Every student taking this course for credit is expected to write a short essay (around 5 pages) on one of the topics listed below. The essays should be thought of as *mini-articles* and have to be structured like one:

- title,
- abstract,
- introduction,
- references.

You are permitted to cite results from textbooks and articles (but not from the lecture notes or class), as long as your essay conveys the main points of the proof of "your main theorem". Advanced topics are marked with an asterisque. References can be requested during the office hour or by e-mail.

1.) (Symmetric powers) Let k be a finite field. For a smooth projective k-variety X, define the symmetric product $X^{(n)}/k$, and prove the formula

$$Z(X,T) = \sum_{n\geq 0} \# X^{(n)}(k) T^n.$$

2.) (Class number formula) For a finite field k we denote by X a smooth projective k-curve. There is a finite group, Cl_X whose elements are degree 0 divisors on X defined over k, modulo principal divisors. Show that one has

$$\mathsf{res}_{T=1}\, Z(X,T) = \frac{\# Cl_X}{q-1}.$$

3.) (ε-factors*) Let k be a finite field. For a smooth projective k-curve X of genus g and an ℓ-adic local system $\mathcal{L}$ of rank 1 on X, show that we have an isomorphism of $\mathbb{Q}_\ell$-vector spaces with Frobenius action

$$\det H^0(\bar{X},\mathcal{L}) \otimes \det H^1(\bar{X},\mathcal{L})^{-1} \otimes \det H^2(\bar{X},\mathcal{L}) \simeq \left(\bigotimes_{x\in X} \mathcal{L}_x^{\otimes \nu_x(\omega)} \right)(1-g),$$

where ω denotes an arbitrary rational 1-form on X, and $\nu_x(\omega)$ the order of ω at x.

4.) (K-equivalence) Define the notion of *K-equivalence* for smooth projective varieties and show (using p-adic integration) that K-equivalent varieties have the same Hodge numbers. Show that a pair of *crepant resolutions* of a variety with quotient singularities yields an example of K-equivalent varieties.

Due on November 27th