

1.) The goal of this exercise is to show that the inverse function theorem fails for algebraic varieties and the Zariski topology.

(a) Let  $\mathbb{G}_{m,k} = \text{MSpec } k[t, t^{-1}]$  and let  $f: \mathbb{G}_{m,k} \rightarrow \mathbb{G}_{m,k}$  be the morphism corresponding to the  $k$ -algebra homomorphism

$$k[t, t^{-1}] \rightarrow k[t, t^{-1}], \quad t \mapsto t^n.$$

Show that  $f$  is étale if  $n$  is coprime to the characteristic of  $k$  (or  $k$  has characteristic 0).

(b) Prove that there do not exist non-empty Zariski open subsets  $U, V \subset \mathbb{G}_m$ , such that  $f(U) = V$  and  $f|_U: U \rightarrow V$  is an isomorphism.

2.) Let  $\mathbb{A}_k^1 \supset U, V$  be non-empty Zariski open subsets. Show that  $U$  and  $V$  are homeomorphic topological spaces.

**Due on Tuesday, October 2nd**