1.) Let M be a smooth **compact and connected** manifold. Define a $\mathbb{Z}$-local system or_M, called the *sheaf of orientations*, such that one has a natural isomorphism

$$H^{\dim M}(M, \mathsf{or}_M) \simeq \mathbb{Z}.$$

2.) Let $\mathcal{A}/\mathbb{F}_q$ be a connected commutative group object in varieties, and let $\mathcal{A} \to \mathcal{A}$ be the Lang isogeny. We denote by $\rho\colon \pi_1^{\text{ét}}(\mathcal{A}, 0) \to \mathcal{A}(\mathbb{F}_q)$ the corresponding group homomorphism. For every $\mathbb{F}_q$-rational point $a \in \mathcal{A}(\mathbb{F}_q)$ we obtain an induced homomorphism $\rho_a\colon \widehat{\mathbb{Z}} \to \pi_1^{\text{ét}}(\mathsf{MSpec}\, \mathbb{F}_q, \mathsf{MSpec}\, \bar{\mathbb{F}}_q) \to \mathcal{A}(\mathbb{F}_Q)$. Compute ρ_a in dependence of a.[1]

Due on Tuesday, November 13th

[1]The formulation of this exercise was slightly changed in order to account for what was covered in class. However the nature of the exercise remains essentially the same.