Underspecification in Degree Operators

Guillaume Thomas*
University of Toronto

Abstract

The goal of this paper is to account for the recurrent homophony between comparison, additivity and continuation cross-linguistically. Building on Roger Schwarzschild’s recent work on comparison in scale segment semantics, I propose that comparative, additive and continuative sentences all assert the existence of a rising scale segment, and differ in terms of (i) the nature of the scale and (ii) the identification of the extremities of the scale segments. At a morphosyntactic level, all three types of sentences involve the combination of a feature that denotes a property of rising scale segments with other features whose denotations constrain the identification of the extremities of the segments in a way that is characteristic of each interpretation (additivity, comparison or continuation). Homophony results from the underspecification of Vocabulary Items in Distributed Morphology.

1 Introduction

The subject matter of this paper is the homophony of comparison, additivity and continuation across languages. To illustrate, English more is interpreted as a comparative operator in (1), while it is interpreted as an additive operator in one interpretation of (2). In this interpretation, the sentence is true even if John ran only one hour on the day of utterance. It can be paraphrased as and in addition, John ran one hour today. By contrast, in its comparative interpretation, (2) is true only if John ran at least three hours on the day of utterance. Other languages display homophony between additivity and continuation. This is the case in German: noch is interpreted as an additive operator in (3), and as a continuation operator in (4):

(1) Today, John ran for one hour more than yesterday.
(2) John ran for two hours yesterday, and he ran for one more hour today.
(3) Otto hat NOCH einen Schnapps getrunken.
   ‘Otto had another schnapps.’
   (Umbach 2012)
(4) Es regnet noch.
   ‘It is still raining.’
   (Umbach 2012)

This pattern is not restricted to Germanic languages. The homophony of comparison and additivity is also attested in Romance languages (Spanish más and Portuguese mais) and in Tupi-Guarani Languages (Guarani -ve, see Thomas 2010a). Homophony between additivity and continuation is attested in Semitic languages (Modern Hebrew od, see Greenberg 2012) and Romance languages (French encore and Italian ancora, see Tovena & Donazzan 2008), among other families. Three-way homophony is attested in Romanian with mai (see Donazzan & Mardale 2010). Finally, there are languages where comparison, additivity and continuation have different exponents, such as Vietnamese. On the other hand, there appears to be no language where comparison and continuation are homophonous, but additivity has a different exponent.

My goal in this paper is to explain why comparison, additivity and continuation should be homophonous in several unrelated languages, and why the homophony of comparison with continuation always entails their homophony with additivity. Reducing these three operations to a single meaning is not desirable, since any two of them can be spelled out differently in some language, and there are languages in which they are not homophonous at all. My strategy is therefore...

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to decompose each operation into distinct features, and to identify one feature \( c \) that is common to all three meanings. Since each operation \( o \) is built by combining \( c \) with some other features specific to \( o \), the homophony of two operations \( o \) and \( o' \) in a given language can be analyzed as a case of underspecification in a realizational theory of morphology, such as Distributed Morphology (Halle & Marantz 1993). In addition, I will argue that continuation is obtained by adding one feature to the feature combination that characterizes additivity. As a consequence, no combination of features can be subsumed under comparison and continuation without being also subsumed under additivity, which explains the aforementioned gap in patterns of homophony cross-linguistically.

My analysis will be formulated in the framework of scale segment semantics, introduced by Schwarzschild (2012, 2013). Scale segment semantics uses an ontology of degrees, like classical analyses of comparison in the tradition of Cresswell (1976) and von Stechow (1984). However, comparison and related notions are expressed through quantification over directed segments of scales of degrees, rather than over degrees themselves. The scale segment analysis of comparison is much younger than more established pointilist analyses, and it does not have quite the same empirical coverage yet. In this respect, a secondary objective of the paper is to provide support for the scale segment analysis by extending its empirical scope.

In the next section, I will discuss the truth and felicity conditions of additive uses of more, since this interpretation may be less familiar to the reader than its comparative interpretation, and I will show how the homophony between comparison, additivity and continuation manifests itself in a wider set of languages. In section 3, I will outline the analysis to be developed in the rest of the paper.

2 Comparison, additivity and continuation

2.1 Additive more

Sentences with more have different truth-conditions in their additive and comparative interpretations. Consider example (5). On its comparative interpretation, the second sentence is true iff it rained for at least six hours on the day of utterance. On its additive interpretation on the other hand, the sentence is true iff it rained for at least two hours on the day of utterance. Likewise, the second sentence in (6) is true on its additive interpretation iff at least two guests arrived in the evening. (5) and (6) also show that additive more has both adverbial and nominal uses, like comparative more. In both uses, more modifies a noun, namely hours in (5) and guests in (6). Yet, in (5) the prepositional phrase for two more hours is used adverbially, while in (6) the noun phrase two more guests is an argument of the verb:

(5) It rained for two hours yesterday. It rained for two more hours today.
(6) Three guests arrived in the afternoon. Two more guests arrived in the evening.

That more really is ambiguous between a comparative and an additive reading is shown by the fact that, in certain contexts, a sentence with more is false under one reading and true under another (see Gillon 2004 for a discussion of this diagnostic of ambiguity). Consider for instance sentence (7), uttered in contexts (a) and (b):

(7) Twenty people died in the church bombing, and ten more people died in the school bombing.
   a. 30 people died in the school bombing.
   b. 10 people died in the school bombing.

In context (a), the sentence with more is true under its comparative reading and false under its additive reading. In context (b), the sentence is true under its additive reading and false under its comparative reading. Examples (8) and (9) illustrate the two readings with further supportive context:

(8) Thirty people died in the attacks: twenty people died in the church bombing, and ten more people died in the school bombing.
(9) The school bombing was deadlier than the church one: twenty people died in the church bombing, but ten more people died in the school bombing.

Note that sentences with more can be disambiguated by the use of an overt standard of comparison expressed with a than-phrase, which blocks the additive interpretation:¹

¹An anonymous reviewer suggests that the following example may have an additive reading:
I ran 5 more miles today than yesterday.

Two more guests arrived in the evening than in the afternoon.

It should also be observed that additive uses of more come with a presupposition. To wit, a speaker who asserts presupposes\(^2\) that John has already had a beer, which explains the infelicity of using more in (13).

John is going to have one more beer.

John hasn’t had a beer yet, but he is going to have one (# more).

More generally, it appears that sentences with additive more have truth-conditions that are equivalent to their alternative without more, whenever they are felicitous. They differ from their bare alternatives only with respect to the presupposition triggered by more in its additive reading. This is shown by the equivalence between sentences (a) and (b) in the following examples:

(14) a. I have run twelve miles in total: I ran seven miles yesterday, and I ran five more miles today.
   b. I have run twelve miles in total: I ran seven miles yesterday, and I run five miles today.

(15) a. Five guests have arrived: three guests arrived in the afternoon, and two more guests arrived in the evening.
   b. Five guests have arrived: three guests arrived in the afternoon, and two guests arrived in the evening.

This supports the intuition that the additive interpretation of more is distinct from its comparative interpretation, since the equivalence displayed above makes it clear that the number of miles or guests mentioned in the last clause of examples (14a) and (15b) is not compared to anything.

Finally, observe that additivity and comparison are expressed with distinct lexical items in some languages. This is the case for instance in German, where additivity is expressed with noch, while comparison is expressed with mehr:\(^3\)


am I NOCH five kilometer run
‘I have run fifteen kilometers in the last days. Yesterday, I ran seven kilometers, and today I ran five kilometers more.’


kilometer MEHR than yesterday run
‘I have a lot of energy today. Yesterday I only ran seven kilometer and today I ran five kilometers more than yesterday.’

When the context forces an additive interpretation, the use of mehr results in a contradiction:

(18) #Ich bin zwölf Kilometer in den letzten Tagen gerannt. Gestern bin ich sieben Kilometer gerannt und heute bin ich fünf Kilometer mehr (als gestern) gerannt.

am I five kilometer MEHR than yesterday run

If the additive interpretation of more were reducible to its comparative interpretation, additivity should also be expressible with German mehr.

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\(^2\)Or at least this is one presupposition that the hearer may try to accommodate. Additive interpretations of more will be discussed in more detail in section 4.

\(^3\)Thanks to Laetitia Klemish and Uli Sauerland for their assistance with and judgments about German examples.
2.2 Homophony across languages

In this section, I give an informal overview of cross-linguistic patterns of homophony between comparison, additivity and continuation, which I will henceforth refer to as CAC operators.

2.2.1 Additivity and comparison

Let us first discuss languages where comparison and additivity are homophonous to the exclusion of continuation, i.e. languages where comparison and additivity have the same exponent, which is different from the exponent of continuation. Some languages in this class are Spanish, Brazilian Portuguese, Guarani and French. The following examples were elicited with native speakers, in translation tasks. The Guarani data were subject to elicitation of truth-value judgments in fieldwork conducted by the author.\(^4\)

Consider first sentences (19) and (20) from Brazilian Portuguese and Spanish.\(^5\) In the (a) sentences, the last clause forces an additive interpretation of the operator \textit{mais/más}, while in the (b) sentences, the presence of an overt standard of comparison forces a comparative interpretation.

(19) \textbf{Brazilian Portuguese}

a. Eu corri duas horas ontem, e corri mais duas horas hoje, logo eu corri quatro horas no total.
   ‘I ran for two hours yesterday, and I ran for two more hours today, therefore I ran for four hours in total.’

b. Hoje, eu corri duas horas a mais do que ontem.
   ‘Today I ran two hours more than yesterday.’

(20) \textbf{Spanish}

a. Corrí dos horas ayer, y corrí dos horas más hoy, así que he corrido quatro horas en total.
   ‘I ran for two hours yesterday, and I ran for two more hours today, therefore I ran for four hours in total.’

b. Hoy, corrí dos horas más que ayer.
   ‘Today I ran two hours more than yesterday.’

In French, the expression \textit{de plus} may be interpreted comparatively or additively. The following headlines were retrieved from the Internet on June 4, 2014. Here again, the (a) sentence must be interpreted additively (as the follow up of the headline makes clear), while the second sentence may only be interpreted comparatively, due to the presence of an overt standard of comparison.

(21) \textbf{French}

a. Ce week end, deux morts de plus sur les routes. Samedi, deux hommes sont morts sur les routes sarthoises.
   ‘This week end two more deaths on the roads. Saturday, two men died on the road sarthoise.’

b. Hoy, corrí dos horas más que ayer.
   ‘Today I ran two hours more than yesterday.’

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\(^4\)Elicitation sessions took place in the community Kañcha Piru, Misiones, Argentina in July 2009, with two adult native speakers.

\(^5\)Many thanks to Maria Cristina Cuervo and Ana Teresa Perez Leroux for sharing their judgments on Spanish, and to Suzi Lima for sharing her judgments on Brazilian Portuguese.

Finally, The ambiguity between comparison and additivity is also attested in Mbyá Guarani, as illustrated by the
following examples:8

\[(22) \text{Mbyá Guarani} \text{ (see also Thomas 2010a)}\]


‘Yesterday, my friend bought eight gourds in my store, and today (s)he bought some more (gourds).’

\text{True if the speaker’s friend bought eight gourds yesterday, and four gourds today.}\]

b. Juan i-tuicha-ve Maria gui.
Juan B3-old-more Maria from
‘Juan is older than Maria.’

\text{True only if Juan’s age is greater than Maria’s.}\]

2.2.2 Additivity and continuation

In a second class of languages, additivity and continuation are homophonous to the exclusion of comparison. This
is the case for instance in German, where noch can express additivity or continuation, and mehr is used to express
comparison. Umbach (2012) distinguishes four interpretations of noch (see also L¨obner 1989; K¨onig 1991; Krifka
2000; Eckardt 2006): temporal, marginal, comparative and additive. In its basic temporal interpretation, noch can
be translated into English as still, as illustrated in (23) and (24). These sentences presuppose that the state that they
describe holds at some time before the time of utterance. If their presupposition is satisfied, they are true if that state
holds up to the time of utterance, and false if that state does not hold at the time of utterance.

\[(23) \text{Hans schl¢ft noch.}\]
‘Hans is still sleeping.’ (K¨onig 1991 p. 136)

\[(24) \text{Hans ist noch ledig.}\]
‘Hans is still single/unmarried.’ (K¨onig 1991 p. 136)

The marginality interpretation of noch is illustrated in (25) and (26). These sentences assert that their predicate
holds of their subject and presuppose that there are other individuals that the predicate holds of to a greater degree. To
illustrate, the first clause of example (25) asserts that Peter is moderate and presupposes that some other individual x is
more moderate than Peter. The use of noch conveys a ranking of inverse moderateness, which starts with this individual
x, progresses to the less moderate individual Peter, and finally reaches Paul, who no longer qualifies as moderate. It is
generally agreed that this interpretation of noch is derived from the more basic temporal interpretation illustrated in (23)
and (24) (see L¨obner 1989). In its temporal interpretation, noch relates a state to a set of times ordered by the relation
of precedence, while in its marginality interpretation, noch relates a property to a set of individuals inversely ordered
by the extent to which they instantiate the property. In this paper, I will also assume that the marginality interpretation
of noch is derived from its temporal interpretation, and I will refer to both as the continuative interpretation of noch.

\[(25) \text{Peter ist noch gem¨assigt, Paul ist schon radikal.}\]
Peter is still moderate, Paul is already radical. (L¨obner 1989 p. 204)

\[(26) \text{Osnabr¨uck liegt (gerade) noch in Niedersachsen.}\]
‘Osnabr¨uck is still in Lower Saxony.’ (Umbach 2012)
In its so-called comparative reading, noch is translated into English as even, as illustrated in (27). I will not discuss comparative readings of noch in this paper. Suffice it to say that the morpheme noch is actually not interpreted as a comparative operator in this use. Umbach (2009) argues that (27) asserts that Berta is taller than Adam, and presupposes that Adam is taller than some salient individual or standard of comparison. The contribution of noch to the truth-conditions of the sentence is mainly to trigger anaphora to this salient individual, while the comparative interpretation is due to suffix -er. The reader is referred to Umbach (2009) for more details.

(27) Berta ist noch grösser als Adam.
   ‘Berta is even taller than Adam.’ (Umbach 2012)

Finally, of particular interest to us is the additive interpretation of noch illustrated in (28) and (29), where capital letters indicate stress. Both sentences are requests for the speaker to have a beer, and presuppose that the speaker already had another drink. In (28), where the stress falls on Bier, the sentence presupposes that the speaker already had some drink other than a beer. In (29), where the stress falls on noch, the sentence presupposes that the speaker already had a beer. The additive interpretation of noch was studied by König (1971, 1991), Eckardt (2006) and Umbach (2009, 2012).

(28) Ich trinke noch ein BIER.
    I will have a beer, too. (König 1991 p. 143)
(29) Ich trinke NOCH ein bier.
    I will have another beer. (König 1991 p. 143)

In Modern Hebrew, continuation is expressed with the particle od:

(30) rina od yeSena
    Rina OD asleep
    ‘Rina is still asleep.’ (Greenberg 2012)

Like German noch, od can also be interpreted additively, as shown in the following examples. The additive interpretation of od is discussed in detail in Greenberg (2012).

(31) etmol axalti 3 tapuzim. ha-yom axalti od (tapuzim).
    yesterday I-ate 3 oranges the-day I-ate od oranges
    ‘Yesterday I ate 3 oranges. Today I ate some more (oranges).’ (Greenberg 2012)
(32) ba-boker rina yaSna kcat. ba-cohorayim hi yaSna od.
    in-the-morning Rina slept a-bit in-the-noon she slept od
    ‘In the morning Rina slept a bit. At noon she slept some more.’ (Greenberg 2012)

The homophony between additivity and continuation is also attested in Romance languages, with Italian ancora:

(33) Maria sta ancora leggendo.
    ‘Mary is still reading.’ (Tovena & Donazzan 2008)
(34) Maria ha letto ancora un libro.
    ‘Mary read one more book.’ (Tovena & Donazzan 2008)

2.2.3 Three-way homophony

There are also languages where all CAC operators have the same exponent. This is the case in Romanian, where mai can be used to express comparison, additivity or continuation. The comparative and continuative interpretations of mai are illustrated in examples (35) and (36). The additive interpretation is illustrated in example (37).

(35) Ion e mai inteligent decât Petre.
    Ion is MAI intelligent than Petre
    ‘Ion is more intelligent than Petre.’ (Donazzan & Mardale 2010)
The following two examples\(^9\) provide more evidence for the homophony of comparison and additivity with Romanian \textit{mai}. The fact that the discourse in (38) is consistent shows that \textit{mai} may be interpreted additively, since a comparative interpretation is incompatible with the last sentence, and the aspectual interpretation of the second sentence (with a past perfective verb) is incompatible with a continuative interpretation. By contrast, an additive interpretation is unavailable in (39), where \textit{mai} is combined with a standard phrase \(^{10}\). This is shown by the fact that the last sentence of the discourse (‘So in total I ran for five hours.’) is infelicitous in the context provided by the two preceding sentences.

\[(38) \text{Am fugit trei ore luni şi am fugit doua ore azi. Deci în total, am fugit cinci ore.}\]  
\text{I ran three hours Monday and have.I ran for two hours more today, so in total I ran for five hours.}

\[(39) \text{Am fugit trei ore ieri. Azi am fugit doua ore mai mult decât ieri. #Deci în total,}\]  
\text{I ran three hours yesterday. Today have.I run two hours more than yesterday so in total am fugit cinci ore.}

\text{I ran five hours.}

\text{‘Yesterday, I ran for three hours. Today, I ran for two more hours than yesterday. #So in total I ran for five hours.’}

### 2.2.4 Languages without CAC homophony

A last class of attested languages consists of those where comparison, additivity and continuation have different exponents. Vietnamese appears to be one such language.\(^{11}\) Comparison is expressed by the morpheme \textit{hơn}, additivity is expressed by the morpheme \textit{nữa}, and continuation is expressed by the morpheme \textit{vẫn}:

\[(40) \text{Hôm qua tôi chạy hai tiếng, và hôm nay tôi chạy (thêm) hai tiếng nữa, vậy tôi đã chạy tổng}\]  
\text{I ran for two hours yesterday, and day present I run additional two hours NUA thus I already run total}\n
\[(41) \text{cộng bốn tiếng.}\]  
\text{four hour}

\text{I ran for two hours yesterday, and I ran for two more hours today, thus I ran for four hours in total.}

\[(42) \text{#Hôm qua tôi chạy hai tiếng, và hôm nay tôi chạy (nhiều) hơn hôm qua hai tiếng, vậy tôi đã chạy}\]  
\text{I ran two hours yesterday, and day present I run many HON day last two hour thus I already run}\n
\[(43) \text{tổng cộng sáu tiếng.}\]  
\text{six hour}

\text{I ran for two hours yesterday, and today I ran for two more hours than yesterday, thus I ran for six hours in total.}

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\(^9\)I am grateful to Anamaria Fallais and Monica Irimia for providing translations and judgments of truth-conditions for Romanian.  
\(^{10}\)I.e. a phrase that introduces the standard of comparison, such as the \textit{than}-PP in the English translation  
\(^{11}\)Many thanks to Tue Trinh for providing the following examples and their glosses.
Examples (41)-(43) show that *hơn cannot be interpreted as an additive operator and *nữa cannot be interpreted as a comparative operator. Example (44) illustrate the expression of continuation with *vẫn and examples (45) and (46) show that *nữa and *hơn cannot be interpreted as continuation operators.

These data from Vietnamese suggest that comparative, additive and continuative particles have different denotations. In particular, if additivity were semantically reducible to either comparison or continuation, languages like Vietnamese should be unattested.

2.2.5 Homophony and negation

In several languages, CAC operators are subject to different patterns of homophony under the scope of negation. The following pairs of examples illustrate this phenomenon in English:

(47) **Comparison**

a. I ate more cookies than Lucie.

b. I did not eat more cookies than Lucie.

(48) **Additivity**

a. I ate two cookies for breakfast, and I ate one more at lunchtime.

b. I ate two cookies this morning, and I did not eat any more cookies after this.

(49) **Continuation**

a. I am still eating cookies.

b. I am not eating cookies anymore.

If we assume that continuative *anymore* is not a morphological atom but is composed of the two morphemes *any* and *more*, we may argue that the three CAC operators are spelled out as *more* under the scope of negation.

While the reader may be reluctant to accept this analysis of *anymore*, three way homophony under negation is also attested in Spanish, Brazilian Portuguese and French, where this complication does not arise:

<table>
<thead>
<tr>
<th>Language</th>
<th>Comparison</th>
<th>Additivity</th>
<th>Continuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>plus</td>
<td>plus</td>
<td><em>toujours</em></td>
</tr>
<tr>
<td></td>
<td>plus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B. Portuguese</td>
<td><em>mais</em></td>
<td><em>mais</em></td>
<td><em>ainda</em></td>
</tr>
<tr>
<td></td>
<td><em>mais</em></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spanish</td>
<td><em>más</em></td>
<td><em>más</em></td>
<td><em>todavía</em></td>
</tr>
<tr>
<td></td>
<td><em>más</em></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1 shows that in all three languages, two-way homophony between comparison and additivity in non-negative sentences is generalized to three-way homophony in the scope of negation. Homophones have been grayed-out in each row of the table.

This generalization of homophony under negation is also attested in German. Remember that in this language, additivity and continuation are spelled out as *noch* in non-negative sentences, while comparison is spelled out as *mehr*. 
Under negation, all three operations are spelled out as *mehr*. This is illustrated in example (50) for continuation (from Löhner 1989), and in example (51) for additivity:

(50) A: Ist das Licht noch an?  
    ‘Is the light still on?’
B: Nein, das Licht ist nicht mehr an.  
    ‘No, the light is not on anymore.’

(51) Heute Morgen habe ich zwei Kekse gegessen, und dann habe ich keine Kekse mehr gegessen.  
    ‘I have eaten two cookies in the morning, and then I have not eaten any more cookies.’

Table 2 summarizes the effect of negation on CAC homophony in these five languages:

<table>
<thead>
<tr>
<th>Language</th>
<th>Comparison</th>
<th>Additivity</th>
<th>Continuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>French</td>
<td>Positive: plus</td>
<td>plus</td>
<td>toujours</td>
</tr>
<tr>
<td></td>
<td>Negative: plus</td>
<td>plus</td>
<td></td>
</tr>
<tr>
<td>B. Portuguese</td>
<td>Positive: mais</td>
<td>mais</td>
<td>ainda</td>
</tr>
<tr>
<td></td>
<td>Negative: mais</td>
<td>mais</td>
<td></td>
</tr>
<tr>
<td>Spanish</td>
<td>Positive: más</td>
<td>más</td>
<td>todavía</td>
</tr>
<tr>
<td></td>
<td>Negative: más</td>
<td>más</td>
<td>más</td>
</tr>
<tr>
<td>English</td>
<td>Positive: more</td>
<td>more</td>
<td>still</td>
</tr>
<tr>
<td></td>
<td>Negative: (any)more</td>
<td>(any)more</td>
<td></td>
</tr>
<tr>
<td>German</td>
<td>Positive: mehr</td>
<td>noch</td>
<td>noch</td>
</tr>
<tr>
<td></td>
<td>Negative: mehr</td>
<td>mehr</td>
<td>mehr</td>
</tr>
</tbody>
</table>

Table 2

These observations show that three way homophony is commonly attested in Germanic and Romance languages, albeit under the scope of negation. Note however CAC homophony is not conditioned by negation in all languages. In Vietnamese for instance, CAC homophony is attested neither in positive sentences nor in negative sentences, as illustrated by the following examples:  

(52) Tôi ăn nhiều bánh hơn Lucie.  
    ‘I ate more cookies than Lucie.’
(53) Tôi không ăn nhiều bánh hơn Lucie.  
    ‘I did not eat more cookies than Lucie.’

(54) Tôi ăn hai cái bánh vào bữa sáng, và tôi ăn (thêm) một cái nữa vào bữa trưa.  
    ‘I ate two cookies for breakfast, and I ate one more at lunchtime.’
(55) Tôi ăn hai cái bánh vào bữa sáng, và tôi không ăn (thêm) một cái nữa vào bữa trưa.  
    ‘I ate two cookies for breakfast, and I didn’t eat one more at lunchtime.’

(56) Trời vẫn mưa.  
    ‘It is still raining.’
(57) Trời không vẫn mưa.  
    ‘It is not raining anymore.’

12Glosses and translations are mine.  
13Thanks to Tue Trinh for providing these examples.
2.3 Unattested language type

Finally, I have been unable to identify a language where comparison and continuation are homophonous, and additivity is realized as a distinct lexical item. This generalization is supported by a study of a modest but diverse set of languages for which elicitation data were obtained, or which were discussed in the previous publications on comparison, additivity and/or continuations:

(58) a. Romance: French, Italian, Brazilian Portuguese, Romanian, Spanish.
    b. Germanic: English, German.
    c. Semitic: Modern Hebrew.
    d. Austroasiatic: Vietnamese.
    e. Tupi: Mbyá Guarani.
    f. Slavic: Russian.
    g. Finno-Ugric: Hungarian.

Table 3 summarizes the attested variation in patterns of homophony between CAC operators in positive sentences:

<table>
<thead>
<tr>
<th>Languages</th>
<th>Comparison</th>
<th>Additivity</th>
<th>Continuation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Romanian</td>
<td>A</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>English</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>French</td>
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</tr>
<tr>
<td>M. Guarani</td>
<td>A</td>
<td>B</td>
<td></td>
</tr>
<tr>
<td>B. Portuguese</td>
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<td>unattested?</td>
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While the lack of two-way homophony between comparison and continuation still has to be confirmed by a full-fledged typological study, these preliminary results are striking. Accordingly, one of the goals of the analysis put forward in this paper is to account for this putative gap in patterns of homophony between CAC operators.

3 Towards a decompositional analysis

3.1 Morphological decomposition

As stated in the introduction, I will defend a decompositional analysis of CAC operators. This analysis will be expressed in Distributive Morphology (Halle & Marantz 1993). In this framework, the terminals of syntactic structures are (bundles of) features without phonological content. We refer to such terminals as morphemes. A morpheme can be made up of a single feature or it can be a bundle of several features. At several points in the derivation, syntactic representations are mapped to phonological form (PF) and to logical form (LF). The phonological realization of morphemes (called vocabulary insertion) occurs in the mapping from syntax to PF. Vocabulary insertion is governed by rules of exponence that pair features with phonological representations. In Distributed Morphology, these rules are called vocabulary items (VIs). Two important principles govern their application:

(59) Rules Apply: (Bobaljik 2015)
A rule applies wherever its structural description is met.
Elsewhere Condition:

Where more than one mutually exclusive rule may apply, (only) the most highly specified rule applies.

In order to see these rules in action, consider that up to three VIs may compete to determine the exponent of a morpheme \([XY]\) composed of two features X and Y:

\[
(61) \quad [\frac{X}{Y}] \leftrightarrow \phi_1 \\
(62) \quad X \leftrightarrow \phi_2 \\
(63) \quad Y \leftrightarrow \phi_3
\]

Given Rules Apply, all three rules should be considered when determining the exponent of \([XY]\) at vocabulary insertion. Yet, the Elsewhere Condition guaranties that (61) will prevail over (62) and (63) since its structural description is more specific than that of the two other rules.

While the mechanism of CAC allomorphy will be discussed in more detail in the rest of the paper, I would like to sketch the proposed analysis in this section, in order to motivate the decompositional analysis of CAC operators. In order to apply the Elsewhere Condition to the analysis of the homophony of CAC operators, we may analyze these operators as bundles of features. One feature, call it RISE, is common to all operators. A second feature, ADD, is common to additivity and continuation. A third feature, CON, is restricted to continuation:

\[
\begin{align*}
(64) & \quad \text{a. Comparison: } \left[ \text{RISE} \right] \\
 & \quad \text{b. Additivity: } \left[ \begin{array}{c} \text{RISE} \\ \text{ADD} \end{array} \right] \\
 & \quad \text{c. Continuation: } \left[ \begin{array}{c} \text{RISE} \\ \text{ADD} \\ \text{CON} \end{array} \right]
\end{align*}
\]

Because the feature RISE is specific to all operators, it will be possible to generate three-way homophony in a given language by formulating an underspecified VI that maps RISE to its exponent E. If the lexicon of the language contains no VI that is more specific, RISE will be spelled out as E. This is the case in Romanian, where all CAC operators are spelled out as mai:

(65) Romanian:

\[ \text{RISE} \leftrightarrow \text{mai} \]

In order to generate two-way homophony, we will also formulate an underspecified VI that matches all three CAC operators. However, the use of this VI to spell out one of these operators will be blocked by adding a VI that matches more features for this operator. The following examples illustrate this mechanism for English and German:

(66) English:

\[
\begin{align*}
(66) & \quad \text{a. RISE } \leftrightarrow \text{ more } \\
 & \quad \text{b. } \left[ \begin{array}{c} \text{RISE} \\ \text{ADD} \\ \text{CON} \end{array} \right] \leftrightarrow \text{ still}
\end{align*}
\]

(67) German:

\[
\begin{align*}
(67) & \quad \text{a. RISE } \leftrightarrow \text{ mehr } \\
 & \quad \text{b. } \left[ \begin{array}{c} \text{RISE} \\ \text{ADD} \end{array} \right] \leftrightarrow \text{ noch}
\end{align*}
\]

Crucially, since the bundle of features that makes up additive operators is a subset of the bundle that makes up continuation operators, it will not be possible to create lexicons where comparison and continuation are homophonous and additivity has its own exponent. To do so, one would need to follow one of three strategies:

1. Formulate a VI that matches all three operators and a more specific one that only matches additive operators.
2. Formulate a VI that matches all three operators and a more specific one that only matches comparative and continuation operators.
3. Formulate a VI that only matches additive operators and another one that only matches comparative and continuation operators.

Strategy 1 is impossible because any VI that matches additive operators also matches continuation operators. Strategies 2 and 3 are impossible because any VI that matches both comparative and continuation operators also matches additive operators.

Finally, we can of course account for lexicons without homophony, as in Vietnamese:
The sensitivity of CAC operators to negation will be discussed in section 5, where we will discuss the morphological analysis in more detail.

3.2 Semantic Decomposition

The morphological analysis sketched in the previous subsection accounts for all attested patterns of CAC homophony and also for the lack of two-way homophony between comparison and continuation. However, the meaning of the different features that have been introduced is still unclear, and I must still explain how they fit in a compositional analysis of CAC operators. In this subsection, I give an informal overview of the semantic analysis of these operators. A full-fledged compositional analysis will be given in the next section.

My analysis of CAC operators is formulated in scale segment semantics, a framework that was developed by Schwarzschild (2012, 2013) for the analysis of comparative constructions, taking inspiration from the vector space semantics of Zwarts (1997) and Faller (2000).

Comparison In scale segment semantics, comparison is understood as a form of quantification over segments of scales of degrees:

\[\text{[John is taller than Mary]}^{e,c} = 1 \text{ iff there is a rising segment on the scale of height that has Mary’s height as its starting point and that has John’s height as its endpoint.}\]

Scale segment semantics allows us to think of scale segments in analogy to events in Neo-Davidsonian event semantics. Consider the truth-conditions in (69). We can define a predicate of scale segments \(\text{RISE}^{e}\), and relate segments to their extremities using relations like \(\text{START}^{e}\) and \(\text{END}^{e}\). The former is akin to a predicate of events like \(\text{run}^{e}\), and the latter to thematic relations like \(\text{SOURCE}^{e}\) and \(\text{GOAL}^{e}\):

\[\text{[John is taller than Mary]}^{e,c} = 1 \text{ iff } \exists \sigma [\mu_{\sigma} = \text{HEIGHT} \land \uparrow^{e} (\sigma) \land \text{START}^{e}(\sigma, \mu_{\sigma}(\text{Mary})) \land \text{END}^{e}(\sigma, \mu_{\sigma}(\text{John}))]\]

How is this relevant to the analysis of CAC operators? This analysis allows us to break down CAC operators into a feature that denotes a property of rising scale segments, and features that specify the extremities of segments. We define the feature \(\text{RISE}^{e}\) as follows:

\[\text{RISE}^{e} = \lambda \sigma. \lambda \sigma. \text{RISE}^{e}(\sigma) = \lambda \sigma. \uparrow^{e} (\sigma)\]

The starting point and endpoint of segments in comparative sentences are specified by the features \(\text{START}^{e}\) and \(\text{END}^{e}\) respectively:

\[\text{[START]}^{e,c} = \lambda x. \lambda \sigma. \text{START}^{e}(\sigma, \mu_{\sigma}(x))\]
\[\text{[END]}^{e,c} = \lambda x. \lambda \sigma. \text{END}^{e}(\sigma, \mu_{\sigma}(x))\]

Insofar as the feature \(\text{START}^{e}\) specifies the starting point of scale segments, it is reasonable to assume that it is spelled out as the preposition \textit{than} in English. The position of the feature \(\text{END}^{e}\) in the syntactic structure of comparative sentences will be discussed in the next section.

The sensitivity of CAC operators to negation will be discussed in section 5, where we will discuss the morphological analysis in more detail.
Additivity  Since I have assumed that the feature RISE is common to all CAC operators, the difference between comparison and additivity will have to be located in the features that specify the extremities of the segments. We may analyze an additive sentence as follows:

\[ \text{I bought more}_{add} \text{ cookies}^e \sigma^c = 1 \text{ iff there is a rising segment } \sigma \text{ of the scale of cardinalities such that the starting point of } \sigma \text{ is the cardinality of some salient set } S \text{ of cookies and the endpoint is the cardinality of the union of this set with the set of cookies that I bought.} \]

In keeping with the analogy between scale segment semantics and event semantics, I propose that the relation of comparison to additivity is a form of argument structure alternation. Additive operators are made up of the same features as comparative operators, with the addition of a feature ADD that recombines the START and END features to meet the following conditions, for some scale segment \( \sigma \):

1. the starting point of \( \sigma \) is the measure of some salient set \( S \),
2. the endpoint of \( \sigma \) is the measure of the union of \( S \) with the set denoted by the constituent that \textit{more} modifies.\(^\text{15}\)

Continuation  Continuation operators will be analyzed as additive operators with an additional presupposition. To do so, I will use the set of initial parts of an eventuality \( e \) (state or event) is ordered by a relation of ‘development,’ which is inspired by Landman’s (1992) concept of stages of events:

\[ \text{An event is a stage of another event if the second can be regarded a more developed version of the first, that is, if we can point at it and say “It’s the same event in a further stage of development.” (Landman 1992 p. 23)} \]

Following Ippolito (2007), I assume that the sentence \textit{it is still raining} asserts that it is raining at the time of utterance and presupposes that this event stretches back to some salient past time. This presupposition can be reformulated using the notion of development as follows (where an initial part \( e' \) of an event \( e \) is less developed than another one \( e'' \) iff \( e'' \) extends further in time than \( e' \)):

\[ \text{It is still raining:} \]
\[ \text{a. Assertion: there is an event } e \text{ of raining whose runtime overlaps the time of utterance.} \]
\[ \text{b. Presupposition: there is a salient event } e' \text{ that precedes the time of utterance and is a less developed initial part of } e \text{ than the sum of } e \text{ and } e', \text{ for some event of raining } e. \]

This presupposition can in turn be reformulated in scale segment semantics, where the relevant scale is a set of real numbers in one-to-one correspondence with the set of initial parts of some event of raining \( e \), ordered by the relation of development:

\[ \text{It is still raining:} \]
\[ \text{a. Assertion: there is an event } e \text{ of raining whose runtime overlaps the time of utterance.} \]
\[ \text{b. Presupposition: there is a rising segment } \sigma \text{ of a scale of development of initial parts of some raining event } e, \text{ such that the starting point of } \sigma \text{ is the degree of development of some salient event of raining } e' \text{ and its endpoint is the degree of development of the sum of } e \text{ and } e'. \]

Note that the content of this presupposition is itself an additive statement. Accordingly, I will argue that the semantic contribution of the feature CON is to turn an additive statement into a presupposition triggered by the continuation operator \textit{still}.

I hope that this partial overview of the analysis to be developed in the next section allows the reader to develop an intuitive understanding of the semantic decomposition of CAC operators. More precisely, it should allow the reader to understand in what sense additivity can be derived from comparison by the introduction of a further feature that manipulates START and END features, and continuation can be derived from additivity by the introduction of a feature that turns an additive statement into a presupposition of the continuation operator.

\(^{15}\)E.g. the set of cookies that the speaker bought, in example (75).
4 An analysis in scale segment semantics

Let us now flesh out the decompositional analysis of CAC operators that was sketched in the previous section. In this section, I put aside the issue of homophony, and I develop a compositional analysis of each operation. I start with comparative operators, then I move on to additive and finally continuation operators.

4.1 Comparison in scale segment semantics

Let a scale be a partially ordered set of degrees in the range of a measure function, then:

\[
\text{(79) } A \text{ scale segment } \sigma = (u, v, >_{\sigma}, \mu_{\sigma}) \text{ such that:} \\
\mu_{\sigma} \text{ is a measure function,} \\
>_{\sigma} \text{ is a partial order on the range of } \mu_{\sigma}, \\
u, v \text{ are in the range of } \mu_{\sigma}. 
\]

Given a scale segment \( \sigma = (u, v, >_{\sigma}, \mu_{\sigma}) \), let \( u \) and \( v \) be the starting point and endpoint of \( \sigma \), respectively, in which case \( \text{START}(\sigma, u) = 1 \) and \( \text{END}(\sigma, v) = 1 \). Any segment has a unique starting point and a unique endpoint. Finally, let \( \not\supset (\sigma) \) be true iff \( v >_{\sigma} u \), in which case we say that \( \sigma \) is a rising scale segment.

The analysis of degrees and unit names that I assume is adapted from Sassoon’s (2010) theory of measurement in natural language. Measure functions like \( \text{HEIGHT} \) map individuals to degrees, which are represented as real numbers. This assumption raises the question of the interpretation of the numerical value that such a function assigns to an individual. Take for instance the Eiffel tower \( e \), whose pinnacle height is 324 m, or equivalently 1,063 ft. Should the measure function \( \text{HEIGHT} \) map \( e \) to 324, to 1,063, or to some other number? In the absence of a specific unit of measurement, there is no clear answer to this question. Yet, when formulating the truth-conditions of a comparative statement like \( \text{The CN tower is taller than the Eiffel tower} \), we might want to leave the choice of unit of measurement underspecified. One way to do this is to allow ourselves to use different mappings of individuals to real numbers \( \text{HEIGHT}_c \) and \( \text{HEIGHT}_{c'} \) in different contexts of utterance \( c \) and \( c' \). In order to capture this contextual dependence of measurement, let us assume with Sassoon that unit names like \( \text{inch} \) are associated with sets of unit objects: to illustrate, the set of inch unit objects is the set of objects whose length is canonically described as ‘one inch.’ I refer to this set as \( \text{INCH} \). Adapting Sassoon’s analysis to scale segment semantics, and simplifying it somewhat, we can then assume that unit names denote relations between scale segments and real numbers. If we let \( f \) be a function that maps any singleton set to its unique member, \( f[\mu_{\sigma,c}(d) : d \in \text{INCH}] \) is the value that the measure function \( \mu_{\sigma,c} \) assigns to any inch unit object in context \( c \), which I abbreviate as \( \text{in}_{\sigma,c} \). Crucially, this analysis is compatible with various mappings \( \mu_{\sigma,c} \) of objects to real numbers, as long as \( \mu_{\sigma,c} \) is a measurement of spatial distance:

\[
\text{(80) } \llbracket \text{inch} \rrbracket^C = \lambda n. \lambda \sigma.n \times f[\mu_{\sigma,c}(d) : d \in \text{INCH}] \\
= \lambda n. \lambda \sigma.n \text{in}_{\sigma,c}
\]

Since the context dependence of measurement will not play a role in my analysis of CAC operators, I will abstract away from it in the notation: I will refer to the measure function associated with the segment \( \sigma \) in a context \( c \) simply as \( \mu_{\sigma} \), and to the denotation of two inches as \( 2 \text{ in}_{\sigma} \).

I assume the following system of types: individuals (type \( e \)), events (type \( v \)), temporal intervals (type \( i \)), scale segments (type \( \sigma \)), degrees (type \( d \)) and truth-values (type \( t \)). Note that scale segment semantics is a form of degree semantics, insofar as the segments that are quantified over are derived from scales, which are ordered sets of degrees. Consequently, the ontology of degrees that is (explicitly or not) adopted in pointilist analyses of comparison in the tradition of von Stechow (1984) and Heim (1985) is preserved in scale segment semantics. In that sense, the adoption of scale segment semantics is not a departure from the ontology of degree semantics.

4.1.1 Comparative form of predicative adjectives

Consider first the comparative form of predicative adjectives. In pointilist degree semantics, one may analyze sentence (81) as a comparison between Mary’s height and John’s height:

\[
\text{(81) } \text{John is taller than Mary is.}
\]

\[
\text{(82) } \llbracket (81) \rrbracket^C = 1 \text{ iff HEIGHT(John)} > \text{HEIGHT(Mary)}
\]
Since the set of degrees of height forms a scale, we may reformulate this analysis in scale segment semantics by quantifying over rising segments of a scale of height. Accordingly, (81) is true iff there is a rising segment on the scale of heights that starts with Mary’s height and that ends with John’s height. In the derivations that follow, I treat $\sigma$ as a parameter of the measure function $\mu_\sigma$: 16

\[(83) \quad \llbracket (81) \rrbracket^e = 1 \text{ iff } \exists \sigma. (\wedge (\sigma) \wedge \mu_\sigma(\text{HEIGHT}) \wedge \text{START}(\sigma, \mu_\sigma(\text{Mary})) \wedge \text{END}(\sigma, \mu_\sigma(\text{John})))\]

I assume with Schwarzschild (2013) that the feature RISE heads a DegP. Building on the analogy between the structure of comparative sentences in scale segment semantics and the argument structure of verbs, I posit that the feature END projects a degP in the extended projection of the RISE feature. The endpoint of the scale is introduced in the Spec of degP, much like the agent argument is introduced in Spec of vP in analyses of argument structure influenced by Kratzer’s (1996) analysis of external arguments (see also Marantz 1984).

Following Lechner (2004), the AP projected by the root $\sqrt{TALL}$ is merged in the specifier of DegP. The standard of comparison is merged as the complement of RISE. It is headed by the feature START, which is spelled out as than.

\[(84) \quad \text{degP} \quad \text{deg}^\prime \quad \text{END} \quad \text{AP} \quad \sqrt{TALL} \quad \text{Deg}^\prime \quad \text{Deg} \quad \text{RISE} \quad \text{PP} \quad \text{START} \quad \sqrt{MARY}\]

Note that while the START feature will end up being spelled-out as the preposition than, the RISE and END features will be spelled out as more after being bundled together by morphological restructuring operations (see section 5). One might therefore wonder why the features are introduced as separate heads to start with. The answer is that this choice simplifies the semantic composition of the sentence. Every feature denotes a function, and assuming that each feature is introduced as a separate head allows us to combine the feature with its sister using principles of function application or predicate modification and generalizations thereof (Heim & Kratzer 1998). If END and RISE were bundled, we would have to introduce new principles of composition in order to be able to interpret bundles of features. This is certainly possible, but I prefer to stick to a more conservative view of the interpretation of syntactic structures, and exploit the restructuring operations of Distributed Morphology to account for discrepancies between abstract syntactic structures and morpho-phonological forms.

There is also a principled reason for introducing the features RISE, END and START at these points of syntactic structure. Pursuing the analogy between event semantics and scale segment semantics, RISE is the head of an extended DegP projection which is analogous to an extended verb phrase. RISE itself denotes a property of rising segments. The endpoints of this segment are introduced by the features START and END in the positions where we would expect thematic heads to introduce arguments in a VP: the PP headed by START is the complement of the Deg head, in a position that is analogous to the theme argument of a verb, and END is located in the extended project of Deg, in a position that is analogous to an agent thematic head.

Let us then discuss the interpretation of this structure. $\sqrt{TALL}$, RISE, END and START are interpreted as follows:

\[(85) \quad \llbracket \sqrt{TALL} \rrbracket^e = \lambda \sigma. \mu_\sigma = \text{HT}\]

\[(86) \quad \llbracket \text{RISE} \rrbracket^e = \lambda \sigma. \nearrow (\sigma)\]

\[(87) \quad \llbracket \text{END} \rrbracket^e = \lambda x. \lambda \sigma. \mu_\sigma(\text{END}(\sigma, \mu_\sigma(x)))\]

16In other words, the definition of $\mu_\sigma$ depends on the value of the variable $\sigma$, which can be bound by a lambda or an existential quantifier. This parametrization of the measure function allows us to refer to the function $\mu_\sigma$ in the lexical entries of END and START without specifying its nature.
\( \text{START}[^{ec}] = \lambda x. \lambda \sigma. \text{START}(\sigma, \mu_x(x)) \)

\( \sqrt{TALL} \) denotes a property of segments on the scale of heights. Information about the starting point of the scale is provided by the PP \( [pp \text{ START } \sqrt{\text{MARY}}] \), which denotes a property of scale segments that have Mary’s measurement \( \mu_x(Mary) \) as a starting point. At this point of the derivation, the nature of \( \mu_x \) is still unspecified, since it is the adjective \( \text{tall} \) that encodes the information about the measure function that is associated with the scale segment \( \sigma \). The PP combines with the feature RISE by Predicate Modification (PM, Heim & Kratzer 1998). The denotations of RISE and AP are then combined by PM to yield a property of rising segments of the scale of height, that start with Mary’s height.

\( \text{PP}[^{ec}] = \lambda \sigma. \text{START}(\sigma, \mu_x(Mary)) \)

\( \text{AP}[^{ec}] = \lambda \sigma. \mu_x = \text{HT} \)

\( \text{DegP}[^{ec}] = \lambda \sigma. \text{RISE}(\sigma) \land \mu_x = \text{HT} \land \text{START}(\sigma, \mu_x(Mary)) \)

The endpoint of the scale segments considered in this sentence is the measurement of the individual John, denoted by the NP in Spec of degP. This individual is related to the scale segment argument of RISE by the deg head END, which combines with DegP by a rule of ‘segment identification’, similar to the rule of ‘event identification,’ which Kratzer (1996) uses to combine VPs with the agent thematic head ‘voice’:

\( \text{degP}[^{ec}] = \lambda \sigma. \text{RISE}(\sigma) \land \mu_x = \text{HT} \land \text{START}(\sigma, \mu_x(Mary)) \land \text{END}(\sigma, \mu_x(\text{John})) \)

\( \exists \sigma \text{ degP}[^{ec}] = 1 \text{ iff } \exists \sigma [ \text{RISE}(\sigma) \land \mu_x = \text{HT} \land \text{START}(\sigma, \mu_x(Mary)) \land \text{END}(\sigma, \mu_x(\text{John})) ] \)

The proposed analysis of the comparative form of adjectives can be extended straightforwardly to their positive form. (96) is true iff there is a rising segment on the scale of heights that starts with a contextually salient ‘neutral’ height, and that ends with John’s height. Following Cresswell (1976), I assume that the contextual standard of comparison is provided by a covert feature POS:

\( \text{POS}[^{ec}] = \lambda x. \lambda \sigma. \text{START}(\sigma, \nu(\mu_x, c)) \land \text{END}(\sigma, \mu_x(x)) \)

Finally, let us discuss differential comparison, since differential modifiers will have a role to play in the analysis of additivity proposed in subsection 4.2. Consider example (100). In a pointilist framework, one could say that this sentence is true if and only if Mary’s height is greater than John’s height and the difference between Mary’s height and John’s height is two inches:

\( \text{deP \ N} \)
Mary is two inches taller than John.

In scale segment semantics, this sentence asserts the existence of a rising segment of the scale of heights starting with John’s height and ending with Mary’s height, such that the difference between the extremities of the segment is two inches. In order to generate these truth-conditions, we introduce a new feature \textit{DIFF}, which relates a scale segment to the difference between its extremities \(\nu[\text{END}(\sigma, v)] \) and \(u[\text{START}(\sigma, u)]\). I will abbreviate this difference as \(\lambda(\sigma)\). The type of the unit name \textit{inch} is lifted to take a relation between numbers and scale segments as an argument:

\[
\begin{align*}
(100) \quad \text{DIFF}^\text{\oe} &= \lambda n. \lambda \sigma. \nu[\text{END}(\sigma, v)] - u[\text{START}(\sigma, u)] \geq n \\
(101) \quad \text{DIFF}^\text{\oe} &= \lambda n. \lambda \sigma. \Delta(\sigma) \geq n \\
(102) \quad \text{inch}^\text{\oe} &= \lambda n. \lambda \sigma. \Sigma(n \text{\oe} in,) (\sigma) \\
(103) \quad [2 \text{ inch DIFF}^\text{\oe} &= \lambda \sigma. \Delta(\sigma) \geq 2 \text{ in,}_r
\end{align*}
\]

I assume that differential phrases are adjoined to degree phrases, with which they combine intersexpctively:

\[
(104) \quad [\text{DegP} \ [ \ [2 \text{ inch}] \text{ DIFF } ] \ [\text{Deg} \ \backslash \ \text{TALL} \ \text{RISE } ] ]
\]

\[
(105) \quad [\langle (104) \rangle^\text{\oe} &= \lambda \sigma. \rightarrow (\sigma) \land \mu_{r} = \text{HT} \land \Delta(\sigma) \geq 2 \text{ in,}_r
\]

In sum, a gradable adjective introduces information about the nature of a scale, encoded as a measure function. Information about the extremities of the scale segment is introduced by the features END and START, while RISE conveys that the scale segment is rising.

### 4.1.2 Amount comparison

Schwarzschild (2012, 2013) does not discuss amount and adverbial comparatives. Yet, it is essential to discuss these constructions, since their analysis will provide the background against which I will analyze additive operators.

In this section, I will implement Hackl’s (2001) analysis of amount comparison in scale segment semantics, and I will extend this analysis to adverbial comparatives. I will make use of a mereological analysis of plurality (see Link 1983; Landman 1989 among others). The domains of individuals \(D_e\) and of events \(D_e\) are structed as join-semilattices, with the mereological sum operation \(\oplus\) as a join. Such a join semi-lattice \(L\) is partially ordered by a part-of relation \(\subseteq\). The supremum of \(L\) is the greatest element of \(L\) with respect to \(\subseteq\). I assume that count nouns take their denotation in a subset of \(D_e\) that is structured as an atomic semi-lattice. For any \(x \in D_e\), \(AT(x)\) is true iff \(x\) is an atomic individual. For any set \(P, \chi P\) is the closure of \(P\) under sum formation. The present proposal does not depend substantially on this analysis of plurality and the denotation of count nouns, which I only adopt for convenience.

Let us begin with amount comparatives. (106) is true iff the number of men who died is greater than the number of women who died:

\[
(106) \quad \text{More men than women died.}
\]

One can express these truth-conditions using quantification over scale segments by adopting a scale of cardinalities. The measure function associated with this scale maps a mereological individual to the number of its atomic parts. The scale is composed of the set of non-negative integers ordered by the greater-than relation. Abstracting away from events for a moment, we say that (106) is true iff there is a rising segment of the scale of cardinalities that starts with the number of individual women who died and that ends with the number of individual men who died:

\[
(107) \quad \Pi(106)\text{ }^\text{\oe} = 1 \iff \exists \sigma \left[ \lambda x, \lambda y. \lt{ AT \wedge \text{START}(\sigma, \mu_{r}(\bigoplus \{ x : \text{woman}(x) \} \cap \{ x : \exists e[\text{die}(e, x)] \})) \land \text{END}(\sigma, \mu_{r}(\bigoplus \{ x : \text{man}(x) \} \cap \{ x : \exists e[\text{die}(e, x)] \})) } \right]
\]

In (107), the measure function \(\mu_{r}\) of the scale \(\sigma\) is \(\|\text{AT}\), which maps an individual to the number of its atomic parts:

\[
(108) \quad \|\text{AT}\ = \lambda x. [ y : y \subseteq x \land \text{AT}(y)]
\]

Now, we need to work our way towards a compositional derivation of the truth-conditions of (106). In this derivation, I will assume that \textit{die} denotes a relation between events and individuals, so that the truth-conditions we will obtain will end up being slightly more complex than those in (107). I analyze the syntactic structure of example (106) as follows:

\[
(107) \quad \Pi(106)\text{ }^\text{\oe} = 1 \iff \exists \sigma \left[ \lambda x, \lambda y. \lt{ AT \wedge \text{START}(\sigma, \mu_{r}(\bigoplus \{ x : \text{woman}(x) \} \cap \{ x : \exists e[\text{die}(e, x)] \})) \land \text{END}(\sigma, \mu_{r}(\bigoplus \{ x : \text{man}(x) \} \cap \{ x : \exists e[\text{die}(e, x)] \})) } \right]
\]

In (107), the measure function \(\mu_{r}\) of the scale \(\sigma\) is \(\|\text{AT}\), which maps an individual to the number of its atomic parts:

\[
(108) \quad \|\text{AT}\ = \lambda x. [ y : y \subseteq x \land \text{AT}(y)]
\]
In this structure, Hackl’s (2001) denotation for many has been decomposed into the two features AMT\(_Q\) and COUNT. AMT\(_Q\) denotes a function that combines with two sets of individuals, a thematic relation like \([\text{END}]^{g,c}\) or \([\text{START}]^{g,c}\) and a variable over scale segments. It forms the intersection of the sets and feeds its mereological fusion to the thematic relation. As a consequence, it is entailed that the intersection of the two sets is not empty (which is the function of AMT\(_Q\) as an existential quantifier) and one of the extremities of the scale segment is identified with the measurement of this intersection. The cardinality measure function COUNT has been factored out of the denotation of many, following recent proposals by Rett (2008) and Solt (2015).

The internal structure of the than-phrase, which is not represented in (109), is given in (110). I assume a clausal analysis of the standard of comparison. The P head START selects a clause (here ignoring the extended projection of VP for clarity) whose predicate is deleted by identity with the main clause predicate. A variable of the same type as START is generated as a sister to AMT\(_Q\) and bound by a lambda abstractor below START. This step is not very elegant but it will allow us to keep a uniform type for AMT\(_Q\) in the main clause and in the than-phrase.

Let us now see how (109) and (110) are interpreted. We will begin with (110). AMT\(_Q\) combines with the variable \(\Sigma_1\) of type \((e, (l, t))\), the type of names of thematic roles for predicates of scale segments:

\[
[\text{AMT}_Q]^{g,c} = \lambda \Sigma.\lambda P.\lambda Q.\lambda \sigma. (\prod (\{ x : P(x) \} \cap \{ x : \exists e [Q(e)(x)] \}) (\sigma))
\]

\(\text{AMT}_Q\) first combines with \(\Sigma_1\), which is a temporary placeholder for the feature START:
(112)  \[ \mathbb{Q}^{I^c} = \lambda P. \lambda Q. \lambda r. g(1)(\bigoplus \{(x : P(x)) \cap \{x : \exists e[Q(e)(x)]\})(\sigma) \]

The resulting constituent then combines successively with its NP and V arguments. This produces a property of scale segments that are related by g(1) (the denotation of \( \Sigma_1 \)) to the set of women who died:

(113)  \[ \mathbb{VP}^{I^c} = \lambda r. g(1)(\bigoplus \{(x : \text{woman}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})(\sigma) \]

Finally, we abstract over the variable \( \Sigma_1 \) and apply the resulting function to the denotation of \texttt{START}. The result is a property of scale segments that have as a starting point the measure of the set of women who died, where the relevant measure function \( \mu_r \) is still unspecified:

(114)  \[ \mathbb{PP}^{I^c} = \lambda r. \text{START}(r, \mu_r(\bigoplus \{(x : \text{woman}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \]

We now move to the interpretation of the main clause in (109). The PP is combined with \texttt{RISE}, and the resulting constituent is combined with the measure function \texttt{COUNT}. This yields a \texttt{DegP} that denotes a property of scale segments that have the cardinality of the set of women who died as their starting point:

(115)  \[ \mathbb{DegP}^{I^c} = \lambda x. \lambda \sigma. \lambda r. \lambda \mu_r = |_{\text{AT}} \wedge \text{START}(\sigma, \mu_r(\bigoplus \{(x : \text{woman}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \]

The endpoint of the scale segment is introduced by the feature \texttt{END}, which combines with the \texttt{DegP} by the rule of segment identification:

(116)  \[ \mathbb{degP}^{I^c} = \lambda x. \lambda \sigma. \lambda r. \lambda \mu_r = |_{\text{AT}} \wedge \text{END}(\sigma, \mu_r(\bigoplus \{(x : \text{woman}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \]

\texttt{AMT}_Q^{I^c} then feeds the intersection of the NP and verb denotations to this function, which specifies the endpoint of the scale segments:

(117)  \[ \mathbb{Q}^{\text{VP}}^{I^c} = \lambda P. \lambda Q. \lambda r. \lambda \mu_r = |_{\text{AT}} \wedge \text{END}(\sigma, \mu_r(\bigoplus \{(x : P(x)) \cap \{x : \exists e[Q(e)(x)]\})) \]

(118)  \[ \mathbb{VP}^{\text{VP}}^{I^c} = \lambda r. \lambda \mu_r = |_{\text{AT}} \wedge \text{END}(\sigma, \mu_r(\bigoplus \{(x : \text{man}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \]

After existential closure, we obtain a proposition that is true if and only there exists a rising segment on a scale of cardinalities that starts with the number of women who died and ends with the number of men who died:

(119)  \[ \exists e_{\text{VP}} \mathbb{VP}^{I^c} = \exists e \left[ \begin{array}{l} (\sigma) \wedge \mu_r = |_{\text{AT}} \\
\text{START}(\sigma, \mu_r(\bigoplus \{(x : \text{woman}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \\
\wedge \text{END}(\sigma, \mu_r(\bigoplus \{(x : \text{man}(x)) \cap \{x : \exists e[\text{die}(e, x)]\})) \end{array} \right] \]

4.1.3 Adverbial comparison

Adverbial comparison is analyzed in essentially the same way. On one of its interpretations, sentence (120) is true iff the total duration of the rain on the day of utterance is greater than the total duration of the rain on the preceding day:

(120)  It rained more today than it did yesterday.

In a scale segment analysis, sentence (120) is true under this reading\footnote{We could also interpret this sentence as a comparison of volume of rain, rather than duration of raining events.} iff there are events of raining today and yesterday and there is a rising segment on a scale of durations, that starts with the duration of the sum of all events of raining on the day that precedes the day of utterance, and that ends with the duration of the sum of all events of raining on the day of utterance. The measure function \( \delta \) maps events to their duration. The truth conditions of sentence (120) are stated formally in (121), where the function \( \tau \) maps events to their runtime (aka temporal trace):

(121)  \[ \mathbb{I}^{\text{(120)}}^{I^c} = 1 \text{ iff } \exists e \left[ \begin{array}{l} (\sigma) \wedge \mu_r = \delta \\
\text{START}(\sigma, \mu_r(\bigoplus \{(e : \text{rain}(e) \wedge \tau(e) \subseteq \text{yesterday}_r)\})) \\
\wedge \text{END}(\sigma, \mu_r(\bigoplus \{(e : \text{rain}(e) \wedge \tau(e) \subseteq \text{today}_r)\})) \end{array} \right] \]
Note that it is important to ensure that the inputs of the duration measure functions be maximal events of raining on each day, since otherwise the sentence would be trivially true if it had rained on each day, independently of the duration of the rain, by virtue of the subinterval property of the predicate of events. Indeed, if there is an event of raining for some duration $d$ today and time is continuous, then there is an infinity of events of raining of any duration $d'$ less than $d$ on that day. We can then trivially find an event of raining yesterday that was shorter than the duration of some event of raining today.

Let us now derive the truth-conditions of this sentence, starting with the than-phrase:

\[
(122) \quad \text{PP}
\]

\[
\text{START} \quad \text{VP}_2
\]

\[
\Sigma_1 \quad \text{AP} \quad \text{VP}_1
\]

\[
\\text{AMT}_A \quad \sqrt{\text{RAIN}} \quad \sqrt{\text{YESTERDAY}}
\]

The adverbial head $\text{AMT}_A$ relates the VP to a thematic relation name. $\text{VP}_1$ denotes a property of events that took place on the day that precedes the day of utterance. $\llbracket \text{AMT}_A \rrbracket^{e,c}$ applies the thematic relation $\llbracket \text{START} \rrbracket^{e,c}$ to this denotation, which yields a property of scale segments that start with the measurement of the set of events of raining yesterday:\(^{18}\)

\[
(123) \quad \llbracket \text{AMT}_A \rrbracket^{e,c} = \lambda \Sigma. \lambda P. \lambda \sigma. \Sigma(\bigoplus \{e : P(e)\})(\sigma)
\]

\[
(124) \quad \llbracket \text{VP}_1 \rrbracket^{e,c} = \lambda e. \text{rain}(e) \land \tau(e) \subseteq \text{yesterday}_c
\]

\[
(125) \quad \llbracket \text{VP}_2 \rrbracket^{e,c} = \lambda \Sigma. \lambda \sigma. \Sigma(\mu_{\sigma}(\bigoplus \{e : \text{rain}(e) \land \tau(e) \subseteq \text{yesterday}_c\}))(\sigma)
\]

\[
(126) \quad \llbracket \text{PP} \rrbracket^{e,c} = \lambda \sigma. \text{START}(\sigma, \mu_{\sigma}(\bigoplus \{e : \text{rain}(e) \land \tau(e) \subseteq \text{yesterday}_c\})))
\]

The syntactic structure of the main clause is given in (127):

\[
(127)
\]

\[
\text{VP}_1
\]

\[
\text{AP}
\]

\[
\text{degP}
\]

\[
\text{VP}_2
\]

\[
\text{AP}
\]

\[
\sqrt{\text{RAIN}}
\]

\[
\sqrt{\text{TODAY}}
\]

\[
\text{TIME}
\]

\[
\text{RISE}
\]

The derivation of the truth-conditions does not differ significantly from that of amount comparatives, except for the fact that adverbial $\text{AMT}_A$ is not interpreted as a quantifier but as a VP modifier:

\[
(128) \quad \llbracket \text{TIME} \rrbracket^{e,c} = \lambda \sigma. \mu_{\sigma} = \delta
\]

\[
(129) \quad \llbracket \text{AP} \rrbracket^{e,c} = \lambda P. \lambda \sigma. \lambda \tau_{\sigma}. (\sigma) \land \mu_{\sigma} = \delta \land \text{START}(\sigma, \mu_{\sigma}(\bigoplus \{e : \text{rain}(e) \land \tau(e) \subseteq \text{yesterday}_c\})))
\]

\[
\land \text{END}(\sigma, \mu_{\sigma}(\bigoplus \{e : P(e)\}))
\]

\(^{18}\)I assume default existential closure of the event argument of the verb (type $v$) at the VP level, which maps the VP denotation of type $(l, (v, t))$ to a property of scale segments of type $(l, t)$.
The bed and the temperature of the water that was spilled on the carpet.

that was spilled on the bed and on the carpet does not equal the sum of the temperature of the water that was spilled on

These truth-conditions are stated formally in (138), where

Firstly, additivity appears to be restricted to extensive measurement. A measure function \( f \) is extensive if and only

and that of another portion of water \( y \) is 30C, the temperature of the portion of water obtained by mixing \( x \) and \( y \) is not 40C. Consequently, the temperature of the water that was spilled on the bed and on the carpet does not equal the sum of the temperature of the water that was spilled on the bed and the temperature of the water that was spilled on the carpet.

Let us now see how the additive interpretation of \textit{more} can be captured in scale segment semantics.

Greenberg identified additional properties of additive operators, which our scale segment analysis must capture. Firstly, additivity appears to be restricted to extensive measurement. A measure function \( f \) is extensive if and only if, given two objects \( x \) and \( y \) that have no parts in common and an appropriate concatenation operation \( \circ \), \( f(x \circ y) = f(x) + f(y) \), see Krantz et al. (1971). A function that maps sets to their cardinality is extensive with respect to the union operation. Likewise, a function that measures the distance of a path in miles is additive, since the length of the concatenation of two paths equals the sum of the length of each path. These are precisely the measure functions that are exploited in examples (131) and (132). Greenberg observed that additive interpretations of \textit{more} are impossible with non-extensive measurement, as illustrated by the contrast between (135) and (136):

\[ (135) \text{3 Liters of water was spilled on the carpet. 2 liters more was spilled on the bed.} \]

\[ (136) \text{30 degree Celsius water was spilled on the carpet. #10 degree Celsius more was spilled on the bed.} \]

4.2 Additivity in scale segment semantics

Background Before we try to analyze additive sentences in scale segment semantics, let us explore the semantic properties of these constructions in more general terms. Greenberg (2012) observes that additive \textit{more} can be used in arguments as well as adjuncts of verbs, as illustrated in (131) and (132) respectively:

\[ (131) \text{Yesterday John spoke with 3 students. Today he spoke with more (students). (Greenberg 2012)} \]

\[ (132) \text{John ran 2 miles in the morning. In the afternoon he ran some more. (Greenberg 2012)} \]

As we observed in section 2, additivity is presuppositional. To illustrate, (131) asserts that John spoke with some students on the day of utterance, and presupposes that he spoke with other students before that time. Greenberg observes that the presupposition is projected outside the scope of negation and questions. Examples (133) and (134) both convey that John had spoken with some students before the day of utterance:

\[ (133) \text{Did John speak with more students today? (Greenberg 2012)} \]

\[ (134) \text{It is not true that John spoke with more students today. (Greenberg 2012)} \]

\[ (135) \text{3 Liters of water was spilled on the carpet. 2 liters more was spilled on the bed.} \]

\[ (136) \text{30 degree Celsius water was spilled on the carpet. #10 degree Celsius more was spilled on the bed.} \]

\[ \text{Measurement of volume is extensive: given two objects } x \text{ and } y \text{ and an appropriate concatenation operation } \circ, \text{ the volume of } x \circ y \text{ is the sum of the volume of } x \text{ and that of } y. \text{ By contrast, measurement of temperature in Celcius is non-extensive. If the temperature of a portion of water } x \text{ is 10C and that of another portion of water } y \text{ is 30C, the temperature of the portion of water obtained by mixing } x \text{ and } y \text{ is not 40C. Consequently, the temperature of the water that was spilled on the bed and on the carpet does not equal the sum of the temperature of the water that was spilled on the bed and the temperature of the water that was spilled on the carpet.} \]

\text{Let us now see how the additive interpretation of more can be captured in scale segment semantics.}

\textbf{Truth-conditions} In its additive interpretation, the second sentence of example (137) only entails that three children danced. Furthermore, it presupposes that some other children engaged in an activity related to the singing:

\[ (137) \text{(4 children danced.) 3 more children sang.} \]

\[ (138) \exists \sigma \{ \exists \tau (\langle \sigma \rangle \wedge \mu_\sigma = \delta \wedge \exists \epsilon (\exists \tau \wedge \mu_\epsilon (\oplus \{ (e : \text{rain}(e) \wedge \tau(e) \subseteq \text{yesterday}_\epsilon, (\tau_\epsilon)))))(\text{END}(\sigma, \mu_\epsilon (\oplus \{ (e : \text{rain}(e) \wedge \tau(e) \subseteq \text{today}_\epsilon, (\tau_\epsilon)))))) \]

\text{One way to capture this interpretation is to assume that this sentence is true iff there is a rising segment of the scale of cardinalities, that starts with the cardinality of some salient plurality of children retrieved by anaphora (in this case, the plurality of children who danced) and that ends with the cardinality of the sum of that plurality with the plurality of children who sang, such that the difference between the starting point and the endpoint of the segment is at least three. These truth-conditions are stated formally in (138), where } g(i) \text{ is a contextually salient (plural or atomic) individual:} \]
Note that these truth-conditions entail that there were at least three children who sang who are not members of the set $g(1)$, i.e. the set that consists of the four children who danced. Otherwise, the difference between the cardinality of $g(1)$ and its union with the set of children who sang would be less than three. This captures the intuition mentioned above that the presupposition of the additive sentence in (137) is about children that are distinct from the three additional children who sang.

Additive interpretations of adverbial *more* are analyzed in the same fashion. In the following formula, the measure function $\delta$ maps events to their duration:

$$\exists \sigma [\sigma \cap \mu_r = \delta \land \text{START} (\sigma, \mu_r (g(1))) \land \text{END} (\sigma, \mu_r (\bigoplus \{ e : \text{rain}(e) \land \tau(e) \subseteq \text{today}_e \} ) \ominus g(1))) \land \Delta (\sigma) \geq 1 \text{hr}_e]$$

### Compositionality

How can we derive the truth-conditions of additive sentences compositionally? Consider sentence (140) again, interpreted as a bare comparative in (140a) and interpreted additively in (140b). These two interpretations differ in the argument structure of the scale segment predication. In the comparative interpretation, the endpoint of the rising scale segment is the cardinality of the intersection of the NP denotation with the VP denotation. In the additive interpretation on the other hand, the endpoint is the cardinality of the union of this set with the set whose cardinality is the starting point of the segment:

(140) 3 more children sang.

a. $\exists \sigma [\sigma \cap \mu_r = |\cdot|_{AT} \land \text{START} (\sigma, \mu_r (g(1))) \land \text{END} (\sigma, \mu_r (\bigoplus \{ x : \exists e [\text{child}(x) \land \text{sing}(e, x)) \} ) ) \land \Delta (\sigma) \geq 3 ]$

b. $\exists \sigma [\sigma \cap \mu_r = |\cdot|_{AT} \land \text{START} (\sigma, \mu_r (g(1))) \land \text{END} (\sigma, \mu_r (\bigoplus \{ x : \exists e [\text{child}(x) \land \text{sing}(e, x)) \} ) \ominus g(1))) \land \Delta (\sigma) \geq 3 ]$

I propose to analyze this difference as an argument structure alternation triggered by an extra functional head ADD, in the extended projection of DegP:

(141) $\llbracket \text{ADD}_i \rrbracket \llbracket e \rrbracket = \lambda \Sigma . \lambda \Sigma'. \lambda x. \lambda \sigma . \Sigma (\sigma)(g(1)) \land \Sigma' (\sigma)(x \ominus \Theta g(1))$

(142)

Note that the location of ADD, in this syntactic structure is motivated by its semantic type. According to its denotation in (141), ADD, combines with two relations between scale segments and individuals. One of these relations is denoted by DegP and relates the scale segment to its starting point, and the other is denoted by the feature END.
ADD, does double duty. Firstly, its index is interpreted as a pro-form that makes anaphoric reference to a (plural) individual mentioned in a preceding utterance. Secondly, ADD, modifies the argument structure of the scale segment predication in a fashion characteristic of additivity. The interpretation of LF (142) proceeds as follows:

(143) \[ \text{ADD}_1 \\
\]

(144) \[ \text{DegP}^e.c = \lambda x, \lambda \sigma. \forall (\sigma) \land \mu_\sigma = |_{\lambda T} \land \text{START}(\sigma, \mu_\sigma(x)) \]

(145) \[ \text{degP'}^e.c = \lambda x, \lambda \sigma. \forall (\sigma) \land \mu_\sigma = |_\lambda T \land \text{START}(\sigma, \mu_\sigma(g(1))) \land \text{END}(\sigma, \mu_\sigma(x \oplus g(1))) \]

(146) \[ \text{AP}_2^c \]

(147) \[ \text{degP}_2^c = \lambda x, \lambda \sigma. \forall (\sigma) \land \mu_\sigma = |_\lambda T \land \text{START}(\sigma, \mu_\sigma(g(1))) \land \text{END}(\sigma, \mu_\sigma(x \oplus g(1))) \land \Delta(\sigma) \geq 3 \]

(148) \[ \text{AMT}_Q^c = \lambda \Sigma. \text{AP}. \lambda Q. \lambda \Sigma. (x : P(x)) \cap \{ x : \exists e[Q(e)(x)] \} (\sigma) \]

(149) \[ \exists x. \text{VP}^e.c = \exists x [ \forall (\sigma) \land \mu_\sigma = |_{\lambda T} \land \text{START}(\sigma, \mu_\sigma(g(1))) \land \text{END}(\sigma, \mu_\sigma(x \oplus g(1))) \land \Delta(\sigma) \geq 3 \]

The structure of sentences with adverbial additive more does not differ significantly from that of adverbial comparatives:

(150) (It rained for two hours yesterday.) It rained for one hour more today.

(151)

\[
\begin{array}{c}
\text{VP}_4 \\
\text{AP}_3 \\
\text{degP}_2 \\
\text{AP}_2 \\
\text{degP}_1 \\
\text{AP}_1 \\
\text{DegP}
\end{array}
\]

\[
\begin{array}{c}
\text{AMT}_d \\
\text{NP} \\
\text{HOUR} \\
\text{DIFF} \\
\text{END} \\
\text{TIME} \\
\text{RISE} \\
\text{START}
\end{array}
\]

\[
\text{VP}_3 \\
\text{V} \\
\text{\sqrt{\text{RAIN}}} \\
\text{\sqrt{\text{TODAY}}}
\]

Absence of additive interpretations with overt standards of comparison  In section 2.1, it was observed that additive interpretations of more are unattested when an overt standard of comparison is used. This restriction is also attested in other languages discussed in section 2. If one takes seriously Schwarczchild’s proposal that END and START are thematic relations for scale segments, the blocking of additive interpretations with overt standards of comparison can be explained by a constraint against the introduction of the same thematic role at two different points of the syntactic structure of a predication, which Carlson called ‘thematic uniqueness’:

(153) \ldots no verbs seems to be able to assign the same thematic role to two or more of its arguments. \quad (\text{Carlson} \ 1984)

In additive sentences, the thematic role START is introduced by the feature ADD, Using a standard of comparison, whose head than spells out the feature START in English, would violate thematic uniqueness, since START would be introduced at two different points of the same predication. This leaves us with the question of how thematic uniqueness should be implemented in the grammar. I will leave this question open. See Williams (2015) chapter 8 for a discussion of possible implementations.
Restriction to extensive measure functions  I have proposed that additive operators form a QP or AP headed by an AMT feature, which corresponds to the parametrized quantifiers much and many of Hackl (2001). Interestingly, the restriction of additive operators to extensive measurement can be argued to follow from this syntactic structure.

Many and much belong to a class of adjectives that Bresnan (1973) called Q-adjectives, along with little and few. Schwarzschild (2006) observed that Q-adjectives and their comparative forms can only express dimensions of measurement that are monotonic on the part-whole structure of the nominal referent:

(154) When a QP is combined with a substance noun, the interpretation is one in which the dimension is monotonic on the relevant part-whole relation in the domain given by the noun. (Schwarzschild 2006)

In other words, the measurement expressed by these QPs must be extensive. Consider for instance example (155). This sentence can convey that the gold in the ring is in excess of weight or volume, which are extensive properties, but it cannot convey that the gold in the ring is in excess of purity or color, which are intensive properties:

(155) He put too much gold in the ring. (Schwarzschild 2006)

Wellwood (2015) observed a similar restriction in the verbal domain: the interpretation of adverbials modifiers headed by much and more make use of extensive measure functions. (156a) for instance cannot mean that Al ran faster than Bill, but it can mean that Al ran for a longer time than Bill, or that he ran a greater distance than Bill:

(156) a. Al ran more than Bill did. (Wellwood 2015)
   b. Al ran as much as Bill did. (Wellwood 2015)

Schwarzschild (2006) argues that this restriction on the interpretation of QPs in the nominal domains is driven by syntax: QPs are introduced as specifiers of a MonP projection, whose head Mon requires an extensive interpretation of the QPs. Be it as it may, it is important to note that the lack of non-extensive measurement is not specific to additive operators, but affects all QPs and APs headed by Q-adjectives and adverbs, including comparative, equative and excessive phrases. As such, it would be undesirable to encode a restriction to extensive measurement in the denotation of features that are specific to additive operators, like ADD.

4.3 Continuation in scale segment semantics

Background  It has often been observed that sill and noch have multiple uses that are related by transposing an operation from a semantic domain to another one (König 1971; Löhner 1989; Ippolito 2007). The following classification is due to Ippolito (2007):

(157) It is still raining. (Aspectual)
(158) Compact cars are still safe. Subcompacts are already dangerous. (Marginal)
(159) John studied all night, and he still failed the test. (Concessive)
(160) It’s still 8 am. (Exclusive)

Still also has a spatial interpretation, which is discussed by Löhner (1989):

(161) Metz is still in France. Saarbrücken is already in Germany. (Spatial)

In this paper, I will put aside the marginal, concessive and exclusive interpretations of still. Although I will only discuss continuation in English, it is hoped that the proposed analysis can be extended to German noch and to continuative operators in other languages without major revisions.

It is generally agreed since Löhner’s (1989) seminal work that the spatial and marginal interpretations of still and noch are derived from their aspectual interpretation. Consequently, I will only discuss this interpretation of still. Aspectual still combines with a stative or imperfective predicate and triggers a presupposition that the very same state that is asserted to hold at the reference time (RT) held at a previous time, as illustrated in (162). Ippolito (2007) shows that still does not presuppose that this state abuts RT, for otherwise (163) would suffer from presupposition failure. Indeed, if the conditional sentence projected the presupposition that the state of John being alive lasted until the time of utterance, this presupposition would be inconsistent with the previous assertion:
(162) John is still alive.
   a. Assertion: a contextually salient state \( s \) of John being alive holds at \( t_e \).
   b. Presupposition: \( s \) held at \( t \), for some time \( t < t_e \).

(163) John\(_j\) died a year ago. If he\(_j\) were still alive (now), he would be a hundred years old.

The lexical entry that Ippolito proposes for still is given in (164). While Ippolito treats presuppositions as conditions on the domain of functions, I assume an analysis of presuppositions in trivalent logic, using the partiality operator \( \partial \) of Beaver & Krahmer (2001).\(^19\)

\[
\text{(164)} \quad \llbracket\text{still}\rrbracket^{e,c} = At.e \forall P. P(e)(t) \land \partial(\exists t' < t \land P(e(t')))
\]

Ippolito argues that the event argument of still is not existentially bound but is rather a free variable whose value is a contextually salient eventuality. Accordingly, we can formulate the truth-conditions of sentence (165) as follows, where \( g(1) \) is a contextually salient event of raining and \( t_e \) is the time of utterance:

(165) It is still raining.

(166) \( \llbracket(165)\rrbracket^{e,c} = 1 \text{ iff } \llbracket\text{rain}\rrbracket(g(1)) \land t_e \subseteq \tau(g(1)) \land \partial(\exists t' < t_e \land \llbracket\text{rain}\rrbracket(g(1)) \land t' \subseteq \tau(g(1))) \]

**Truth-conditions** My goal is to decompose the continuative operator still in such a way that it will include the set of features that make up additive operators. As a consequence, I will have to depart from Ippolito’s (2007) analysis of aspectual continuation. At the same time, I would like to preserve the main insights of Ippolito’s analysis.

How can we factor a form of additivity into the truth conditions in (166)? What we want to preserve is the intuition that sentence (165) asserts the existence of an event of raining \( e_2 \) whose temporal trace includes the time of utterance, and presupposes that this event of raining is the continuation of a salient event of raining \( e_1 \) that occurred in the past of the time of utterance. This presupposition can be formulated as an additive statement, if we think of \( e_1 \) and \( e_2 \) as parts of an event of raining \( e \), such that \( e_1 \) is a less developed part of \( e \) than \( e_1 \oplus e_2 \). The relation ‘less developed part of \( e \) than’ is represented as ‘\( \prec_e \)’ and is defined as follows:

(167) For any eventualities\(^20\) \( e, e' \) and \( e'' \), \( e' \prec_e e'' \) iff:

1. \( e' \) and \( e'' \) are initial parts of \( e \)
2. \( \tau(e') \subseteq \tau(e'') \)

Given any eventuality \( e \), the set of initial parts of \( e \) is totally ordered by the relation \( \prec_e \). Consequently, we can define a function \( \text{DEV}_e \) that measures the development of initial parts of \( e \) as follows:

(168) For any eventualities \( e \), let \( S_e \) be the set of initial parts of \( e \). A measure of development of initial parts of \( e \) is a function \( \text{DEV}_e \) such that:

1. \( \text{DEV}_e \) is a function from \( S_e \) to \( [0, 1] \) and,
2. for any two eventualities \( e \) and \( e' \) in \( S_e \), \( \text{DEV}_e(e) \prec \text{DEV}_e(e') \) if and only if \( e \prec_e e' \).\(^23\)

Using these notions, we can to factor additivity into the truth-conditions of sentence (165) as follows, where \( \text{INIT}(e, e') \) means that \( e \) is an initial part of \( e' \):

\[
\exists e \llbracket\text{rain}(e) \land t_e \subseteq \tau(e) \land \partial(\exists \sigma | \exists \mu \in [0, 1] \land \mu = \text{DEV}_e(e) \land \text{START}(\sigma, \mu, \text{rain}(e)) \land \text{END}(\sigma, \mu, \text{rain}(e) \oplus e)\rrbracket
\]

\(^19\)Following Bach (1986), I use ‘eventuality’ to refer to states and events.

\(^20\)That is to say, \( e' \) and \( e'' \) are parts of \( e \), and \( e' \) and \( e'' \) have the same starting time as \( e \).

\(^22\)\( e' \) is temporally included in \( e'' \).

\(^23\)I assume that the order \( \prec_e \) on \( S_e \) is dense.
This formula states that there is an event $e$ of raining that includes the time of utterance, and presupposes that $e$ is a non-initial part of some event of raining $\epsilon$, such that the salient event $g(1)$ is a less developed initial part of $\epsilon$ than the sum of $e$ and $g(1)$. The requirement that $e$ is not an initial part of $\epsilon$ guarantees that the onset of $g(1)$ precedes that of $e$. (169) correctly entails that $g(1)$ starts before $e$, and that both $g(1)$ and $e$ are parts of a single event of raining that overlaps the time of utterance and stretches back into the past.

This analysis accounts for the conditions of use of still identified by Ippolito. Sentence (165) is correctly predicted to be infelicitous if there is no salient event of raining in the past of the time of utterance. Since neither the salient event $g(1)$ nor the sum $g(1) \oplus e$ is presupposed to overlap with the time of utterance, the analysis accounts for the felicity of discourse (170), repeated from example (163):

(170) John, died a year ago. If he were still alive (now), he would be a hundred years old.

Finally, if we assume that the perfective aspect creates properties of events that are maximal events in the extension of the VP (see Koenig & Muansuwan 2000; Filip 2008; Altshuler 2014), we predict that continuation is incompatible with the perfective. Indeed, an event cannot be both a maximal event of raining, and a part of a larger event of raining.

Note that the notion of ‘less developed part of an eventuality’ that I used in this section was inspired by Landman’s (1992) notion of ‘stages’ of events:

(171) An event is a stage of another event if the second can be regarded a more developed version of the first, that is, if we can point at it and say “It’s the same event in a further stage of development.” Landman (1992).

However, Rothstein (2004) has argued that only dynamic eventualities have stages. Since continuative particles may modify stative predicates, we cannot analyze their denotation in terms of the notion of stage thus interpreted. We could of course assume that states have stages after all, pace Rothstein (2004), and define the function $\text{DEV}_{\epsilon}$ as a mapping from the set of initial stages of $\epsilon$ to $[0, 1]$. I decided not to do so to avoid confusion. Note also that Greenberg (2012) used Landman’s notion of stage in her analysis of additive particles. See section 6.2.1 for a discussion of Greenberg’s analysis.

**Compositionality** Continuation is derived from additivity by adding a feature CON in the extended projection of the RISE feature:

(172)

$\begin{array}{l}
TP \\
\quad \downarrow \text{PRES} \\
\quad \exists_e \quad \text{AspP} \\
\quad \downarrow \text{AP} \\
\quad \quad \text{degP}_2 \\
\quad \quad \quad \text{deg}_2 \quad \downarrow \text{END} \\
\quad \quad \quad \text{deg}_1 \quad \downarrow \text{degP} \\
\quad \quad \quad \quad \quad \quad \quad \quad \text{ADD}_1 \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \text{Deg} \quad \downarrow \text{PP} \\
\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \text{RISE} \quad \downarrow \text{START} \\
\end{array}$

$[\text{CON}]^{\epsilon,c}$ combines with the relation between segments and events denoted by its degP sister, and with the relations between events and times denoted by the intermediate projection of the aspect head:

(173) $[\text{CON}]^{\epsilon,c} = \lambda \Sigma. \lambda R_{(\forall \sigma)} \lambda e^\epsilon. \lambda \text{At}_t. R(e)(t) \land \partial (\exists \sigma \exists e \exists \sigma'. [R(e)(t') \land \neg \text{INIT}(e, \epsilon) \land \mu_\sigma = \text{STAGE}_\epsilon \land \Sigma(e)(\sigma)'].$
Note that the quantification over the time variable \( t' \) was not present in the formula in (169). We will see that the final truth-conditions can be simplified by eliminating this variable.

CON combines with degP2, which denotes a relation between entities \( e \) and rising scale segments \( \sigma \), with the measurement of \( g(1) \) as a starting point and that of the sum of \( e \) and \( g(1) \) as an endpoint. Note that the denotation of \( \text{degP2} \) constrains neither the type of \( e \) nor the nature of the measure function \( \mu_e \). Both are specified by the denotation of CON:

\[
\text{degP2}^{e,c} = \lambda e.\lambda \sigma. \overrightarrow{R}((\sigma) \land \text{START}(\sigma, \mu_e(g(1)))) \land \text{END}(\sigma, \mu_e(e \oplus g(1)))
\]

The resulting AP denotes a modifier of relations \( R \) between events and times, which combines with the imperfective AspP:

\[
\text{AP}^{e,c} = \lambda R.\lambda e.\lambda t.R(e)(t) \land \partial(\exists \sigma \exists \lambda \exists e)\left[ R(e)(t') \land \neg \text{INIT}(e, \lambda) \land \overrightarrow{R}((\lambda) \land \mu_e = \text{STAGE}_e) \land \text{END}(\lambda, \lambda, (\sigma, g(1)) \oplus e) \right]
\]

\[
\text{AspP}^{e,c} = \lambda e.\lambda t. \text{rain}(e) \land t \subseteq \tau(e)
\]

In the final truth-conditions, the presupposition states that some interval \( t \) is included in the runtime of \( e \), but no other constraints is put on \( t \):

\[
\text{TP}^{e,c} = \exists e[\text{rain}(e) \land t \subseteq \tau(e) \land \partial(\exists \sigma \exists e)\left[ \text{rain}(e) \land t \subseteq \tau(e) \land \neg \text{INIT}(e, \lambda) \land \overrightarrow{R}((\lambda) \land \mu_e = \text{STAGE}_e) \land \text{END}(\lambda, (\sigma, g(1)) \oplus e) \right]]
\]

Since the runtime of any event includes some temporal interval, the statement that \( \exists t(t \subseteq \tau(e)) \) is trivially true, and we can simplify the truth-conditions as follows:

\[
\text{TP}^{e,c} = \exists e[\text{rain}(e) \land t \subseteq \tau(e) \land \partial(\exists \sigma \exists e)\left[ \text{rain}(e) \land \neg \text{INIT}(e, \lambda) \land \overrightarrow{R}((\lambda) \land \mu_e = \text{STAGE}_e) \land \text{END}(\lambda, (\sigma, g(1)) \oplus e) \right]]
\]

This closes our discussion of the semantic decomposition of CAC operators. In this section, I proposed a decomposition of CAC operators into atomic features that are interpreted in scale segment semantics, and I showed how these features fit inside a compositional analysis of comparison, additivity and continuation. In the next section, I show how the features that make up each operator are bundled together by post-syntactic operations, and how the resulting bundles of features are spelled out by rules of Vocabulary Insertion.

## 5 Generating homophony

In the previous section, we have decomposed CAC operators into several features that are introduced at different points of the syntactic structure. Yet, in order to apply the morphological analysis that was sketched in section 3.1, these features must be combined into bundles that will serve as input to rules of vocabulary insertion. In the model of grammar that I have assumed, the structures that were proposed in the previous section correspond to the output of syntactic derivations that have not yet been sent to the level of Phonological Form (PF), and morphological analysis is part of the mapping of these structures to PF. I propose that it is at this post-syntactic stage that the features of CAC operators are reorganized into bundles.

This reorganization of syntactic structure will rely on the operations of Merger and Fusion (Halle & Marantz 1993). Merger joins a head with the head of its complement XP. I assume that Merger can result in upward movement of a head, as well as in downward movement. Consequently, merger can act as a form of head movement at PF (Matushansky 2006; Gribanova & Harizanov 2016):

(179) Merger:

\[ [x \ F_1 \ F_2] \rightarrow [x \ [F_1 \ F_2]] \]

\[ [x \ [F_1 \ F_2]] \rightarrow [x \ F_1 \ F_2] \]

After merger has applied, adjacent heads can be bundled together by the operation of Fusion, which reconfigures a complex head formed of two morphemes into a simpler head with a complex morpheme:

(180) Fusion:

\[ [x \ F_1 \ F_2] \rightarrow [x \ [F_1 \ F_2]] \]
5.1 Spelling out CAC operators in English

I propose that degree heads in the extended projection of RISE undergo head movement at PF. The resulting complex head is then fused into a single feature bundle, which serves as input to a rule of Vocabulary insertion, as illustrated by the following examples:

(181) Comparison:

```
degP
  \_ deg
    \_ END
      \_ ... deg
        \_ deg
          \_ RISE

degP
  \_ deg
    \_ RISE
      \_ END

\rightarrow
```

(182) Additivity:

```
degP
  \_ deg
    \_ END
      \_ ... deg
        \_ ADD
          \_ deg
            \_ RISE
              \_ PP
                \_ START

degP
  \_ deg
    \_ RISE
      \_ ADD

\rightarrow
```

(183) Continuation:

```
degP
  \_ deg
    \_ END
      \_ ... deg
        \_ ADD
          \_ deg
            \_ RISE
              \_ PP
                \_ START

degP
  \_ deg
    \_ RISE
      \_ ADD

\rightarrow
```

The most specific structural description that matches both comparative operators and additive operators only includes the features RISE and END. On the other hand, the Vocabulary Item that spells out continuative operators includes all the features that we introduced in our discussion of CAC operators:

(184) \[ \text{RISE} \text{ END} \leftrightarrow \text{more} \]

(185) \[ \text{RISE} \text{ END} \text{ START} \text{ ADD} \text{ CON} \leftrightarrow \text{still} \]

The reader will have noticed that the feature START only undergoes merger with additive \textit{more} and continuative \textit{still}. In the expression of comparison, START is spelled out \textit{in situ} as the prepositional head \textit{than}. We must therefore add the following Vocabulary Item to our lexicon:

(186) \[ \text{START} \leftrightarrow \text{than} \]

How can we block the merger of START in comparative sentences? In the syntactic structure of additive and continuative operators, START lacks a complement. Indeed, it was argued in section 4 that the starting point of the
scale segment with additive more and still was provided by the index on the feature ADD₁. With comparison on the other hand, the starting point of the scale is denoted by the complement of START. I assume that merger is ruled out in this case, since it would result in the production of a headless prepositional phrase that is filtered out at PF:

(187)  "Rosa is more intelligent Fred is."

We must still account for the morphological realization of CAC operators under negation:

(188)  I did not eat more cookies than Lucie.
(189)  I ate two cookies this morning, and I did not eat any more cookies after this.
(190)  I am not eating cookies anymore.

Still is a positive polarity item (PPI). Although I wish to remain agnostic with respect to theories of positive polarity (see Progovac 1994; Szabolcsi 2004; Homer 2012), I will assume that PPIs bear an uninterpretable feature uPOS, which results in ungrammaticality when the PPI occurs in a negative domain. I propose that continutative or additive anymore and comparative any more are formed by a negative polarity item any and the relevant CAC operator. Since still is a PPI, the only VI that can spell out CAC features in a negative domain is more:

(191)  Vocabulary Items of English:

|   | a. RISE END ↔ more | b. RISE END START ADD CON uPOS ↔ still | c. [START] ↔ than |

5.2 Cross-linguistic variation

Cross-linguistic variation in patterns of CAC homophony is captured at vocabulary insertion. Portuguese-type languages are like English, except insofar as the three operations are clearly homophonous under negation:

(192)  Portuguese type:

|   | a. RISE END ↔ mais | b. RISE END START ADD CON uPOS ↔ ainda | c. RISE END ↔ mais |

In German-type languages, additivity and continuation are homophones outside the scope of negation, and all three notions are homophones under negation:

(193)  German type:

|   | a. RISE END ↔ mehr | b. RISE END START ADD CON uPOS ↔ noch | c. RISE END ↔ mehr |

Finally, we can also account for the existence of languages without homophony, as well as languages where all three operators are homophones:

(194)  Vietnamese type:

|   | a. RISE END ↔ hơn | b. RISE END START ADD CON uPOS ↔ nũa | c. RISE END START ADD CON uPOS ↔ vẫn |
Important, the present analysis predicts that comparison cannot be homophonous with continuation, without also being homophonous with additivity, as we discussed in section 3.

6 Comparison to previous analyses of CAC operators

Comparison, additivity and continuation have received unequal attention in formal semantics. Comparison and continuation have certainly been the most studied operations, while additivity has not been discussed systematically until recently, and is still very much understudied. In this section, I will try to situate the scale segment analysis developed in this paper in the literature on each operation. We will conclude that, while there are many detailed analyses of each notion, no analysis has attempted to explain the homophony between CAC operators. As a consequence, existing analyses of comparison appear to be unrelated to analyses of continuation, and additivity has also been treated as distinct from comparison and continuation.

6.1 Comparison

Analyses of comparison in formal semantics can be split into two families: degree based analyses (Cresswell 1976; von Stechow 1984; Heim 1985; Beck 2011 among others) and delineation analyses (McConnell-Ginet 1973; Klein 1980, 1982; Burnett 2014 among others). The former posit an ontology of degrees and quantification over degrees in the object language. One may for instance argue that the degree adjective tall maps an individual $x$ to the set of degrees $d$ such that $x$ is at least $d$-tall. The truth conditions of sentence (196) could then be formulated as in (196b), which boils down to the statement that John’s height (i.e. the maximal degree $d$ such that John is at least $d$-tall) is greater than Mary’s height. This is essentially Heim’s (2000) analysis:

(196) John is taller than Mary.

a. $\llbracket \text{tall} \rrbracket^{g,c} = \lambda x. \lambda d. \text{HEIGHT}(x) \geq d$

b. $\max(d : \text{HEIGHT}(\text{John}) \geq d)) > \max(d : \text{HEIGHT}(\text{Mary}) \geq d))$

By contrast, delineation analyses do not posit quantification over degrees, but rely instead on the intuition that the extension of a degree predicate like tall is context sensitive (Klein 1980, 1982; Burnett 2017). The following presentation of a delineation semantics of comparison is adapted from Burnett (2014). Adjectives are interpreted simply as functions from individuals to truth-values, whose extension depends on a comparison class $C(c)$ determined in a context $c$:

(197) For any context $c$ and comparison class $C(c)$:

a. $\llbracket \text{tall} \rrbracket^{g,c} \subseteq C(c)$

b. $\llbracket \text{John is tall} \rrbracket^{g,c}(x) = 1$ iff $\text{John} \in \llbracket \text{tall} \rrbracket^{g,c}$

John is tall $\llbracket \text{John is tall} \rrbracket^{g,c}(x) = 0$ iff $\text{John} \in C(c) - \llbracket \text{tall} \rrbracket^{g,c}$

John is tall $\llbracket \text{John is tall} \rrbracket^{g,c}(x)$ is undefined otherwise.

Comparison is analyzed as a form of quantification over comparison classes, as illustrated in the following example, where $c[X]$ is a context that is identical to $c$ except for the fact that the comparison class $X$ is used in place of $C(c)$:

(198) $\llbracket \text{John is taller than Mary} \rrbracket^{g,c} = 1$ iff $\exists X$ such that $\llbracket \text{John is tall} \rrbracket^{g,c[X]} = 1$ and $\llbracket \text{Mary is tall} \rrbracket^{g,c[X]} = 0$.

The analysis of comparatives in scale segment semantics that I have adapted from Schwarzschild (2012, 2013) belongs to the family of degree based analyses, insofar as adjectives are analyzed as properties of scale segments, which are themselves derived from scales of degrees. Aside from this ontological common ground, it is not trivial to tell how scale segment semantics compares to other forms of degree semantics in terms of its potential to solve classical issues in the semantics of comparison, such as restrictions on the scope of quantifiers in comparative clauses.
The most important property of scale segment semantics for the analysis of CAC homophony, and what distinguishes it from other degree based analyses of comparison, is that it allows us to introduce the two terms of a comparative relation through the functional heads END and START, which are clearly dissociated from the comparative relation itself (RISE) and from the measure function contributed by adjectives and other degree predicates. This has allowed us to identify a feature that is common to all three CAC operators, and to relate comparison to additivity through a form of argument structure alternation, by manipulating END and START independently from each other and from RISE. By contrast, analyses of comparison both in degree semantics and in delineation semantics tend to assume that the expression that corresponds to the individual argument of END is an argument of the degree predicate that the comparative operator modifies. Of course, it might be possible to recast the decompositional analysis that was proposed in this paper in a pointilist version of degree semantics, but it is a virtue of Schwarzchild’s analysis that the analogy between Neo-Davidsonian event semantics and scale segment semantics that it stresses provides a conceptual motivation for this decomposition.

6.2 Additivity

6.2.1 Event-based analyses

Additive interpretations of more have been discussed by Greenberg (2009, 2010b,a, 2012) and Thomas (2010b, 2011a,b). Thomas’s discussion of more originates in his earlier analysis of additive interpretations of the comparative suffix -ve in Guarani (Thomas 2010a), and Greenberg also discusses additive interpretations of the modern Hebrew continuative particle od. Thomas’ and Greenberg’s analyses of additivity are based on very similar intuitions and are also implemented in a similar fashion. Consequently, I will only discuss Greenberg’s analysis.

Greenberg’s analysis of additivity is based on Nakanishi’s (2007) notion of derived measure functions. Such functions are obtained by composing a homomorphism from events to their participants or to some event parameter like a spatial path, and a measure function whose domain includes the range of this homomorphism. We may for instance implement in a similar fashion. Consequently, I will only discuss Greenberg’s analysis.

Greenberg’s analysis of additivity is based on Nakanishi’s (2007) notion of derived measure functions. Such functions are obtained by composing a homomorphism from events to their participants or to some event parameter like a spatial path, and a measure function whose domain includes the range of this homomorphism. We may for instance implement in a similar fashion.

There are two events $e_2$ and $e_3$ and two (plural) individuals $y$ and $z$ such that:

i. $e_2$ and $y$ stand in some salient relation $P_2$ (namely, $e_2$ is an event of $y$ singing),

$P_2(y)(e_2)$

ii. $e_2$ does not follow $e_1$,

$\tau(e_2) \leq \tau(e_1)$

iii. $z$ are children,

$\text{children}(z)$

iv. $z$ is the sum of $x$ and $y$,

$z = x \oplus y$

v. $e_3$ and $z$ stand in some relation $P_3$,

$P_3(z)(e_3)$

vi. $e_3$ is the sum of $e_1$ and $e_2$,

$e_3 = e_1 \oplus e_2$

vii. $e_3$ is more developped than $e_2$,

$e_3 >_{\text{developed}} e_2$

viii. the number of atomic parts of the agent of $e_2$ is some degree $d_2$,

$\mu(h(e_2)) = d_2$

ix. the number of atomic parts of the agent of $e_3$ is $d_2 + 3$.

$\mu(h(e_3)) = d_2 + d_3$
Let us unpack these complex truth-conditions. The non-presupposed information conveyed by an assertion of (199b) is simply the proposition that three children danced. The presupposition of (199b) is more interesting. Greenberg argues that additive sentences presuppose the existence of an event $e_2$ that stands in an additive relation to the event $e_1$ described by the non-presupposed content of the sentence. The presupposed event $e_2$ is required in (200b-ii) not to follow $e_1$. This requirement accounts for the fact that an additive interpretation is attested in (201) but not in (202):

(201) (Yesterday John interviewed three students). Today he interviewed more (students).

(202) Today John interviewed three students. Yesterday he interviewed more (students).

By contrast, Greenberg (2012) observes that an additive interpretation is possible when the two events are simultaneous:

(203) This morning Danny interviewed 3 students in his office. At that time Susan interviewed some more students in the library.

While the lack of additive interpretation of more in (202) should be explained, I believe that the lexical entry of additive operators should not encode restrictions on the temporal order of events. Indeed, the very interpretation that Greenberg wishes to block is actually attested in naturally occurring examples, like (204):

(204) There is credible reporting that Israel and South Africa may have conducted some tests in the late 1970s, in the Indian Ocean off the African Coast. We caught one test in 1979, apparently because the weather cleared ahead of schedule. Our ability to detect nuke tests was less exacting in those days, and there’s belief that Israel and South Africa may have conducted two more (before 1979) that we missed.

http://formerspook.blogspot.com/2006/12/worlds-worst-kept-secret.html

In (200b-vi) and (200b-vii) the sum of $e_1$ and $e_2$ is required to form an eventuality $e_3$ that is more developed than $e_2$. Greenberg (2012) revised this part of her analysis to require that $e_2$ be a stage of $e_3$, which was meant to account for the putative unavailability of additive interpretations of more in discourses like (205) out-of-the-blue, the idea being that the two events described in this discourse are not easily conceived of as two different stages of a bigger eventuality.

(205) I baked 3 cakes for my son’s birthday party. A woman I know in New York baked more cakes for her son’s party.

Note that $e_1$, $e_2$ and $e_3$ are not required to be the same sorts of events. This accounts for the felicity of discourses like (206), in which the first sentence describes an event of buying and the second describes an event of baking:

(206) John bought two stollens. Mary baked one more cake.

Finally, the nominal additivity of (199b) is captured in (200b-viii) and (200b-ix), which require that the number of atomic parts of the patient of $e_3$ be the sum of the number of atomic parts of the patients of $e_1$ and $e_2$.

Greenberg’s analysis has a number of desirable consequences. First, it explains the incompatibility of additive more with non-extensive measure functions. Indeed, it predicts that (207) suffers from presupposition failure, since the sum of the temperatures of the water that was spilled in each event is not equal to the temperature of the total water that was spilled in the plural event (assuming this measure is even defined):

(207) 30 degree Celsius water was spilled on the carpet. #10 degree Celsius more was spilled on the bed.

For essentially the same reasons, Greenberg’s analysis captures the fact that discourse (208) entails that John and Mary spoke with a total of 7 different students. That is to say, the discourse should suffer from presupposition failure otherwise:

(208) Yesterday John spoke with 4 students. Today Mary spoke with 3 more students.

Greenberg (2010a) gives different lexical entries to nominal and adverbial additive more.\(^{25}\)

---

\(^{25}\)Greenberg discusses this example in Modern Hebrew.

\(^{25}\)In the following formulas, I assume that $e_2$, $P_2$ and $d_2$ are free variables whose value is resolved by anaphora. That is to say, whenever the variable $e_2$ is used, what is really meant is $g(e_2)$, etc. I decided to omit the assignment function to avoid making the formulas more cluttered than they already are. Note that in Greenberg’s original formulation, the variables $e_2$, $P_2$ and $d_2$ are existentially bound. I believe that it is better to assume that these expressions are anaphoric to expressions mentioned in the previous discourse, since otherwise the presupposition would be too weak: in example (199), only clearly infers that number of children who danced must be added to the number of children who sang. In other words, the value of $e_2$, $P_2$ and $d_2$ has to be resolved by anaphora to contextually salient antecedents.
(209) \[ \langle \text{more}^\text{add} \rangle c^c = \lambda d_1. \lambda Q. \lambda P_1. \lambda e_1. \exists x (Q(x) \land P_1(x)(e_1) \land \mu(h(e_2)) = d_1 \land \delta(\exists y [P_2(y)(e_2) \land Q(y) \land \mu(h(e_2)) = d_2 \land \tau(e_2) \leq \tau(e_1) \land \exists z [P_3(z)(e_3) \land e_3 = e_1 \oplus e_2 \land Q(z) \land z = x \oplus y \land \mu(h(e_3)) = d_1 + d_2 \land e_3 \succeq_{\text{developed}} e_2])]) \]

(210) \[ \langle \text{more}^\text{add} \rangle c^c = \lambda d_1. \lambda P_1. \lambda e_1. \exists x (P_1(x)(e_1) \land \mu(h(e_1)) = d_1 \land \delta(\exists y [P_2(y)(e_2) \land \mu(h(e_1)) = d_2 \land \tau(e_2) \leq \tau(e_1) \land \exists z [P_3(z)(e_3) \land e_3 = e_1 \oplus e_2 \land \mu(h(e_3)) = d_1 + d_2 \land e_3 \succeq_{\text{developed}} e_2])]) \]

While \text{more}^\text{add} is a quantifier with an additional degree argument, \text{more}^\text{add} is an adverbial modifier.

In sum, Greenberg and Thomas argue that additivity is hardwired in the denotation of additive operators. In both analyses, additivity is captured through a form of derived measurement of events.

### 6.2.2 Focus sensitive analyses

Eckardt (2006) and Umbach (2012) analyze additive \textit{noch} in German as a focus-sensitive particle in alternative semantics. For Eckard, \textit{noch} signals that one of its focus-induced alternatives was asserted in the previous discourse:

(211) \textit{noch} + S associates with focus.

Let A be the focused element in S. The sentence presupposes that:

a. Alt(A) is a restricted and fixed reference domain under debate,

b. one or more alternatives \( q \in \llbracket S \rrbracket \) were asserted in the last utterances in discourse,

c. there is a specific order on Alt(A) such that for all \( A', A'' \leq A \), the assertion \( \llbracket S \rrbracket (A'/A) \) was made before \( \llbracket S \rrbracket (A''/A) \) iff \( A' < A'' \),

d. there is some alternative C such that \( C \leq A' \) iff \( \lnot \llbracket S \rrbracket (A'/A) \) holds true.\(^{26}\)

The sentence asserts its content under ordinary semantic evaluation. \((\text{Eckardt 2006 ex. (14)})\)

To illustrate, example (212) asserts that the speaker knows a man who speaks Russian, and presupposes that the proposition that the speaker knows a man who speaks X, for some alternative X to ‘Russian’, has been asserted in the previous discourse:

(212) Ich kenne noch einen Mann, der RUSSICH spricht.

I know ‘yet another’ man, (one) who can speak Russian.

Eckardt defends a unified analysis of additive and temporal (i.e. continuative) interpretations of \textit{noch}, but she does not explore the similarities between the two interpretations in detail.

Umbach’s (2012) analysis of additive \textit{noch} is similar to Eckardt’s. Her analysis is expressed using structured mean-

ings (von Stechow 1990), where the meaning of an expression \( <B,F> \) that consists of a background B and a focus F is obtained by applying B to F:

\[ \langle \text{alt} \rangle c^c = \lambda d_1. \lambda P_1. \lambda e_1. \exists x (P_1(x)(e_1) \land \mu(h(e_1)) = d_1 \land \delta(\exists y [P_2(y)(e_2) \land \mu(h(e_1)) = d_2 \land \tau(e_2) \leq \tau(e_1) \land \exists z [P_3(z)(e_3) \land e_3 = e_1 \oplus e_2 \land \mu(h(e_3)) = d_1 + d_2 \land e_3 \succeq_{\text{developed}} e_2])]) \]

(213) \textit{noch} \( \langle <B,F> \rangle \) iff \( <B,F> \) where Alt(F) is ordered such that the order is aligned with the order of mentioning \(<_m\) on the subset of mentioned alternatives Alt\(_m\)(F), and F is maximal in Alt\(_m\)(F) presupposing that \( \exists x \in \text{Alt}_m(F) \) such that \( x \neq F, x <_m F, \) and \( <B,x> \).

\(^{26}\)If I understand Eckardt’s analysis correctly, condition (d) should be read as ‘there is an alternative C such that every alternative A’ that is not mentioned before C is such that the result of substituting A by A’ in S is a false sentence.’ It is meant to capture Eckard’s assumption that additive \textit{noch} signals the existence of a ‘negative phase,’ an assumption which is motivated by the unacceptability of the following example:

(1) # Tick kann schwimmen, und TRICK kann noch schwimmen, und TRACK kann noch schwimmen.

Tick can swim, and Trick can still swim, and Track can still swim

‘Tick can swim, and Trick can still swim, (but) Track cannot swim.’

Eckardt argues that the last conjunct must be false, since the alternative TRACK is ordered after the focused element TRICK in the set of alternative. It is unclear to me how the analysis in (211) derives this result. Regardless, the following example from Umbach (2012) shows that not all instances of \textit{noch} signals the existence of a negative phase in the sense of Eckardt (glosses and translations are mine):

(2) Tick kann schwimmen, und TRICK kann noch schwimmen, und TRACK kann auch noch schwimmen.

Tick can swim, and Trick can still swim, and Track can too still swim

‘Tick can swim, and Trick can still swim, and Track can still swim too.’

\(^{26}\)
Umbach keeps from Eckardt the idea that *noch* manipulates a set of focus-induced alternatives that are ordered according to their time of mention in discourse.

An interesting feature of Umbach’s and Eckardt’s analyses is that they point to a possible unification of continuation and additivity in alternative semantics. Unfortunately, it is unclear how this analysis could account for the homophony between comparison and additivity.

### 6.2.3 Varieties of additivity

In the literature on focus sensitive particles, the term *additive* is used to describe particles like *too*, which do not fall under the type of additivity that was described in this paper. Heim (1990) and Kripke (2009) analyze *too* as a focus sensitive particle, which triggers a presupposition that the backgrounded part of the sentence holds of some salient alternative to the focused expression associated with *too*:

\[(\exists F' \neq F [ \ldots F' \ldots ])\]

An interesting feature of Umbach’s and Eckardt’s analyses is that they point to a possible unification of continuation and additivity in alternative semantics. Unfortunately, it is unclear how this analysis could account for the homophony between comparison and additivity.

#### (214) BILL was at the party too.

- Assertion: Bill was at the party.
- Presupposition (for some salient individual \(x\)): \(x\) is not Bill and \(x\) was at the party.

Additive particles like *too* are part of a broader class of focus sensitive grading particles, along with scalar particles and exclusive particles (see König 1991; Krifka 1999). Krifka represents the meaning of these grading particles schematically as follows, where \(F\) stands for an expression in focus, \(F'\) is one of its alternatives, the presupposition of each particle is represented between parentheses, and \(<_{\text{likely}}\) means ‘less likely’:

\[(215)\]

\[\begin{align*}
\text{a. [ADD}_1 [ \ldots F_1 \ldots ]] & : [ \ldots F_2 \ldots ] (\exists F' \neq F [ \ldots F' \ldots ]) \\
\text{b. [EXCL}_1 [ \ldots F_1 \ldots ]] & : \neg\exists F' \neq F [ \ldots F' \ldots ] (\exists F \ldots ) \\
\text{c. [SCAL}_1 [ \ldots F_1 \ldots ]] & : [ \ldots F_2 \ldots ] (\neg\exists F' \neq F [ \ldots F' \ldots ] <_{\text{likely}} [ \ldots F \ldots ] )
\end{align*}\]

To illustrate, the sentence *Only JOHN arrived*, with the exclusive particle *only*, asserts that no alternative \(F'\) to John is such that \(F'\) arrived and presupposes that John arrived.

The meaning of additive uses of *more* and *noch* is clearly similar to that of focus sensitive additive particles like *too*. As illustrated by the following pair of examples, both *too* and additive *more* can be used to convey that a predication (namely, *I had \(x\)*) holds of some alternative to an entity that was made salient in the previous discourse:

#### (216) I had a beer. Then I had one more beer.

#### (217) I had a beer. Then I had a SCHNAPS too.

However, Greenberg (2012) and Umbach (2012) observed differences between these particles, which suggest that they have different meanings. While the use of additive *more* in (218) entails that some of the students that John spoke with today are different from the students that he spoke with yesterday, example (219) with *too* does not exclude that John spoke with the same students on each occasion:

\[(218)\] (Yesterday John spoke with 3 students). Today he spoke with more (students).  \hspace{1cm} (Greenberg 2012)

\[(219)\] (Yesterday John interviewed three students). Today he interviewed students too.  \hspace{1cm} (Greenberg 2012)

In German, Umbach observed that *noch* can associate with a deaccented focus even in the absence of a contrastive topic, unlike *auch*:

\[(220)\] Otto hat einen Schnaps getrunken. Und du glaubst es nicht:

- ‘He had a schnaps. And you won’t believe it:’
- ‘He had another schnaps.’
- ‘He had a schnaps, too.’

For the sake of clarity, let us call particles like *more* and *noch* incremental additive particles. Whether these particles form a natural class with non-incremental additive particles is still an open question. In any case, insofar as non-incremental additive particles are not homophonous with comparison and continuation, the study of the relation of incremental additivity to non-incremental additivity falls outside the scope of this paper.

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6.2.4 Wrapping up

The analysis of additivity that I have proposed is inspired by Greenberg’s and Thomas’s event based analyses. A limitation of these earlier analyses is that they did not address the homophony between CAC operators. The focus sensitive analyses of noch of Eckardt and Umbach were more promising in this respect, although they did not address the homophony of additivity and comparison.

6.3 Continuation

Two influential analyses of the continuation particle still are due to Löbner (1989) and Krifka (2000).

Löbner’s analysis In Löbner’s analysis, nochlerst (‘still’) belongs to a class of phase quantifiers that also include the particle schon (‘already’) as well as the negated forms nicht mehr (‘not anymore’) and noch nicht (‘not yet’). Löbner identifies three types of uses of phase quantifiers, depending on the aspect of the predicate that the particle modifies, and its association with focus:

(221) Type 1; noch modifies an imperfective/stative predicate and associates with broad focus:

Das Licht ist noch an.
the light is still on
‘The light is still on.’

(222) Type 2; noch associates with a focus on a degree expression:

Es ist erst EINS.
it is only one
‘It is only ONE.’

(223) Type 3; noch associates with a focus on a temporal frame adverbial:

Sie kommt erstUM ZWEI.
she comes only at two
She won’t be coming until two.

Löbner analyzes type 1 uses of still and noch as follows, where $\phi(t)$ means that $\phi$ is true throughout an interval $t$ and $t' \propto t$ means that the interval $t'$ begins before and abuts $t$:

(224) $\llbracket \text{still} \rrbracket^{\epsilon}(\phi)(t)$

assertion: $\phi(t)$

presupposition: $\exists t' \propto t[\phi(t')]$

$\llbracket \text{Still} \rrbracket^{\epsilon}(\phi)(t)$ asserts that $\phi$ holds at $t$ and presupposes that $\phi$ was true throughout an interval that precedes and abuts $t$. In type 2 and type 3 uses, noch is replaced by erst and still by only or until. The relation of type 2 and type 3 uses to type 1 uses has been a matter of debate in subsequent literature, see among others Krifka (2000), Condoravdi (2002), von Stechow & Penka (2006).

Besides proposing a unified analysis of all three types of uses of phase quantifiers, Löbner (1989) also argued that these particles are logically related by internal and external negation. According to this analysis, still and already are duals, not anymore is the outer negation of still and not yet is its inner negation.

Krifka’s analysis Krifka (2000) proposes an analysis of phase quantifiers, which he calls aspectual particles, using an alternative semantics for focus. All uses of aspectual particles are focus sensitive, including type 1 uses. The set of focus alternatives $A$ that aspectual particles manipulate is ordered by a relation $\leq_A$. In their type 1 uses, the focus is on the whole sentence, and the only alternative under consideration is the negation of the proposition denoted by the sentence:

(225) It is still raining.

27Focused constituents are written in capital letters.
28This presentation of Löbner’s analysis is due to Krifka (2000).
Focus: $[\text{it is raining}]^{\text{fc}}$
Alternative: $[\text{it is not raining}]^{\text{fc}}$

Still presupposes that its focus is ranked lower than its alternatives with respect to the relation $\leq_A$. Its temporal interpretation, in its type 1 use, is obtained by aligning $\leq_A$ to the order of times, which in this example means that the proposition that it is not raining becomes true after the proposition that it is raining does. In this analysis, still does not trigger a past oriented presupposition. This is problematic since, as Ippolito (2007) points out, it is unclear how Krifka’s analysis accounts for examples like (226), which suggests that John is still unemployed presupposes that John was already unemployed at an earlier time:

(226) John was never unemployed. So far he has had a steady job.
#If John were still unemployed, his wife and kids would leave him.

**Ippolito’s analysis** The analysis of continuation that I have proposed in this paper is a reformulation of Ippolito’s (2007) analysis of temporal uses of *still* in scale segment semantics, which is repeated in example (227) and (228):

(227) John is still alive.
   a. Assertion: a contextually salient state $s$ of John being alive holds at $t_c$.
   b. Presupposition: $s$ held at $t$, for some time $t < t_c$.

(228) $[\text{still}]^{\text{fc}} = \lambda t.\lambda e.\lambda P. P(e)(t) \land \partial(\exists t'[t' < t \land P(e)(t')])$

If we restrict our attention to ‘continuation’ uses of *still*, Ippolito’s analysis can be interpreted as a refinement of Lönner’s analysis, with a weaker formulation of the presupposition of *still*. Ippolito’s analysis diverges from Lönner’s more significantly in her account of concessive and exclusive uses of *still*, illustrated in (229) and (230), which she analyzes as scalar and exclusive focus particles respectively:

(229) John studied all night, and he still failed the test. (Concessive)
(230) It’s still 8 am. (Exclusive)

**Taking stock** Neither Lönner (1989) nor Krifka (2000) addressed the question of the homophony of continuation with comparison and additivity. While the analysis of continuation that I proposed in this paper is largely compatible with Ippolito’s (2007) revision of Lönner’s analysis of type 1 uses of *still* and *noch*, it is unclear how the adoption of scale segment semantics should affect our analyses of type 2 and type 3 uses of phase quantifiers, and of the putative duality of *still* and *already*. It is also unclear how the reformulation of Ippolito’s analysis of continuation in scale segment semantics bears on her proposal that the different interpretations of *still* involve the use of different focus sensitive grading particles.

### 6.4 Summary and open issues

My analysis of CAC operators is based on Schwarzschild’s analysis of comparison in scale segment semantics and on a reformulation of previous analyses of additivity and continuation in this framework. While this change of framework allowed us to describe a lexico-grammatical system consisting of comparative, additive and continuation operators, the adoption of scale segment semantics also raises a number of questions which will remain unanswered in this paper.

Concerning comparison, it is still unclear how the adoption of scale segment semantics bears on the analysis of a number of classical issues in the grammar of comparative constructions, such as restrictions on the scope of quantifiers in comparative clauses, the role of comparison in decompositional analyses of superlatives, and the interaction of comparison with negation.

With respect to continuation, I have stressed the integration of the temporal use of aspectual particles like *still* and *noch* in a paradigm that also includes comparison and additivity. However, a complete analysis of these particles should also account for their duality with *already* and *schon*, and more generally for Lönner’s (1989) aspectual square of opposition, and it should account for the putative membership of these particles in a wider class of focus sensitive grading particles. It is unclear how the adoption of scale segment semantics in the analysis of continuation operators affects these two dimensions of the analysis of aspectual particles.
Similar questions arise for the analysis of incremental additive particles like *more* and *noch*. In this paper, I proposed an analysis of their homophony with comparison and continuation operators, but it has also been proposed that these particles are part of a broader class of additive focus sensitive particles in the sense of *Krifka* (2000). The relation between these two forms of additivity is still largely unexplored, despite the seminal work of *Eckardt* (2006), *Tovena & Donazzan* (2008) and *Umbach* (2012).

7 Conclusion

I have presented an integrated analysis of comparison, additivity and continuation in scale segment semantics, which accounts for patterns of homophony attested in several languages. There is of course much more to be said about each operation and its proper analysis in terms of quantification over scale segments. In the context of this paper, the most important advantage of scale segment semantics is the possibility it offers to express a rigorous decompositional analysis of comparison, additivity and continuation that throws light on patterns of homophony that would otherwise be formally inscrutable.

References


