that the optimal policy requires some market power but not full monopoly profits. In each of these papers the focus is on the trade-off between using one reward mechanism versus another; we focus on possible complementarities between the two reward methods.[2]

The empirical literate on prizes has focused on their efficacy, and the way the structure of the prize impact the innovation level. Boudreau et al (2011) focus on how the number of competitors influences incentives. Boudreau et al (2016) studies the impact of competition across different types of competitors, as well as the impact of different details in the structure of the prize itself. Boudreau and Lakhani (2015) focus on the impact of intermediate disclosure on the tournament incentives. We complement these papers by pointing out how the contest rules might interact with other IP features, like patents.

Our paper takes a mechanism design approach to studying the optimal reward. This follows in the tradition of Scotchmer (1999) and Cornelli and Schankerman (1999). These papers have the feature that an information friction prevents the planner from paying for innovations; instead innovator chooses

[2] This literature on patents vs. prizes has led to papers that have continued to study the trade-off between patents and prizes, but attempted to see whether external signals might allow prizes to dominate patents. Kremer (1998) designs an auction to elicit information to design the prize. Chari, Golosov and Tsyvinski (2012) compare prizes and patents when the planner can observe market signals over time. Their main finding is that patents are necessary if the innovator can manipulate market signals. Galasso, Mitchell and Virag (2016) show how a patent buyout that exploits information from market outcomes as a guide to the payment amount can be effective at determining both marginal and total willingness to pay of consumers and can generate the right innovation incentives.

from a menu of protections, at various prices. Hopenhayn and Mitchell (2001) study how to eliminate the payment through menus of breadth and length, but they continue to focus on the case where information frictions prevent the planner from awarding prizes. Hopenhayn, Llobet and Mitchell (2006) study the optimal patent design in a model where innovation is cumulative, involving contributions of multiple innovators. In their model the optimal reward might include payments between innovators but typically not from the planner to the innovators, like a prize.

When benefits can be measured, Loeb and Magat (1979) show that a reward based on consumer surplus is sufficient to get the first best. Our model incorporates this idea on one dimension with the information friction typically used to justify patents on another dimension. The paper is therefore related to the literature on multi-tasking and its applications to innovation. Devatripont et al. (2000) provide a survey of multitask agency problems. Lazear's (1989) seminal work shows that in a multi-tasking environment firms may benefit from providing low-incentive payments to their employees because such compensation scheme reduces uncooperative behavior. Holmstrom and Milgrom (1991) study a principle-agent model in which the agent's task has several dimensions and some of them cannot be perfectly measured. They show that providing incentives in one activity can shift effort toward measurable activities and sometime a fixed wage may be better than incentives.

In the intersection of the multi-tasking literature and innovation incentives, Hellman and Thiele (2011) study the incentive contract problem in a multi-tasking model about innovation. In their model, employees privately observe innovation opportunities that fall outside of the performance metrics. They show that incentive compensation provided by the firm depends on the firmspecificity of innovation. Their paper is broadly similar in that it describes how the different dimensions impact incentives.

Contractibility is often viewed as an important determinant of private contracts, and therefore our focus on contractibility as a driving force behind optimal rewards is related to the literature on private contracts for research. Lerner and Malmendier (2010) discuss the issue of contractibility in research licensing agreements. Their question is related in that it makes the point about things being partially contractible in the way that is assumed here. They show that contractibility affects contract design, to avoid that researchers use their funding to subsidize other projects, an option contract is optimal. They provide support of their theory in a sample of biotechnology research contracts. Our paper follows the line that contractibility is fundamental to the design of institutions.

Another related literature is the labor market tournament literature starting with Lazear and Rosen (1981) and Rosen (1986).[3] Labor market tournaments are closely related to a prize with multiple innovators. Taylor (1995) adapts a model of a tournaments to a research setting in which a sponsor pays a prize to the innovation of highest quality. He shows that the use of an entry fee is crucial to minimize duplication of research and maximize the post-entry research effort levels. In the version of our model with multiple innovators, a similar force is at work. We show how a minimum technical standard for a prize winner can serve a similar function to a fixed entry cost.

Our results on minimum technical standards are similar to the theory of the optimal "minimum inventive step" for a patent, for instance in Scotchmer and Green (1990), La Manna (1992) and O'Donoghue (1998). Those papers typically view the innovation contribution as observable, however, and therefore better

[3] Connelly et al (2014) provide a review of the tournament literature.

rewards (like a prize) would dominate patents. Our paper explicitly models the notion that technical standards can be described, but only imperfectly, giving rise to the Grand Innovation Prize.

3 Single innovator and complete information

We begin our analysis with a model in which there is only one innovator who makes a one-time choice of innovative effort (as in Arrow, 1962; Wright, 1983; and Scotchmer, 2004). We introduce competition between multiple inventors in Section 4. As in Holmstrom and Milgrom (1991) we allow effort to be a two-dimensional vector (x, y) and we refer to each component of the vector as an 'innovation activity'. Effort (x, y) leads to an innovation that generates consumer surplus $U(x, y) = \theta(x + y)$ where θ is a parameter capturing the value of the innovative effort for society.

The central assumption of our model is that innovation activity x is measurable and contractible whereas activity y is not contractible. The idea is that a precise technical metric is required to compare the various innovations, but the future impact of the technology may be related to characteristics that are hard to specify ex-ante with unambiguous criteria.[4]

The y variable in our model captures a variety of aspects of the innovation, which are hard to include into the ex-ante technical specifications of the prize but are important components of the welfare generated by the technology. First, y includes various features of the innovation related to consumer desirability. For example, among organizers of PIAXP there was the concern of rewarding hyper-efficient cars which look like 'rolling coffins' (Murray et al., 2012). Second, the variable captures aspects related to the manufacturability of the products which

[4] Murray et al (2012) document how the ex-ante development of technical specifications for the winning technologies was the most challenging component of the PIAXP prize design.

embed the new technology. The technical specifications in the prize may not include features which are important for cost-effective large scale production. Finally, y captures aspects of the innovation which spur future improvements of the technology. Prize organizers may not specify materials, design and processes which are likely to attract large interests by follow-on innovators and generate greater future improvements.

3.1 Benchmark Model: Independent Costs

We assume that the cost of innovative effort is

$$C(x, y) = \frac{x^2}{2} + \frac{y^2}{2}.$$

In this formulation, costs of the two dimensions do not interact, and the model can focus on the impact of the GIP on each dimension separately. The full model considers the impact of cost complementarities.[5] Gross consumer surplus is

$$U(x, y) = \theta(x + y),$$

and thus welfare is equal to

$$W(x, y) = U(x, y) - C(x, y).$$

Under these assumptions, the first best effort levels equal to $x^* = y^* = \theta$ and the corresponding welfare is equal to $W^* = \theta^2$.

This simple specification builds on a long-standing tradition in the economics of innovation literature. Once we drop the measurable dimension, x, our setting collapses into the one-dimensional technology typically used to examine patents and other innovation policy instruments (see Cornelli and Schankerman, 1999; Shavell and Van Ypersele, 2001; and Hopenhayn et al, 2006). The

[5] A common assumption in this class of models is that the R&D process is deterministic. One can easily extend the results to the case in which the returns to R&D are stochastic.

multi-dimensional version of this classic approach was developed by Weyl and Tirole (2012) in the absence of R&D effort (the innovator chooses which ideas to develop, but not how much R&D to do). Our setting extends Weyl and Tirole (2012) introducing multiple research investments with different degrees of contractibility.

We consider a social planner aiming at maximizing welfare with one of three alternative policy instruments. To develop the intuition, we assume symmetric information between the innovator and the planner. We discuss the asymmetric information case in Section 5.

The first mechanism is the *prize system* in which the innovator receives a reward P if the innovative activity observed by the planner meets a specific effort target $x = \underline{x}$ and the innovation is sold in a competitive market.[6] In this case the profits for the innovator are

$$P - C(x,y) \quad \text{if} \quad x = \underline{x}$$
$$-C(x,y) \quad \text{else}$$

with solution that we label x^P, y^P. Because $C_x > 0$ the optimal strategy for the innovator is to set $x^P = \underline{x}$ and $y^P = 0$. The problem for the planner is

$$\max_{\underline{x}} W(\underline{x}, 0) = \theta \underline{x} - \frac{\underline{x}^2}{2}$$

with solution $x^P = \theta$ and $y^P = 0$. Note that this level of effort can be achieved by choosing any $P \geq C(\theta, 0)$. The welfare with the prize regime is equal to

$$W^P = \frac{\theta^2}{2}.$$

[6] One can replace this assumption with the more realistic alternative in which the planner specifies a minimum effort target $x \geq \underline{x}$. The two assumptions are equivalent in our case given that the planner has complete information and x is contractable.

The second mechanism is the *patent system* in which the innovator obtains intellectual property rights on the technology. Following the innovation literature we assume that granting patent rights generates a dead-weight-loss through monopoly prices which reduces consumer surplus to $\gamma U(x, y)$ with $\gamma \in [0, 1]$. Moreover, we assume that patents allow the monopolist to appropriate a fraction $\beta \in [0, 1]$ of the welfare generated. This formulation can be microfunded in a variety of ways. For example, in a linear demand model in which $p = a - bq$ and marginal cost is zero, the distortions induced by monopoly pricing imply $\gamma = 3/4$ and $\beta = 2/3$. In the case of perfect price discrimination $\beta = \gamma = 1$. In the presence of a negotiation between the innovator and a single user γ captures the surplus loss due to bargaining frictions whereas β captures the bargaining power of the innovator. In Appendix A1 we discuss in more detail how our theoretical set-up maps to alternative families of demand functions, to models of price discrimination and to different bargaining structures.

Our more general formulation implies that the problem for the innovator is

$$\max_{x,y} \beta\gamma U(x, y) - C(x, y)$$

which implies $x^{IP} = y^{IP} = \beta\gamma\theta$ with corresponding welfare equal to

$$W^{IP} = \gamma U(x^{IP}, y^{IP}) - C(x^{IP}, y^{IP}) = \theta^2 \beta \gamma^2 (2 - \beta).$$

Comparing the welfare in the prize and patent regimes we get the following result.

Proposition 1 *Patents dominate prizes if and only if* $\gamma \geq (2\beta(2 - \beta))^{-1/2}$.

This result illustrates a basic trade-off between patents and prizes that is central to our model. Patents have the advantage of rewarding innovative effort both for contractible and non-contractible research activities. Prizes, instead, only reward effort on the contractible activity. Nonetheless, prizes have the advantage of allowing for large rewards that can exceed the profit that an innovator can obtain through the patent rights. Moreover, patents induce monopoly pricing which generates product market distortions. Proposition 1 shows that patents are preferable to prizes only when the distortion effect of patents is small enough (β is low) and monopoly pricing allows the patentees to appropriate a substantial fraction of the surplus generated (γ is high).

The third regime is the *grand innovation prize* (GIP), which combines the two previous systems. The innovator is awarded a patent on the technology, but also receives a prize if some technical specifications are met. This implies that the innovator appropriates a fraction β of the welfare and she obtains a prize P if $x = \underline{x}$. The problem for the innovator is

$$\max_{x,y} \Pi(x,y) = \begin{cases} \beta\gamma\theta(x+y) - \frac{x^2}{2} - \frac{y^2}{2} + P \text{ if } x = \underline{x} \\ \beta\gamma\theta(x+y) - \frac{x^2}{2} - \frac{y^2}{2} \text{ else} \end{cases} .$$

If the prize is large enough, the innovator meets the target so $x^{GIP} = \underline{x}$, and the optimal level of y is $y^{GIP}(\underline{x}) = \theta\beta\gamma$. Anticipating this reaction the planner will chose $\underline{x}$ to maximize

$$\gamma U(\underline{x}, y(\underline{x})) - C(\underline{x}, y(\underline{x})) = \gamma\theta(\underline{x} + \theta\beta\gamma) - \frac{\underline{x}^2}{2} - \frac{(\theta\beta\gamma)^2}{2},$$

which implies that

$$x^{GIP} = \underline{x} = \gamma\theta.$$

Given these level of efforts, the innovator will meet the target if $P \geq \frac{1}{2}\theta^2\gamma^2(1-\beta)^2$ and the corresponding welfare will be equal to:

$$
\begin{aligned}
W^{GIP} &= \gamma\theta(\gamma\theta + \beta\gamma\theta) - \frac{(\gamma\theta)^2}{2} - \frac{(\beta\gamma\theta)^2}{2} \\
&= \frac{1}{2}\theta^2\gamma^2\left(1 + 2\beta - \beta^2\right).
\end{aligned}
$$

Notice that the effort level for $\underline{x}$ is above the level with patents. The effort on y is equivalent to the one with patents. This suggests that the welfare created by GIP is at least as large as the one with a patent. In fact, it is straightforward to show that $W^{GIP} - W^{IP} = \frac{1}{2}\theta^2\gamma^2(1-\beta)^2$, thus we have the following result:

Proposition 2 *$W^{GIP} > W^{IP}$, a grand innovation prize yields higher social welfare than a patent when $\beta < 1$.*

The result in proposition 2 is not surprising because GIPs nest patents as a special case (when the cash transfer is equal to zero), so one can think of GIPs as improved patents. On the other hand, GIPs do not contain prizes as a special case because the consumer price is set at the monopoly level with GIPs and at the competitive level with simple prizes. The next proposition compares prizes and grand innovation prizes:

Proposition 3 *GIPs dominate simple prizes if $\gamma \geq \left(1 + 2\beta - \beta^2\right)^{-1/2}$.*

Figure 1 below provides a graphical illustration of proposition 3. To analyze the trade-offs, we first note that the GIP system generates larger innovation effort by attacking two types of under-investments in innovation activities. The under-investment in measurable activities (x) that arises with the patent system is reduced thanks to the award that the innovator obtains if the effort target is met.

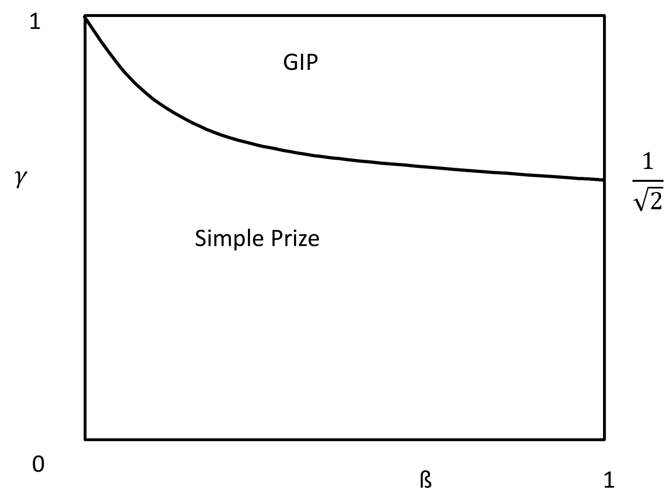

Figure 1: GIPs vs Simple Prizes

The under-investment in unmeasurable activities (y) that arises with prizes is reduced because the innovator obtains a fraction of the welfare generated, which depends on the entire set of innovative efforts. As a result of these comparisons, one can show (as illustrated in figure 1) that GIPs perform better than simple prizes when the distortion generated by monopoly pricing is small (γ close to 1) and when the innovator is able to appropriate a large fraction of the surplus created (β close to 1). In these cases, the positive investment in the y activity under GIP overcomes the monopoly pricing distortion that is present in GIP but not with prizes.[7]

Proposition 3 illustrates how complementarity between rewards and patents

[7] One can extend our model introducing asymmetric weights for activities x and y. For example, we can assume welfare to be equal to

$$W(x, y) = \theta(x + wy) - \frac{x^2}{2} - w\frac{y^2}{2}$$

with $w \geq 0$. One can think of this extension as a setting with a continuum of innovation activities with a fraction $1/(1 + w)$ that are measurable. It is possible to show that simple prizes perform better than GIPs for small values of w and that GIP dominate simple prizes for large values of w.

can arise in the presence of multiple research efforts with different degrees of contractibility. This is in stark contrast with the literature where they are typically viewed as substitute instruments. For clarity of exposition, this result is shown here in a simple model. The next sections enrich the framework introducing additional features of multitasking and embedding the trade-off in a canonical model of patent race.

3.2 Interaction between Contractible and non-Contractible Dimensions

Next we consider the full model, incorporating complementarities in innovation by using the cost function from Benabou and Tirole (2016). Assume that the cost of innovative effort is

$$C(x, y) = \frac{x^2}{2} + \frac{y^2}{2} + \rho xy$$

with $-1 < \rho < 1$. Notice that when $\rho < 0$ the two efforts are complement (i.e. $C_{xy} < 0$) and when $\rho > 0$ the two efforts are substitute (i.e. $C_{xy} > 0$). Complementarity may arise when investments toward non-contractible features of the technology (e.g. ergonomics, look, noise level) generate knowledge which help addressing the contractible aspects of the technology (e.g. fuel consumption). Conversely, substitutability is likely to arise when the innovator faces tight resource constraints and investments toward one feature of the technology render more challenging the development of other features. In this alternative setup, the first best effort levels are $x^* = y^* = \theta/(1 + \rho)$ with corresponding welfare equal to $W^* = \theta^2/(1 + \rho)$. The ordering of GIP and prize will depend on both the relative size of β and γ, as in the last section, and the degree of complementarity ρ, and therefore refines the basic result of proposition 3, and shows how grand innovation prizes are related to complementarities.

We first study patents. Using similar steps as in the baseline case, we obtain $x^{IP} = y^{IP} = \beta\gamma\theta/(1+\rho)$ with corresponding welfare equal to

$$W^{IP} = \frac{\beta\gamma^2\theta^2(2-\beta)}{1+\rho}.$$

With simple prizes it is optimal for the innovator to invest $x^P = \underline{x}$. For research activity y, we need to distinguish between substitute and complement efforts. When efforts are substitute ($\rho > 0$) the innovator will set $y^P = 0$ but when they are complements ($\rho < 0$) she will exert effort on both activities choosing $y^P = -\rho\underline{x}$. Given these strategies, the problem for the planner is to choose $\underline{x}$ such that welfare is maximized conditional on a $y^P = \max\{0, -\rho\underline{x}\}$. In the case of substitute efforts this gives $x^P = \theta$ with a corresponding welfare of $W^P = \theta^2/2$.

With complementary efforts the planner solves

$$\max_{\underline{x}} W(\underline{x}, y(\underline{x})) = \theta(\underline{x} - \rho\underline{x}) - \frac{\underline{x}^2}{2} - \frac{(\rho\underline{x})^2}{2} + (\rho\underline{x})^2,$$

and the solution involves $x^P = \theta/(1+\rho)$ and $y^P = -\rho\theta/(1+\rho)$. This implies that the welfare with the prize regime is equal to

$$W^P = \max\left\{\frac{\theta^2}{2}\frac{1-\rho}{1+\rho}, \frac{\theta^2}{2}\right\}.$$

Comparing the welfare in the prize and patent regimes, we obtain the following result:

Proposition 4 *$W^{IP} > W^P$ if*

$$\gamma \geq \sqrt{\frac{1+|\rho|}{2\beta(2-\beta)}}.$$

Together, Propositions 1 and 4 illustrate how introducing complementarity and substitutability in efforts increases the set of parameters for which prizes are preferable to patents. The intuition for the result is as follows. In our baseline model, the main drawback of prizes is that they generate incentives to invest in x but not in y. When efforts are substitute, the welfare cost from asymmetric investments is reduced because it is more costly to invest both in x and in y. Conversely, when efforts are complement, innovators invest in both research activities even if their rewards are only conditional on x. As complementarities increase, investment in y with prizes increase and the welfare generated exceeds the one of the patent regime.

With *GIPs*, the optimal level of y is $y^{GIP}(\underline{x}) = \max\{\theta\gamma\beta - \rho\underline{x}, 0\}$. Anticipating this reaction, the planner chooses $\underline{x}$ to maximize

$$\max_{\underline{x}} W(\underline{x}) = \gamma\theta(\underline{x} + y(\underline{x})) - \frac{(\underline{x})^2}{2} - \frac{(y(\underline{x}))^2}{2} - \rho\underline{x}y(\underline{x}).$$

Any optimum where $y > 0$ must satisfy $x^{GIP} = \gamma\theta/(1+\rho)$, $y^{GIP} = \gamma\theta(\beta - \rho(1-\beta))/(1+\rho)$. The welfare generated in the GIP regime when the optimum satisfies $y^{GIP} > 0$ is equal to:

$$W^{GIP} = \frac{1}{2}\frac{\gamma^2\theta^2}{(1+\rho)}(1 + 2\beta\rho + 2\beta - \rho - \beta^2 - \beta^2\rho). \tag{1}$$

Finally, when $y^{GIP} = 0$, the planner sets $x^{GIP} = \gamma\theta$ with welfare equal to $\theta^2\gamma^2/2$.

The following lemma illustrates that the planner implements asymmetric efforts ($y^{GIP} = 0$) when substitutability in efforts is high.

Lemma 5 *If* $\rho \geq \frac{\beta(2-\beta)}{2-\beta(2-\beta)}$ *then* $x^{GIP} = \gamma\theta$ *and* $y^{GIP} = 0$.

In appendix A5 using lemma 5, we show that $W^{GIP} > W^{IP}$ holds when $\beta < 1$. The simple intuition is that a GIP combines patents with another policy control (prizes), and thus it should do better than simple patents. More interestingly, we obtain the following comparison between GIP and simple prizes:

Proposition 6 $W^{GIP} > W^{P}$ *if* $\gamma > \left(1 + 2\beta - \beta^2\right)^{-1/2}$ *and*

$$\frac{\gamma^2(1+\beta(2-\beta))-1}{\gamma^2(1-\beta(2-\beta))+1} > \rho > -\frac{\gamma^2(1+\beta(2-\beta))-1}{1-\gamma^2(1-\beta(2-\beta))}.$$

Proposition 6 shows that Proposition 3 is robust to environments in which efforts are complement or substitutes. In particular, when ρ is small in absolute value the above condition on ρ holds when $\gamma > \left(1 + 2\beta - \beta^2\right)^{-1/2}$. As in Proposition 3, a necessary condition for GIPs to perform better than simple prizes is that the distortion generated by monopoly pricing is small (γ is close to 1) and the innovator is able to appropriate a large fraction of the surplus created (β close to 1). But in this extended model, simple prizes may be preferable in the presence of very high levels of complementarity or very high levels of substitutability between the innovation activities. With high substitutability, the loss in welfare generated by investing only in the contractible activity becomes very small. Conversely, when efforts are complements, innovators invest in both research activities even if the prize is only conditional on x. When complementarities are large enough, the under-investment in y with prizes becomes negligible and the welfare generated exceeds the one with the GIP regime.

The main trade-offs of our baseline model are also present when complementarities affect consumer surplus rather than the cost, for example if we assume $U(x, y) = \theta(x + y) + \rho xy$. It is possible to show that in this case complementarities tend to reduce the appeal of simple prizes. This is because in the simple prize regime the reward only depends on x and any complementarity between x and y cannot be internalized by the innovator. Conversely, complementarities will be internalized by the innovator with GIPs or patents because the reward for the innovator depends on the consumer surplus it generates.

4 Grand innovation prizes in an innovation race

Our baseline model assumed the existence of only one innovator and that the technology was instantaneously developed after investment in x and y. In this section, we relax these assumptions and extend the model in two directions. First, following Loury (1979) and De Nicolò (2000), we assume that the innovator can invest in a third (not-contractable) effort type which is the speed of development. Second, we consider the presence of multiple innovators racing for the prize (or patent).

In Section 4.1, we show that when there is a single innovator all of the previous comparisons are unchanged when speed is an added choice. In Section 4.2, we show that when there are multiple innovators, patents may perform particularly poorly because firms may be incentivized to produce small innovations very rapidly; a grand innovation prize, that specifies some minimum standard on at least some dimension, works against this incentive and therefore has an additional benefit not described in the model of Section 3. In Section 4.3, we explore the impact of free entry on the results.

4.1 Introducing innovation speed in the one innovator case

We start by analyzing the case in which only one innovator participates in the innovation process. We assume that completion time of the project follows a Poisson process which depends on the investment in speed, ϕ and that the interest rate is r. We assume that the overall cost for the innovation is

$$\left(\frac{x^2}{2}+\frac{y^2}{2}\right)\phi$$

where the multiplicative specification captures the idea that it is more costly to anticipate the development of high quality projects.

To reach the first best, the maximization problem of the planner is

$$\max_{\phi,x,y} \int_0^\infty e^{-(\phi+r)t}\phi\theta(x+y)dt - \left(\frac{x^2}{2}+\frac{y^2}{2}\right)\phi$$

which can be rewritten as

$$\max_{\phi,x,y} \frac{\phi}{r+\phi}\theta(x+y) - \left(\frac{x^2}{2}+\frac{y^2}{2}\right)\phi.$$

Solving this maximization problem, we obtain

$$\phi^* = r, \tag{2}$$

and

$$x^* = y^* = \frac{\theta}{2r}. \tag{3}$$

Formulas (2) and (3) imply that the first best welfare is $W^* = \theta^2/4r$.

A prize P with target $\underline{x}$ leads to the following problem for the innovator:

$$\max_{\phi,y} \frac{\phi}{r+\phi}P - \left(\frac{\underline{x}^2}{2}+\frac{y^2}{2}\right)\phi.$$

The level of investment in y is zero, so the first order condition in ϕ is

$$\frac{r}{(r+\phi)^2}P = \underline{x}^2/2. \tag{4}$$

Crucially, the above calculations show that by choosing P in an appropriate manner, the planner can implement any innovation speed ϕ respecting $\underline{x}$, the target for activity x. Therefore, to maximize welfare, the planner can directly control x and ϕ, maximizing welfare by solving

$$\max_{\phi,x} \frac{\phi}{r+\phi}\theta x - \frac{x^2}{2}\phi,$$

which yields $\underline{x} = \theta/2r$ and $\phi = r$. The corresponding welfare is $W^P = \theta^2/8r$.[8]

[8] Notice that the planner can implement her preferred ϕ because the first order condition for the innovator allow to compute the prize to implement ϕ given $\underline{x}$. Specifically: $P = \frac{1}{r}(r+\phi)^2\left(\frac{1}{2}\underline{x}^2 + \frac{1}{2}\theta^2\beta^2\frac{\gamma^2}{(r+\phi)^2} - r\theta\beta\gamma\frac{\underline{x}+\theta\beta\frac{\gamma}{r+\phi}}{(r+\phi)^2}\right)$.

We now consider the patent reward. The firm solves

$$\max_{\phi,x,y} \frac{\phi}{r+\phi}\beta\gamma\theta(x+y) - \left(\frac{x^2}{2}+\frac{y^2}{2}\right)\phi,$$

which yields $x^{IP} = y^{IP} = \beta\gamma\theta/2r$, and $\phi^{IP} = r$. The resulting welfare is

$$W^{IP} = \frac{(2-\beta)}{4r}\theta^2\beta\gamma^2.$$

Finally, we turn to GIP. Variable y is chosen by the innovator to maximize

$$\frac{\phi}{r+\phi}(P+\theta\beta\gamma(y+\underline{x})) - \left(\frac{\underline{x}^2}{2}+\frac{y^2}{2}\right)\phi,$$

which yields

$$y = \frac{\theta\beta\gamma}{r+\phi}. \tag{5}$$

Plugging this into the welfare and maximizing with respect to ϕ and $\underline{x}$ we obtain $\phi^{GIP} = r$, and $x^{GIP} = \theta\gamma/2r$. Therefore, $y^{GIP} = \theta\gamma\beta/2r$ by (5). The corresponding welfare is

$$W^{GIP} = \frac{1}{8r}\theta^2\gamma^2\left(1+2\beta-\beta^2\right).$$

Therefore, all the welfare levels, W^P, W^{IP}, W^{GIP} are equal to those of the static case divided by $4r$. This implies that the comparison between the instruments is the same as in the original static model.

4.2 A race with n innovators

Introducing multiple innovators leads to a contests with multi-tasking. While in other contexts this poses several technical challenges and requires various simplifying assumptions (DeVaro and Gürtler, 2016), our model extends naturally to a race with n participants.

Let us start considering simple prizes. Because investment in y is zero, each innovator solves

$$\max_{\phi} \frac{\phi}{r+\phi+(n-1)\widetilde{\phi}} P - \phi \underline{x}^2/2$$

where $\widetilde{\phi}$ is the speed employed by the other $n-1$ innovators and the prize P is paid only to the first innovator reaching target $\underline{x}$. Using the first order condition, and exploiting symmetry ($\widetilde{\phi} = \phi$) yields

$$P = \frac{\underline{x}^2 \left(r+n\phi\right)^2}{2\left(r+(n-1)\phi\right)}, \tag{6}$$

which is the prize that implements speed ϕ given target $\underline{x}$.

Formula (6) shows that just like in the case with a single innovator, by choosing P in an appropriate manner, the planner can implement any innovation speed ϕ respecting $\underline{x}$, the target for activity x. Therefore, to maximize welfare, the planner can directly control x and $\Phi = n\phi$, and maximize welfare by solving

$$\max_{\Phi, x} \frac{\Phi}{r+\Phi}\theta x - \Phi \frac{x^2}{2}.$$

This implies $x^P = \theta/2r$ and $\Phi^P = r$, which leads to the same welfare as in the one innovator case.

We now consider the patent reward. The firm solves the following problem

$$\max_{\phi, x, y} \frac{\phi}{r+\phi+(n-1)\widetilde{\phi}} \beta\gamma\theta(x+y) - \left(\frac{x^2}{2} + \frac{y^2}{2}\right)\phi.$$

The first order conditions yield

$$y = x = \frac{\beta\gamma}{r+\phi+(n-1)\widetilde{\phi}}\theta,$$

and

$$x^2 = \frac{r+(n-1)\widetilde{\phi}}{\left(r+\phi+(n-1)\widetilde{\phi}\right)^2} 2\beta\gamma\theta x.$$

However, these conditions imply that $\phi = r + (n-1)\widetilde{\phi}$, so a symmetric equilibrium does not exist as each firm wants to adopt an innovation speed higher than its rivals. To avoid this technical complication, let us assume that the policy space is such that $\phi \in [0, \overline{\phi}]$ must hold for some high value $\overline{\phi}$. Then a symmetric equilibrium exists, and is characterized by

$$y^{IP} = x^{IP} = \frac{\beta\gamma}{r + n\overline{\phi}}\theta,$$

and

$$\phi^{IP} = \overline{\phi}.$$

If the upper bound $\overline{\phi}$ is taken to be large, then y^{IP} and x^{IP} are close to zero. This result differs substantially from the one innovator case. The race induces firms to develop only very small innovations (i.e. technologies with very small values of x and y) and to 'rush' to the patent office to obtain the patent right. The intuition for this finding is that the multi-dimensional nature of the problem induces firms to undercut each other both in speed and in innovation quality, and innovators find it more profitable to invest in speed rather than in technology value.

Consider now a grand innovation prize. The innovator solves

$$\max_{y} \frac{\phi}{r + \phi + (n-1)\widetilde{\phi}} \left(P + \theta\beta\gamma(\underline{x} + y)\right) - \left(\frac{\underline{x}^2}{2} + \frac{y^2}{2}\right)\phi,$$

which yields $y = \theta\beta\gamma/(r + \Phi)$. Plugging this into the welfare we obtain

$$W = \frac{\Phi}{r + \Phi}\theta\gamma(\underline{x} + \frac{\theta\beta\gamma}{r + \Phi}) - \Phi\frac{1}{2}(\frac{\theta\beta\gamma}{r + \Phi})^2 - \Phi\frac{\underline{x}^2}{2}.$$

The first order conditions with respect to Φ and $\underline{x}$ yield[9] $\phi^{GIP} = r$ and $x^{GIP} =$

[9] Also in this case the planner can implement her preferred ϕ given a level of x by setting $P = \frac{1}{r+\phi(n-1)}(r + n\phi)^2 \left(\frac{1}{2}x^2 - \left(\theta^2\beta^2\frac{\gamma^2}{r+n\phi} + x\theta\beta\gamma\right)\frac{r+\phi(n-1)}{(r+n\phi)^2} + \frac{1}{2}\theta^2\beta^2\frac{\gamma^2}{(r+n\phi)^2}\right)$.

$\theta\gamma/2r$, $y^{GIP} = \theta\gamma\beta/2r$. The corresponding welfare is equal to

$$W^{GIP} = \frac{1}{8r}\theta^2\gamma^2\left(1 + 2\beta - \beta^2\right),$$

which is the same as in the one innovator case.

The results in this section illustrate that in a racing model the welfare generated by simple prizes and GIPs is similar to our baseline case but simple patent mechanisms perform much worse when there are at least two firms racing for the patent. Because in our framework research effort is multi-dimensional, patent races generate high level of inefficiencies, which go beyond the duplication of effort commonly emphasized in the literature. GIPs can address the under-investment that takes place with patents by avoiding the undercutting in the measurable dimension of the innovation investment.

These results have implications for patent policy. In particular, they suggest that by setting high inventive step and non-obviousness requirements - which in our model translate to $\underline{x}$ - patent offices may reduce innovators' incentives to rush to patent technologies of low value.

4.3 Robustness to endogenous entry

It is also interesting to explore the robustness of these findings in the case where the number of firms is endogenously determined by a zero profit condition. In particular, suppose that in order to participate in the discovery process a firm needs to sustain a fixed entry cost, F, and in equilibrium the number of firms is such that each entering firm makes zero profits. We analyze this case formally in appendix A2. We exploit this alternative model to study whether the properties of prizes, patents and GIPs derived for a fixed number of participants extend to the case of endogenous entry, especially in the limiting case in which F tends to zero.

Appendix A2 describes the properties of the optimal policy (P, $\underline{x}$) for prizes and grand innovation prizes in the case of endogenous entry. The main result is that at the limit (i.e. as F tends to zero) the welfare generated by this alternative model is exactly the same as in the case in which the number of innovators is fixed. Therefore, the comparison between these two mechanisms appear robust to endogenous entry. At the same time, the predictions of the endogenous entry model differ from the fixed n model in the case of patents. Because of the undercutting process described in Section 4.2, in the endogenous entry model the number of firms racing is always strictly below 2 for any $F > 0$.[10] Intuitively, entry costs mitigate the dissipation effect by inducing only one firm to enter the patent race. This implies that the comparison between the three regimes becomes identical to the case in which only one innovator is active, and all the findings from our baseline model of Section 3 hold.[11]

5 Asymmetric information and costly public funds

The prior results can be generalized in many ways and in some (but not all) cases GIPs are superior to prizes and patents. In this section, we describe two important extensions that are often studied in the innovation literature in more details.

5.1 Asymmetric information

Assuming that the innovator has private information about market demand or cost variables is standard in the mechanism design approach to IP policy (Scotchmer, 2004). We show that our basic results are unaffected by the presence

[10] This result is subject to the technical caveat about equilibrium existence that we highlighted in Section 4.2. The precise statement with free entry is that given any fixed cost $F > 0$, if the upper bound on the speed of innovation $\overline{\phi}$ is large enough, then profits are negative if more than one firm enters.

[11] This also suggests that application fees may also be used by patent offices to generate entry costs and mitigate undercutting incentives. See our discussion in appendix A2.

of asymmetric information because the main effects at work depend on the multidimensionality of effort (multitasking), and on the assumption that one of the dimensions cannot be contracted on. In appendix A3, we make this point by introducing a simple extension of our baseline model which combines screening with multi-tasking.[12]

Specifically, we extend our basic framework by assuming that there is a cost parameter τ which is private information for the single innovator. The cost parameter τ is distributed on $[\underline{\tau}, \overline{\tau}]$ with probability density f. With this extension, the social welfare function is[13]

$$W = \theta(x + y) - \tau(\frac{x^2}{2} + \frac{y^2}{2}).$$

We first consider simple mechanisms, that is, mechanisms where the planner decides in advance (before receiving any message from the innovator about τ) whether to have a prize, a patent or a grand innovation prize. In a prize regime a mechanism is a pair $(x(\tau), P(\tau))$. In appendix A3 we show that with an appropriate payment scheme any allocation is implementable in which x is weakly decreasing in τ. Therefore, the complete information welfare where $x^P = \theta/\tau$ is attainable for all τ, and thus

$$W^P = \int_{[\underline{\tau},\overline{\tau}]} \frac{\theta^2}{2\tau} f(\tau) d\tau = E_\tau[\theta^2/2\tau].$$

In the patent regime, research efforts are $x^{IP} = y^{IP} = \beta\gamma\theta/\tau$. This implies that welfare will be equal to:

$$W^{IP} = E_\tau[\theta^2 \beta\gamma^2 (2 - \beta) / 2\tau] = \beta\gamma^2 (2 - \beta) E_\tau[\theta^2/2\tau].$$

[12]See Benabou and Tirole (2014) for another recent paper combining screening and multi-tasking in a different application.

[13]The reason why we assume that the private information comes in on the cost side, is that private information about θ is not payoff relevant for the innovator in the prize regime. Therefore, when a simple prize is offered it is impossible for the planner to screen the innovator based on the innovator's private information about θ, a degenerate case.

Thus the comparison between prizes and patents is the same as in the case under complete information:

$$W^{IP} \geq W^{P} \Longleftrightarrow \beta\gamma^2 (2-\beta) \geq 1.$$

In the grand innovation prize regime a mechanism is a vector $(x(\tau), P(\tau))$ because $y(\tau)$ cannot be contracted on as it is chosen by the innovator. The same argument as in the case of simple prizes means that grand innovation prizes are not affected by asymmetric information either because any mechanism where $x(\tau)$ is decreasing is possible to implement. Therefore, the welfare of grand innovation prizes is similar to the level under complete information:

$$W^{GIP} = E_\tau[\theta^2\gamma^2(1+2\beta-\beta^2)/2\tau] = \gamma^2(1+2\beta-\beta^2)E_\tau[\theta^2/2\tau].$$

Therefore, the welfare comparison between the three mechanisms is the same as under complete information.

These results can also be used to examine the case in which the planner can choose the reward mechanism depending on the reported cost type τ. It is easy to see that the optimal mechanism (i.e. P vs GIP) does not depend on the actual cost type τ. In fact, the planner will choose a GIP if $\gamma^2(1+2\beta-\beta^2) > 1$, otherwise a prize will be chosen.

5.2 Costly public funds

The assumption that society does not incur a loss when raising revenue (to pay the prize) is a typical assumption in the economics of innovation literature (Chari et al., 2012; Weyl and Tirole, 2012). The assumption is justifiable if prizes are conducted by philanthropic foundations and are not associated with distortionary taxation. However, it is useful to extend our baseline model considering the case in which the planner finances prize P at a cost κP where $\kappa > 0$

denotes the cost of public funds due to the deadweight loss associated with taxation (as in Laffont and Tirole, 1993). The planner faces a trade-off between two types of welfare distortion: the cost of raising money through public taxation, κ, and the surplus losses due to market power arising from patents.

While the welfare from simple patents is not affected by the cost of public funds, the welfare from prizes becomes

$$W^P = \frac{\theta^2}{2(\kappa+1)}.$$

Similarly, the welfare generated with a GIP becomes

$$W^{GIP} = \frac{\theta^2\gamma^2}{2(\kappa+1)}\left(2\beta + 4\kappa\beta - \beta^2 - 2\kappa\beta^2 + 1\right).$$

In this environment, simple patents dominate prizes when κ is very large. More interestingly, the welfare from GIP is less sensitive to κ compared to the welfare with prizes because in a GIP part of the reward is funded by monopoly profits, while with prizes the whole reward comes from public funds.

In appendix A4, using a simple two-type model, we explore the link between asymmetric information and costly public funds. We argue that for simple prizes under asymmetric information it is optimal for the planner to distort the schedule $x(\tau)$ such that $x(\tau) < x^P(\tau)$ for all types except the most efficient ($\tau = \underline{\tau}$) so that the planner can reduce the prize $P(\tau)$ paid out to all types $\tau \in [\underline{\tau}, \overline{\tau}]$. Similarly, with Grand Innovation Prizes the optimal schedule of x and y are distorted below the complete information optimum when $\kappa > 0$.

6 Discussion and implications for policy

Our theoretical results have interesting implications for innovation policy and can provide guidance to government agencies and philanthropists on how to design effective innovation prizes.

The first message of our analysis is that the joint use of IP rights and cash rewards - commonly observed in a variety of successful innovation contests - should not be considered as a divide between the practice and the policy prescriptions of economic theory. Our model, which extends classic prize and patent theories by introducing multidimensional and partially contractible research investments, provides an economic rationale for the complementary use of patents and prizes by a socially oriented prize organizer

Our theory emphasizes a previously overlooked problem with innovation prizes when they require technologies to be placed in the public domain. When prize organizers can only describe a subset of technical specifications of the winning technology, innovators may overlook the technological dimensions not specified by the organizers. This is particularly likely when innovators face tight budget constraints, as in the case of small start-up firms. We show that this problem can be corrected by combining prize rewards with patents and thus letting the inventors obtain profits which depend on all the features of the technology that are valued by the market, and not only by those specified by the prize organizers.

The second insight from our theory is that cash prizes with technologies placed in the public domain can be very effective for competitions in which the solution can be described in great detail, such as the software development contests examined in Boudreu et al. (2011). At the same time our framework shows that broader, more difficult to describe situations call for a GIP in which cash transfers and monopoly rights are jointly awarded. An additional implication is that policy makers designing innovation prizes may prefer to focus on few core technical specifications that can be clearly articulated and measured and assure access to strong IP protection, rather than listing vague and hard to measure requirements for the winning technology.

Our results also show that two key policy-relevant parameters required to assess the effectiveness of GIPs are the profits that can be extracted from the patent and the deadweight loss generated by monopoly pricing. GIPs dominate simpler prize rewards when (i) the profits that can be extracted from the patent are small relative to the social benefit of the technology, and (ii) the distortionary effects of monopoly rights are limited.

In settings where ideas are commonly held, so that there is the potential for a patent race, GIPs can mitigate the incentive for innovators to substitute speed of arrival for quality of the innovation. In a sense the GIP has a similar effect to enacting stricter patentability requirements (i.e. inventive step and non-obviousness), and therefore can be a tool for a policymaker who wants to target particular innovation areas for more substantial contributions. Moreover, it can do so without the need to specify the details of what constitutes a large enough step. Therefore GIPs can be useful in areas where a substantial innovation is needed, but difficult to fully describe, as is common with substantial innovations.

Outside the context of our model there are a variety of additional reasons why policy makers may consider the use of GIPs. GIPs may provide a useful coordination device to focus the attention to specific social problem or new technological developments. In fact, one of the goals of the XPrize foundation is to "inspire the formation of new industries and revitalize markets that are currently stuck due to existing failures" (Murray et al., 2012). Moreover, as suggested by the National Academy of Engineering (NAE, 1999) with the right media exposure GIP may attract young people into science and engineering and inspire public support toward research investments. Finally, in some technology areas GIPs may speed up the cumulative innovation process by spurring the development of the initial version of a technology which is then is used as input into subsequent innovation (Hopenhayn, Llobet and Mitchell, 2006; Galasso and Schankerman, 2015).

In combination, our results identify new properties of GIPs and examine their relevance for the design of effective innovation policies. The trade-offs emphasized in our theory can inform and offer guidance to policy makers as they attempt to increase R&D investments with new incentive mechanisms.

7 Conclusions

The economics of innovation literature has typically viewed prizes and patents as substitute ways to reward innovators and to encourage research investments. The management literature has documented that Grand Innovation Prizes allow participants to keep the IP rights on their technologies. This paper developed a model which fills this gap between theory and practice. Guided by the case study analysis in Murray et al (2012), our model assumed that some performance targets can be described but a full description of a solution is unavailable. This hybrid of the usual assumptions justifying prizes and patents, respectively, makes the use of both in conjunction a natural outcome.

Our analysis compares three different reward structures: patents, prizes and GIPs. We show that often GIPs generate larger innovation incentives than patents or prizes, because partial contractibility of the technological features generates a complementarity between monopoly rights and cash rewards. Specifically, the under-investment in measurable activities that arises with the patent system is reduced thanks to the award that the innovator obtains if the effort target is met. The under-investment in unmeasurable activities that arises with prizes is reduced because the innovator obtains a fraction of the welfare generated, which depends on the entire set of innovative efforts. We discuss robustness of our model to complementary and substitute research investments, costly public funds, incomplete information and competition between multiple innovators.

References

[1] Arrow, Kenneth (1962), "Economic Welfare and the Allocation of Resources for Inventions," in Richard Nelson, ed., *The Rate and Direction of Inventive Activity: Economic and Social Factors* (Princeton: Princeton University Press): 609–625

[2] Bénabou, Roland, and Jean Tirole (2016) "Bonus culture: Competitive pay, screening, and multitasking," *Journal of Political Economy* 124: 305-370

[3] Bergemann, Dirk, Benjamin Brooks, and Stephen Morris (2015) "The limits of price discrimination," *American Economic Review* 105: 921-957.

[4] Boudreau, Kevin, Nicola Lacetera and Karim Lakhani (2011) "Incentives and problem uncertainty in innovation contests: an empirical analysis," *Management Science* 57, 843-886

[5] Boudreau, Kevin and Karim Lakhani (2015) ""Open" disclosure of innovations, incentives and follow-on reuse: Theory on processes of cumulative innovation and a field experiment in computational biology." *Research Policy* 44: 4-19.

[6] Boudreau, Kevin, Karim Lakhani, Michael Menietti (2016) "Performance Responses To Competition Across Skill-Levels In Rank Order Tournaments: Field Evidence and Implications For Tournament Design," *RAND Journal of Economics* 47, 140-165

[7] Chari V. V., Mikhail Golosov and Aleh Tsyvinski (2012) “Prizes and Patents: Using Market Signals to Provide Incentives for Innovations," *Journal of Economic Theory* 147: 781-801

[8] Connelly, Brian, Laszlo Tihanyi, Russell Crook, Ashley Gangloff (2014) "Tournament Theory: Thirty Years of Contests and Competitions," *Journal of Management* 40: 16-47

[9] Cornelli, Francesca, and Mark Schankerman (1999) "Patent renewals and R&D incentives," *RAND Journal of Economics* 30: 197-213.

[10] Denicolò, Vincenzo (2000) "Two-stage patent races and patent policy," *RAND Journal of Economics* 31: 488-501

[11] Dewatripont, Mathias, Ian Jewitt, and Jean Tirole (2000) "Multitask agency problems: Focus and task clustering," *European Economic Review* 44: 869-877

[12] DeVaro Jed and Oliver Gürtler (2016) "Strategic Shirking in Promotion Tournaments," *Journal of Law and Economic Organizations*, 32: 620-651

[13] Galasso, Alberto, Matthew Mitchell and Gabor Virag (2016) "Market Outcomes and Dynamic Patent Buyouts," *International Journal of Industrial Organization 48:* 207-243

[14] Galasso, Alberto, and Mark Schankerman (2015) "Patents and cumulative innovation: Causal evidence from the courts," *Quarterly Journal of Economics* 130: 317-369

[15] Gans, Joshua and Scott Stern (2017) "Endogenous Appropriability," *American Economic Review* 107: 317-321

[16] Hellmann, Thomas and Veikko Thiele (2011) "Incentives and Innovation: A Multitasking Approach," *American Economic Journal: Microeconomics* 3:78-128

[17] Holmstrom, Bengt and Paul Milgrom (1991) "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership and Job Design," *Journal of Law Economics and Organization* 7:24-52

[18] Hopenhayn, Hugo, Gerard Llobet and Matthew Mitchell (2006) "Rewarding Sequential Innovations: Prizes, Patents, and Buyouts," *Journal of Political Economy* 114: 1041-1068

[19] Hopenhayn Hugo and Matthew Mitchell (2001) "Innovation variety and patent breadth," *RAND Journal of Economics* 32: 152-166

[20] Kay, Luciano (2011) "The effect of inducement prizes on innovation: evidence from the Ansari X Prize and the Northrop Grumman Lunar Lander Challenge," *R&D Management* 41: 360-377

[21] Kremer, Michael (1998) "Patent buyouts: A mechanism for encouraging innovation," *Quarterly Journal of Economics* 113: 1137-1167

[22] La Manna, Manfredi (1992) "Optimal patent life vs optimal patentability standards," *International Journal of Industrial Organization* 10: 81-89

[23] Lazear, Edward (1989) "Pay Equality and Industrial Politics," *Journal of Political Economy* 97:561-580

[24] Lazear, Edward and Sherwin Rosen (1981) "Rank-Order Tournaments as Optimum Labor Contracts," *Journal of Political Economy* 89: 841-864

[25] Lerner, Josh, and Ulrike Malmendier (2010) "Contractibility and the Design of Research Agreements," *American Economic Review* 100: 214-46

[26] Loeb, Martin, and Wesley Magat (1979) "A decentralized method for utility regulation," *Journal of Law and Economics* 22: 399-404

[27] Loury, Glenn (1979) "Market structure and innovation," *Quarterly Journal of Economics* 93: 395-410

[28] McKinsey (2009) "“And the winner is . . . ” Capturing the promise of philanthropic prizes". Sidney, Australia: McKinsey and Co.

[29] Murray, Fiona, Scott Stern, Georgina Campbell and Alan MacCormack (2012) "Grand Innovation Prizes: A theoretical, normative and empirical evaluation," *Research Policy* 41: 1779-1792

[30] National Academy of Engineering (1999), Report concerning federally sponsored inducement prizes in engineering and science, https://www.nap.edu/catalog/9724/concerning-federally-sponsored-inducement-prizes-in-engineering-and-science

[31] O'Donoghue, Ted (1998) "A Patentability Requirement for Sequential Innovation," *RAND Journal of Economics* 29: 654–679

[32] Scotchmer, Suzanne (1999) "On the Optimality of the Patent Renewal System," *RAND Journal of Economics* 30: 181-196

[33] Scotchmer, Suzanne (2004) *Innovation and Incentives*, the MIT Press, Cambridge MA

[34] Scotchmer, Suzanne and Green, Jerry, (1990) "Novelty and Disclosure in Patent Law," *RAND Journal of Economics*, 21:131-146

[35] Shavell, Steven and Tanguy Van Ypersele (2001), "Rewards versus Intellectual Property Rights," *Journal of Law and Economics* 44: 525-547

[36] Sherwin Rosen (1986) "Prizes and Incentives in Elimination Tournaments," *American Economic Review*, 76: 701-715

[37] Taylor, Curtis (1995) "Digging for Golden Carrots: An Analysis of Research Tournaments," *American Economic Review* 85: 872-890

[38] Weyl, Glenn and Jean Tirole (2012) "Market Power Screens Willingness to Pay," *Quarterly Journal of Economics* 127: 1971-2003

[39] Williams, Heidi (2012) "Innovation Inducement Prizes: Connecting Research to Policy," *Journal of Policy Analysis and Management* 31: 1-25

[40] Wright, Brian (1983) "The Economics of Invention Incentives: Patents, Prizes and Research Contracts," *American Economic Review* 73: 691-707

Appendix

A1. Microfounding β and γ

In this Appendix, we discuss a variety of alternative micro-foundations for the parameters β and γ and relate them to features of the underlying demand function and the contracting environment.

A1.1 A parameterized family of demand functions

First, we provide a family of demand functions for which β and γ depend on a single parameter k, and discuss when the conditions in Proposition 1, and Proposition 3 hold. This exercise helps us in relating our findings in Proposition 1 and 3 to familiar properties of demand functions. In particular, we consider the demand family

$$q = \sqrt[k+1]{\theta(x+y)(k+1)} - p^{\frac{1}{k}}.$$

Notice that consumer surplus when $p = 0$ is equal to

$$\int_0^{\sqrt[k+1]{\theta(x+y)(k+1)}} \left(\sqrt[k+1]{\theta(x+y)(k+1)} - q \right)^k dq =$$

$$\left| -\frac{\left(\sqrt[k+1]{\theta(x+y)(k+1)} - q \right)^k}{k+1} \right|_0^{\sqrt[k+1]{\theta(x+y)(k+1)}} = \theta(x+y). \tag{7}$$

Moreover, the equilibrium profits for a monopolist which are obtained from maximizing

$$\max_p p \left(\sqrt[k+1]{\theta(x+y)(k+1)} - p^{\frac{1}{k}} \right)$$

are equal to

$$\theta(x+y)(\frac{k}{k+1})^k. \tag{8}$$

Under monopoly the total welfare generated is equal to

$$\theta(x+y)\left(1 - (\frac{k}{k+1})^{k+1}\right). \tag{9}$$

Together, (7), (8) and (9) provide parameterized values for γ and β

$$\gamma = 1-(\frac{k}{k+1})^{k+1} \tag{10}$$

$$\beta = \frac{k^k/(k+1)^k}{1-(\frac{k}{k+1})^{k+1}}. \tag{11}$$

Using, (10) and (11), the key condition in Proposition 1 $\gamma \geq (2\beta(2-\beta))^{-1/2}$ boils down to $k \leq 1$. This implies that in our parametrized family of demand functions patents yield higher welfare than prizes if and only if $k \leq 1$. In our environment this condition holds exactly when the demand function is concave in p which corresponds to the case where demand declines relatively slowly at low prices but more quickly at high prices. In other words, there are only a few consumers who value the innovation a lot, and there are a lot of consumers who value the innovation little. In the opposite case where relatively many consumers place a high value on the innovative product, prizes offer higher welfare than patents.

Proposition 3 compares GIP and prizes, and finds that a GIP performs better if $\gamma \geq (1+2\beta-\beta^2)^{-1/2}$. Using the formulas above, this condition rewrites as $k \leq 1.32$ (derived from a simple numerical analysis with Scientific Workplace). The interpretation is only slightly different from the case of Proposition 1: GIP performs better than simple prizes if and only if the demand function is not too convex. More importantly the set of parameters in which Proposition 3 holds exceeds the set of parameters for which Proposition 1 holds.

Notice that in the special case when $k=1$ the demand is linear and $\gamma = \frac{3}{4}$ and $\beta = \frac{2}{3}$. In this case Proposition 1 implies that prizes and patents generate the same welfare whereas Proposition 3 implies that GIPs perform better than simple prizes.

The family of demand functions analyzed in the subsection A1.1 suggests that β and γ are one-to-one at least for standard demand functions. However, this is not the case for at least two reasons. The first reason is that since the demand function is known the parameters β and γ are known real constants. It is easy to see that β and γ can vary independently if we choose arbitrary demand functions. To drive this point home and to provide robustness checks, such an analysis is provided in subsection A1.2 showing that different combinations of β and γ arise when the family of constant elasticity demand functions is used.

The second reason is that under different bargaining and pricing protocols (i.e., if price discrimination is allowed or if profits and welfare are determined in a dynamic game of innovation) the monopolist earns different profits, and total welfare also varies in a flexible manner. Section A1.3 illustrates two formulations of this reasoning building on Bergemann et al. (2015) and Gans and Stern (2017).

A1.2 The constant elasticity family

One can consider a different family of demand functions, those with constant demand elasticity. Let $q = p^{-\varepsilon}$ with $\varepsilon > 1$ and assume that the firm faces an arbitrary marginal cost equal to $c > 0$. Tirole (1988, pages 67 and 88) shows that in this case the monopoly profit is $\pi^m = \frac{c^{1-\varepsilon}}{(\varepsilon-1)^{1-\varepsilon}}\varepsilon^{-^{\varepsilon}}$, the welfare under monopoly price is $W^m = \frac{2\varepsilon-1}{\varepsilon-1}(\frac{\varepsilon}{\varepsilon-1})^{-\varepsilon}\frac{c^{1-\varepsilon}}{\varepsilon-1}$, and the competitive welfare is $W^c = \frac{c^{1-\varepsilon}}{\varepsilon-1}$. Therefore,

$$\beta = \frac{\pi^m}{W^m} = \frac{(\varepsilon-1)^{1-\varepsilon}}{2\varepsilon-1},$$

and

$$\gamma = \frac{W^m}{W^c} = (2\varepsilon-1)\varepsilon^{-\varepsilon}(\varepsilon-1)^{\varepsilon-1}.$$

These values do not depend on c, so we characterized another family of demand functions where β, γ depend only on a single parameter. However, the map of how β corresponds with γ is different from the one induced by the demand family that was parametrized by k studied in subsection A1.1.[14] Therefore, β and γ do not have a one-to-one correspondence in general.

Also, for this parametrized family the condition in Proposition 1 holds if and only if $\varepsilon \leq 1.59$, which corresponds to the case where demand is not too elastic. So, patents perform better than simple prizes if and only if the demand is not too elastic because in this case the deadweight loss from monopoly pricing is not too large. Similarly, the condition in Proposition 3 holds if and only if $\varepsilon \leq 1.75$. Therefore, the optimal GIP performs better than a simple prize if the demand elasticity is not too high. The intuition is the same as above.

A1.3 Decoupling β and γ

In A1.1 and A1.2 we examined how β and γ change across demand functions keeping constant other features of the pricing and contracting environment. An alternative approach to microfounding β and γ is to compare alternative contractual structures for a specific demand function. With this approach, β and γ may move independently even for a fixed demand specification.

To see this, consider a linear demand function. The analysis in A1.1 has shown that monopoly pricing in the absence of price discrimination gives $\gamma = \frac{3}{4}$ and $\beta = \frac{2}{3}$. Suppose now that the innovator can perfectly price discriminate with a two-part tariff. In this case we will have that $\gamma = \beta = 1$, for the very same demand function. Bergemann et al. (2015) show that with additional patterns of segmentation and price discrimination the innovator can achieve a large number of alternative combinations of producer and consumer surplus.[15]

[14] This is easy to see by letting $\varepsilon = 1$, and noting that $\beta = \gamma = 1$ in this case. This first-best configuration is not possible to reach in the demand family parametrized by k.

[15] Bergemann et al. (2015) show that depending on how much information a price discriminating monopolist has any combination of total welfare, consumer surplus and profits is possible to support in equilibrium subject to the constraint that the monopolist earns at least

One can also obtain different values of β and γ for the same demand function introducing more structure in the patent grant process and the bargaining between innovator and technology users. To see this consider the following example which builds on Gans and Stern (2017). The gross consumer surplus generated by the innovation is

$$U(x,y) = \frac{(x+y)}{1-\delta}.$$

By setting $\theta = 1/(1-\delta)$ we consider an innovation which lasts for an infinite horizon with discount factor δ. There is only one consumer which negotiates with the patent holder. Moreover, it takes T periods for the patent office to process a patent application and negotiation between the innovator and the consumer can only take place after time T (Gans and Stern, 2017). Thus, processing delay at the patent office lead to

$$W^{IP} = \gamma U(x,y)$$

where $\gamma = \delta^T$. The surplus W^{IP} will be split between the patentee and the consumer through a bargaining process, which gives the patentee $\beta\gamma U(x,y)$. Thus, one may interpret β as the share of surplus appropriated by the patentee which depends on their bargaining power. Notice how, in this set-up γ depends on patent office delays and β on the relative bargaining power of the patentee. These two forces are logically independent from each other, which implies that β and γ are decoupled from each other.

A2. Race with endogenous entry

In this appendix, we study the case of free entry as discussed in Section 4.3. The model is the same as with n innovators except that n is not fixed but it is characterized by a zero-profit condition. Letting V_n be value of entering when

as much in profits as in the case where he cannot price discriminate.

n firms are present in the market (including the new entrant), it must hold that

$$V_n = F + \phi \frac{x^2 + y^2}{2}, \tag{12}$$

that is fixed entry cost plus variable innovation costs are equal to the expected producer surplus enjoyed by the entrant on the product market. Notice how the above condition does not impose an integer constraint on the number of entrants n. Such an integer constraint would be easy to formally recognize but would not substantially change our analysis, except for the case of patents which will discuss in greater detail below.

Prizes

We start by considering prizes. First, we derive the equilibrium level of innovation effort (ϕ) and the number of entrants (n) given the level of innovation x, y. We then characterize the optimal policy of the planner and show that the welfare optimum is the same as with any any fixed n (including $n = 1$).

Calculating n, ϕ for free entry for given x, y

Let us first consider the Bellman-equation for the value function if n firms are present with a fixed n and study optimality conditions for ϕ. By standard reasoning,

$$rV_n(\phi) = \phi(P - V_n) - (n-1)\widetilde{\phi} V_n \tag{13}$$

holds where $\widetilde{\phi}$ is the effort level of the other firms. To explain this, note that a firm becomes successful at rate ϕ, in which case she obtains the prize P, or another firm becomes successful (with success rate $(n-1)\widetilde{\phi}$) in which case the firm makes a zero surplus. Rearranging (13), we obtain

$$V_n = \frac{\phi P}{r + \phi + (n-1)\widetilde{\phi}}. \tag{14}$$

Taking a derivative with respect to ϕ, we obtain

$$V_n'(\phi) = \frac{1}{r+n\phi}(1 - \frac{\phi}{r+n\phi})P. \tag{15}$$

In a symmetric equilibrium where $\widetilde{\phi} = \phi$ we have

$$V_n = \frac{\phi}{r+n\phi}P. \tag{16}$$

Using the zero-profit constraint (12) with $y = 0$ and (16),

$$V_n = \frac{\phi}{r+n\phi}P = F + \phi x^2/2, \tag{17}$$

or

$$\frac{1}{r+n\phi} = \frac{F+\phi x^2/2}{\phi P}. \tag{18}$$

Therefore,

$$\begin{aligned} V_n'(\phi) &= \frac{F+\phi x^2/2}{\phi P}(1 - \phi\frac{F+\phi x^2/2}{\phi P})P = \\ &= \frac{F+\phi x^2/2}{\phi}\frac{P-F-\phi x^2/2}{P}. \end{aligned} \tag{19}$$

The optimality condition with respect to ϕ is to set $\phi = \arg\max_z V_n(z) - \phi z^2/2$, which implies

$$V_n'(\phi) = x^2/2. \tag{20}$$

Using (19) and (20),

$$x^2/2 = \frac{F+\phi x^2/2}{\phi}\frac{P-F-\phi x^2/2}{P}.$$

One can solve the last quadratic equation to obtain

$$\phi = \frac{\sqrt{PF}-F}{x^2/2}. \tag{21}$$

By (18) and (21),

$$r + n\phi = \frac{\phi P}{F+\phi x^2/2} = \frac{\phi P}{\sqrt{PF}}. \tag{22}$$

Then (21) implies

$$r + n\phi = \frac{\phi P}{\sqrt{PF}} = \frac{\sqrt{PF} - F}{x^2/2} \frac{P}{\sqrt{PF}} = \frac{P - \sqrt{PF}}{x^2/2}. \tag{23}$$

Optimal prizes

Letting $\Phi = n\phi$, note that by (17) total innovation costs can be written as

$$n(F + \phi x^2/2) = \frac{n\phi}{r + n\phi} P = \frac{\Phi}{r + \Phi} P.$$

Total expected social benefit from innovation can be written as $\frac{\Phi}{r+\Phi}\theta x$. Therefore, social welfare can be written as

$$W = \frac{\Phi}{r + \Phi}(\theta x - P).$$

Substituting in for $\Phi = n\phi$ from (23), then the maximization problem becomes

$$\max_{x,P} (1 - \frac{rx^2}{2(P - \sqrt{PF})})(\theta x - P).$$

The first-order condition with respect to x yields

$$\theta(2(P - \sqrt{PF}) - rx^2) = 2rx(\theta x - P).$$

The condition with respect to P is

$$2(P - \sqrt{PF}) = \theta x(1 - \frac{\sqrt{F}}{2\sqrt{P}}) \tag{24}$$

after some algebra. Comparing the last two formulas yields,

$$2(P - \sqrt{PF}) = \theta x(1 - \frac{\sqrt{F}}{2\sqrt{P}}) = \frac{2rx(\theta x - P)}{\theta} + rx^2,$$

which yields

$$x = \frac{\theta^2(1 - \frac{\sqrt{F}}{2\sqrt{P}}) + 2rP}{3\theta r}. \tag{25}$$

Comparing then (24) and (25) yields,

$$\theta(1 - \frac{\sqrt{F}}{2\sqrt{P}})(\theta^2(1 - \frac{\sqrt{F}}{2\sqrt{P}}) + 2rP) = 6\theta r(P - \sqrt{PF}).$$

After further algebra, we obtain

$$2P(\theta^2 - 4rP) + \theta^2 F = -\sqrt{PF}(10rP - \theta^2). \tag{26}$$

While this condition can be solved for P explicitly, we are mainly interested in the $F \to 0$ case, so a general solution is not provided here. Then (26) implies that $P = \theta^2/4r$ and (25) implies $x = \theta/2r$. Therefore, by (23), and $F = 0$ together with $\Phi = n\phi$,

$$r + \Phi = \frac{P}{x^2/2} = 2r.$$

This implies that the welfare is the same as with $n = 1$,

$$W^{PR} = \frac{\Phi}{r+\Phi}(\theta x - P) = \frac{\theta^2}{8r}. \tag{27}$$

Patents

With patents the planner does not have any choice variable. Therefore, our analysis has simply to derive equilibrium values of x, y, n, ϕ when a simple patent is offered. It is clear that $x = y$ holds, so we only need to solve for n, x, ϕ. The following Lemma characterizes equilibrium levels of x and $\Phi = n\phi$:

Lemma 7 *For the case of patents,*

$$x = \frac{2F}{\theta\beta\gamma}, \tag{28}$$

and

$$r + \Phi = \frac{\theta\beta\gamma}{x} = \frac{(\theta\beta\gamma)^2}{2F}. \tag{29}$$

Proof. First, by free-entry,

$$V_n = F + \phi\frac{x^2 + y^2}{2} = F + \phi x^2.$$

Second, by construction,

$$V_n = \frac{\phi}{r+\Phi}\theta\beta\gamma(x+y) = \frac{\phi}{r+\Phi}\theta\beta\gamma * 2x.$$

By optimization

$$\frac{\partial V_n}{\partial \phi} = x^2,$$

$$\frac{\partial V_n}{\partial x} = 2\phi x.$$

Also,

$$rV_n = \phi(2\theta\beta\gamma x - V_n) - (n-1)\widetilde{\phi}V_n.$$

Taking a derivative and collecting terms yields,

$$(r+\Phi)\frac{\partial V_n}{\partial \phi} = 2\theta\beta\gamma x - V_n$$

Using the above formulas,

$$(r+\Phi)x^2 = (r+\Phi)\frac{\partial V_n}{\partial \phi} = 2\theta\beta\gamma x - V_n = 2\theta\beta\gamma x(1 - \frac{\phi}{r+\Phi}). \qquad (30)$$

We can use the formula (22) that was developed for prizes to obtain

$$\frac{\phi}{r+\Phi} = \frac{\sqrt{2\theta\beta\gamma x F} - F}{2\theta\beta\gamma x - \sqrt{2\theta\beta\gamma x F}} = \frac{\sqrt{F}}{\sqrt{2\theta\beta\gamma x}}. \qquad (31)$$

To see this, note that all we did is to sub in $2\theta\beta\gamma x$ for the reward for the winner and used (22) otherwise with dividing through in (22). Again, we can do this because we took x, y as given for these calculations. Therefore, using (30) and (31),

$$(r+\Phi)x^2 = 2\theta\beta\gamma x(1 - \frac{\phi}{r+\Phi}) = 2\theta\beta\gamma x(1 - \frac{\sqrt{F}}{\sqrt{2\theta\beta\gamma x}}) = 2\theta\beta\gamma x - \sqrt{2\theta\beta\gamma x F}.$$

Now, we substitute $x = \frac{\beta\gamma\theta}{r+\Phi}$ in to obtain

$$\beta\gamma\theta x = 2\theta\beta\gamma x - \sqrt{2\theta\beta\gamma x F},$$

or

$$\beta\gamma\theta x = \sqrt{2\theta\beta\gamma x F},$$

which implies

$$x = \frac{2F}{\beta\gamma\theta}.$$

Therefore,

$$r + \Phi = \frac{\beta\gamma\theta}{x} = \frac{(\theta\beta\gamma)^2}{2F},$$

which completes the proof. **Q.E.D.**

We are ready to obtain the welfare generated by patents. Total benefit from innovation is $\theta\gamma(x+y)\frac{\Phi}{r+\Phi} = 2\theta\gamma x\frac{\Phi}{r+\Phi}$ by construction. Using the free-entry condition, $x = y$, and $\Phi = n\phi$, total innovation costs can be written as follows,

$$TC = n(F + \phi x^2) = nV_n = n\frac{\phi}{r+\Phi}\theta\beta\gamma(x+y) = 2\theta\beta\gamma x\frac{\Phi}{r+\Phi}$$

and social welfare can be written as

$$W^{PAT} = \frac{\Phi}{r+\Phi}(2\theta\gamma x - 2\theta\beta\gamma x),$$

or using Lemma 7,

$$W^{PAT} = \frac{\Phi}{r+\Phi}2\theta\gamma x(1-\beta) = (1 - \frac{r}{r+\Phi})\theta\gamma(1-\beta)\frac{4F}{\theta\beta\gamma} =$$

$$= \left(1 - \frac{2rF}{(\theta\beta\gamma)^2}\right)\theta\gamma(1-\beta)\frac{4F}{\theta\beta\gamma}.$$

This formula implies that

$$\lim_{F\to 0} W^{PAT} = 0.$$

This finding suggest a role for application fees. Suppose that an application fee of $\widetilde{F}$ can be levied. Then, we can implement the $n = 1$ allocation for patents if we choose $\widetilde{F} = \frac{(\theta\beta\gamma)^2}{4r}$ because then using (28), and (29) we obtain $x = \frac{\gamma\beta\theta}{r}$ and $\Phi = r$ just as in the case where $n = 1$. Therefore, with application fees, patents can also get back to the second-best ($n = 1$), and the comparison between patents and prizes is the same as when $n = 1$.

The next result characterizes the level of entry n:

Lemma 8 *For all $F > 0$, $\phi(2-n) = r$ and thus $n < 2$. Also, $\lim_{F \to 0} n = 2$.*

Proof. Given a level of entry n, in the main text we derive the optimal level of innovation size x, y. In particular,

$$x = y = \frac{\beta\gamma\theta}{r + n\phi},$$

and

$$x^2 = \frac{r + (n-1)\phi}{(r + n\phi)^2} 2\beta\gamma\theta x.$$

These formulas imply that

$$\phi(2-n) = r. \tag{32}$$

By Lemma 7, $\lim_{F \to 0} \phi = 0$, so (32) implies

$$\lim_{F \to 0} n = 2,$$

which completes our proof. **Q.E.D.**

Lemma 8 implies that for any positive entry cost there is a single firm entering if we impose an integer constraint on the number of firms present and assume that firms enter sequentially. Therefore, if a free-entry condition is imposed then patents deliver the same welfare as when $n = 1$. This observation implies that the negative welfare consequences of patents with $n > 1$ fixed are mitigated when free-entry is modelled in a precise manner.

Grand Innovation Prizes

Preliminary analysis

Just like in the case of prizes, we start the analysis by studying n and ϕ (taking x and y as given). Let

$$\pi = \theta\beta\gamma(x + y) \tag{33}$$

denote the product market profit from winning the innovation race. First, we argue that for GIP,

$$r+\Phi=\frac{P+\pi-\sqrt{F(P+\pi)}}{\frac{x^2+y^2}{2}}. \tag{34}$$

A formal proof is not necessary as the argument is very similar to the one that establishes (23) for simple prizes, with just two differences: (i) the value of winning is now $P+\pi$ and not just P and (ii) the cost of providing the innovation is now $\frac{x^2+y^2}{2}$ and not just $x^2/2$. Next, we derive x and y. Variable y is chosen by the innovator, solving

$$\max \frac{\phi}{r+\Phi}\theta\beta\gamma y-\phi\frac{y^2}{2}.$$

The FOC yields

$$y=\frac{1}{r+\Phi}\theta\beta\gamma. \tag{35}$$

To choose x for a given level of ϕ, the planner solves

$$\max \frac{\phi}{r+\Phi}\theta\gamma x-\phi\frac{x^2}{2},$$

which yields

$$x=\frac{1}{r+\Phi}\theta\gamma. \tag{36}$$

For reasons of tractability, we focus on the case of small fixed costs. By similar arguments as in the case of prizes and patents, social welfare can be written as

$$W^{GIP}=\frac{\Phi}{r+\Phi}(\theta\gamma(x+y)-(P+\pi)).$$

Upon substituting from (33), (34), (35), and (36),

$$W^{GIP}=\frac{\Phi}{r+\Phi}(\frac{\theta^2\gamma^2(1+\beta)}{r+\Phi}-(r+\Phi)\frac{\theta^2\gamma^2(1+\beta^2)}{2(r+\Phi)^2})=$$

$$\frac{\Phi\theta^2\gamma^2}{(r+\Phi)^2}\left(1+\beta-\frac{1+\beta^2}{2}\right).$$

Therefore, one needs to solve

$$\max \frac{\Phi}{(r+\Phi)^2},$$

which yields

$$\Phi = r.$$

Therefore, the optimum involves

$$x = \theta\gamma/2r,$$

and

$$y = \theta\gamma\beta/2r.$$

Substituting in,

$$W^{GIP} = \frac{\Phi\theta^2\gamma^2}{(r+\Phi)^2}\left(1+\beta-\frac{1+\beta^2}{2}\right) = \frac{\theta^2\gamma^2}{4r}\left(1+\beta-\frac{1+\beta^2}{2}\right) =$$

$$= \frac{\theta^2}{8r}\gamma^2\left(2+2\beta-\left(1+\beta^2\right)\right) = \frac{\theta^2}{8r}\gamma^2\left(1+2\beta-\beta^2\right).$$

Therefore, the welfare comparison is exactly the same as with $n = 1$ or with any fixed n (see also (27)):

$$W^{GIP} \geq W^{PR} \Longleftrightarrow \gamma^2\left(1+2\beta-\beta^2\right) \geq 1. \tag{37}$$

GIP with a $P \geq 0$ constraint

The above analysis allows for prizes that are negative. This assumption may be valid in some cases where the planner can restrict the availability of simple patents for firms that enter a GIP contest. In other cases, it is interesting to study a constraint that requires the planner to set a positive prize, $P \geq 0$. Substituting in from the results above we have

$$r+\Phi = \frac{P+\pi}{\frac{x^2+y^2}{2}} = \frac{2P(r+\Phi)^2 + 2(r+\Phi)\theta^2\gamma^2\beta(1+\beta)}{\theta^2\gamma^2(1+\beta^2)},$$

or

$$1 = \frac{2P(r+\Phi)}{\theta^2\gamma^2(1+\beta^2)} + \frac{2\beta(1+\beta)}{1+\beta^2}.$$

This is consistent with a positive prize ($P \geq 0$) if and only if

$$\frac{2\beta(1+\beta)}{1+\beta^2} \leq 1,$$

or $\beta \leq \sqrt{2}-1$. Therefore, when $\beta \leq \sqrt{2}-1$ our analysis is valid even under a $P \geq 0$ constraint. When $\beta > \sqrt{2}-1$ the results no longer apply under a $P \geq 0$ constraint. In this case, one needs to set $P = 0$ and the GIP boils down to having a constraint on x.[16] Then

$$r + \Phi = \frac{2\pi}{x^2+y^2} = \frac{2\theta\beta\gamma(x + \frac{1}{r+\Phi}\theta\beta\gamma)}{x^2 + \left(\frac{1}{r+\Phi}\theta\beta\gamma\right)^2},$$

or

$$x^2(r+\Phi)^2 = 2\theta\beta\gamma x(r+\Phi) + \theta^2\beta^2\gamma^2.$$

This yields

$$\omega = x(r+\Phi) = \frac{2\theta\beta\gamma + \sqrt{(2\theta\beta\gamma)^2 + 4\theta^2\beta^2\gamma^2}}{2} = \theta\beta\gamma(1+\sqrt{2}).$$

The welfare is

$$W^{GIP} = \frac{\Phi}{r+\Phi}(\theta\gamma(x+y) - \pi) = \frac{\Phi}{r+\Phi}\theta\gamma(x+y)(1-\beta), \tag{38}$$

and upon substituting in $x = \frac{\theta\beta\gamma(1+\sqrt{2})}{r+\Phi}$, $y = \frac{1}{r+\Phi}\theta\beta\gamma$ this still yields $\Phi = r$ because the maximization problem still boils down to $\max \frac{\Phi}{(r+\Phi)^2}$. Therefore, the optimum involves

$$x = \theta\beta\gamma(1+\sqrt{2})/2r,$$

and

$$y = \theta\gamma\beta/2r.$$

[16] This implies that the planner should be able to prevent low value innovations to be patented otherwise the innovators would surely opt out of the GIP as it conveys no advantage to them if patenting without participating in the GIP is an option.

Note, that x is distorted upward now compared to the second-best level where $\theta\gamma/2r$.[17] This is because the second-best level would require $P < 0$ to implement. Therefore,

$$W^{GIP} = \frac{\theta^2\gamma^2\beta(2+\sqrt{2})(1-\beta)}{4r}.$$

Welfare from an optimal prize: $P = \frac{\theta^2}{4r}$, $x = \frac{\theta}{2r}$ and $y = 0$. Also,

$$r + \Phi = \frac{2P}{x^2} = 2r,$$

so $\Phi = r$. Therefore,

$$W^{PR} = \frac{\Phi}{r+\Phi}(\theta x - P) = \frac{\theta^2}{8r}.$$

Thus

$$W^{GIP} \geq W^{PR} \Longleftrightarrow \gamma^2\beta(2+\sqrt{2})(1-\beta) \geq 1/2.$$

This holds if γ is high and β is close to 1/2.

The main novelty when $P \geq 0$ is that GIP mechanisms may perform poorly compared to prizes when the monopoly can extract a high proportion of the rents (that is, β is close to 1). The reason is rent dissipation. In particular, (38) implies that when $\beta = 1$ and $P \geq 0$ then $W^{GIP} = 0$ because all rents are dissipated under free entry. Rent dissipation is caused by excess entry, and can be fixed only when the value of the prize is negative.

A3. Asymmetric information

In this appendix, we provide a formal analysis to support the results in Section 5. First, we show that the presence of asymmetric information does not change our main results. Second, we present a simple example with two types to show how asymmetric information coupled with costly public funds changes our analysis.

[17] This is because $\beta(1+\sqrt{2}) > 1$ for all $\beta > \sqrt{2} - 1$.

We consider the case of simple prizes, the analysis for the other mechanisms are similar. Let $x^P = \theta/\tau$ be the optimal level of x when τ is known by the planner. We show that this level can be still implemented under asymmetric information. We use the revelation principle, and (without loss of generality) restrict attentions to mechanisms where the innovator reports τ and the planner sets $(x(\tau), P(\tau))$. Crucially, the planner offers a prize schedule $P(\tau)$ such that it is incentive compatible for the innovator to report truthfully. Formally, incentive compatibility requires that for all $\tau, \tau' \in [\underline{\tau}, \overline{\tau}]$

$$P(\tau) - \tau\frac{x^2(\tau)}{2} \geq P(\tau') - \tau\frac{x^2(\tau')}{2}. \tag{39}$$

We complete the proof by solving for a schedule $P(\tau)$ such that (39) holds for all $\tau, \tau' \in [\underline{\tau}, \overline{\tau}]$ if $x(\tau) = \theta/\tau$ and $x(\tau') = \theta/\tau'$. Let

$$P(\overline{\tau}) = \overline{\tau}\frac{x^2(\overline{\tau})}{2},$$

so that the highest type (and thus all other types as well) is willing to participate in the prize mechanism. The optimization problem of the innovator with type τ when deciding what type τ' to report is

$$\max_{\tau'} P(\tau') - \tau\frac{x^2(\tau')}{2}.$$

Truth-telling to be optimal requires that

$$\tau = \arg\max_{\tau'}(P(\tau') - \tau\frac{x^2(\tau')}{2}).$$

Therefore, the first order condition becomes

$$P'(\tau) = \tau x(\tau)x'(\tau)$$

with

$$x(\tau) = x^P(\tau) = \theta/\tau.$$

Therefore,

$$P'(\tau) = -\theta^2/\tau^2,$$

and thus

$$P(\tau) = P(\overline{\tau}) + \int_{\tau}^{\overline{\tau}} \theta^2/z^2 dz = \frac{\theta^2}{\tau} - \frac{\theta^2}{2\overline{\tau}}.$$

It can be shown that global optimality conditions for the problem of the innovator are satisfied.[18]

Therefore, mechanism $(x = \frac{\theta}{\tau}, P = \frac{\theta^2}{\tau} - \frac{\theta^2}{2\overline{\tau}})$ implements the complete information optimum.

A4. Asymmetric information with costly public funds

Here we show that the optimal schedule for the efforts does change when public funds our costly. It is sufficient for our purposes to illustrate this change with a simple two-type example for prizes. The same insight would be true when there are more than two types or if we considered a Grand Innovation Prize.

Imagine a case with two possible types with $\tau_h > \tau_l$. We let the planner's objective be $\theta(x+y) - \tau(\frac{x^2}{2} + \frac{y^2}{2}) - \kappa P$. With prizes, $y = 0$ and this objective function is reduced to $\theta x - \tau\frac{x^2}{2} - \kappa P$. Under complete information the planner offers a prize value such that the innovator is indifferent between entering or not as public funds are costly, that is, $\tau\frac{x^2}{2} = P$. Therefore, the planner maximizes

[18]Consider $P(\tau') - \tau\frac{x^2(\tau')}{2} = \theta^2(\frac{1}{\tau'} - \frac{\tau}{2\tau'} - \frac{1}{\overline{\tau}})$, and thus

$$\frac{\partial}{\partial\tau'}(P(\tau') - \tau\frac{x^2(\tau')}{2}) = \theta^2(\frac{\tau}{(\tau')^3} - \frac{1}{(\tau')^2}) =$$

$$= \frac{\theta^2}{(\tau')^3}(\tau - \tau').$$

Therefore, $\frac{\partial}{\partial\tau'}(P(\tau') - \tau\frac{x^2(\tau')}{2}) \geq 0$ if and only $\tau' < \tau$, which means that $\tau = \arg\max_{\tau'}(P(\tau') - \tau\frac{x^2(\tau')}{2})$.

$\theta x - \tau(1+\kappa)\frac{x^2}{2}$ and thus the complete information optimum is to induce

$$x^P(\tau) = \theta/[\tau(1+\kappa)].$$

In what follows, we show that the optimal x value for τ_l is still $x^P(\tau_l) = \theta/[\tau_l(1+\kappa)]$ but the optimal x value for τ_h is less than $x^P(\tau_h) = \theta/[\tau_h(1+\kappa)]$. Letting μ_h and μ_l be the probability of the two types (with $\mu_h + \mu_l = 1$), the planner's problem becomes

$$\max_{x_l, x_h, P_l, P_h} \mu_h(\theta x_h - \tau_h \frac{x_h^2}{2} - \kappa P_h) + \mu_l(\theta x_l - \tau_l \frac{x_l^2}{2} - \kappa P_l)$$

such that

$$P_h \geq \tau_h \frac{x_h^2}{2} \tag{40}$$

$$P_l \geq \tau_l \frac{x_l^2}{2}, \tag{41}$$

and

$$P_h - \tau_h \frac{x_h^2}{2} \geq P_l - \tau_h \frac{x_l^2}{2}, \tag{42}$$

$$P_l - \tau_l \frac{x_l^2}{2} \geq P_h - \tau_l \frac{x_h^2}{2}. \tag{43}$$

The first two constraints are the participation constraints for the two types while the last two constraints are the incentive constraints, so that the two types do not wish to mimic each other.

We start the analysis with the following result:

Lemma 9 *Constraints (41) and (42) do not bind.*

Proof. Constraint (43) and the fact that $x_l \geq x_h$ imply

$$\tau_h \frac{x_l^2}{2} - \tau_h \frac{x_h^2}{2} > \tau_l \frac{x_l^2}{2} - \tau_l \frac{x_h^2}{2} \geq P_l - P_h,$$

which implies that (42) does not bind. It is straightforward to show that (41) does not bind because the low type can always mimic the high type and make a positive rent:

$$P_l - \tau_l \frac{x_l^2}{2} \geq P_h - \tau_l \frac{x_h^2}{2} \geq P_h - \tau_h \frac{x_h^2}{2} \geq 0,$$

which completes the proof of the Lemma. Q.E.D.

Therefore, the two constrains to obey are $P_l - \tau_l \frac{x_l^2}{2} \geq P_h - \tau_l \frac{x_h^2}{2}$ and $P_h \geq \tau_h \frac{x_h^2}{2}$. It is clear that both constraints need to bind otherwise the planner could reduce P_l and P_h without changing x_l, x_h and still obey the incentive constraints reducing the payments and thus improving the planner's payoffs. To see this, suppose that $P_h > \tau_h \frac{x_h^2}{2}$. Then reducing P_h slightly does not violate any of the two relevant constraints. Given this discussion, the problem can be rewritten as

$$\max_{x_l, x_h, P_l, P_h} \mu_h (\theta x_h - \tau_h \frac{x_h^2}{2} - \kappa P_h) + \mu_l (\theta x_l - \tau_l \frac{x_l^2}{2} - \kappa P_l)$$

such that

$$P_h = \tau_h \frac{x_h^2}{2}$$

and

$$P_l - \tau_l \frac{x_l^2}{2} = P_h - \tau_l \frac{x_h^2}{2}.$$

Upon substitutions, this problem becomes

$$\max_{x_l, x_h} \mu_h (\theta x_h - \tau_h (1+\kappa) \frac{x_h^2}{2}) + \mu_l (\theta x_l - \tau_l \frac{x_l^2}{2} - \kappa (\tau_h \frac{x_h^2}{2} - \tau_l \frac{x_h^2}{2} + \tau_l \frac{x_l^2}{2})).$$

This problem is a simple concave problem, and the sufficient first order conditions yield that

$$x(\tau_l) = \theta / [\tau_l (1+\kappa)],$$

and $\mu_h(\theta - \tau_h(1+\kappa)x_h) - \mu_l\kappa(\tau_h - \tau_l)x_h = 0$, which implies that

$$x(\tau_h) < \theta/[\tau_h(1+\kappa)]$$

as we claimed.

A5. Omitted proofs for Section 3

Proof of Proposition 1 The result directly follows from $W^{IP} - W^P = \theta^2\beta\gamma^2(2-\beta) - \frac{\theta^2}{2} = \frac{\theta^2}{2}(4\beta\gamma^2 - 2\beta^2\gamma^2 - 1)$.

Proof of Proposition 3 The result follows because $W^{GIP} - W^P = \frac{1}{2}\theta^2\gamma^2\left(1+2\beta-\beta^2\right) - \frac{\theta^2}{2} = \frac{1}{2}\theta^2\left(2\beta\gamma^2 + \gamma^2 - 1 - \beta^2\gamma^2\right)$ which is positive if

$$\gamma \geq \frac{1}{\sqrt{1+2\beta-\beta^2}}.$$

Proof of Proposition 4 In the case of substitute efforts ($\rho > 0$),

$$\begin{aligned} W^{IP} - W^P &= \frac{\beta\gamma^2\theta^2(2-\beta)}{1+\rho} - \frac{\theta^2}{2} \\ &= -\frac{1}{2}\frac{\theta^2}{\rho+1}\left(2\beta^2\gamma^2 - 4\beta\gamma^2 + \rho + 1\right), \end{aligned}$$

which is positive if $\gamma \geq \sqrt{\frac{1+\rho}{2\beta(2-\beta)}}$. In the case of complementary efforts,

$$\begin{aligned} W^{IP} - W^P &= \frac{\beta\gamma^2\theta^2(2-\beta)}{1+\rho} - \frac{\theta^2}{2}\frac{1-\rho}{1+\rho} \\ &= \frac{1}{2}\frac{\theta^2}{\rho+1}\left(-2\beta^2\gamma^2 + 4\beta\gamma^2 + \rho - 1\right), \end{aligned}$$

which is positive if $\gamma \geq \sqrt{\frac{1-\rho}{2\beta(2-\beta)}}$.

Proof of Lemma 5 It is optimal for the planner to require $y^{GIP} = 0$ if this provides higher welfare. This occurs if $\frac{\gamma^2\theta^2}{2} > \frac{1}{2}\frac{\gamma^2\theta^2}{(1+\rho)}(1+2\beta\rho+2\beta-\rho-\beta^2-\beta^2\rho)$ or

$$\rho > \frac{\beta(2-\beta)}{2-\beta(2-\beta)} \tag{44}$$

Notice that $y^{GIP}=0$ is the optimal response of the innovator given $x^{GIP}=\gamma\theta$ if $y^{GIP}=\theta\gamma\beta-\rho x^{GIP}\leq 0$ or $\rho\geq\beta$. Because $\dfrac{\beta(2-\beta)}{2-\beta(2-\beta)}\geq\beta$ the innovator will set $y^{GIP}=0$ any time condition (44) is satisfied.

Proof that $W^{GIP}>W^{IP}$ From (1), it follows that

$$W^{GIP}-W^{IP}=\frac{1}{2}\theta^2\gamma^2\left(1-\beta\right)^2\frac{1-\rho}{1+\rho}. \tag{45}$$

when $y^{GIP}>0$. Exploiting Lemma 5, we can show that

$$W^{GIP}-W^{IP}=\frac{1}{2}\theta^2\frac{\gamma^2}{\rho+1}(2\beta^2-4\beta+\rho+1) \tag{46}$$

when $y^{GIP}=0$. The result for the case where $y^{GIP}>0$ follows directly from (45). The case where $y^{GIP}=0$ follows from (46) using that in this case $\rho\geq\frac{\beta(2-\beta)}{2-\beta(2-\beta)}$ by Lemma 5, and thus $2\beta^2-4\beta+\rho+1\geq 2\beta^2-4\beta+\frac{\beta(2-\beta)}{2-\beta(2-\beta)}+1>0$ if $\beta<1$.

Proof of Proposition 6 In the case of substitute efforts ($\rho>0$), $y^{GIP}=0$ holds, and thus,

$$W^{GIP}-W^{P}=\frac{\theta^2\gamma^2}{2}-\frac{\theta^2}{2}<0.$$

This implies that GIP are dominated by simple prizes if ρ is too large. Consider now the case of substitute efforts ($\rho>0$) and $y^{GIP}>0$. In this case,

$$W^{GIP}-W^{P}=\frac{1}{2}\frac{\gamma^2\theta^2}{(1+\rho)}(1+2\beta\rho+2\beta-\rho-\beta^2-\beta^2\rho)-\frac{\theta^2}{2},$$

which is positive if

$$\rho<\frac{\gamma^2(1+\beta(2-\beta))-1}{\gamma^2(1-\beta(2-\beta))+1}.$$

The cutoff is positive if $\gamma\geq\left(1+2\beta-\beta^2\right)^{-1/2}$ and that it is more stringent than the one required in lemma 5. In the case of complements ($\rho<0$),

$$W^{GIP}-W^{P}=\frac{1}{2}\frac{\gamma^2\theta^2}{(1+\rho)}(1+2\beta\rho+2\beta-\rho-\beta^2-\beta^2\rho)-\frac{\theta^2}{2}\frac{1-\rho}{1+\rho},$$

which is positive if

$$\gamma^2(1+\beta(2-\beta))-1 > \rho(\gamma^2(1-\beta(2-\beta))-1).$$

Notice that $\gamma^2(1-\beta(2-\beta))-1 \leq 0$. This implies that when $\gamma^2(1+\beta(2-\beta))-1 < 0$ the inequality cannot be satisfied. If $\gamma^2(1+\beta(2-\beta))-1 > 0$ the condition is satisfied if

$$-\rho < \frac{\gamma^2(1+\beta(2-\beta))-1}{1-\gamma^2(1-\beta(2-\beta))}.$$