Competing Through Information Provision*

Jean Guillaume Forand†

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Abstract

This paper studies the symmetric equilibria of a two-buyer, two-seller model of directed search in which sellers commit to information provision. More informed buyers have better differentiated private valuations and extract higher rents from trade. I establish how the characteristics of exogenously fixed sale mechanisms determine equilibrium information provision and show that competition generates complementarities between allocative and informational efficiency. Information provision is higher under competition than under monopoly, yet partial information is provided for many natural sale mechanisms. In contrast, when sellers commit to both information provision and sale mechanisms, I identify simple conditions under which every equilibrium has full information. Sellers capture the efficiency gains from increased information and compete only over non-distortionary rents offered to buyers.

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1 Introduction

> *(Christie's and Sotheby's) embarked on cutthroat competition to get goods for sale (... and) provide ever more luxurious services. Catalogues became ever fatter, printed in colour, on glossy art paper. (...) On the inside page of Sotheby's catalogue of the Old Master paintings sale held in London on Dec. 13 (2001), six "specialists in charge" are listed. (...) They identify the paintings, research them, know which world specialist on this or that painter needs to be contacted, and, more mundanely, which client is most likely to be interested in what painting, etc.*[1]

Competing sellers are typically thought of as proposing prices to buyers, or more generally sale mechanisms. However, as the quality of buyers' information about goods affects their gains from trade, sellers may try to attract buyers by offering better information. This paper considers a

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†Department of Economics, University of Toronto, 150 St. George Street, Toronto, Ontario, M5S 3G7. jg.forand@utoronto.ca

[1] International Herald Tribune, 12/01/2002.

market in which sellers post levels of information provision that are observed by potential buyers before they choose which seller to visit. When considering how much information to reveal to buyers, sellers trade off market share against the cost of selling goods to buyers with better private information.

Privately informed buyers gain informational rents through trade. Conceptually, a buyer's information about his valuation for a good has two elements; the private knowledge of some personal attributes, along with an understanding of how these characteristics relate to the good's properties. Sellers cannot influence the first kind of knowledge, but controlling the information about their goods affects the second kind. As noted by Bergemann and Pesendorfer (2007), by providing less information to buyers *before* trading, sellers give out fewer informational rents *during* the exchange process. Restrictions on information come at a cost, since in the presence of more than one buyer higher information provision increases surplus by better identifying the buyer that most prefers the good. Furthermore, and this is the novel insight of this paper, if sellers compete for buyers, the latter may shun low-information selling sites.

I show that the effect of information provision depends critically on its role in competition. If sellers choose information provision independently of sale mechanisms, competition is channelled only through the level of information, which depends on the characteristics of the sale mechanisms. Sellers prefer mechanisms that soften competition and these have inefficient allocations and high rents. On the other hand, when sellers choose sale mechanisms and information provision jointly, they channel competition away from inefficient restrictions on information and into redistributive rent transfers to buyers. They provide full information and allocate goods efficiently based on that information.

The case of the auction houses of Christie's and Sotheby's, related at the beginning of the section, provides a good example of competitive information provision. In that industry, the services surrounding an auction play a critical role in allowing potential customers to better evaluate an object's worth to them. In the early 1990's, competition between the auction houses stiffened considerably, and expanding the services that provide information to buyers became an important competitive tool. Furthermore, later in the decade Phillips, a minor auction house, tried to break the Christie's-Sotheby's duopoly. It did so by providing high guarantees to sellers who consigned objects there, but it also tried to match the bigger auction houses' superior capacity to inform buyers by luring away some of their teams of experts.[2] However, eventually Phillips became "less willing to provide lavish guarantees and loans. It emerged that Phillips's cash, rather than its expertise, had lured sellers of high-quality art; they returned to Christie's and Sotheby's."[3] Another example is the website of Multiple Listing Service, mls.com, which allows real-estate agents to advertise houses for sale by posting pictures and descriptions. Rival agents adopt different strategies and the quality of the information revealed in the advertisements

[2]The Economist, 01/03/2001.
[3]The Economist, 21/02/2002.

varies widely. Some agents post a bare-bones description of the house along with a picture of the house's exterior. Others provide pictures of some of the rooms, some even post a full slide show. Bergemann and Pesendorfer (2007) provide other examples of both monopolistic and competitive markets where information provision decisions are important.

In this paper, I present a model of directed search in which two sellers with unit supplies compete for the unit demands of two buyers by promising information.[4] Sellers commit to information structures and sale mechanisms, after which buyers choose the seller to visit and sales take place. As in Peters and Severinov (1997), sorting occurs ex ante; buyers obtain their private information only once they choose a seller. If fully informed, buyers either have (independent) high or low valuations for either sellers' objects. However, buyers' information is mediated by the information structures offered by sellers. Information structures, as in Bergemann and Pesendorfer (2007), map signals controlled by sellers into buyers' inferences about their valuations for goods. Sellers cannot observe signals' effects on buyers' estimates of their valuations, but instead control ex post distributions of values. By providing more information, sellers release private signals that allow buyers to differentiate their private values from the public expectation, interpreted as reflecting that pool of public knowledge about the goods' ex ante characteristics accessed by any potential buyer. As in Damiano and Li (2007), Ganuza and Penalva (2006), Johnson and Myatt (2007) and Ivanov (2009), I consider information structures ordered by the precision with which they allow buyers to access their true valuations. In my model, information structures have a simple correlated structure; sellers choose the probability with which all buyers get access to their valuations for their good upon visiting their site. Ex post, all buyers visiting a particular seller are informed or uninformed.

In the subgame following the sellers' announcements, I assume that buyers sort into sale sites according to that subgame's unique symmetric mixed strategy equilibrium. This restriction, common in directed search, rules out equilibrium coordination among buyers and ensures smooth responses in sellers' profits to changes in their announcements. In equilibrium, sellers face a random demand, whose distribution they affect through their choice of strategy. I consider two variants for the sellers' strategy sets. In the first, sellers only commit to information provision, while in the second they commit both to information and to sale mechanisms. In the first case, information provision is determined independently of sale mechanisms, which can be set by previous competitive outcomes, industry standards or regulation. This centers attention on the effects of competition through information when it is layered onto pre-existing terms of trade. In the second case, information provision and sale mechanisms are determined jointly. In both cases, I restrict attention to symmetric equilibria of the game between the sellers.

In Section 3, sale mechanisms are exogenously fixed and common to both sites, and sellers can attract buyers only by promising more information. Information provision increases buyer

[4]Following Moen (1997) see Burdett, Shi and Wright (2001), Coles and Eeckhout (2003), Peters (2009), Shi (2001) and Shimer (2005). See Shi (2006) for a recent survey of directed search and Delacroix and Shi (2009) for a model in which posted prices act as informative signals about good quality.

rents across demand states (i.e., when one buyer or two buyers are present) and generates a novel version of a trade-off well known in models of directed search and competing auctioneers;[5] higher information attracts more traffic yet decreases profits-per-head. Fixing the mechanism determines the shape of this trade-off, which, through competition, determines equilibrium information provision. Furthermore, in the presence of more than one buyer, higher information provision increases surplus as it to identify the buyer who most values the good.

I establish a number of comparative statics results for equilibria under *regular* mechanisms, which include common mechanisms such as auctions and prices. Under regular mechanisms a monopolist would not release information, so that any gains in informational efficiency are due solely to competition. First, equilibrium information provision is increasing in the efficiency of the sale mechanism's allocations in informed states. That is, competition creates a complimentarity between allocative and informational efficiency. High-surplus mechanisms increase sellers' gains from information provision and lead to traffic-stealing and more intense competition. Second, equilibrium information provision is decreasing in the rents offered to buyers, since increased rents soften competition between the sellers. Third, sellers' equilibrium profits are always lower under mechanisms with higher allocative efficiency. Fourth, sellers' equilibrium profits are not monotone in the rents offered to buyers by sale mechanisms. Profits always increase if a mechanism offers higher rents in the one-buyer demand state, while they may drop if a mechanism offers higher rents in the two-buyer demand state. Higher rents in the one-buyer state makes the two-buyer state relatively unattractive and stiffens the competition between the buyers, while higher rents in the two-buyer demand state reduces buyers' aversion to meeting at a site.

In Section 4, sellers commit to both sale mechanisms and information provision. When the effects of information are no longer mediated by the characteristics of exogenously fixed mechanisms, sellers can disentangle their rent and information provision decisions. Under a no-exclusion assumption for informed low-valuation types, I characterise a class of symmetric equilbria in which sellers capture the efficiency benefits of increased information. In these equilibria, sellers provide full information, hold auctions and compete over the rents offered to buyers by setting appropriate reserve prices. Closely related to Coles and Eeckhout (2003), who present a two-buyer, two-seller model of directed search with sale mechanisms under perfect information, a continuum of symmetric equilibria exist that are differentiated by the sharing of a fixed level of surplus between buyers and sellers.[6] In all equilibria, competition drives the marginal buyer's rents to its contribution to site surplus. The full information result exploits the ex ante nature of rent and information promises; profiles in which sellers do no offer full information are vulnerable to deviations in which they provide more information, adjust buyers' rents through transfers to keep their visit decisions constant, and pocket the extra surplus.

Recent work in mechanism design, auctions and optimal pricing has found that when given

[5]See Burguet and Sàkovics (1999), McAfee (1993), Hernando-Veciana (2005), Peters and Severinov (1997) and Virag (2009).

[6]See Section 4 for a more detailed discussion of the two papers' results.

some means of doing so, monopolists often substantially alter the informational attributes of their customers. In a model in which a seller designs a sale mechanism ex post, Bergemann and Pesendorfer (2007) characterise optimal information structures, which take a discrete monotone partitional form. Ganuza and Penalva (2006) study information provision in second-price auctions when buyers' ex post distributions of valuations are ordered by dispersion,[7] and show that the seller's incentive to limit buyers' information vanishes as the number of buyers grows and the competition between them for the good wipes out their informational rents (on this see also Board (2009)). In contrast, when the seller designs a mechanism ex ante and hence can 'sell' information to buyers, Esö and Szentez (2006) show that the seller can capture all rents accruing from the information it controls and provides full information.[8] In a model of monopoly pricing, Johnson and Myatt (2006) assume that sellers' information provision orders buyers' ex post distributions of valuations by sequences of rotations.[9] In a result recalling that of Lewis and Sappington (1994), they find conditions under which a seller's optimal choice of information provision is to release either all or none of the available signals. Bergemann and Valimäki's (2006) survey provides more references to related literature.

The question of how the incentives to provide information extend to a competitive market has received little attention to date. Damiano and Li (2007) present a model of two-seller competition with information provision and ex post price competition which generalises that of Moscarini and Ottaviani (2001). With a single buyer and price competition, information does not enhance surplus and in equilibrium sellers provide information to differentiate goods ex post and soften competition. Ivanov (2009) studies a related model with any number of sellers and continuous type distributions and shows that as the number of sellers increases there is a unique symmetric equilibrium with full information provision.

2 Model

Sellers: Two sellers, a and b, have a single good for sale.
Buyers: Two buyers have unit demands. An informed buyer's valuation for either seller's good is either θ_H or θ_L, with $\theta_H > \theta_L$. The sellers' goods are ex ante similar to buyers; the prior distribution of buyer valuations for either good is (p_H, p_L). The expected value of any good for a buyer is $\bar{\theta} = p_L\theta_L + p_H\theta_H$.
Information Provision: In the first stage of the game, sellers commit to information provision. Information structures are as follows: seller k posts a probability π_k with which information about the good is revealed at site k to all buyers that choose to attend it. Ex post, either all

[7]For random variables X and Y with distribution functions F and G, Y is said to be *more dispersed* than X if $F^{-1}(\beta) - F^{-1}(\alpha) \leq G^{-1}(\beta) - G^{-1}(\alpha)$ for all $0 < \alpha \leq \beta < 1$. See Shaked and Shanthikumar (2007).

[8]See Section 4.1 for a discussion of my full information result and its relation to that of Esö and Szentez (2006).

[9]Continuous distribution function G is said to obtained from distribution F *by (clockwise) rotation around* z if $F(x) \leq G(x)$ for all $x \leq z$ and $F(x) \geq G(x)$ for all $x \geq z$.

buyers at site k are informed or all are uninformed. Informed buyers' values for goods are private and uninformed buyers have known expected value $\bar{\theta}$ for any good.

Remarks: The two-seller, two-buyer setup is restrictive but counters well-known equilibrium existence and tractability issues in finite directed search and competing auctions, which explains why Peters and Severinov (1997), following McAfee (1993), focus on large economies in which a seller's impact on market conditions vanishes. Burguet and Sàkovics (1999) prove existence of a symmetric equilibrium in a 2-seller, n-buyer framework, but their characterisation is difficult to work with. See Hernando-Veciana (2005) and Virag (2009) for existence results in finite competing auctions, and Galenianos and Kircher (2009) along with Galenianos, Kircher and Virag (2009) for directed search equilibria in finite markets.

Having sellers choose the probability of providing information and not directly choosing some ex post distribution of types simplifies the model by reducing the ex post information states to two; informed and uninformed. However, the essential feature is that choices of (π_a, π_b) differentiate the sites with respect to information ex ante. In fact, the information structures of my model can be seen to be discrete examples of those of Johnson and Myatt (2006). Consider ex post distribution of valuations F^{π} for a single buyer over valuation space $\{\theta_L, \bar{\theta}, \theta_H\}$ generated by the information structure of my model with probability π. $\bar{\theta}$ is a rotation point for the family of distributions $\{F^{\pi}\}$ since for $\pi > \pi'$, $F^{\pi}(x) \geq F^{\pi'}(x)$ for all $x < \bar{\theta}$ and $F^{\pi}(x) \leq F^{\pi'}(x)$ for all $x \geq \bar{\theta}$.

Demand and Information States: Once sorted into selling sites, buyers either receive information about the good or not, learn the realisation of the demand state, and take part in the sale mechanism. Let $\eta \in \{1, 2\}$ denote the *demand state* of a sale site and $\tau \in \{i, u\}$ its *information state*, where i stands for informed and u for uninformed. The *state* of a sale site is given by $(\eta, \tau) \in \{1, 2\} \times \{i, u\}$.

Sale Mechanisms: How goods are delivered to the buyers attending site k may be exogenously fixed or committed to by the seller in tandem with π_k. Terms of trade at site k are given by direct incentive compatible mechanisms.[10] These mechanisms specify allocations and transfers as functions of reported types for all information and demand states of the market and are constrained to be anonymous. Also, I assume that sellers cannot charge entry fees to buyers prior to the state of the site being realized. That is, all buyer participation decisions are ex post. A complete, and standard, presentation of the sale mechanisms, and corresponding payoffs for buyers and sellers, is reported to Appendix A. Let Γ be the set of direct incentive compatible mechanisms for my model. Importantly, any mechanism $\gamma_k \in \Gamma$ at site k induces *ex ante rents for buyers in state* (η, τ), $R_k^{\eta,\tau}$, in each state (η, τ). These rents are computed before buyers learn their types and hence, in informed states, ex ante rents are the average of θ_H and θ_L-type rents. Denote the *ex ante surplus at site k in state* (η, τ) *under mechanism* γ_k as $\mathcal{S}_k^{\eta,\tau}$. The ex ante

[10] As is known from the literature on common agency (see Epstein and Peters (1999), Martimort and Stole (2002) and Peters (2001)), restricting sellers to direct mechanisms is not without loss of generality.

surplus is obtained by averaging total gains from trade in state (η, τ) over buyer types, and it depends on mechanisms' allocation rules. Let $\bar{\mathcal{S}}_k^{\eta,\tau}$ be the maximal available surplus at site k in state (η, τ).

Seller Strategies: To focus on competition in information provision, in Section 3 sale mechanism are fixed and a strategy for seller k is a probability $\pi_k \in [0, 1]$. In Section 4, sellers compete by promising both information and mechanisms, and a strategy for seller k is $(\pi_k, \gamma_k) \in [0, 1] \times \Gamma$, a probability π_k along with a mechanism γ_k.

Buyers' Subgame: Given sellers' sale mechanisms and information offers $(\pi_a, \gamma_a, \pi_b, \gamma_b)$, buyers simultaneously choose which site to visit. A strategy for a buyer is $q : ([0, 1] \times \Gamma)^2 \to [0, 1]$, where q denotes the probability with which the buyer visits seller a. The buyers' subgame has a large number of equilibria; I consider symmetric mixed strategy equilibria. It has been argued, notably by Levin and Smith (1994) in the context of a single auction with entry and by Burdett, Shi and Wright (2001) in a directed search model, that the equilibria with symmetric mixed strategies by buyers and random demand are more appealing than asymmetric pure strategy equilibria which generate fixed demand. Burdett, Shi and Wright (2001) show that there exist many equilibria with pure actions on the equilibrium path in which sellers' equilibrium offers are supported by buyers' threats to revert to the mixed strategy equilibrium in the buyers' subgame. In such equilibria coordination improves buyers' payoffs relative to the mixed strategy equilibrium but yields behaviour that is not relevant for the questions studied here.

Buyers' Subgame Equilibrium: Given strategy (π_a, γ_a) for seller a and a visit probability q for buyers, a buyer attending site a expects rents

$$\begin{aligned}\mathcal{R}_a(\pi_a, \gamma_a, q) &= \mathbf{E}_\eta \mathbf{E}_\tau R_a^{\eta,\tau} \\ &= q\Big[\pi_a R_a^{2,i} + (1 - \pi_a) R_a^{2,u}\Big] \\ &\quad + (1 - q)\Big[\pi_a R_a^{1,i} + (1 - \pi_a) R_a^{1,u}\Big]. \end{aligned} \tag{1}$$

The first expectation above is with respect to the binomial distribution with parameter q of the number of opponents faced by a buyer present at site a, and the second with respect to the binomial distribution with parameter π_a over information states at site a. Similarly, given strategy (π_b, γ_b) for seller b and visit probability q, a bidder attending auction site b expects rents

$$\begin{aligned}\mathcal{R}_b(\pi_b, \gamma_b, q) &= \mathbf{E}_\eta \mathbf{E}_\tau R_b^{\eta,\tau} \\ &= (1 - q)\Big[\pi_b R_b^{2,i} + (1 - \pi_b) R_b^{2,u}\Big] \\ &\quad + q\Big[\pi_b R_b^{1,i} + (1 - \pi_b) R_b^{1,u}\Big]. \end{aligned}$$

Buyers' visit decisions depend on whether or not sellers' mechanisms generate *congestion effects*, that is, whether their rents at a given site decrease when the other buyer visits it more frequently. Site k's mechanism generates congestion effects if $\pi_k R_k^{1,i} + (1 - \pi_k) R_k^{1,u} \geq \pi_k R_k^{2,i} + (1 - \pi_k) R_k^{2,u}$.

It this is the case, in the unique symmetric mixed strategy equilibrium of the buyers' subgame the visit probability must satisfy[11]

$$q \begin{cases} = 0 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \geq \mathcal{R}_b(\pi_b, \gamma_b, 0), \\ = 1 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 1) \leq \mathcal{R}_b(\pi_b, \gamma_b, 0), \end{cases}$$

while if both $\mathcal{R}_a(\pi_a, \gamma_a, 1) < \mathcal{R}_b(\pi_b, \gamma_b, 0)$ and $\mathcal{R}_a(\pi_a, \gamma_a, 1) > \mathcal{R}_b(\pi_b, \gamma_b, 0)$, $q \in (0, 1)$ is the unique solution to

$$\mathcal{R}_a(\pi_a, \gamma_a, q) = \mathcal{R}_b(\pi_b, \gamma_b, q). \tag{2}$$

Natural sales mechanisms, such as posted prices and auctions, always generate congestion effects and hence (2) pins down buyer behaviour uniquely for these mechanisms. All exogenous mechanisms considered in this paper, as well as the equilibrium endogenous mechanisms, will generate congestion effects. However, as in Coles and Eeckhout (2003), since off the equilibrium path sellers can commit to mechanisms that do not generate congestion effects, it is necessary to determine buyers' behaviour for such mechanisms.

If some sellers' sale mechanisms does not generate congestion effects, visit probability q satisfies

$$q \begin{cases} = 0 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 0) < \mathcal{R}_b(\pi_b, \gamma_b, 0) \text{ and } \mathcal{R}_a(\pi_a, \gamma_a, 1) < \mathcal{R}_b(\pi_b, \gamma_b, 1), \\ = 1 & \text{if } \mathcal{R}_a(\pi_a, \gamma_a, 0) > \mathcal{R}_b(\pi_b, \gamma_b, 0) \text{ and } \mathcal{R}_a(\pi_a, \gamma_a, 1) > \mathcal{R}_b(\pi_b, \gamma_b, 1). \end{cases}$$

However, if either $\mathcal{R}_a(\pi_a, \gamma_a, 0) \geq \mathcal{R}_b(\pi_b, \gamma_b, 0)$ and $\mathcal{R}_a(\pi_a, \gamma_a, 1) \leq \mathcal{R}_b(\pi_b, \gamma_b, 1)$ or $\mathcal{R}_a(\pi_a, \gamma_a, 0) \leq \mathcal{R}_b(\pi_b, \gamma_b, 0)$ and $\mathcal{R}_a(\pi_a, \gamma_a, 1) \geq \mathcal{R}_b(\pi_b, \gamma_b, 1)$, then both $q = 1$ and $q = 0$ are equilibria, along with any q satisfying (2). That is, when mechanisms do not generate congestion effects, buyers have an incentive to coordinate onto a common site, and the strategies allowing for this coodination are symmetric. Hence symmetry alone does not yield a unique equilibrium. I assume that in such cases the equilibrium selected is the mixed strategy equilibrium satisfying (2). It is possible to justify this selection by noting that in the symmetric pure strategy equilibrium one seller receives no visits and makes no profits, and hence has an incentive to offer a different mechanism at the offer stage.[12]

Equilibrium: With the equilibrium in the buyers' subgame fixed, buyer behaviour is characterised by q. When interior, its responses to information provision and mechanisms is given by (2). In the rest of the paper, an equilibrium refers to a subgame perfect equilibrium of the full

[11] To lessen notation, the visit probability generated by $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ will simply be denoted by q, with its dependence on information provision and mechanisms understood.

[12] Coles and Eeckhout (2003) give a different justification for ignoring pure strategy symmetric coordination equilibria. They note that since the mixed strategy equilibrium is always determined by (2), a seller that wishes to induce the mixed strategy outcome can always change his mechanism to induce congestion effects without varying rents and hence have the mixed equilibrium be the unique symmetric equilibrium of the subgame.

game with buyer strategies given by q. Throughout the paper, I consider symmetric equilibria in the sellers' strategies.

Sellers' Profits: Profits of seller k, given strategy profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, can be expressed as surplus less rents as

$$\mathcal{P}_k(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k}) = \mathbf{E}_\eta \mathbf{E}_\tau \left[\mathcal{S}_k^{\eta,\tau} - \eta R_k^{\eta,\tau} \right]. \tag{3}$$

The first expectation is with respect to the binomial distribution with parameter q (if $k = a$) or $1 - q$ (if $k = b$) of demand at site k, and the second with respect to the binomial distribution with parameter π_k over information states at site k.

A Characterisation of Incentive Compatible Mechanisms: Note that buyers' sorting decisions, as expressed by (2), depend only on information provision and expected rents $R_k^{\eta,\tau}$. In particular, buyer decisions are not affected by how rents are shared between types conditional on being informed. This ex ante feature of rent promises allows a useful characterisation of incentive-compatible mechanisms, which simplifies sellers' strategy sets. Crucially, as Lemma 6 in Appendix A.3 illustrates, we can restrict θ_H-type incentive-compatibility constraints to be binding in states $(1, i)$ and $(2, i)$. This is without loss of generality since any incentive-compatible mechanism at site k that achieves rents $\left\{R_k^{\eta,\tau}\right\}_{\substack{\eta \in \{1,2\} \\ \tau \in \{i,u\}}}$ with non-binding θ_H-type incentive constraints can be replaced by an incentive compatible mechanism that achieves the same levels of expected rents with the same allocations, but in which these constraints bind. Under this new mechanism, profits are unchanged and all traffic and information provision incentives are preserved. The proof is simple: given an incentive compatible mechanism in which the incentive constraint of θ_H-types in state (η, i) does not bind, we can increase θ_L-type rents and decrease θ_H-type rents through transfers until the constraint binds, while ensuring that the expected rents in demand state (η, i) are unchanged.

Denote by $\tilde{\Gamma}$ the set of incentive compatible mechanisms with binding θ_H-type incentive compatibility constraints. My result shows that restricting sellers to offering mechanisms in $\tilde{\Gamma}$ does not alter the set of equilibrium outcomes of the game, that is, information provision, allocations, rents and visit probabilities. Denote *low-type rents under mechanism* γ_k *in state* (η, τ) by $r_k^{\eta,\tau}$. These are the rents offered to θ_L-types in informed states and to the uninformed otherwise. Let $I_k^{\eta,i}$ be the expected informational rents to θ_H-types in state (η, i). Lemma 7 in Appendix A shows that mechanisms $\gamma_k \in \tilde{\Gamma}$ are characterised by monotone allocation probabilities, $r^{\eta,\tau} \geq 0$ for all states (η, τ) and expected rents

$$R_k^{\eta,u} = r_k^{\eta,u} \text{ for } \eta \in \{1, 2\},$$
$$R_k^{1,i} = r_k^{1,i} + I_k^{1,i},$$
$$R_k^{2,i} = r_k^{2,i} + I_k^{2,i}.$$

3 Fixed Mechanisms

In this section, sale mechanisms are exogenously fixed and common to both sale sites and sellers commit solely to information provision. This centers attention on information's impact on competition in which terms of trade have already been determined. Exogenous mechanisms constrain the rent offers sellers can extend to buyers through their choice of information provision. The goal of this section is to understand how sale mechanisms affect sellers' trade-off between traffic and profit-per-buyer, and through this equilibrium information provision.

3.1 Second-Price Auctions

I start with an example in which sellers hold second-price auctions without reserve prices irrespective of how many buyers visit them. As Board (2009) and Ganuza and Penalva (2006) derive the optimal information structures for monopolists in a second-price auction with two buyers, this example constitutes a useful benchmark to gauge the effects of competition.

With second-price auctions, buyers obtain the good for free in the one-buyer state, and capture the full surplus $\bar{\theta}$. In the two-buyer state, to bid their best estimate of their true value is a weakly dominant strategy for buyers. When uninformed, this best estimate is $\bar{\theta}$.

A buyer that attends site a, given π_a and q, expects rents

$$\mathcal{R}_a(\pi_a, q) = q\pi_a p_H p_L(\theta_H - \theta_L) + (1-q)\bar{\theta},$$

while a bidder attending site b, given π_b and q, expects rents

$$\mathcal{R}_b(\pi_b, q) = (1-q)\pi_b p_H p_L(\theta_H - \theta_L) + q\bar{\theta}.$$

In the mixed strategy equilibrium of the buyers' subgame, the probability with which buyers visit site a is given by

$$q = \frac{\bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}{\bar{\theta} - \pi_a p_H p_L(\theta_H - \theta_L) + \bar{\theta} - \pi_b p_H p_L(\theta_H - \theta_L)}. \tag{4}$$

The profits of seller a, given (π_a, π_b) and the resulting q, are given by

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= q^2\left[\pi_a\Big(p_H^2\theta_H + (1-p_H^2)\theta_L\Big) + (1-\pi_a)\bar{\theta}\right] \\ &= q^2\big[\bar{\theta} - \pi_a p_H p_L(\theta_H - \theta_L)\big]. \end{aligned} \tag{5}$$

The term in the brackets of (5) is the expected price paid by the buyer who obtains the good in the two-buyer state. This price decreases in π_a, since the seller then gives away a higher share of the surplus as informational rents. Denote this price by $w_a(\pi_a)$. Suppose a single second-price auctioneer faced a fixed set of two buyers, then its profits given information provision π would be $w(\pi)$.

Proposition 1. (No Information under Monopoly) *A second-price auctioneer with no reserve price facing two buyers maximises profits by setting* $\pi = 0$.

This result is known from Board (2009) and Ganuza and Penalva (2006). Returning to my model, note that (4) can be rewritten as

$$q = \frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}. \tag{6}$$

Since buyers get all the surplus if alone, q depends only on how much profits sellers get from demand states with two buyers. Thus (5) becomes

$$\begin{aligned}\mathcal{P}_a(\pi_a, \pi_b) &= \left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)}\right]^2 w_a(\pi_a) \\ &= w_b(\pi_b)\left[\frac{w_b(\pi_b)}{w_a(\pi_a) + w_b(\pi_b)} \cdot \frac{w_a(\pi_a)}{w_a(\pi_a) + w_b(\pi_b)}\right] \\ &= w_b(\pi_b) q(1-q). \end{aligned} \tag{7}$$

Clearly, seller a's choice of information influences profits in (7) only through its effect on $q(1-q)$, which attains a maximum when $q = \frac{1}{2}$. Seller a can attain this maximum by setting $\pi_a = \pi_b$. This leads to the following result.

Proposition 2. (Any Level of Information under Competition) *When the sale mechanism is a second-price auction with no reserve price,* (π_a, π_b) *is an equilibrium if and only if* $\pi_a = \pi_b$.

This surprising result states that a seller's best-response to any information offer by an opponent is to match that promise. Rewriting the rents of a buyer attending site a yields

$$\mathcal{R}_a(\pi_a, q) = \bar{\theta} - q w_a(\pi_a). \tag{8}$$

That is, it is as though seller a gives an entering buyer an 'attendance fee' $\bar{\theta}$, but imposes a 'congestion charge' of $w_a(\pi_a)$ when the other buyer is also present. Rents can be rewritten in this particular form since the sale mechanism is a second-price auction with no reserve price, yet this does not depend on my assumptions about buyers' types and sellers' information structures. Suppose buyers' true valuations were instead given by some continuous random variable Y with mean $\bar{\theta}$. Denote by $Y_{1:2}$ and $Y_{2:2}$ the expected values of the first and second order statistics of Y, then $Y_{1:2} + Y_{2:2} = 2\bar{\theta}$. Rewriting rents as in (8) uses the discrete version of this identity, which in turn allows the representation of profits in (7). Similarly, this result is not due to my special correlated information structures. If instead π indexed ex post valuations Y^π with $\mathbf{E}Y^\pi = \bar{\theta}$ for all π, then $Y^\pi_{1:2} + Y^\pi_{2:2} = 2\bar{\theta}$ for all π, and the result of Proposition 2 still follows. Hence while the result of Proposition 2 is not due to my model's special information structures, it does depend critically on there being only two buyers and two sellers.[13]

[13]The result of Proposition 2 also depends critically on the other assumptions of the model, for example that information provision is costless. Say providing information required a cost of c. Then seller a's profits would be given by (7), less some cost term that depends on q and c. Thus at symmetric profiles marginal profits are negative, so that the only symmetric equilibrium has no information provision.

3.2 Equilibrium with Regular Mechanisms

The case of second price auctions, while special, demonstrates that the set of equilibria in information provision for any exogenous incentive compatible mechanisms will be difficult to deal with in general. Here, I introduce a class of mechanisms, called *regular mechanisms*, which impose restriction on mechanisms' allocations and rent offers to uninformed types.[14]

Definition 1. (No Waste) *A mechanism $\gamma_k \in \Gamma$ has no waste if and only if the good is always delivered to some buyer.*

In a mechanism with no waste, the full surplus ($\bar{\theta}$) is realized in the one-buyer and uninformed states, while in state $(2, i)$ the full surplus is realized only when a θ_L-type never obtains the good when a θ_H-type is present.

Definition 2. (Regular Mechanisms) *An incentive compatible mechanism γ is regular if and only if*

i. (Exploiting the uninformed) $R^{1,u} = R^{2,u} = 0$.

ii. (Congestion effects) $R^{1,i} > R^{2,i}$.

iii. γ *has no waste.*

Property *i* states that in uninformed states a regular mechanism fully exploits the buyers' lack of information. Sellers benefit from restricting buyers' information through an easing of incentive constraints and in regular mechanisms, sellers capture all gains from trade when buyers have no private information. Property *ii* states that regular mechanisms generate *congestion effects* and a buyer strictly prefers being alone at a selling site. Finally, that regular mechanisms have *no waste* is a sufficient condition for expected surplus in the two-buyer state to be increasing in information provision, that is, $\mathcal{S}^{2,i} \geq \bar{\theta}$. Total available surplus in the two-buyer state always increases in information provision, yet the sale mechanism's allocation rules may sufficiently restrict delivery of the good in informed states that realized surplus decreases in information provision.

Regular mechanisms combine the properties that make the study of ex ante competition through information provision interesting: sellers extract more rents from poorly informed buyers; buyers, who compete for goods, dislike the presence of other buyers and; information provision does not solely redistribute rents, but enhances total surplus. In informed states, standard mechanisms that always deliver the good to some buyer, such as auctions with reserve prices lower than θ_L, or a uniform price (independent of demand state) less than θ_L, can be components of regular mechanisms when combined with take-it-or-leave-it offers of $\bar{\theta}$ in uninformed states.

[14]All definitions that involve mechanism allocations are stated formally in Appendix A.4.

Under a regular mechanism γ, seller a's profits are

$$\mathcal{P}_a(\pi_a,\gamma,\pi_b,\gamma) = q^2\left[\pi_a\mathcal{S}^{2,i} + (1-\pi_a)\bar{\theta} - 2\pi_a R^{2,i}\right] + 2q(1-q)\left[\bar{\theta} - \pi_a R^{1,i}\right]. \tag{9}$$

Seller a's profits in the one-buyer state, $\bar{\theta} - \pi_a R^{1,i}$, are clearly decreasing in π_a. Furthermore, as shown in Appendix A.4, that γ has no waste implies that seller a's profits in the two-buyer state, $\pi_a\mathcal{S}^{2,i} + (1-\pi_a)\bar{\theta} - 2\pi_a R^{2,i}$, are also decreasing in π_a. Note that this implies that, as in the case of second price auctions, a monopolist facing two buyers under regular mechanisms would never provide information. Hence, any information provision achieved in equilibrium with regular mechanisms is due to competition.

At symmetric profiles, the market is shared equally between the two sellers. In particular, an equally split market maximises the probability that a seller is visited by a single buyer ($2q(1-q)$), which means that marginal shifts in information provision at symmetric profiles have no effect on this probability.[15] This simplifies the expression for marginal profits at symmetric profiles under regular mechanism γ, which is given by

$$\left.\frac{\partial\mathcal{P}_a(\pi_a,\gamma,\pi_b,\gamma)}{\partial\pi_a}\right|_{\pi_a=\pi_b=\pi} = \left.\frac{\partial q}{\partial\pi_a}\right|_{\pi_a=\pi_b=\pi}\left[\pi\mathcal{S}^{2,i} + (1-\pi)\bar{\theta} - 2\pi R^{2,i}\right] + \frac{1}{4}\left[\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i}\right] - \frac{1}{2}R^{1,i}. \tag{10}$$

The first term of (10) is the *increased traffic effect* of an increase in information provision, which says that seller a gains two-buyer state profits more often. The two last terms are the *decreased profit-per-head effect*, since seller a now hands over more rents to all visiting buyers in each state. Since the right-hand side of (10) can cross 0 at most once, regular mechanisms produce a unique symmetric equilibrium candidate profile.

Lemma 1. (Unique Candidate for Symmetric Equilibrium) *In games with regular mechanisms, there is a unique candidate profile for symmetric equilibrium in information provision, given by*

$$\pi^* \equiv \begin{cases} \frac{-(R^{1,i}+R^{2,i})\bar{\theta}}{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))} & \text{if } 2R^{1,i} > \bar{\theta} \text{ and } R^{1,i}+R^{2,i} > \frac{2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})}{2R^{1,i}-\bar{\theta}}, \\ 1 & \text{otherwise.} \end{cases} \tag{11}$$

Clearly, with regular mechanisms, no equilibrium with $\pi = 0$ can exist, as uninformed buyers get no rents and any deviation by some seller from such a profile to any $\pi' > 0$ would attract all buyers. Hence relative to monopoly, competition always improves informational efficiency. Lemma 1 depends on the fact that the decreased profit-per-head effect is negative and does not

[15] This observation, often useful in the the rest of the paper, is due to the binomial distribution of demand at sale sites. That is, if $X \sim B(n,q)$ then $\frac{\partial \Pr(X=k)}{\partial q} > 0$ whenever $k > qn$, where qn is the mean state of X. If qn is an integer, then $\frac{\partial \Pr(X=qn)}{\partial q} = 0$. That is, if q is increased marginally, states above the mean state become more likely and states below the mean less likely, while the probability of the mean state is unchanged.

depend on π_a by the linearity of the information structures. Also, I show in Appendix B that $\frac{\partial q}{\partial \pi_a}\big|_{\pi_a=\pi_b=\pi}$ is decreasing in π, the symmetric level of information provision. This, along with the fact that under regular mechanisms sellers' profits in the two-buyer state are decreasing in π, implies that the increased traffic effect, though positive, is decreasing in π. That is, buyers are less sensitive to information provision when in a high-information environment, and also in such environments the profits generated by more frequent buyer visits are lower.

Since the profit function in (9) is not concave in π_a, (10) alone is not sufficient to establish the existence of a symmetric equilibrium. In fact, the behaviour of (9) in π_a is complex. In Appendix B, I present conditions on mechanisms' rents that guarantee that seller a's profit function is single-peaked around $\pi_a = \pi^*$ when $\pi_b = \pi^*$ and $\pi^* < 1$.[16] In the same way, it is possible to derive sufficient conditions for the existence of full-information equilibria when $\pi^* = 1$. I focus on interior symmetric equilibria in order to derive comparative statics results that describe how varying the features of regular mechanisms affect equilibrium levels of information provision and seller profits.

First, I consider shifts in the allocative efficiency of the mechanisms that leave rents unchanged. These changes can be implemented by changing mechanisms' allocations and adjusting rents through transfers. I then consider shifts in rents that leave expected surplus unchanged. Such shifts can be implemented through changes in transfers, without affecting allocations. Let Ψ be the set of regular mechanisms γ such that: (i) The information provision game between sellers with mechanism γ has a unique symmetric equilibrium (π^*, π^*) with $\pi^* < 1$, and; (ii) There exists a neighbourhood N of γ in the space of regular mechanisms such that any $\hat{\gamma} \in N$ induces a unique symmetric equilibrium in information provision $(\hat{\pi}^*, \hat{\pi}^*)$ with $\hat{\pi}^* < 1$. The proof of Proposition 3 shows Ψ to be nonempty.

Proposition 3. (Information Provision Increases in Surplus and Decreases in Rents) *For a regular mechanism $\gamma \in \Psi$, the symmetric equilibrium information provision π^* is such that*

$$\frac{\partial \pi^*}{\partial \mathcal{S}^{2,i}} > 0,$$

and

$$\frac{\partial \pi^*}{\partial R^{1,i}} < \frac{\partial \pi^*}{\partial R^{2,i}} \leq 0,$$

with $\frac{\partial \pi^*}{\partial R^{2,i}} = 0$ if and only if $\mathcal{S}^{2,i} = \bar{\theta}$.

Higher surplus in the two-buyer state increases the gains to information provision, which leads to increased competition between sellers and more information provision. Competition generates a complimentarity between allocative and informational efficiency. More efficient mechanisms, by realising higher surplus, lead sellers to attempt to capture more of it by providing information.

[16] See the proof of Proposition 3.

Similarly, more generous mechanisms lead to lower equilibrium information provision, since higher rents dampen the competition between sellers by increasing the cost of attracting more buyers. However, the drop in equilibrium information provision is more pronounced when rents in the one-buyer state rather than in the two-buyer state are increased. Rewrite buyer rents from attending site a in (1) as

$$\mathcal{R}_a(\pi_a, \gamma, q) = \pi_a \left[R^{1,i} - q(R^{2,i} - R^{1,i})\right]. \tag{12}$$

That is, when attending site a and conditional on being informed, it is as though a buyer is paid a 'attendance fee' of $R^{1,i}$, while he suffers a 'congestion charge' of $R^{2,i} - R^{1,i}$ whenever the other buyer is also present. An increase in $R^{1,i}$ affects buyer rents to attending site a in two ways as both the attendance fee and the congestion charge increase. The second effect reduces buyers' incentives to visit a deviating seller with higher probability, as this increases their chance of meeting at the same site. This buyer inertia further softens the competition between sellers. On the other hand, an increase in $R^{2,i}$ reduces the congestion charge suffered by a buyer at site a. By making buyers less averse to meeting their opponents at a site, this increases sellers' incentives to deviate from symmetric profiles and hence intensifies competition between them.

Proposition 4. (Profits Decrease in Surplus and Nonmonotone in Rents) *For a regular mechanism γ, suppose conditions i and ii of Proposition 3 hold. Then*

$$\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial \mathcal{S}^{2,i}} < 0.$$

Furthermore,

$$\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial R^{1,i}} > 0$$

$$\text{and} \quad \frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial R^{2,i}} \geq (<)\ 0 \ \text{if} \ R^{1,i} + R^{2,i} \leq (>) \frac{\mathcal{S}^{2,i} - \bar{\theta}}{\sqrt{2}}.$$

Since equilibrium profits are decreasing in two-buyer state surplus, sellers' preferred mechanisms generate inefficiencies in allocations. This is the more negative implication of the complementarity between allocative and informational efficiency; given a choice, sellers would lower both to soften their competitive environment. If sellers could collude and commit to sales mechanisms while anticipating future competition in information they would protect themselves against its effects by selecting mechanisms with inefficient allocations.

According to Proposition 4, when rents in the one-buyer state increase, the drop in the equilibrium level of information provision raises profits enough to compensate for the rent increase, while this is not always the case for increases in rents in the two-buyer state. For example, note that under any mechanism in which $\mathcal{S}^{2,i} = \bar{\theta}$, we have that $\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial R^{2,i}} < 0$. Since changes in rents are achieved through transfers, the different mechanisms considered in the second result of Proposition 4 have the same allocative efficiency. However, the mechanisms preferred by sellers may foster inefficient outcomes by leading to low levels of information provision.

3.3 Examples

3.3.1 Ex Post Optimal Mechanisms

The results of the previous section can be used to study the situation in which sellers commit to levels of information provision but cannot commit to sale mechanisms. In that case, once buyers have chosen sale sites, sellers deliver their good through each states' *ex post optimal mechanisms.* When buyers are uninformed, sellers optimally make take-it-or-leave-it offers of $\bar{\theta}$. When buyers are informed, the optimal mechanisms for both the one and two-buyer states depend on whether or not sellers prefer to exclude θ_L-types and sell only to θ_H-types. For both demand states, a seller strictly prefers to sell to θ_L-types whenever $\theta_L > p_H\theta_H$. When θ_L-types are excluded, sellers extract all informational rents from θ_H-types. In that case, buyers expect no rents from any demand state regardless of the level of information provision. The interesting case is when $\theta_L > p_H\theta_H$ and informed θ_H-types obtain rents.

Assumption 1. (No Exclusion under Ex Post Optimal Mechanisms) $\theta_L > p_H\theta_H$.

Note also that under Assumption 1, the ex post optimal mechanisms are regular and can be described by rent levels for low types $r^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$ and expected informational rents $I_k^{1,i} = p_H(\theta_H - \theta_L)$ and $I_k^{2,i} = \frac{1}{2}p_Lp_H(\theta_H - \theta_L)$. By Lemma 1, there is a unique candidate π^* for symmetric equilibrium and since under ex post optimal mechanisms $R^{1,i} = p_H(\theta_H - \theta_L)$, it follows that

$$\begin{aligned} 2R^{1,i} - \bar{\theta} &= p_H\theta_H - p_L\theta_L - 2p_H\theta_L \\ &= p_H\theta_H + p_L\theta_L - 2\theta_L \\ &< \theta_L(p_L - 1) \\ &< 0, \end{aligned}$$

where the first inequality follows from $\theta_L > p_H\theta_H$. Thus, by (11), under optimal sale mechanisms, the only candidate for symmetric equilibrium is full information provision, which can be shown to constitite an equilibrium.

Proposition 5. (Full Information with no Commitment to Mechanisms) *Under Assumption 1 and ex post optimal mechanisms, the unique symmetric equilibrium has full information provision.*

To show that full information provision is indeed a symmetric equilibrium, I show that seller a's profits are increasing in π_a when $\pi_b = 1$. When buyers face the optimal mechanisms once sorted, expected rents are low. This increases the sensitivity of their sorting decisions to shifts in information provision and enhances sellers' traffic-stealing incentives. Sellers achieve their favoured ex post outcomes, yet competition leads them to make their most costly ex ante information commitments.

3.3.2 Pricing Mechanisms

Consider *pricing mechanisms*, where $t^{\eta,\tau}$ is the price charged by the sellers in state (η, τ). Note that, as in Coles and Eeckhout (2003), I allow sellers to set prices that vary across demand states. When two buyers are present at the same site and both their values exceed the relevant price, each obtains the good with equal probability. Such a pricing mechanism is regular if

i. *(Exploiting the uninformed)* $t^{\eta,u} = \bar{\theta}$ *for* $\eta \in \{1, 2\}$.

ii. *(Congestion effects)* $\bar{\theta} - t^{1,i} > \frac{1}{2}\left(\bar{\theta} - t^{2,i}\right)$.

iii. *(No waste)* $t^{1,i}, t^{2,i} \leq \theta_L$.

Thus, by Lemma 1, under pricing mechanisms respecting *i*, *ii* and *iii* there is a unique candidate π^* for symmetric equilibrium. Furthermore, if $\pi^* < 1$, this candidate profile is indeed a symmetric equilibrium.[17] From (11), $\pi^* < 1$ if

$$t^{1,i} < \frac{\bar{\theta}}{2}. \tag{13}$$

The second condition of (11) is always satisfied for pricing mechanisms since $\mathcal{S}^{2,i} = \bar{\theta}$. Thus, for $t^{1,i}$ and $t^{2,i}$ satisfying *ii*, *iii* and (13) the level of information provision in symmetric equilibrium is given by

$$\pi^* = \frac{\bar{\theta}}{2(\bar{\theta} - t^{1,i})} < 1,$$

which does not depend on $t^{2,i}$ and is increasing in $t^{1,i}$ (decreasing in $R^{1,i}$). Applying Proposition 4 to this example, it follows that

$$\frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial t^{1,i}} < 0$$
$$\text{and} \quad \frac{\partial \mathcal{P}_a(\pi^*, \gamma, \pi^*, \gamma)}{\partial t^{2,i}} > 0.$$

That is, a seller's preferred pricing mechanism charges a minimal price in state $(1, i)$ and a maximal price in state $(2, i)$, while still respecting *ii*, *iii* and (13). This happens when $t^{1,i} = 0$ and $t^{2,i} = \theta_L$. In this pricing mechanism, sellers give away the good when one buyer is present but charge the highest price that leads to no exclusions in the two-buyer state. Equilibrium information provision is $\pi^* = \frac{1}{2}$. The sellers' favoured pricing mechanism has low information provision and makes buyers very averse to meeting one another at the same site by providing large rents to a buyer who is alone. While regular pricing mechanisms are equally efficient with respect to informed allocations, the mechanism most preferred by sellers is the least informationally efficient.

[17] This follows by (18) in Appendix B.

4 Endogenous Mechanisms

In this section, sellers commit jointly to information provision and sale mechanisms. Before stating my main result, the following definition provides further properties of mechanisms' allocations.

Definition 3. (Partial and Full Allocative Efficiency) *A mechanism $\gamma_k \in \Gamma$ has partial allocative efficiency (PAE) if and only if the good is always sold to some buyer in uninformed states, and to a θ_H-type in informed states if such a type is present.*

A mechanism $\gamma_k \in \Gamma$ has full allocative efficiency (FAE) if and only if it has partial allocative efficiency and the good is always sold to a θ_L-type in informed states if no θ_H-type is present.

Under FAE, the surplus in state $(2, i)$ is maximized and denoted it by $\bar{\mathcal{S}}^{2,i}$. A mechanism with PAE may exclude θ_L-types.[18]

Proposition 6. (Symmetric Equilibrium with Endogenous Mechanisms) *Under Assumption 1, $(\pi, \gamma, \pi, \gamma) \in ([0,1] \times \Gamma)^2$ is a symmetric equilibrium if and only if $\pi = 1$, γ has full allocative efficiency, $R^{2,i} \leq R^{1,i}$ and $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$.*

Proposition 6 characterises symmetric equilibria under Assumption 1. While this assumption guarantees allocative efficiency in monopoly, efficient mechanisms also lead monopolists not to provide information. This does not happen here as sellers manage to disentangle information and rent provision decisions even in the presence of competition. Sellers' incentives to do so stem from the fact that while providing rents is solely redistributive, providing information enhances efficiency. Sellers post auctions and take advantage of their allocative efficiency by providing full information. Competition then determines non-distortionary rents.

There is a continuum of equilibria that are ranked from the most favourable to sellers (with rents $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and $r^{2,i} = 0$) to the most favourable to buyers (with rents $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and $R^{2,i} = R^{1,i}$). All mechanisms have congestion effects and as seen in Section 4.3, the condition that $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ has the interpretation that the seller equates the rents owed the marginal buyer ($R^{1,i}$) to its contribution to site surplus ($\frac{\bar{\mathcal{S}}^{2,i}}{2}$). The equilibria differ in how the surplus is shared between buyers and sellers, yet full information, FAE and symmetric seller strategies ensure that outcomes are (constrained) efficient.[19] Profits are not driven to zero in any equilibrium. In the one-buyer state, profits are positive since they are given by $\bar{\theta} - \frac{\bar{\mathcal{S}}^{2,i}}{2}$ and it is the case that $2\bar{\theta} > \bar{\mathcal{S}}^{2,i}$. In the two-buyer state, profits are $\bar{\mathcal{S}}^{2,i} - 2R^{2,i}$, which is positive except in the equilibrium most favourable to buyers. That sellers do not compete away all profits in the

[18]To relate this to earlier definitions, any mechanism with FAE has no waste, but a mechanism with no waste may allocate the good to a θ_L-buyer in the presence of a θ_H-buyer in state $(2, i)$. A mechanism with PAE need not have no waste.

[19]In the presence of coordination among buyers, the efficient distribution of buyers across sale sites has one of them with each seller. In the absence of coordination, efficiency requires maximising the likelihood of having one buyer at each site, which happens when $q = \frac{1}{2}$.

presence of traffic effects has been noted in the literature on competing auctions.[20] Congestion effects and mixed strategies by buyers smooth out jumps in demand induced by changes in rent offers and competition between sellers is not as fierce as in Bertrand competition.

The continuum of rent levels supported in equilibrium is closely related to Coles and Eeckhout (2003). Adjusting for the fact that with high and low-type buyers surplus levels vary across demand states and that incentive constraints imply that buyers cannot be made to expect zero rents, the rent levels pinned down by Proposition 6 mirror theirs. In their paper with known valuations, a mechanism consists of demand state-dependent prices which are all equally efficient. In my model, information provision, allocations and rent levels are interdependent and must be determined simultaneously. The benefits of screening between types imply that in my model auctions have an efficiency advantage. A by-product of my model's setup is that it yields a clear interpretation of why competition fixes rents only in the one-buyer state, which is simply a consequence of equating marginal rents to marginal contributions to site surplus.

4.1 Equilibrium Information Provision

This section derives necessary conditions for full information provision in equilibrium.

Lemma 2. (Full or No Information in Equilibrium) *Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is an equilibrium, that $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_a^{\eta,\tau}$ is strictly increasing (decreasing) in π_a, and that it is not the case that γ_a and γ_b are the ex post optimal mechanisms. Then $\pi_a = 1$ ($\pi_a = 0$).*

Intuitively, as information provision increases the potential size of the surplus, it allows Pareto-improving deviations for sellers from any profile with less than full information. Fixing a profile of mechanisms, information provision also has a distributive effect through rents as it shifts probability among information states within and across demand states. However, since sellers commit ex ante to both state-contingent rents and information, consider a deviation from a strategy profile with less than full information in which a seller increases information provision and offsets its effect on buyer rents through transfers. In this way, buyers' sorting decisions are unaffected and sellers pocket the newly generated surplus. The one proviso to the above argument is that the initial mechanisms must be such that more information actually increases the expected surplus at site k, $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_k^{\eta,\tau}$. If some buyer types are excluded by the mechanism, this need not be the case. However, in this case, reduced information provision will generate constrained efficiency gains that the seller can capture through transfers.

The proof shows that given an equilibrium in which surplus is increasing in information provision with $\pi_a < 1$, unless it is the case that buyers' rents are at a minimum (i.e. at the ex post optimal mechanisms), seller a can always increase information provision and adjust transfers

[20] In Peters and Severinov (1997) where, as is the case here, buyers sort into sites before observing their values, the (unique) symmetric equilibrium in reserve prices of second-price auctions is bounded away from cost. This is true also in the duopolistic model of Burguet and Sákovics (1999) where, however, buyers know their values before sorting.

so as to keep buyer rents constant. Allocations are unchanged and so, by assumption, higher surplus is generated and seller profits increase, contradicting the fact that π_a can be a component of an equilibrium.

Full-information symmetric equilibria could arise for some of the exogenous mechanisms of Section 3. However, as sellers could not commit to sale mechanisms, the rationale for their existence was quite different. There, increasing information provision was profitable only if the increase in traffic generated compensated the seller for the higher rents now offered to buyers and full-information equilibria exist under mechanisms that generate more incentives for traffic-stealing. With ex ante commitments to mechanisms, Lemma 2 shows that sellers can deviate to a full information profile and capture its efficiency benefits without concerning themselves with traffic effects, since they control state-contingent rents. Although information provision is also efficient when mechanisms are exogenously fixed, sellers lack the tools to exploit it.

The intuition that sellers can exploit efficiency gains through ex ante offers is very general. The result of Esö and Szentez (2007), while apparently similar, presents significant differences. When the seller controls the release of signals but does not observe their realizations, Esö and Szentez (2007) show that it can achieve the same allocation and profits as under the optimal mechanism in the case in which it directly observes the signals by suitably controlling the entry fees paid by buyers before they get access to the new information. In my paper, sellers compete for buyers and cannot charge entry fees. Given any strategy profile for sellers with $\pi_k < 1$, I need to check whether seller k has a profitable deviation that involves an increase in information provision, which is the case when expected surplus is increasing in information provision. The reason why such a deviation cannot be guaranteed for any profile of mechanisms is that, as put by Bergemann and Pesendorfer (2007), buyers' participation constraints must hold ex post.

4.2 Equilibrium Allocations

This section presents results on the efficiency of equilibrium allocations in the game with endogenous mechanisms. The first result shows that holding auctions is weakly dominant.[21]

Lemma 3. (No Exclusions of θ_H or Uninformed Types) *A strategy (π_k, γ_k) for seller k in which γ_k does not have partial allocative efficiency is weakly dominated.*

More specifically, for any profile in which seller k posts a mechanism that does not have PAE, I can find an alternative mechanism with PAE that leaves buyer rents and hence visit decisions unchanged and yields strictly higher profits to seller k, whenever buyers visit seller k with positive probability. This result states that not only will equilibrium mechanisms have PAE, but that it is without loss of generality when searching for equilibria to consider deviations from candidate profiles that have PAE.

[21] This is as in McAfee (1993), where, however, the focus is on large markets. In small markets, arguments must consider the effect of a change in any seller's mechanism on market-wide rents and profits.

The proof deals with θ_H-type and uninformed allocations separately, and mirrors analogous results in the monopoly framework. It shows that profits can be increased and θ_L-types made less willing to mimic θ_H-types if seller k increases θ_H-type allocations and transfers simultaneously, keeping θ_H-types at the same level of rents.[22] Similarly, a profile in which uninformed buyers are excluded with positive probability is vulnerable to a deviation where a seller increases both allocation probabilities and transfers, keeping buyers at the same level of rents.

In my model, the classic arguments from the monopoly case that determine θ_L-type allocations cannot be applied directly due to their competitive effects on traffic across sale sites. In the monopoly case, Assumption 1 determines whether sellers excludes θ_L-types in either demand state, since in any mechanism in which θ_L-types are excluded with some probability, the seller can increase profits by increasing both θ_L-types' allocation probabilities and transfers, keeping their rent level constant, even if this increases θ_H-type rents (through the binding incentive compatibility constraint for θ_H-types). This increases rents expected over informed types. The problem with this argument in my framework is that an increase in rents in any state increases traffic but may decrease the likelihood of the one-buyer state (when $q > \frac{1}{2}$), and hence its effect on total profits may depend on the relation between profits in the one-buyer and two-buyer states. The next result, unlike Lemma 3, presents only a necessary condition on θ_L-type allocations in symmetric equilibria.

Lemma 4. (No Exclusion of θ_L-types in Symmetric Equilibrium) *Under Assumption 1, if $(\pi, \gamma, \pi, \gamma)$ is a symmetric equilibrium, then γ has full allocative efficiency.*

From Lemma 3, PAE is necessary for any equilibrium in which both sellers are visted with positive probability, and in a symmetric equilibrium $q = \frac{1}{2}$. To show that under Assumption 1, θ_L-types always receive the good in the absence of θ_H-types in a symmetric equilibrium, the proof applies the argument for the monopoly case outlined above to find a deviation from any symmetric equilibrium that violates FAE. The difficulty mentioned above is dealt with by the fact that at a symmetric profile small increases in traffic have a negligible effect on the probability of the one-buyer state. The proof of Lemma 4 also guarantees that profits in the two-buyer state are nonegative.

Without Assumption 1, a seller wants to exclude θ_L-types to depress θ_H-type rents. Marginally, whether this is profitable depends on whether the increased profits from θ_H-types compensate the drop in traffic in the two-buyer state. This traffic-rents trade-off will also involve the level of information provision. Without Assumption 1, it is difficult to derive a simple necessary condition on θ_L-type allocations which, as above, does not depend on information provision.

[22] In the two-buyer state, it may be the case that θ_L-types receive the good even in the presence of θ_H-types, and that the ressource constraint binds, so that the seller cannot allocate the good more often to θ_H-types without allocating it less often to θ_L-types. But then the seller can simply 'free up' allocation probabilities by delivering the good less often to θ_L-types and keep their rents constant by decreasing their transfers.

4.3 Equilibrium Rents

This section derives necessary conditions on equilibrium rents under Assumption 1.

Lemma 5. (Equilibrium Rents) *Under Assumption 1, if $(\pi, \gamma, \pi, \gamma)$ is a symmetric equilibrium, then $R^{2,i} \leq R^{1,i}$ and $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$.*

Under the regular mechanisms of Section 3, buyers faced congestion effects and preferred being alone at a sale site when informed. Lemma 5 confirms that a seller will always impose congestion effects in a symmetric equilibrium. The intuition for this is as follows. As in (12), rewrite a buyer's expected rents at site a from a symmetric profile with $\pi = 1$ as

$$R^{1,i} + q(R^{2,i} - R^{1,i}), \tag{14}$$

that is, as an 'attendance fee' of $R^{1,i}$ along with a 'bonus' ('congestion charge') of $R^{2,i} - R^{1,i}$ when another buyer attends and $R^{2,i} > R^{1,i}$ ($R^{2,i} \leq R^{1,i}$). If $R^{2,i} > R^{1,i}$, decreasing $R^{2,i}$ lowers the bonus, but buyers remain indifferent between attending sites a and b only if this bonus is handed out more often, i.e., if q increases. As sellers can decrease rents while increasing traffic, profiles with $R^{2,i} > R^{1,i}$ admit a profitable deviation.

The condition $R^{1,i} = \frac{\bar{\mathcal{S}}^{2,i}}{2}$ states that the marginal buyer attending a site is awarded his marginal contribution to site surplus.[23] To see this, note that seller a's profits at symmetric profiles with FAE are marginally increasing in $R^{1,i}$ (or $R^{2,i}$) whenever $R^{1,i} < \frac{\bar{\mathcal{S}}^{2,i}}{2}$.[24] A marginal buyer drawn to site a by a marginal change in rents receives $R^{1,i}$, its 'attendance fee', from seller a. On the other hand, this marginal buyer brings its share of the surplus when another buyer is also present, $\frac{\bar{\mathcal{S}}^{2,i}}{2}$, to site a. Since the probability of the one-buyer state is unaffected by small changes in q at a symmetric profile, a marginal buyer brings nothing to that state. A seller will want to attract a marginal buyer whenever his contribution exceeds the cost of luring him. Similarly, if $\frac{\bar{\mathcal{S}}^{2,i}}{2} < R^{1,i}$, a seller can gain by shedding a marginal buyer through a decrease in rents.

4.4 Sufficiency

The proof of Proposition 6 follows from the results of the previous sections. The necessity of FAE for symmetric equilibrium has been established in Lemma 4. Under FAE, information provision increases the surplus available at a selling site since two buyers generate more surplus when informed than when uninformed, as $\mathcal{S}^{2,i} = \bar{\mathcal{S}}^{2,i} > \bar{\theta}$ and $\mathcal{S}^{1,i} = \bar{\theta}$, and hence Lemma 2 states that $\pi = 1$ is necessary for symmetric equilibrium unless both sellers commit to the ex post optimal mechanisms. The necessity of full information under Assumption 1 for ex post optimal mechanisms follows from Proposition 5. Lemma 5 provides the conditions for equilibrium rents.

[23] Interpret the marginal buyer as the mass involved in a marginal increase in q.
[24] This follows from (26) in Appendix B.

Note that $R^{2,i} \leq R^{1,i} = \frac{\bar{S}^{2,i}}{2}$ implies that $2R^{2,i} \leq \bar{S}^{2,i}$ and hence that profits in the two-buyer state are nonegative. The sufficiency argument is direct; taking a profile satisfying the conditions of the proposition, I show that no deviation can be profitable.

5 Conclusion

This paper has analysed the strategic interactions of sellers who compete for buyers by committing to information provision. When mechanisms are exogenously fixed and sellers compete solely through offers of information, they may prefer to compete in high-rent environments, as these lessen the intensity of competition and lead to lower information provision. Furthermore, as higher surplus mechanisms increase sellers' competitive incentives to provide information, they prefer to compete in environments with low allocative efficiency, and hence low information provision. When sellers commit to both information provision and mechanisms, under a no-exclusion assumption, all symmetric equilibria have full information provision. However, a variety of rent levels are supported in equilibrium as a result of different equilibrium offers of mechanisms. In a sense, this result shows that sellers prefer to compete through mechanisms rather than through information provision. By doing so they maximize the available surplus, and competition determines the equilibrium share of this surplus going to buyers.

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A Appendix: Sale Mechanisms

A.1 Definitions

Let $\Psi^{\eta,\tau}$ denote the set of *report profiles* that can be received by the seller in state (η, τ). That is,

$$\Psi^{\eta,\tau} = \begin{cases} \{(\theta_m, \theta_n)\}_{(m,n)\in\{L,H\}^2} & \text{if } \eta = 2 \text{ and } \tau = i, \\ \{\theta_m\}_{m\in\{L,H\}} & \text{if } \eta = 1 \text{ and } \tau = i, \\ \emptyset & \text{if } \tau = u. \end{cases}$$

An *anonymous direct mechanism* for seller k is a collection of functions

$$\Big\{\{x_k^{\eta,\tau} : \Psi(\eta,\tau) \to [0,1], y_k^{\eta,\tau} : \Psi(\eta,\tau) \to \mathbf{R}\}\Big\}_{\substack{\eta\in\{1,2\}\\ \tau\in\{i,u\}}},$$

where $x_k^{\eta,\tau}(\psi)$ and $y_k^{\eta,\tau}(\psi)$ are, respectively, the probability a buyer obtains the good and the transfer he must pay to seller k when the report profile is $\psi \in \Psi^{\eta,\tau}$ in state (η, τ). Since no report is necessary when buyers are uninformed, I write probabilities and transfers as $x_k^{\eta,u}$ and $y_k^{\eta,u}$, respectively, for $\eta \in \{1, 2\}$. Also, since mechanisms are anonymous, define $x_k^{2,i}(\theta_m, \theta_n)$ as the probability that a buyer reporting θ_m obtains the good when the other buyer reports θ_n. A similar remark holds for the transfer function $y_k^{2,i}(\theta_m, \theta_n)$. The allocation probabilities satisfy

$$\begin{aligned} &x_k^{1,\tau}(\psi) \le 1 \quad \text{for } \psi \in \Psi^{1,\tau} \text{ and } \tau \in \{i,u\}, \\ &x_k^{2,u} \le \frac{1}{2}, \\ &x_k^{2,i}(\theta_m,\theta_n) + x_k^{2,i}(\theta_n,\theta_m) \le 1 \quad \text{for } (m,n) \in \{L,H\}^2. \end{aligned} \tag{15}$$

In state $(2, i)$ at site k, each buyer only knows his own valuation. For $j \in \{H, L\}$, define the *reduced form* transfers and winning probabilities as $X_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} x_k^{2,i}(\theta_j, \theta_{-j})$ and $Y_k^{2,i}(\theta_j) = \mathbf{E}_{\theta_{-j}} y_k^{2,i}(\theta_j, \theta_{-j})$. *Incentive-compatible direct mechanisms* respect a set of state-contingent incentive and participation constraints. When no information is released at site k, no incentive constraints apply. The relevant participation constraints are

$$x_k^{1,u}\bar{\theta} - y_k^{1,u} \ge 0, \tag{PC$_k^{1,u}$}$$
$$x_k^{2,u}\bar{\theta} - y_k^{2,u} \ge 0. \tag{PC$_k^{2,u}$}$$

In state $(1, i)$ at site k, the set of constraints is given by

$$x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) \ge x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L), \tag{IC$_k^{1,i}(\theta_H)$}$$
$$x_k^{1,i}(\theta_L)\theta_L - y_k^{1,i}(\theta_L) \ge x_k^{1,i}(\theta_H)\theta_L - y_k^{1,i}(\theta_H), \tag{IC$_k^{1,i}(\theta_L)$}$$
$$x_k^{1,i}(\theta_L)\theta_L - y_k^{1,i}(\theta_L) \ge 0. \tag{PC$_k^{1,i}(\theta_L)$}$$

As is well known, the participation constraint of the θ_H-type, ($\mathrm{PC}_k^{1,i}(\theta_H)$), is satisfied whenever ($\mathrm{IC}_k^{1,i}(\theta_H)$) and ($\mathrm{PC}_k^{1,i}(\theta_L)$) hold.

The constraints that need to be satisfied in state $(2,i)$ at site k are given by

$$X_k^{2,i}(\theta_H)\theta_H - Y_k^{2,i}(\theta_H) \geq X_k^{2,i}(\theta_L)\theta_H - Y_k^{2,i}(\theta_L), \qquad (\mathrm{IC}_k^{2,i}(\theta_H))$$

$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq X_k^{2,i}(\theta_H)\theta_L - Y_k^{2,i}(\theta_H), \qquad (\mathrm{IC}_k^{2,i}(\theta_L))$$

$$X_k^{2,i}(\theta_L)\theta_L - Y_k^{2,i}(\theta_L) \geq 0. \qquad (\mathrm{PC}_k^{2,i}(\theta_L))$$

As in the single buyer case, the participation constraint of the θ_H-type, ($\mathrm{PC}_k^{2,i}(\theta_H)$), is satisfied whenever ($\mathrm{IC}_k^{2,i}(\theta_H)$) and ($\mathrm{PC}_k^{2,i}(\theta_L)$) hold. The class of incentive compatible direct mechanisms for this problem is denoted by Γ, and a particular mechanism at site k by γ_k.

A.2 Rents and Profits

Given mechanism γ_k, at site k, exptected rents are given by

$$\begin{aligned}
R_k^{\eta,u} &= x_k^{\eta,u}\bar{\theta} - y_k^{\eta,u} \qquad \text{for } \eta \in \{1,2\}, \\
R_k^{1,i} &= \mathbf{E}_\theta \left[x_k^{1,i}(\theta)\theta - y_k^{1,i}(\theta) \right], \\
R_k^{2,i} &= \mathbf{E}_\theta \left[X_k^{2,i}(\theta)\theta - Y_k^{2,i}(\theta) \right],
\end{aligned}$$

and the surplus is given by

$$\begin{aligned}
\mathcal{S}_k^{1,u} &= x_k^{1,u}\bar{\theta}, \\
\mathcal{S}_k^{2,u} &= 2x_k^{2,u}\bar{\theta}, \\
\mathcal{S}_k^{1,i} &= p_H x_k^{1,i}(\theta_H)\theta_H + p_L x_k^{1,i}(\theta_L)\theta_L h \\
\mathcal{S}_k^{2,i} &= 2\left[p_H X_k^{2,i}(\theta_H)\theta_H + p_L X_k^{2,i}(\theta_L)\theta_L \right].
\end{aligned}$$

Given strategy profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$, the profits of seller k are given by

$$\mathcal{P}_k(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k}) = \mathbf{E}_\eta \mathbf{E}_\tau \mathbf{E}_\psi \left[\eta y_k^{\eta,\tau}(\psi) \right].$$

The first expectation is taken with respect to the binomial distribution with parameter q (if $k = a$) or $1 - q$ (if $k = b$) of demand at site k, and the second with respect to the binomial distribution with parameter π_k over information states at site k. The final expectation is taken with respect to the distribution of truthful reports in state (η, τ).

A.3 A Characterisation of Incentive-Compatible Mechanisms

Lemma 6 shows that it is without loss of generality to restrict sellers to offering mechanisms in which θ_H-type incentive-compatibility constraints are binding in states $(1,i)$ and $(2,i)$.

Lemma 6. (θ_H-Type Incentive-Compatibility Constraints Bind) *Given any strategy profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$ for sellers, there exists a mechanism $\tilde{\gamma}_k \in \Gamma$ in which $(\widetilde{IC}_k^{1,i}(\theta_H))$ and $(\widetilde{IC}_k^{2,i}(\theta_H))$ are binding, allocations are as in γ_k and such that under profile $(\pi_k, \tilde{\gamma}_k, \pi_{-k}, \gamma_{-k})$ buyers' rents and sellers' profits are the same as under profile $(\pi_k, \gamma_k, \pi_{-k}, \gamma_{-k})$.*

Proof: Consider an incentive compatible mechanism γ_k at site k such that $(\text{IC}_k^{1,i}(\theta_H))$ is slack. In particular, say

$$x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + C,$$

with $C > 0$. Consider an alternative mechanism $\tilde{\gamma}_k$ identical to γ_k except that

$$\begin{aligned}\tilde{y}_k^{1,i}(\theta_H) &= y_k^{1,i}(\theta_H) + p_L C \\ \tilde{y}_k^{1,i}(\theta_L) &= y_k^{1,i}(\theta_L) - p_H C.\end{aligned}$$

In that case,

$$\begin{aligned}\tilde{x}_k^{1,i}(\theta_H)\theta_H - \tilde{y}_k^{1,i}(\theta_H) &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - p_L C \\ &= x_k^{1,i}(\theta_H)\theta_H - y_k^{1,i}(\theta_H) - C + p_H C \\ &= x_k^{1,i}(\theta_L)\theta_H - y_k^{1,i}(\theta_L) + p_H C \\ &= \tilde{x}_k^{1,i}(\theta_L)\theta_H - \tilde{y}_k^{1,i}(\theta_L).\end{aligned}$$

Thus, $\widetilde{IC}_k^{1,i}(\theta_H)$ binds. Since under $\tilde{\gamma}_k$ the transfer of type θ_L has been decreased, $\widetilde{PC}_k^{1,i}(\theta_L)$ is satisfied. Since both $\widetilde{IC}_k^{1,i}(\theta_H)$ and $\widetilde{PC}_k^{1,i}(\theta_L)$ hold, then so does $\widetilde{PC}_k^{1,i}(\theta_H)$. Finally, under $\tilde{\gamma}_k$ θ_H-types are worse off and θ_L-types are better off, so that $\widetilde{IC}_k^{1,i}(\theta_L)$ holds. Hence $\tilde{\gamma}_k$ is incentive compatible.

Profits for seller k in state $(1, i)$ under mechanism $\tilde{\gamma}_k$ are given by

$$\begin{aligned}p_H\tilde{y}_k^{1,i}(\theta_H) + p_L\tilde{y}_k^{1,i}(\theta_L) &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L) + p_H p_L C - p_L p_H C \\ &= p_H y_k^{1,i}(\theta_H) + p_L y_k^{1,i}(\theta_L),\end{aligned}$$

where the last line is profits under γ_k in state $(1, i)$. Profits in other states are also unaffected. The proof for the case in which $IC_k^{2,i}(\theta_H)$ is slack is identical, with reduced-form mechanisms replacing the mechanisms. To that end, note that in state $(2, i)$, profits under mechanism γ_k are given by

$$\begin{aligned}&p_H^2\left[2y_k^{2,i}(\theta_H, \theta_H)\right] + 2p_L p_H\left[y_k^{2,i}(\theta_H, \theta_L) + y_k^{2,i}(\theta_L, \theta_H)\right] + p_L^2\left[2y_k^{2,i}(\theta_L, \theta_L)\right] \\ &= 2\left[p_H Y_k^{1,i}(\theta_H) + p_L Y_k^{1,i}(\theta_L)\right].\end{aligned}$$

□

As the proof manipulates mechanisms in different demand states independently, given an original profile where the incentive compatibility constraints of θ_H-types in both demand states are slack, one could find a rent and profit-equivalent mechanism with incentive constraints binding in both states by the same procedure.

Denote $\tilde{\gamma}_k$ as the *$IC(\theta_H)$-equivalent* of γ_k. Similarly, denote by $\tilde{\Gamma}$ the set of IC(θ_H)-equivalent mechanisms. Given information provision (π_a, π_b), a game with mechanisms $(\gamma_a, \gamma_b) \in (\Gamma \setminus \tilde{\Gamma})^2$ generates the same distribution over outcomes as a game with mechanisms $(\tilde{\gamma}_a, \tilde{\gamma}_b)$, where $\tilde{\gamma}_k$ is the $IC(\theta_H)$-equivalent mechanism of γ_k. That is, excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibria in terms of information provision. On the other hand, when sellers also choose mechanisms, it is not the case that equilibrium mechanisms must belong to $\tilde{\Gamma}$. However, Lemma 6 states that excluding mechanisms in $\Gamma \setminus \tilde{\Gamma}$ does not reduce the set of equilibrium allocations, traffic levels and payoffs. In what follows, incentive compatible mechanisms refers to mechanisms in $\tilde{\Gamma}$.

Given mechanism γ_k at site k, we can rewrite the expected rents promised at site k as

$$R_k^{\eta,u} = r_k^{\eta,u} \text{ for } \eta \in \{1,2\},$$
$$R_k^{1,i} = r_k^{1,i} + p_H x_k^{1,i}(\theta_L)(\theta_H - \theta_L),$$
$$R_k^{2,i} = r_k^{2,i} + p_H X_k^{2,i}(\theta_L)(\theta_H - \theta_L).$$

Furthermore, Lemma 6 justifies the use of the following well-known result, whose proof is standard and omitted.

Lemma 7. (Characterisation of IC(θ_H)-Equivalent Mechanisms) *$\gamma_k \in \tilde{\Gamma}$ if and only if $x_k^{1,i}(\theta_H) \geq x_k^{1,i}(\theta_L)$, $X_k^{1,i}(\theta_H) \geq X_k^{1,i}(\theta_L)$, $r_k^{\eta,\tau} \geq 0$ for all $\eta \in \{1,2\}$ and $\tau \in \{i,u\}$ and θ_H-type rents are given by $x_k^{1,i}(\theta_H - \theta_L)$ in state $(1,i)$ and $X_k^{2,i}(\theta_H - \theta_L)$ in state $(2,i)$.*

A.4 Properties of Allocations

I give the formal definitions, in terms of the underlying mechanisms, of the properties of allocations used in the text.

Definition 1. (No Waste) *A mechanism $\gamma_k \in \Gamma$ has no waste if and only if*

$$x_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L) = x_k^{1,u} = 1,$$
$$x_k^{2,u} = \frac{1}{2},$$
$$x_k^{2,i}(\theta_H, \theta_H) = x_k^{2,i}(\theta_L, \theta_L) = \frac{1}{2}$$
$$x_k^{2,i}(\theta_H, \theta_L) + x_k^{2,i}(\theta_L, \theta_H) = 1.$$

As noted in Section 3.2, that γ has no waste implies that seller a's profits in the two-buyer state decrease in π_a, since the term in the first brackets of (9) is linear in π_a and

$$\begin{aligned}
\mathcal{S}^{2,i} - \bar{\theta} - 2R^{2,i} &= \mathcal{S}^{2,i} - \bar{\theta} - 2\Big(r^{2,i} + X^{2,i}(\theta_L)p_H(\theta_H - \theta_L)\Big) \\
&\leq \mathcal{S}^{2,i} - \bar{\theta} - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= \theta_H \left[2p_H X^{2,i}(\theta_H) - p_H\right] + \theta_L \left[2p_L X^{2,i}(\theta_L) - p_L\right] - 2X^{2,i}(\theta_L)p_H(\theta_H - \theta_L) \\
&= (\theta_H - \theta_L)\left[p_L - 2X^{2,i}(\theta_L)\right] \\
&\leq 0. \qquad (16)
\end{aligned}$$

The second line follows since $r^{2,i} \geq 0$, the fourth since $p_H X_k^{2,i}(\theta_H) + p_L X_k^{2,i}(\theta_L) = \frac{1}{2}$ under no waste, and the last since $X^{2,i}(\theta_L) \geq \frac{p_L}{2}$ under no waste.

As noted in Section 3.2, under Assumption 1, the ex post optimal mechanisms are regular, and are described by allocation probabilities

$$\begin{aligned}
x_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_L) = x_k^{1,u} &= 1, \\
x_k^{2,u} &= \frac{1}{2}, \\
X_k^{2,i}(\theta_H) &= \frac{p_H}{2} + p_L, \\
X_k^{2,i}(\theta_L) &= \frac{p_L}{2},
\end{aligned}$$

and rent levels for low types $r^{\eta,\tau} = 0$ for all $\eta \in \{1,2\}, \tau \in \{i,u\}$.

Definition 2. (Partial and Full Allocative Efficiency) *A mechanism $\gamma_k \in \Gamma$ has partial allocative efficiency (PAE) if and only if*

$$\begin{aligned}
&x_k^{1,i}(\theta_H) = x_k^{1,u} = 1, \\
&x_k^{2,u} = \frac{1}{2}, \\
&x_k^{2,i}(\theta_H, \theta_L) = 1, \text{ and } x_k^{2,i}(\theta_H, \theta_H) = \frac{1}{2}.
\end{aligned}$$

A mechanism $\gamma_k \in \Gamma$ has full allocative efficiency (FAE) if and only if it has partial allocative efficiency and also

$$\begin{aligned}
&x_k^{1,i}(\theta_L) = 1, \\
&x_k^{2,i}(\theta_L, \theta_L) = \frac{1}{2}.
\end{aligned}$$

B Appendix: Proofs

Proof of Lemma 1: Setting (10) equal to zero and checking the conditions for which $\pi < 1$, we obtain the expression for π^*. By the argument in the text, all that needs to be shown is that $\frac{\partial q}{\partial \pi_a}\Big|_{\pi_a=\pi_b=\pi}$ is decreasing in the symmetric probability π. By (2) and using the fact that $R^{1,u} = R^{2,u} = 0$ for regular mechanisms, we have

$$q = \frac{\pi_a R^{1,i} - \pi_b R^{2,i}}{(R^{1,i} - R^{2,i})(\pi_a + \pi_b)},$$

and it can be verified that

$$\frac{\partial q}{\partial \pi_a}\Big|_{\pi_a=\pi_b=\pi} = \frac{R^{1,i} + R^{2,i}}{4\pi(R^{1,i} - R^{2,i})},$$

which is decreasing in π.

□

Proof of Proposition 3: The first part of the proof is the following lemma which provides sufficient conditions for the existence of interior symmetric equilibria.

Lemma 8. *Given a regular mechanism γ that generates rents such that*

i. $2R^{1,i} > \bar{\theta}$ *and* $R^{1,i} + R^{2,i} > \frac{2R^{1,i}(\mathcal{S}^{2,i} - \bar{\theta})}{2R^{1,i} > \bar{\theta}}$.

ii. $2R^{1,i}(R^{2,i})^2 - 6(R^{1,i})^2 R^{2,i} + 8(R^{1,i})^3 - \bar{\theta}(4(R^{1,i})^2 + R^{1,i}R^{2,i} - (R^{2,i})^2) \leq 0.$

the symmetric equilibrium of the game between sellers is $\pi^* = \frac{-(R^{1,i}+R^{2,i})\bar{\theta}}{2R^{1,i}(\mathcal{S}^{2,i} - \bar{\theta} - (R^{1,i}+R^{2,i})} < 1.$

Proof: Point i of the statement ensures that $\pi^* < 1$. Consider a candidate symmetric profile (π, π) and a deviation by seller a to $\pi + \lambda$ for $\lambda \in (-\pi, 1-\pi]$, which induces traffic level $q^\lambda \in (0, 1]$. Then we have that

$$\begin{aligned} q^\lambda &= \frac{\pi(R^{1,i} - R^{2,i}) + \lambda R^{1,i}}{(R^{1,i} - R^{2,i})(2\pi + \lambda)} \\ &= \frac{1}{2} + z, \\ \text{with } z &= \frac{\lambda(R^{1,i} + R^{2,i})}{2(R^{1,i} - R^{2,i})(2\pi + \lambda)}. \end{aligned} \tag{17}$$

Also,

$$\begin{aligned}
\mathcal{P}_a(\pi+\lambda,\pi)-\mathcal{P}_a(\pi,\pi) &= z(z+1)\left[\pi\mathcal{S}^{2,i}+(1-\pi)\bar{\theta}-2\pi R^{2,i}\right]-2z^2\left[\bar{\theta}-\pi R^{1,i}\right]\\
&\quad+(\frac{1}{2}+z)^2\left[\mathcal{S}^{2,i}-\bar{\theta}-2R^{2,i}\right]-2\lambda(\frac{1}{2}+z)(\frac{1}{2}-z)R^{1,i}\\
&=\frac{\lambda^2}{D}\Bigg[4R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})\Bigg[(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})\bar{\theta}\\
&\quad-2\lambda(R^{1,i})^2(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))\Bigg]\\
&\quad+(R^{1,i}+R^{2,i})^2\bar{\theta}\Bigg[\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})(5R^{1,i}-R^{2,i})\\
&\quad+(R^{1,i}+R^{2,i})^2\Bigg]\Bigg]\\
&\leq F\Bigg[(\mathcal{S}^{2,i}-\bar{\theta})(4(R^{1,i})^2+R^{1,i}R^{2,i}-(R^{2,i})^2)\\
&\quad-2R^{2,i}(R^{1,i}+R^{2,i})(2R^{1,i}-R^{2,i})\Bigg] \qquad (18)\\
&< H\Bigg[2R^{1,i}(R^{2,i})^2-6(R^{1,i})^2R^{2,i}+8(R^{1,i})^3\\
&\quad-\bar{\theta}(4(R^{1,i})^2+R^{1,i}R^{2,i}-(R^{2,i})^2)\Bigg]. \qquad (19)
\end{aligned}$$

Where $D,\ F,\ H>0$ are functions of parameters. The second equality follows from setting $\pi=\pi^*$ and rearranging terms. The first inequality follows from the fact that $q^\lambda\leq 1$ when $\lambda\leq\frac{\bar{\theta}(R^{1,i}+R^{2,i})(R^{1,i}-R^{2,i})}{-2R^{1,i}R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))}$. The last inequality follows since $\pi^*<1$ when $\mathcal{S}^{2,i}-\bar{\theta}<\frac{(R^{1,i}+R^{2,i})(2R^{1,i}-\bar{\theta})}{2R^{1,i}}$.

□

To show that the set of regular mechanisms Ψ is nonempty, note that (18) implies that under any mechanism in which $\mathcal{S}^{2,i}=\bar{\theta}$ (a pricing mechanism in the two-buyer state), deviations from the symmetric profile (π^*,π^*) are strictly not profitable for seller a. Hence given any regular mechanism γ with $X^{2,i}(\theta_L)=X^{2,i}(\theta_H)=\frac{1}{2}$ and rents that satisfy condition i of Lemma 8, there is a neighbourhood N of γ in the space of regular mechanisms such that for all $\hat{\gamma}\in N\ \gamma$, $\hat{\gamma}$ satisfies condition i of Lemma 8 and the term inside the brackets of (18) is negative. Thus all such $\hat{\gamma}$ induce a unique symmetric equilibrium (π^*,π^*) with $\pi^*<1$.

Finally, the derivatives mentioned in the proposition can be computed directly to yield

$$\begin{aligned}\frac{\partial \pi^*}{\partial R^{1,i}} &= -\frac{2\bar{\theta}R^{1,i}(R^{1,i}+R^{2,i}) - 2\bar{\theta}R^{2,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\ &< 0, \\ \text{and} \quad \frac{\partial \pi^*}{\partial R^{2,i}} &= -\frac{2\bar{\theta}R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\ &\leq 0.\end{aligned}$$

From these it can be checked that

$$\begin{aligned}\frac{\partial \pi^*}{\partial R^{1,i}} - \frac{\partial \pi^*}{\partial R^{2,i}} &= \frac{-2\bar{\theta}(R^{1,i}+R^{2,i})(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+2R^{2,i}))}{(-2R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2} \\ &< 0.\end{aligned}$$

□

Proof of Proposition 4: The profits of both sellers at a symmetric equilibrium with a regular mechanism γ are given by

$$\frac{1}{4}\left[\pi^*(\mathcal{S}^{2,i}-\bar{\theta}-2R^{2,i})+\bar{\theta}\right] + \frac{1}{2}\left[\bar{\theta}-\pi^* R^{1,i}\right]. \tag{20}$$

Direct computation yields

$$\begin{aligned}\frac{\partial \mathcal{P}_a(\pi^*,\pi^*)}{\partial R^{1,i}} &= \frac{\bar{\theta}}{8(R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2}\Big[R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta})(R^{1,i}+R^{2,i}) \\ &\qquad + R^{2,i}(\mathcal{S}^{2,i}-\bar{\theta}-2(R^{1,i}+R^{2,i}))(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i}))\Big] \\ &> 0 \\ \frac{\partial \mathcal{P}_a(\pi^*,\pi^*)}{\partial R^{2,i}} &= \frac{\bar{\theta}R^{1,i}}{2(R^{1,i}(\mathcal{S}^{2,i}-\bar{\theta}-(R^{1,i}+R^{2,i})))^2}\Big[(\mathcal{S}^{2,i}-\bar{\theta})^2-2(R^{1,i}+R^{2,i})^2\Big].\end{aligned}$$

□

Proof of Proposition 5: I show that if γ is the ex post optimal mechanism under Assumption 1, then $\mathcal{P}_a(\pi_a,\gamma,1,\gamma)$ is increasing in π_a. Given $\pi_a \leq 1$, $q \leq \frac{1}{2}$, and if π_a is such that $q > 0$, then

$$\begin{aligned}\mathcal{P}_a(\pi_a,\gamma,1,\gamma) &= \left(\frac{\pi_a - \frac{p_L}{2}}{(1+\pi_a)(1-\frac{p_L}{2})}\right)^2\bar{\theta} + 2\left(\frac{(\pi_a-\frac{p_L}{2})(1-\frac{\pi_a p_L}{2})}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\left(\bar{\theta}-\pi_a p_H(\theta_H-\theta_L)\right) \\ &= \left(\frac{\pi_a-\frac{p_L}{2}}{((1+\pi_a)(1-\frac{p_L}{2}))^2}\right)\left(\bar{\theta}(p_H\pi_a+2-\frac{p_L}{2})+2(1-\frac{\pi_a p_L}{2})\pi_a p_H(\theta_H-\theta_L)\right) \\ &\equiv A(\pi_a)\big(B(\pi_a)+C(\pi_a)\big)\end{aligned}$$

Where $B(\pi_a)$ is clearly increasing in π_a, while it can be shown that $A(\pi_a)$ and $C(\pi_a)$ are increasing whenever $\pi_a \leq 1 + p_L$ and $\pi_a \leq \frac{1}{p_L}$, respectively, which is always true.

□

Proof of Lemma 2: Lemma 2, stated in terms of $IC(\theta_H)$-equivalent mechanisms, requires that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$. This condition simply states that it is always possible, for at least one state, to decrease transfers in an incentive compatible way. Any mechanism $\gamma_a \in \Gamma$ that satisfies this last property would have its $IC(\theta_H)$-equivalent mechanism satisfy the property that it not be the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ (through Lemma 6). The following proof then applies to all incentive compatible mechanisms that are components of some equilibrium, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Suppose that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is an equilibrium, that $\mathbf{E}_\eta \mathbf{E}_\tau \mathcal{S}_a^{\eta,\tau}$ is increasing in π_a, that it is not the case that $r_a^{2,i} = r_a^{2,u} = r_a^{1,i} = r_a^{1,u} = 0$ and that $\pi_a < 1$. Consider a deviation by seller a to a profile in which

$$\begin{aligned} \hat{\pi}_a &= \pi_a + \lambda \\ \hat{r}_a^{\eta,\tau} &= r_a^{\eta,\tau} - \delta^{\eta,\tau}, \end{aligned}$$

where $\lambda \in (0, 1-\pi_a]$ and $\delta^{\eta,\tau} < r_a^{\eta,\tau}$ for all (η,τ). For this deviant profile not to affect buyers' visit decisions (or expected rents), we need

$$\begin{aligned} & q\left[(\pi_a+\lambda)\left[r_a^{2,i} - \delta^{2,i} + z_a^{2,i}\right] + (1-\pi_a-\lambda)\left[r_a^{2,u} - \delta^{2,u}\right]\right] \\ &\quad + (1-q)\left[(\pi_a+\lambda)\left[r_a^{1,i} - \delta^{1,i} + z_a^{1,i}\right] + (1-\pi_a-\lambda)\left[r_a^{1,u} - \delta^{1,u}\right]\right] \\ &\quad = q\left[\pi_a\left[r_a^{2,i} + z_a^{2,i}\right] + (1-\pi_a)\left[r_a^{2,u}\right]\right] + (1-q)\left[\pi_a\left[r_a^{1,i} + z_a^{1,i}\right] + (1-\pi_a)\left[r_a^{1,u}\right]\right], \end{aligned}$$

or

$$\begin{aligned} & (\pi_a+\lambda)\left[q\delta^{2,i} + (1-q)\delta^{1,i}\right] + (1-\pi_a-\lambda)\left[q\delta^{2,u} + (1-q)\delta^{1,u}\right] \\ &\quad = \lambda\left[q\left[r_a^{2,i} + z_a^{2,i} - r_a^{2,u}\right] + (1-q)\left[r_a^{1,i} + z_a^{1,i} - r_a^{1,u}\right]\right], \end{aligned} \tag{21}$$

where $z_a^{1,i} = r_a^{1,i} + p_H x_a^{1,i}(\theta_L)(\theta_H - \theta_L) \geq 0$ and $z_a^{2,i} = r_a^{2,i} + p_H X_a^{2,i}(\theta_L)(\theta_H - \theta_L) \geq 0$ are the expected informational rents given the allocations of the original mechanism. The sign of the right-hand side (RHS) of (21) is given by the properties of the mechanism at site a. It is positive if buyers prefer, on average, to be informed at the site, and negative if buyers prefer, on average, to be uninformed.

Suppose $\pi_a > 0$. Suppose $RHS(\lambda) > 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta,\tau) \neq (2,u)$ and $\delta^{2,u} > 0$. Then

$$\begin{aligned} \lim_{\lambda\to 0} LHS((\delta^{\eta,\tau}),\lambda) &> \lim_{\lambda\to 0} RHS(\delta) \\ &= 0 \end{aligned}$$

since $\pi_a < 1$. Also

$$\begin{aligned} LHS((\delta^{\eta,\tau}), 1-\pi_a) &= 0 \\ &< RHS(1-\pi_a). \end{aligned}$$

Hence there exists $\hat{\lambda} \in (0, 1-\pi_a]$ such that $LHS((\delta^{\eta,\tau}), \hat{\lambda}) = RHS(\hat{\lambda})$.

Suppose $RHS(\lambda) < 0$. Suppose $\pi_a > 0$. By assumption, there exists some $r_a^{\hat{\eta},\hat{\tau}} > 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta, \tau) \neq (\hat{\eta}, \hat{\tau})$ and $\delta^{\hat{\eta},\hat{\tau}}$ such that

$$LHS((\delta^{\eta,\tau}), 1-\pi_a) > RHS(1-\pi_a) \tag{22}$$

Fix $\hat{\lambda} \in (0, 1-\pi_a]$ such that $LHS((\delta^{\eta,\tau}), \hat{\lambda}) = RHS(\hat{\lambda})$. Such a $\hat{\lambda}$ exists by (22) and since

$$\begin{aligned} \lim_{\lambda \to 0} RHS(\lambda) &= 0 \\ &> \lim_{\lambda \to 0} LHS(\delta^{\eta,\tau}, \lambda) \end{aligned}$$

as $\pi_a > 0$ and $\hat{\delta}^{\hat{\eta},\hat{\tau}} < 0$.

Suppose $r_a^{2,u} = r_a^{1,u} = 0$. Then buyers get no rents from visiting seller a in equilibrium. Either $q = 0$ and seller a makes no profits in equilibrium, or buyers get no rents from either site in equilibrium. The first case cannot occur in equilibrium, as any deviation for sellers that ensure positive profits and visit probabilities is profitable, and such deviations always exist. In the second case, any seller could deviate by offering marginally more rents and capturing all buyer visits, another contradiction. Hence there is some $\hat{\eta}$ with $r_a^{\hat{\eta},u} > 0$. Set $\delta^{\eta,\tau} = 0$ for all $(\eta, \tau) \neq (\hat{\eta}, u)$ and $\delta^{\hat{\eta},u} < 0$ Set $\hat{\lambda}$ such that $LHS((\delta^{\eta,\tau}), \hat{\lambda}) = RHS(\hat{\lambda})$. Such a $\hat{\lambda}$ exists since $LHS((\delta^{\eta,\tau}), 1) = 0 > RHS(1)$ and

$$\begin{aligned} \lim_{\lambda \to 0} RHS(\lambda) &= 0 \\ &< \lim_{\lambda \to 0} LHS(\delta^{\eta,\tau}, \lambda) \end{aligned}$$

as $\hat{\delta}^{\hat{\eta},u} < 0$.

Finally, if $RHS(\lambda) = 0$, buyers are indifferent between informed and uninformed states at site a and a seller can increase information provision without shifting traffic by setting $\delta^{\eta,\tau} = 0$ for all $\eta \in \{1, 2\}, \tau \in \{i, u\}$.

In all cases, the arguments above yield a deviation for seller a which keeps rent payouts unchanged and strictly increases the surplus available at site a. This implies that $(\pi_a, \gamma_a, \pi_b, \gamma_b)$ is not an equilibrium.

□

Proof of Lemma 3: My argument proceeds with mechanisms in $\tilde{\Gamma}$. However, if a mechanism in $\Gamma \setminus \tilde{\Gamma}$ without PAE were a component of an equilibrium, applying the following proof to its

IC(θ_H)-equivalent (through Lemma 6) would yield a contradiction, since a best response to a IC(θ_H)-equivalent mechanism is also a best-response to the original mechanism.

Consider an incentive compatible mechanism γ_k at site k such that $x_k^{1,i}(\theta_H) < 1$. Consider an alternative mechanism $\hat{\gamma}_k$ identical to γ_k except that

$$\hat{x}_k^{1,i}(\theta_H) = x_k^{1,i}(\theta_H) + \epsilon$$
$$\hat{y}_k^{1,i}(\theta_H) = y_k^{1,i}(\theta_H) + \epsilon\theta_H,$$

where $\epsilon \in (0, 1 - x_k^{1,i}]$. We have $\hat{x}_k^{1,i}(\theta_H) > x_k^{1,i}(\theta_H) \geq \hat{x}_k^{1,i}(\theta_L) > x_k^{1,i}(\theta_L)$ and $\hat{r}^{1,i} = r^{1,i} \geq 0$ since $\gamma_k \in \tilde{\Gamma}$, and so $\hat{\gamma}_k \in \tilde{\Gamma}$. Note that $\hat{R}^{1,i} = R^{1,i}$ and hence buyer rents and visit decisions are unaffected. However, seller k's profits are higher under $\hat{\gamma}_k$ than under γ_k if buyers sometimes visit k since θ_H-type transfers in the one-buyer state are higher.

As noted in the text, the proof needs to be modified in the two-buyer state if $X_k^{2,i}(\theta_H) < p_L + \frac{1}{2}p_H$ and if the constraint $x^{2,i}(\theta_H, \theta_L) + x^{2,i}(\theta_L, \theta_H) \leq 1$ from (15) is binding under the original mechanism γ_k. If this is not the case, then the previous proof applies to the reduced-form mechanisms. If not, it must be that $x^{2,i}(\theta_L, \theta_H) > 0$, that is, a θ_L-type is sometimes allocated the good in the presence of a θ_H-type. Consider an alternative mechanism $\hat{\gamma}_k$ identical to γ_k except that

i. θ_L-types never get preference over θ_H-types, $\hat{x}^{2,i}(\theta_H, \theta_L) = 1$ and $\hat{x}^{2,i}(\theta_L, \theta_H) = 0$, so that

$$\hat{X}^{2,i}(\theta_L) = X^{2,i}(\theta_L) - p_H x_k^{2,i}(\theta_L, \theta_H)$$
$$\hat{X}^{2,i}(\theta_H) = X^{2,i}(\theta_H) + p_L x_k^{2,i}(\theta_L, \theta_H).$$

ii. Transfers are adjusted so that rents to both types are unchanged

$$\hat{Y}^{2,i}(\theta_L) = Y^{2,i}(\theta_L) - \theta_L(X_k^{2,i}(\theta_L) - \hat{X}^{2,i}(\theta_L))$$
$$\hat{Y}^{2,i}(\theta_H) = Y^{2,i}(\theta_H) + \theta_H(\hat{X}_k^{2,i}(\theta_H) - X^{2,i}(\theta_H)).$$

By condition i and since $\gamma_k \in \tilde{\Gamma}$, we have that $\hat{X}^{2,i}(\theta_H) > X^{2,i}(\theta_H) \geq X^{2,i}(\theta_L) > \hat{X}^{2,i}(\theta_L)$. Along with condition ii, this implies that $\hat{\gamma}_k \in \tilde{\Gamma}$.

Profits to seller k in the two-buyer state under $\hat{\gamma}_k$ are given by

$$2\left[p_L\hat{Y}^{2,i}(\theta_L) + p_H\hat{Y}^{2,i}(\theta_H)\right] = 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H) + p_H p_L(\theta_H - \theta_L)x^{2,i}(\theta_L, \theta_H)\right]$$
$$> 2\left[p_L Y^{2,i}(\theta_L) + p_H Y^{2,i}(\theta_H)\right],$$

where the last expression is profits to seller k in the two-buyer state under γ_k. The inequality follows since by hypothesis $x^{2,i}(\theta_L, \theta_H) > 0$. Thus seller k gains by offering $\hat{\gamma}_k$ if buyers visit k with positive probability since traffic and one-buyer state profits are unchanged and two-buyer state profits are higher. Furthermore, under $\hat{\gamma}_k$ it is the case that $\hat{X}_k^{2,i}(\theta_H) = p_L + \frac{1}{2}p_H$.

Similarly, for uninformed allocations, consider an incentive compatible mechanism γ_k at site k such that $x_k^{\eta,u} < 1$ for some $\eta \in \{1,2\}$. Consider an alternative mechanism $\hat{\gamma}_k$, identical to γ_k except that in state (η, u)

$$\hat{x}_k^{\eta,u} = x_k^{\eta,u} + \epsilon$$
$$\hat{y}_k^{\eta,u} = y_k^{\eta,u} + \epsilon\bar{\theta},$$

where $\epsilon \in (0, 1 - x^{\eta,u}]$. Thus buyer rents are the same under both mechanisms but seller k's profits are higher in state (η, u) if buyers visit seller k with positive probability since the good is sold more often at higher prices.

□

Proof of Lemma 4: Consider an incentive compatible mechanism γ_k at site k such that $x_k^{1,i}(\theta_L) < 1$ and the level of rents provided to type θ_L is given by $r^{1,i} \geq 0$.[25] Then

$$y_k^{1,i}(\theta_L) = \theta_L x_k^{1,i}(\theta_L) - r^{1,i}, \tag{23}$$

and, by Lemmas 6 and 3

$$y_k^{1,i}(\theta_H) = \theta_H - x_k^{1,i}(\theta_L)(\theta_H - \theta_L) - r^{1,i}. \tag{24}$$

By (23) and (24), write seller k's profits conditional on $(\mathrm{IC}_k^{1,i}(\theta_H))$ binding and type θ_L receiving rents $r^{1,i}$ as

$$x_k^{1,i}(\theta_L)(\theta_L - p_H\theta_H) + p_H\theta_H - r^{1,i}. \tag{25}$$

These are increasing in $x_k^{1,i}(\theta_L)$ whenever $\theta_L > p_H\theta_H$. Since $x^{1,i}(\theta_H) = 1$ by Lemma 3, an increase in $x^{1,i}(\theta_L)$ maintains incentives compatibility so seller k can increase profits in state $(1, i)$ by doing so. This increases traffic to site k (since rents to θ_H-types increase). But at a symmetric equilibrium $q = \frac{1}{2}$ and marginal changes in traffic have negligeable effects on the probability of the one-buyer state $(2q(1-q))$, so that profits of seller k increase with marginal changes in $x^{1,i}(\theta_L)$ if profits in the two-buyer state are assumed to be nonnegative. However, note that this argument ensures that profits in the two-buyer state must be nonnegative in a symmetric equilibrium. If not, a seller could marginally increase transfers in the two-buyer state without affecting traffic significantly in the one-buyer state, while both traffic and losses per buyer would decrease in the two-buyer state.

□

[25] I need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.

Proof of Lemma 5: To show that $R^{2,i} \leq R^{1,i}$, consider a symmetric equilibrium with $\pi = 1$, FAE and a mechanism[26] γ such that $R^{1,i} < R^{2,i}$. This last fact implies that $r^{2,i} > 0$. Consider a mechanism $\hat{\gamma}_k$ for seller k identical to γ except that $\hat{r}^{2,i} = r^{2,i} - \Delta$. By (2) and the argument in the text for $\Delta \approx 0$, $\hat{\gamma}_k$ leads to an infinitesimal increase in the number of buyers visiting site k. Locally, moving away from a symmetric profile does not change the probability of the one-buyer state, while it increases that of the two-buyer state, where rents are now lower. This deviation is thus profitable given that profits in the two-buyer state are nonnegative (see proof of Lemma 4).

To show that $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$, consider marginal variations in $R^{1,i}$ and $R^{2,i}$ that leave $\pi = 1$ and allocative efficiency unchanged. Assume for now that $r^{1,i} > 0$ and $r^{2,i} > 0$ to ensure that it is always possible to effect such marginal changes through transfers. Profits for seller a are given by

$$\mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b) = q^2[\bar{S}^{2,i} - 2R_a^{2,i}] + 2q(1-q)[\bar{\theta} - R_a^{1,i}].$$

At a symmetric profile, the marginal changes in the term $q(1-q)$ can be ignored and thus

$$\frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{1,i}} = 2q\left[\frac{\partial q}{\partial R_a^{1,i}}(\bar{S}^{2,i} - 2R_a^{2,i}) - (1-q)\right], \tag{26}$$

where, at a symmetric profile with $\pi = 1$ we have $q = \frac{1}{2}$ and $\frac{\partial q}{\partial R_a^{1,i}} = \frac{1}{4(R_a^{1,i} - R_a^{2,i})}$. Thus

$$\begin{aligned} \frac{\partial \mathcal{P}_a(\pi_a, \gamma_a, \pi_b, \gamma_b)}{\partial R_a^{1,i}} &= \left(\frac{1}{4}\right)\frac{\bar{S}^{2,i} - 2R_a^{2,i}}{R_a^{1,i} - R_a^{2,i}} - \frac{1}{2} \\ &= 0 \quad \text{only when } R^{1,i} = \frac{\bar{S}^{2,i}}{2}. \end{aligned}$$

In the same way, it can be computed that $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{2,i}} = 0$ only when $R^{1,i} = \frac{\bar{S}^{2,i}}{2}$. That is, the same condition holds for marginal changes in expected rents in both one-buyer and two-buyer states. Since $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{2,i}} = 0$ and $\frac{\partial \mathcal{P}_a(\pi_a,\gamma_a,\pi_b,\gamma_b)}{\partial R_a^{1,i}} = 0$ yield the same condition, we need to worry about the existence of derivatives only when $r^{1,i} = r^{2,i} = 0$. But then an argument considering deviations $R_a^{1,i} + \Delta$ or $R_a^{2,i} + \Delta$ yields the result.

□

Proof of Proposision 6: Fixing some profile that satisfies the assumptions of the proposition, I will first show that with $\pi = 1$ and FAE, no deviation consisting of either individual or joint shifts (not necessarily local) in $R^{1,i}$ and $R^{2,i}$ can achieve higher profits. Since the candidate profile has full information and FAE, considering changes in rents where surplus in both states is maximized gives an upper bound on the profitability of deviations that involve the same changes in rents but that include a decrease in information provision and/or allocative efficiency.

[26] I need only consider mechanisms in $\tilde{\Gamma}$, by the remark in the proof of Lemma 3.

Consider some profile with $\pi = 1$ and associated rents $R^{1,i} \geq R^{2,i}$. Consider a deviation profile for seller a in which

$$\hat{R}_a^{1,i} = R^{1,i} + \Delta^1$$
$$\hat{R}_a^{2,i} = R^{2,i} + \Delta^2,$$

where Δ^η for $\eta \in \{1, 2\}$ need not be positive. Clearly, seller a cannot profitably deviate to any mechanism for which $\hat{q} = 0$. Also, the most profitable deviation to some mechanism such that $\hat{q} = 1$ is such that any less generous mechanism leads to $\hat{q} < 1$. Hence we can restrict attention to pairs (Δ^1, Δ^2) such that the level of traffic $\hat{q} \in (0, 1]$ is given by (2). Hence $\hat{q}$ is given by

$$\begin{aligned}\hat{q} &= \frac{(R^{1,i} - R^{2,i}) + \Delta^1}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2} \\ &= \frac{1}{2} + z \\ &\quad \text{with } z = \left(\frac{1}{2}\right) \frac{\Delta^1 + \Delta^2}{2((R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2}.\end{aligned}$$

The difference in profits can be written as

$$\begin{aligned}\mathcal{P}_a(1, \hat{\gamma_a}, 1, \gamma_b) - \mathcal{P}_a(1, \gamma_a, 1, \gamma_b) &= \left[\bar{S}^{2,i} - 2R^{2,i}\right](x(x+1)) - 2\left[m - R^{1,i}\right]x^2 \\ &\quad - 2\Delta^2\left(\frac{1}{2} + x\right)^2 - 2\Delta^1\left(\frac{1}{2} + x\right)\left(\frac{1}{2} - x\right) \\ &= C\Bigg[\left[\bar{S}^{2,i} - 2R^{2,i}\right]\left(4((R^{1,i} - R^{2,i})) + 3\Delta^1 - \Delta^2\right)\left(\Delta^1 + \Delta^2\right) \\ &\quad - 2\left[\bar{\theta} - R^{1,i}\right]\left(\Delta^1 + \Delta^2\right)^2 \\ &\quad - 8\left((R^{1,i} - R^{2,i})\right)\left((R^{1,i} - R^{2,i}) + \Delta^1\right)\left(\Delta^1 + \Delta^2\right)\Bigg],\end{aligned}$$

where $C = \left(\frac{1}{4}\right)\left[\frac{1}{2(R^{1,i} - R^{2,i})) + \Delta^1 - \Delta^2}\right]^2 > 0$. Set the original candidate profile as

$$R^{1,1} = \frac{\bar{S}^{2,i}}{2}$$
$$R^{2,i} = \frac{\bar{S}^{2,i}}{2} - \epsilon, \text{ for } \epsilon \geq 0.$$

simplifying the profit difference yields

$$\begin{aligned}\mathcal{P}_a(1, \hat{\gamma_a}, 1, \gamma_b) - \mathcal{P}_a(1, \gamma_a, 1, \gamma_b) &= C\left[(\Delta^1 + \Delta^2)^2(-2\epsilon - (2\bar{\theta} - \bar{S}^{2,i}))\right] \\ &< 0 \text{ for any } (\Delta^1, \Delta^2), \text{ since } \epsilon > 0 \text{ and } 2\bar{\theta} > \bar{S}^{2,i}.\end{aligned}$$

Thus no deviations are profitable.

□