Coordination and Bargaining Power in Contracting with Externalities

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Abstract

Building on Genicot and Ray (2006) we develop a model of non-cooperative bargaining that combines the two main approaches in the literature of contracting with externalities: the offer game (in which the principal makes simultaneous offers to the agents) and the bidding game (in which the agents make simultaneous offers to the principal). Allowing for agent coordination, we show that the outcome of our bargaining procedure may differ remarkably from those of the offer and the bidding games. In particular, we find that bargaining can break agents' coordination and that the principal's payoff can be decreasing in his own bargaining power.

1 Introduction

When a single principal interacts with several agents multilateral externalities may arise. For example, in Rasmusen, Ramseyer and Wiley (1991) a buyer signing an exclusive dealing contract with an incumbent monopolist imposes a negative externality on other buyers. In Katz and Shapiro (1986) a firm purchasing a cost-reducing technology imposes a negative externality on competing firms. Segal (1999, 2003) and Genicot and Ray (2006) provide various additional examples in which similar externalities arise.

In these settings, agents may find it profitable to coordinate their actions and to reduce trade with the principal to a minimum. This incentive to coordinate has been studied in Segal (2003) and Genicot and Ray (2006). More specifically, Segal

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(2003) shows that agent-coordination tends to reduce inefficient trade. Genicot and Ray (2006), developing a dynamic model, show that a principal can partially prevail agent-coordination by employing a mix of simultaneous and sequential contracting.

In both of these papers the principal makes take-it-or-leave-it offers to the agents. This is not surprising because extreme allocations of bargaining power are common in the literature of contracting with externalities. More specifically, Segal and Whinston (2003) noticed that two approaches have been widely adopted: the *offer-game* (Segal (1999, 2003), Möller (2007) and Genicot and Ray (2006)) in which the principal makes take-it-or-leave-it offers to the agents and the *bidding game* (Bernheim and Whinston (1986), Martimort and Stole (2003), Prat and Rustichini (1998) and Bergermann and Valimaki (2003)) in which the agents make simultaneous offers to the principal.

The objective of this paper is to analyze the impact of agent-coordination relaxing these extreme assumptions on the allocation of bargaining power. To this end, building on Genicot and Ray (2006), we develop a simple dynamic framework where a principal interacts with two agents and no-trade is the efficient outcome. In our set-up the outcomes of the offer and the bidding games differ dramatically. In particular, there is no trade if the agents make simultaneous offers to the principal and there is inefficient trade if the principal is endowed with the entire contractual power.

To analyze more intermediate allocations of bargaining power, we develop a bargaining game à la Rubinstein and we analyze its finite and infinite horizon equilibria. We show that the equilibrium outcomes of our bargaining procedure may differ remarkably from those of the offer and the bidding games.

In particular, our analysis indicates that the payoff of the principal may be larger in the bargaining game than in the offer game. This happens because the negotiation process allows him to break agents' coordination and to trade at better terms. In other words, we find cases in which the payoff of the principal is decreasing in his own bargaining power. Moreover, for large values of the discount factor, we show that in finite horizon bargaining games in which the principal is the last mover, agents fail completely to coordinate and the surplus extracted by the principal is maximum.

The paper is organized as follows. In section 2 we present the model and we discuss its static implications. In section 3 we study the bargaining problem considering both a finite and an infinite horizon. Section 4 concludes.

2 The Model

We develop a simplified version of the setting in Genicot and Ray (2006) where a principal (or seller) trades with two agents (or buyers) that we label as A and B. As in Genicot and Ray (2006) we consider reduced-form versions of contracts: a contract specifies the payoff t that an agent will receive after trading with the principal. An agent is defined as "contracted" if he has agreed on the terms of trade with the principal. Conversely it is said to be "free" if no trade takes place between him and the principal. A free agent receives a payoff r_k where k is the number of free agents (counting himself).

We indicate the bilateral surplus between one buyer and the seller as F, i.e. the surplus generated by trade is going to be equal to F if only one agent is contracted and equal to $2F$ if both agents are contracted[1]. For instance, if A accepts a contract x_A and B accepts a contract x_B then the payoff of the seller is going to be $2F - x_A - x_B$ and x_A and x_B are going to be the payoffs for the two buyers.

Finally we assume that the following condition is satisfied:

$$r_2 > 2F - r_2 > r_1. \tag{1}$$

Assumptions (1) captures three important features of the environments we want to analyze. First, there are negative externalities from trade: $r_2 > r_1$. Second, trade is not efficient because it does not maximize the surplus of the vertical structure. In fact, because $2r_2 > 2F$, total surplus is maximized without trade. Third, the total surplus generated by trade is greater than the sum of the two outside options r_2 and r_1, i.e. $2F - r_2 - r_1 > 0$. As we will see shortly, this implies that that the principal can make positive profits by trading with the agents.

2.1 The offer and the bidding game

As Segal and Whinston (2003) point out, various games have been proposed to study this environment of contracting with externalities. In the two most studied games, the bargaining power is concentrated on one side of the market. More specifically in the *offer game* the principal makes take-it-or-leave-it offers to the agents. In the *bidding game* the agents make simultaneous offers to the principal. We analyze now the outcome of an *offer game* in our environment. Following Segal (1999) we study the case in which the seller publicly commits to a vector of transfers $t = (t_A, t_B)$ and each buyer either accepts or rejects the offer. More specifically, we consider a game between buyers described by the following bimatrix:

		Buyer B: Accept	Buyer B: Reject
Buyer A	Accept	t_A, t_B	t_A, r_1
	Reject	r_1, t_B	r_2, r_2

This game has an equilibrium in which the principal offers transfers that render each buyer indifferent between accepting and rejecting the contract as long as the other buyer accepts. In fact, if $t = (r_1, r_1)$ then (*Accept, Accept*) is a Nash equilibrium of the game between buyers. Moreover this vector of transfer is the cheapest way for the seller to implement (*Accept, Accept*) as a Nash Equilibrium.

[1] This simplifies greatly the notation. It is possible to generalize our results to various settings in which the surplus generated with two agents, $F(2)$, differs from 2F.

Nevertheless, this equilibrium is not unique. Segal's analysis focuses only on this equilibrium point because he assumes that the seller can coordinate buyers on his preferred equilibrium. This is equivalent to impose coordination failure between the two buyers. *(Reject, Reject)* is another Nash Equilibrium of the game. Moreover, from the buyers perspective, this no-trade Nash Equilibrium Pareto dominates the equilibrium with trade so there is an incentive for them to coordinate on it[2].

In this paper, following Genicot and Ray (2006) and Segal (2003) we allow buyers to coordinate on their preferred Nash equilibrium. To implement (*Accept, Accept*) in the presence of buyers coordination, the seller must offer at least one buyer, say A, a transfer that induces him to accept even if he expects the other buyer not to. Simultaneously he can offer B a transfer that induces him to trade if he expects A to be contracted. Segal (2003) calls this strategy "*divide and conquer*". In our model it implies the vector of transfers $t = (r_2, r_1)$. Notice that it is profitable for the seller to offer such contracts because (1) implies $2F - r_2 - r_1 > 0$.

We now turn to the *bidding game* in which both buyers make take-it-or-leave-it offers to the seller. To analyze this case, we follow Martimort and Stole (2003) and we assume that the contractual space of this game is exactly the same as the one in the offer game described above: each buyer $i \in \{A, B\}$ proposes a contract t_i to the principal. Also the bidding game has two set of Nash Equilibria. In one of these equilibria both buyers ask for F and the seller trades with both of them. The other one is the set of equilibria in which both buyers ask for a transfer greater than F and trade does not occur. This second set of Nash Equilibria is Pareto superior and buyers have an incentive to coordinate on it.

From the previous analysis, we notice how the allocation of bargaining power affects dramatically the final outcome. More precisely, in the offer game, where the principal has the entire bargaining power, the principal implements inefficient trade despite agent coordination. Conversely, in the bidding game, where agents make take-it-or-leave it offers, the efficient no-trade outcome is obtained.

3 Bargaining

The analysis of the previous section points out that the efficiency of the outcome depends crucially on the assumptions on bargaining power. This observation raises the question of how the outcome will change with less extreme forms of bargaining power. If bargain power is not concentrated on one side, will the principal be able to implement trade? And at what terms? To answer these questions, we develop a non-cooperative bargaining game of alternating offers. In particular, we adapt Rubinstein's bargaining game to the framework introduced in section 2.

[2] A comparison of the two equilibria shows that only the second is coalition proof in the sense of Bernheim, Peleg and Whinston (1987) and Bernheim and Whinston (1987).

3.1 A Simple Bargaining Game

We turn now to the analysis of a dynamic bargaining game. To do this we need to move from a static to a dynamic framework. The time horizon is infinite and actions take place at times $T = 1, 2, 3, ...$ As in Genicot and Ray (2006) we consider now F, r_2 and r_1 as per period payoffs. Therefore, in each period a free agent will receive a one-period payoff of r_1 or r_2 depending on the total number of free agents at that date. Conversely, a contracted agent will receive the payoff specified in the contract t_i. We assume that all players have a common discount factor δ.

In the static games studied in section 2 we ruled out agents coordination failure. Genicot and Ray (2006) propose a dynamic version of this assumption:

Buyer Coordination: *Only perfect equilibria satisfying the following restriction are considered: there is no date and no subset of buyers who can change their responses at that date and all be strictly better off, with the additional property that the changed responses are also individual best responses, given the equilibrium continuation from that point on.*

Let us analyze, in this dynamic setting, the case in which the seller makes a **unique take-it-or-leave-it offer** in the first period. In this case, if both buyers reject the offer, they remain free for an infinite horizon and obtain a payoff of $r_2/(1-\delta)$. If only one of the two rejects the contract he obtains $r_1/(1-\delta)$. Therefore, the divide and conquer strategy for the seller implies the following payoffs[3]:

$$t_A = \frac{r_2}{1-\delta} \quad \text{and} \quad t_B = \frac{r_1}{1-\delta}.$$

If we normalize lifetime payoffs by multiplying them by $(1-\delta)$ we obtain a result equivalent to the one of the corresponding static game. The same applies to the case in which buyers make a take-it-or-leave-it offer: each of them asks a (normalized) payoff greater than F and trade does not occur.

Consider now the following **two period bargaining** model. At $T = 1$ both buyers, simultaneously and non-cooperatively, propose transfers to the seller. Having observed these offers, the seller can accept both of them, only one or none. The buyer whose offer has been accepted receives the transfer, whereas the one whose offer has been rejected receives the outside option for one period. At $T = 2$ the seller is going to propose a transfer to the uncontracted buyers. A free buyer either accepts the offer or he obtains the outside option for an infinite horizon. Solving this game we obtain the following result:

Proposition 1 *There exists a $\widehat{\delta} \in [0, 1]$ such that, if $\delta \geq \widehat{\delta}$, there exists a unique subgame-perfect Nash equilibrium in which both buyers trade in T=1. As δ tends to one the payoffs of the buyers tend to r_1.*

[3] In the following analysis we impose a kind of lexicographic preference for the seller: whenever he has to *divide and conquer* he is going to offer the largest transfer to A. It is easy to relax this assumption (but the notation becomes more complicated), the proof is available from the author upon request.

Proof. See Appendix. ■

We provide now a simple intuition for this result. Consider the case in which both buyers coordinate and ask for a very large transfer in period 1. If this happens, they both receive $r_2(1-\delta)$ in period one and in period two A will be offered r_2 and B will be offered r_1. If the discount factor is large enough, first period payoffs have little weight on agent lifetime payoffs. In this case, it is possible to show that B has an incentive to deviate and being contracted in the first period. In fact, in the first period B can offer the principal a payoff equivalent to receiving F for one period and r_2 for the rest of the play. In the appendix we show that there exists a threshold $\widehat{\delta}$ for which this deviation is profitable. Moreover, the principal will accept this offer and will contract A in the second period offering him r_1.

Interestingly, this profitable deviation triggers competition between buyers. Indeed A, knowing that B is going to deviate, has an incentive to propose a payoff lower than the one announced by B in order to be the one contracted first. This Bertrand-style competition implies that the only possible equilibrium for $\delta \geq \widehat{\delta}$ involves payoffs that are both accepted immediately by the seller. In the appendix we show that these payoffs are:

$$t_A = t_B = F(1-\delta) + \delta r_1.$$

The striking feature of this two period bargaining game is that, despite agent coordination, for δ close to one the unique subgame perfect Nash equilibrium payoff tends to **the coordination failure payoff of the one shot game**. Counterintuitively, the principal obtains a larger payoff entering into a two stage bargaining game than making a take-it-or-leave-it offer. This happens because the negotiation process permits the principal to break buyers' coordination and to trade at better terms. In other words, each of the agents anticipates that in period two the principal will divide and conquer them. This generates an incentive for them to be contracted in the first period trying to avoid being the "conquered agent" in period two.[4]

What if buyers move second? Using backward induction we know that if both buyers are uncontracted in the second period then they can guarantee for themselves a payoff of r_2. If only one of them is uncontracted, he can guarantee for himself a payoff of F since this is the maximum amount that renders the seller indifferent between contracting the free buyer or not. Therefore, in the first period the seller must offer a transfer equal to r_2 to A and equal to $r_1(1-\delta) + \delta F$ to B. Notice that this contract is profitable for the seller only if $2F - r_2 - r_1(1-\delta) - \delta F > 0$ that implies:

[4] It is possible to generalize this example considering the case in which the seller makes the offer in the second period with probability p. Also in this case bargaining triggers competition among buyers. Nevertheless there will be inefficient trade only if $p > (r_2 - F)/(r_2 - r_1)$ and the limit equilibrium payoffs will tend to $(pr_1 + (1-p)r_2)$.

$$0 \leq \delta \leq \delta' = \frac{2F - r_2 - r_1}{F - r_1}.$$

In this case, for large values of δ, the outcome of the bargaining procedure does not differ from the one of the one shot game in which the buyers propose a transfer to the seller. Summing up, if the principal moves first the outcomes are quite similar to those of the one shot games described in section 2. Indeed if $\delta \geq \delta'$ we have efficiency. For $\delta \leq \delta'$ trade is going to occur and agents are divided and conquered. The outcomes for $\delta = 0$ and $\delta = 1$ correspond exactly to those of the one shot games.

3.2 General Bargaining Games

In this section we extend the previous bargaining games to various time horizons. In the following two propositions, we describe equilibrium outcomes for finite horizons bargaining games.

Proposition 2 *For any bargaining game of finite length $T > 2$ in which the seller offers in the last period there exists a $\widehat{\delta} \in [0,1]$ such that if $\delta \geq \widehat{\delta}$ in the unique Subgame Perfect Nash Equilibrium both buyers trade in the first period. Moreover:*

$$\lim_{\delta \to 1} t_A(\delta) = \lim_{\delta \to 1} t_B(\delta) = r_1 \ .$$

Proof. See Appendix. ■

The intuition behind this result is quite simple. As δ gets large, waiting becomes costless and the seller can threaten the buyers to wait until period $T - 2$ where he will contract both of them with transfers arbitrarily close to r_1. A similar extension is possible for games in which buyers are last movers as next proposition describes.

Proposition 3 *For any bargaining game of finite length $T > 2$ in which the buyers offer in the last period and δ is large enough there is a unique and efficient Subgame Perfect Nash Equilibrium.*

Proof. Suppose that this is not the case and that both buyers are contracted with transfers $t_A(\delta)$ and $t_B(\delta)$. Then as $\delta \to 1$ we need that $t_A(\delta) \to r_2$ and that $t_B(\delta) \to F$ because these are the payoffs that each buyer can guarantee to himself at time T and waiting is almost costless. But by assumption (1) we have that $2F - r_2 - F < 0$ which implies that trade is not profitable for the seller as δ gets large. Therefore we have a contradiction. ■

From the previous propositions we conclude that, in a finite horizon bargaining game with a discount factor close to one, the identity of the last mover determines the efficiency of the outcome. Indeed, trade occurs if the seller is the last mover and it does not occur if the buyers offer in the last period. This result is not surprising since also in Rubinstein(1982) two-player bargaining game if the horizon is finite and $\delta = 1$ the last mover obtains the entire surplus. What is striking different from Rubinstein(1982) is that in our model the payoff obtained by the principal in a bargaining game in which he is the last mover differs from what he gets in the one shot game where he makes a take it or leave it offer. More specifically, also in these general bargaining games, doing the last offer allows the principal to break coordination between the two buyers obtaining a transfer that corresponds to the *coordination failure* outcome of the one shot game.

We now turn to the analysis of the infinite horizon game. In this setting the last mover advantage no longer exists. The following proposition shows that, if the discount factor is large enough, there exists an inefficient SPNE in which trade occurs.

Proposition 4 *For δ large enough, there exists a Subgame Perfect Nash Equilibrium in which trade occurs. If the seller makes the first offer transfers are:*

$$t_A(\delta) = r_2(1-\delta) + \delta F(1-\delta) + \delta^2 \left(\frac{r_1 + \delta F}{1+\delta}\right) \tag{2}$$

$$t_B(\delta) = \frac{r_1 + \delta F}{1+\delta}. \tag{3}$$

If buyers make the first offer the SPNE implies:

$$t_A(\delta) = t_B(\delta) = \frac{F + \delta r_1}{1+\delta}.$$

Proof. See Appendix. ■

It is easy to see that the payoffs described in Proposition 4 satisfy the one-deviation principle [Fudenberg and Tirole (1998)]. Consider for example the case in which the principal is the player offering in period one. If A rejects the offer, he obtains r_2 for one period (given coordination between buyers) but in future periods he does not obtain more than $(F + \delta r_1)/(1+\delta)$ and therefore he has no incentive to deviate.

Notice that in the infinite horizon offer game, the normalized equilibrium payoffs for the two buyers are r_2 and r_1 (this happens because buyer A is "pivotal" and it is shown in Genicot and Ray (2006)). Moreover, in the infinite horizon bidding game payoffs are r_2 and r_2. Therefore, without loss of generality, we can compare the limiting payoffs of the infinite horizon game with those of the one shot games. It is important to notice that, as the discount factor gets large, equilibrium payoffs do not tend to the coordination failure payoffs any more. At the limit they now tend to $(F + r_1)/2$ that is the average between payoffs of the dominated equilibria of the two one-shot games.

Surprisingly, the seller gets a larger profit in the infinite horizon game than in the one shot offer game. In fact, at the limit, the principal pays $F + r_1$ that is definitely less than what he pays in the one-shot game: $r_2 + r_1$. Therefore, the counter-intuitive conclusion remains: the seller is better off entering a negotiation than making a take it or leave it offer. In the next proposition we show that, if inefficiencies are not too large, this Subgame Perfect Nash Equilibrium is the unique equilibrium of the game.

Proposition 5 *Consider an infinite horizon game in which the following condition is satisfied:*

$$2F - r_2 - \frac{r_1 + F}{2} > 0. \tag{4}$$

Then, for δ large enough, the Subgame Perfect Nash Equilibrium in which trade occurs is the unique SPNE of the game.

Proof. See Appendix. ■

4 Conclusion

In this paper we have shown how departing from extreme assumptions on bargaining power may affect predictions for environments with multilateral externalities. These results have implications for the way we think about negotiation with multiple parties. In many settings a negotiator may expect his multiple counterparts to coordinate their actions. Our simple model suggests that in these cases simple bargaining mechanisms (as take-it-or-leave-it offers) are not necessarily an optimal choice. In fact, our analysis suggests that a negotiator may increase his payoff by adopting more sophisticated negotiation techniques.

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Appendix

Proof of Proposition 1

At T=2, if both buyers are not contracted, the seller offers $t = (r_2, r_1)$ and trades with both. If only one is free, he offers him r_1 and the buyer accepts. Suppose there exists a pair of transfers (t_A, t_B) for which the only buyer served at T=1 is buyer B. The profit the principal gets from B's offer has to be larger than the one he gets rejecting both offers: $(F - t_B)(1-\delta) + \delta\,(2F - t_B - r_1) \geq \delta(2F - r_1 - r_2)$ or $t_B \leq F(1-\delta) + \delta r_2$. Moreover, rejecting A's offer has to be better than accepting it, and this implies $t_A > F(1-\delta) + \delta r_1$. To have the seller choosing B instead of A we need $t_A > t_B$. In addition, we need to show that no buyer has an incentive to deviate from these transfers. Notice that the payoff of A is simply r_1, therefore A has an incentive to deviate offering $t_A = F(1-\delta) + \delta r_1$ that is going to be accepted by the principal. Using a similar argument it is easy to see that there does not exist a pair of transfers for which the only agent served at T=1 is A.

We now show that there exists an equilibrium in which both buyers are served in T=1. In this case, to accept both transfers has to be better than to reject all of them, more specifically: $2F - t_A - t_B \geq \delta\left(2F - r_1 - r_2\right)$ or

$$t_A + t_B \leq 2F(1-\delta) + \delta\left(r_1 + r_2\right). \tag{5}$$

In addition, the principal should not prefer to deviate serving one buyer only which implies: $t_A \leq F(1-\delta) + \delta r_1$ and $t_B \leq F(1-\delta) + \delta r_1$. These two conditions imply (5), therefore the natural candidate transfers are $t_A = t_B = t^* = F(1-\delta) + \delta r_1$. Notice that no deviation is profitable for the buyers. Asking a $\widetilde{t} < t^*$ implies a payoff of $\widetilde{t}$ and is not optimal. Asking a $\widetilde{t} > t^*$ implies a payoff of r_1 which is less than t^*. These payoffs tend to r_1 as $\delta \to 1$. An equilibrium in which both buyers trade in T=2 exists only if the seller prefers not to trade with both buyers in T=1, this implies:

$$t_A + t_B \geq 2F(1-\delta) + \delta\left(r_1 + r_2\right). \tag{6}$$

Moreover, the seller must not prefer to trade with only one buyer, this implies both $t_A \geq F(1-\delta) + \delta r_2$ and $t_B \geq F(1-\delta) + \delta r_2$ that in turn guarantee (6). Therefore a candidate Nash equilibrium is given by any $t^* > F(1-\delta) + \delta r_2$. If buyers ask for higher transfers their payoffs do not change. If they ask for lower transfers their payoffs change as long as transfers are lower than $\widehat{t} = F(1-\delta) + \delta r_2$. For A is never optimal to ask $\widehat{t}$. For B asking $\widehat{t}$ is not optimal as long as $\widehat{t} = F(1-\delta) + \delta r_2 \leq r_2(1-\delta) + \delta r_1$ that implies $\delta \leq \widehat{\delta} = (r_2 - F)/(r_2 - F + r_2 - r_1)$.

Proof of Proposition 2

For any even number $\tau \geq 2$ we define:

$$\underline{r}^{\tau}(\delta) = \begin{cases} F(1-\delta) + \delta r_1 & \text{if } \tau = 2 \\ (1-\delta)\sum_{t=0}^{\frac{\tau}{2}-1} \delta^{2t} F + (1-\delta)\sum_{t=0}^{\frac{\tau}{2}-2} \delta^{2t+1} r_1 + \delta^{\tau-1} r_1 & \text{if } \tau > 2\,. \end{cases}$$

Notice that for a given τ, $\underline{r}^{\tau}(\delta)$ is continuous in δ and converges to r_1 as $\delta \to 1$. We want to show that for any game of length $T > 2$ in which the seller makes the last offer the equilibrium transfers are:

$$\widehat{t}_A^T = r_2(1-\delta) + \delta \underline{r}^{T-1}(\delta) \tag{7}$$
$$\widehat{t}_B^T = r_1(1-\delta) + \delta \underline{r}^{T-1}(\delta) \tag{8}$$

if T is odd and $\widehat{t}_A^T = \widehat{t}_B^T = \underline{r}^T(\delta)$ if T is even.

From Proposition 1 we know that, if the game has only two periods and $\delta \geq \widehat{\delta}$ trade is going to occur with transfers $t_A = t_B = \underline{r}^2(\delta) = F(1-\delta) + \delta r_1$. Using this result we show that these transfers characterize the subgame perfect Nash Equilibrium of any bargaining game with length $T = t+2$ for any $t \in \{1,2,3,.....\}$. The proof proceeds by induction. Consider $t = 1$ so $T = 3$. The seller can trade with both buyers in period 3 implementing a divide and conquer strategy with $\widehat{t}_A^3 = r_2(1-\delta) + \delta \underline{r}^2(\delta)$ and $\widehat{t}_B^3 = r_1(1-\delta) + \delta \underline{r}^2(\delta)$. These transfers render each buyer indifferent between accepting and rejecting. The principal has no incentive to delay trade or to contract only one of the buyers. We now assume that the equilibrium transfers are those specified for a general t and we show that these relations still hold for $t+1$. To assume that the property is true for a general t is equivalent to assume that it holds for a general $T > 2$. Therefore we need to show that it holds for games of length $T+1$. If T is even the principal can contract both buyers in $T+1$ offering $\widehat{t}_A^{T+1} = r_2(1-\delta) + \delta \underline{r}^T(\delta)$ and $\widehat{t}_B^{T+1} = r_1(1-\delta) + \delta \underline{r}^T(\delta)$. If T is odd B has an incentive to be contracted in $T+1$ if $F(1-\delta) + \delta \widehat{t}_A^T > r_2(1-\delta) + \delta \widehat{t}_B^T$. Because for every T odd we have that $\widehat{t}_A^T - \widehat{t}_B^T = (r_2 - r_1)(1-\delta)$, the previous condition can be rewritten as $\delta > \widehat{\delta} = (r_2 - F)/(r_2 - r_1)$. Therefore, for δ large enough, competition between buyers induces transfers $\widehat{t}_A^{T+1} = \widehat{t}_B^{T+1} = \underline{r}^{T+1}(\delta)$. Furthermore the seller will accept these transfers if:

$$2F - 2\underline{r}^{T+1}(\delta) > \delta 2F - \delta(1-\delta)r_2 - \delta(1-\delta)r_1 - 2\delta^2 \underline{r}^{T-1}(\delta)$$

that is satisfied for any value of δ. We can therefore conclude that if $\delta \in \left[\widehat{\delta}, 1\right]$ trade occurs immediately with transfers $(\widehat{t}_A)$ and $(\widehat{t}_B)$ that tend to r_1 as $\delta \to 1$.

Proof of Proposition 4

Jun (1989) characterizes the equilibrium of a similar game in which a firm bargains with two unions. Our proof extends his framework introducing coordination between agents and externalities. We start our analysis providing some basic results that will be useful to prove Proposition 4 and Proposition 5. Suppose that the seller offers in $t = 0$. In this case the outside option payoff for a buyer not contracted, given that the other has accepted the contract at $t = 0$, corresponds to the following transfer:

$$\underline{r}(\delta) = \lim_{T \to \infty} \left[(1-\delta) \sum_{t=0}^{\frac{T-1}{2}} \delta^{2t+1} F + (1-\delta) \sum_{t=0}^{\frac{T-1}{2}} \delta^{2t} r_1 \right] = \frac{r_1 + \delta F}{1+\delta}.$$

The proof of the following lemma is immediate and follows directly from Rubinstein (1982).

Lemma A1 *Consider a subgame played by the seller and one buyer starting in the period after an agreement between the seller and the other buyer has been reached. In this case the agreement is going to be reached immediately and the transfer is going to be* $\underline{r}(\delta)$ *if the seller offers the contract and* $F(1-\delta) + \delta \underline{r}(\delta)$ *if the buyer offers it.*

In fact, once one of the two buyers has reached an agreement, our game becomes identical to a Rubinstein (1982) bargaining model with an outside option of r_1. This result allows us to narrow down the possible range of equilibrium payoffs. To characterize the outcome when the seller is the first mover we adopt an argument similar to the one used by Sutton (1986) and Jun (1989) and classify the equilibria according to when and who accepts the offer for the first time. There are five possible types of equilibria:

1. Both buyers accept in $t = 2n$ $\quad n = 0, 1, 2, ...$

2. Only one buyer accepts in $t = 2n$ $\quad n = 0, 1, 2, ...$

3. The seller accepts both offers in $t = 2n + 1$ $\quad n = 0, 1, 2, ...$

4. The seller accepts only one offer in $t = 2n + 1$ $\quad n = 0, 1, 2, ...$

5. No offer is ever accepted.

We are now ready to prove Proposition 4 and describe a pair of subgame perfect equilibrium strategies that generates an inefficient equilibrium of type 1.

Seller's Strategy: the seller offers (2) and (3) when both buyers are not contracted and offers (3) if only one buyer is not contracted. He accepts both offers if both t_A and $t_B \leq (F+\delta r_1)/(1+\delta)$ and he accepts $\min\{t_A,\ t_B\}$ if t_A or $t_B \geq (F+\delta r_1)/(1+\delta)$ and $\min\{t_A, t_B\} \leq \widetilde{t} = F(1-\delta)+\delta\left[r_2(1-\delta) + \delta F(1-\delta) + \delta^2(F+\delta r_1)/(1+\delta)\right]$ accepting A's offer if $t_A = t_B$. He rejects both offers if $\min\{t_A, t_B\} > \widetilde{t}$. Finally, if there is only one buyer uncontracted, he accepts $t \leq (F + \delta r_1)/(1 + \delta)$.

Buyers' Strategies: A accepts if both buyers are free and $t_A \geq r_2(1-\delta)+\delta F(1-\delta) + \delta^2(F+\delta r_1)/(1+\delta)$ whereas B accepts if offered $t_B \geq (F + \delta r_1)/(1 + \delta)$ and A does not reject. If both buyers are free, buyer A offers $t_A > r_2$ if $t_B > \widetilde{t}$, he offers t_B if $(F + \delta r_1)/(1 + \delta) < t_B \leq \widetilde{t}$ and offers $(F + \delta r_1)/(1 + \delta)$ if $(F + \delta r_1)/(1 + \delta) \geq t_B$. Buyer B offers $\widetilde{t}$ if $t_A > r_2$, offers $t_A - \varepsilon$ if $(F + \delta r_1)/(1 + \delta) < t_A \leq \widetilde{t}$ and offers $(F + \delta r_1)/(1 + \delta)$ if $(F + \delta r_1)/(1 + \delta) \geq t_A$. If only one buyer is not contracted he is going to accept (3) and offer $(F + \delta r_1)/(1 + \delta)$.

Following the approach in Fernandez Glazer (1991) it is easy to check that these strategies are subgame perfect. The second part of the proposition follows from the following lemma.

Lemma A2 *When buyers offer at the first period the seller does not prefer to wait one period.*

Proof. We need to show that

$$\delta^{t+1}\left[2F - r_2(1-\delta) - r_1(1-\delta) - 2\delta(F+\delta r_1)/(1+\delta)\right] < \delta^t\left[2F - 2(F+\delta r_1)/(1+\delta)\right]$$

that rewrites as $2F - \delta r_2 - \delta r_1 < 2\left[F + \delta r_1\right]$ that is always satisfied. ■

Proof of Proposition 5

Consider equilibria of type 1 and define the suprema payoffs that the buyers can achieve from the offer of the manufacturer as $\overline{v}_A$ and $\overline{v}_B$. Let us consider now the case of equality: $\overline{v}_A = \overline{v}_B = \overline{v}$. For δ large enough we have that both buyers in $t = 1$ are going to ask a large transfer obtaining a payoff of $r_2(1-\delta) + \delta\overline{v}$ and this implies that the maximum payoff that A can obtain in $t = 0$ is going to be $\overline{v} = (1-\delta)r_2 + \delta r_2(1-\delta) + \delta^2\overline{v}$ which implies $\overline{v} = r_2$. But then, if the seller offers r_2

to A, A is going to accept the offer and this allows the seller to contract immediately B offering him $\underline{r}(\delta)$ and contradicting the assumed equality. Notice that he can do this for values of δ large enough because of (4). Therefore the only possible equilibrium of type 1 is the one in which $\overline{v}_A \neq \overline{v}_B$. Let us focus on the subgame starting at time 2 and suppose that $\overline{v}_A > \overline{v}_B$. Notice that this can happen only if both buyers are contracted at some point in time. If contracts are accepted at $t = 2$ there is an incentive for B to deviate in $t = 1$ if $F(1-\delta) + \delta\overline{v}_A > (1-\delta)r_2 + \delta\overline{v}_B$ that is satisfied for δ large enough. This implies that the supremum payoffs in $t = 0$ are $\overline{v}_A = (1-\delta)r_2 + \delta F(1-\delta) + \delta^2\overline{v}_B$ and $\overline{v}_B = (1-\delta)r_1 + \delta F(1-\delta) + \delta^2\overline{v}_B$. Solving we obtain $\overline{v}_A = (1-\delta)r_2 + \delta F(1-\delta) + \delta^2(r_1 + \delta F)/(1+\delta)$ and $\overline{v}_B = (r_1 + \delta F)/(1+\delta)$. It is possible to repeat the same exercise for the infimum rather than the supremum obtaining the same result. Therefore, there exists a unique SPNE payoff and it corresponds to trading with the transfers specified in Proposition 4. We now show that equilibria of type 2 and equilibria of type 4 cannot exist.

Lemma A3 *There cannot be an Equilibrium of type 2.*

Proof. For this equilibrium we need that $(F - t_A)(1-\delta) + \delta[2F - t_A - F(1-\delta) - \delta\underline{r}(\delta)] \geq 2F - t_A - \underline{r}(\delta)$ that can be written as $\delta^2[F - \underline{r}(\delta)] \geq F - \underline{r}(\delta)$ that is a contradiction. ■

Lemma A4 *There cannot be Equilibria of type 4.*

Proof. The conditions to have such an equilibrium are that $(F - t_A)(1-\delta) + \delta(2F - t_A - \underline{r}(\delta)) \geq 2F - t_A - t_B$,that $t_B \geq F(1-\delta) + \delta\underline{r}(\delta)$ and $t_B > t_A$. In this case the payoff of B is going to be $r_1(1-\delta) + \delta\underline{r}(\delta)$. In this case B can profitably deviate asking for $F(1-\delta) + \delta\underline{r}(\delta)$ and being contracted. ■

Let us now turn to equilibria of type 3. The following lemma shows, using a procedure similar to the one used to study equilibria of type 1, that if both buyers are not contracted, they will reach an agreement offering to the seller the same transfer.

Lemma A5 *If both retailers are free and t is odd an agreement is reached with* $t_A = t_B = (F + \delta r_1)/(1+\delta)$.

Proof. Suppose there exist equilibria in period $t+2$ in which the offers of both buyers are accepted and their supremum payoffs are $\overline{v}_A$ and $\overline{v}_B$. The maximum payoffs they can achieve in period $t+1$ are $(1-\delta)r_2 + \delta\overline{v}_A$ and $(1-\delta)r_2 + \delta\overline{v}_B$ and they are obtained rejecting any offer of the seller. Therefore the corresponding suprema of period t are $\overline{v}_A = \overline{v}_B = r_2$. But if these are the suprema then in period $t+1$ the seller can profitably deviate offering r_2 to A and $\underline{r}(\delta)$ to B and these transfers are going to be accepted. Therefore the two suprema to consider in period $t+1$ are r_2 and $\underline{r}(\delta)$. This discrimination is source of competition in period t and implies equilibrium transfers $t_A = t_B = F(1-\delta) + \delta\,\underline{r}(\delta)$ that can be rewritten as these specified in the lemma. ■

Given that it is possible to reach an agreement both if the seller offers first and if buyers make an offer we will now show that if the seller is first mover he is not going to wait one period.

Lemma A6 *If the seller is the first to offer he prefers to contract immediately* *rather than to postpone to next period.*

Proof. We need to compare the payoffs of the principal contracting and waiting: $\delta^t [2F - r_2(1-\delta) - r_1(1-\delta) - 2\delta(F + \delta r_1)/(1+\delta)] > \delta^{t+1} [2F - 2(F + \delta r_1)/(1+\delta)]$ that implies $2F - r_2 - r_1 > 0$ which is true by assumption. ■

Notice that this lemma implies that the seller is going to contract the two buyers immediately. Therefore nonexistence of equilibria of type 5 can be seen as a corollary of this lemma. We have therefore shown that when condition (4) is satisfied there exists a unique SPNE. The SPNE is of type 1 when the seller is the first mover and it is of type 3 when the buyers are first movers.