1.) Show that $\mathsf{Pic}(\mathbb{P}^1_k) \simeq \mathbb{Z}$.[1]

2.) Let k be a perfect field, and n a positive integer which is invertible in $\bar{k}$.[2]

(a) Compute $H^2(\mathbb{P}^1_k, \mu_n)$ using the Kummer sequence.

(b) Assume that k is algebraically closed, and that ℓ is a prime number, which is invertible in k. Consider the profinite-group A given by the inverse limit $\lim_i \mu_{\ell^i}$, where the transition maps $\mu_{\ell^{i+1}} \to \mu_{\ell^i}$ are given by $\lambda \mapsto \lambda^\ell$. Show that there is a natural isomorphism

$$H^2_{\text{ét}}(\mathbb{P}^1_k, A) \simeq \mathbb{Z}_\ell.$$

(c) My apologies. I decided to remove (c), as it uses terminology which will only be introduced next week.

Due on Tuesday, November 6th

[1]You may use without proof that $\mathsf{Pic}(\mathbb{A}^1_k) = 0$.
[2]That is, n is coprime to the characteristic p of $\bar{k}$.