Statement of Current Research Interests

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Introduction

My research interests are focused in general in geometric topology, and more specifically in knot homologies. I am currently using Category Theory, and the theory of Planar Algebras to study the Khovanov homology of alternating tangles. I plan to use the same techniques to study the Khovanov homology of some specific non-alternating link diagrams that are constituted from alternating tangles.

At the end the XX Century, Khovanov [8] opened new prospects in knot theory associating a bigraded chain complex to a diagram D of a link L. The homology of that complex is an invariant of links. A remarkable property of this homology is that its Euler characteristic is the Jones polynomial $J(L)$ of the link. Subsequently, Bar-Natan [2] showed how to compute $Kh(L)$ and found that it is a stronger invariant than the Jones polynomial. The Khovanov's theory is an amazing application of Category Theory in Knot Theory. First of all, Khovanov's construction uses a functor from the category of 2-dimensional cobordisms between 1-dimensional manifolds to the category of graded $\mathbb{Z}$-modules. Finally, the invariant resulting from this construction is functorial. This fact was proved separately in [3], [7] and [9].

In [3] Bar-Natan presented a new way of seeing the Khovanov homology. In his approach, formal chain complex is assigned to every tangle, that is to say, to each part of a link. This formal chain complex, regarded within a special category, is an (up to homotopy) invariant of the tangle. In this way we arrive to a "divide an conquer" strategy in which we study properties or compute invariants of "half-links" (tangles), and then combine them together to obtain properties or compute invariants of the link as a whole.

In his work, Bar-Natan defined a category whose objects are smoothings (simple curves in the plane) and whose morphisms are cobordisms between these curves. He also used the concept of planar algebra as a tool for gluing together not only the objects of that category, but also the morphisms, and by these means understand many of the properties of the Khovanov homology for links. This divide and conquer method has many computational and theoretical advantages and has been used in different ways: it was used to prove the invariance of the Khovanov homology, see [3]; it was applied in [4] to make a local algorithm which provides a faster computation of the Khovanov homology of a link. Moreover, it was used in [5] to give a simple proof of Lee's result stated in [12], about the dimension of the Lee variant of the Khovanov homology.

Current Research

In [6], I used the divide and conquer technique to extend to tangles a celebrated result of Thistlethwaite [14] about the Jones Polynomial of alternating Links. Now, my current research deals mainly with the use of this technique to extend to tangles a Lee's theorem [11] about the Khovanov homology of alternating links.

To accomplish that, I define a certain category $\mathcal{C}ob^3_o$ of "oriented cobordisms", in the same spirit as the category $\mathcal{C}ob^3_{/l}$ of [3]. The objects of $\mathcal{C}ob^3_o$ are "oriented smoothing"; the orientations of the strands in an oriented smoothing S allow me to define an integer parameter associated to it, its "rotation number" $R(S)$. Specifically, for degree-shifted smoothings $S\{q\}$ I define $R(S\{q\}) := R(S) + q$. I further use this "degree-shifted rotation number" to define a special class of chain complexes of the form

$$\Omega : \qquad \cdots \longrightarrow \left[S^r_{j_r}\right] \longrightarrow \left[S^{r+1}_{j_{r+1}}\right] \longrightarrow \cdots .$$

Each $S^r_{j_r}$ is a vector of smoothings which satisfies that the difference between twice its homological degree r and the degree-shifted rotation number of its smoothings is a constant. We call this type of chain complexes "diagonal complexes". Furthermore, the diagonal complexes whose partial closure are also diagonal are called "coherently diagonal". Finally, I used the concept of "alternating planar algebra", introduced in [6]. These alternating planar algebra are defined in a similar way as planar algebras, but using only diagrams that are oriented in an alternating way. These alternating diagrams are perfect tools for composing alternating tangles, oriented smoothings, and diagonal complexes.

With these definitions I state two conjectures, which eventually will bring the generalization of the fore-mentioned Lee's result. I am working in the proof of these conjectures.

Conjecture 1 . *Coherently diagonal complexes form an alternating planar algebra.*

The second conjecture follows immediately from the first. Its proof is reduced to the simple task of verifying that the Khovanov homologies of the one-crossing tangles (⤫) and (⤬) which are of course alternating are coherently diagonal complexes.

Conjecture 2 *Let T be a non-split alternating $2k$-boundary tangle ($k > 0$), then the Khovanov homology $Kh(T)$ can be interpreted as a coherently diagonal complex.*

It is a simple matter to verify, in the same way I did it in [6] with a Thistlethwaite's result, that in the case of alternating tangles with no boundary, i.e., in the case of alternating links, this result reduces to Lee's theorem on the Khovanov homology of alternating links.

Future Research Goal

Looking to the future, I would find it exciting if eventually this could be brought to bear on certain non-alternating link diagrams that are constituted from alternating tangles. For example, the *adequate* diagrams introduced by Lickorish-Thistlethwaite in [13], or the almost alternating links introduced in [1]. I am very interested in applying the results of my present research to the Khovanov homology of these type of Links.

Another goal is to explore the conjecture saying that Khovanov homology is invariant under knot mutation. I already wrote an algorithm in Mathematica that constructs pairs of mutant knots, and have applied a Bar-Natan program [10] to compute the Khovanov homology of a great number of mutant knot pairs. Since two tangles are implied directly in the mutation of a knot, it seems that the knowledge of the divide and conquer technique applied to the Khovanov homology is a good start to pursue that conjecture.

References

[1] C. Adams, J. Brock, J. Bugbee, T. Comar, K. Faigin, A. Huston, A. Joseph, D. Pesikoff, *Almost alternating links.* Topology Appl., **46** (1992), 151165

[2] D. Bar-Natan, *On Khovanov's categorification of de Jones polynomial*, Alg. Geom. Top., **2** (2002), 337-370.

[3] D. Bar-Natan, *Khovanov's homology for tangles and cobordisms*, Geometry & Topology, **9** (2005), 1443–1499.

[4] D. Bar-Natan, *Fast Khovanov homology computations*, arXiv:math.Gt/0606318 (2006)

[5] D. Bar-Natan and S. Morrison, *The Karoubi envelope and Lee's degeneration of Khovanov homology*, Algebraic & Geometry Topology, **6** (2006), 1459–1469.

[6] H. Burgos Soto, *The Jones polynomial and the planar algebra of alternating links*, Journal of Knot Theory and Its Ramifications, To appear, arXiv:math.Gt/0807.2600v1 (2008)

[7] M. Jaccobson, *An invariant of link cobordism from Khovanov homology*, Algebr. Geom. Topol **4** (2004) 1211-1251

[8] M. Khovanov, *A categorification of the Jones Polynomial*, Duke Math J., **101(3)** (1999), 359–426.

[9] M. Khovanov, *A functor-valued invariant of tangles*, Algebr. Geom. Topol **2** (2002) 665-741

[10] KnotTheory', a knot theory mathematica package, http://katlas.math.toronto.edu/wiki/ KnotTheory.

[11] E. S. Lee, *The support of the Khovanov's invariants for alternating links*, arXiv:math.GT/0201105, v1, (2002).

[12] E. S. Lee, *A Khovanov invariant for alternating links*, arXiv:math.GT/0210213, v2, (2002).

[13] W. B. R. Lickorish and M. B. Thistlethwaite, *Some links with non-trivial polynomials and their crossing numbers*, Comment. Math. Helv. **63** (1988), 527539.

[14] M. B. Thistlethwaite, *Spanning tree expansion of the Jones polynomial,* Topology **26** (1987) 297–309.