Quantum Point Contacts for Neutral Atoms

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We show that the conductance of atoms through a tightly confining waveguide constriction is quantized in units of $\lambda_{\mathrm{dB}}^2/\pi$, where λ_{dB} is the de Broglie wavelength of the incident atoms. Such a constriction forms the atom analog of an electron quantum point contact and is an example of quantum transport of neutral atoms in an aperiodic system. We present a practical constriction geometry that can be realized using a microfabricated magnetic waveguide, and discuss how a pair of such constrictions can be used to study the quantum statistics of weakly interacting gases in small traps.

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Quantum transport, in which the center-of-mass motion of particles is dominated by quantum mechanical effects, has been observed in both electron and neutral-atom systems. Pioneering experiments demonstrated quantum transport in periodic structures. For example, Bloch oscillations and Wannier-Stark ladders were observed in the conduction of electrons through superlattices [1] with an applied electric field, as well as in the transport of neutral atoms through accelerating optical lattices [2,3]. Further work with neutral atoms in optical lattices has utilized their slower time scales (kHz instead of THz) and longer coherence lengths to observe a clear signature of dynamical Bloch band suppression [4], an effect originally predicted for but not yet observed in electron transport [5].

Quantum transport also occurs in aperiodic systems. For example, a quantum point contact (QPC) is a single constriction through which the conductance is always an integer multiple of some base conductance. The quantization of electron conductance in multiples of $2e^2/h$, where e is the charge of the electron and h is Planck's constant, is observed through channels whose width is comparable to the Fermi wavelength λ_F. Experimental realizations of a QPC include a sharp metallic tip contacting a surface [6] and an electrostatic constriction in a two-dimensional electron gas [7,8]. Electron QPC's have length-to-width ratios less than 10 because phase-coherent transport requires that channels must be shorter than the mean-free path between scattering events, ℓ_{mfp}. Geometric constraints are the limiting factor in the accuracy of quantization in an electron QPC [9].

In this Letter, we present an experimentally realizable system that forms a QPC for neutral atoms—a constriction whose ground state width b_0 is comparable to $\lambda_{\mathrm{dB}}/2\pi$, where λ_{dB} is the de Broglie wavelength of the atoms. The "conductance," as defined below, through a QPC for atoms is quantized in integer multiples of $\lambda_{\mathrm{dB}}^2/\pi$. The absence of frozen-in disorder, the low rate of interatomic scattering ($\ell_{\mathrm{mfp}} \sim 1$ m), and the availability of nearly monochromatic matter waves with de Broglie wavelengths $\lambda_{\mathrm{dB}} \sim 50$ nm [10,11] offer the possibility of conductance quantization through a cylindrical constriction with a length-to-width ratio of $\sim 10^5$. This new regime is interesting because deleterious effects such as reflection and intermode nonadiabatic transitions are minimized [12], allowing for accuracy of conductance quantization limited only by finite-temperature effects. Furthermore, the observation of conductance quantization at new energy and length scales is of inherent interest.

If a QPC for neutral atoms were realized, it would provide excellent opportunities for exploring the physics of small ensembles of weakly interacting gases. For instance, the transmission through a series of two QPC's would depend on the energetics of atoms confined in the trap between the two constrictions. The physics of such a "quantum dot" for atoms is fundamentally different from that of electrons, since the Coulombic charging energy that dominates the energetics of an electron quantum dot [13] is absent for neutral particles. The quantum statistics of neutral atoms energetically restricted to sub-three-dimensional spaces has already aroused theoretical interest in novel effects such as fermionization [14] and the formation of a Luttinger liquid [15].

Recently, several waveguides have been proposed [16–18] whose confinement may be strong enough to meet the constraint $b_0 \lesssim \lambda_{\mathrm{dB}}/2\pi$ for longitudinally free atoms. In this Letter, we will focus on the example of a surface-mounted four-wire electromagnet waveguide for atoms [18] (see Fig. 1) which exploits recent advances in microfabricated atom optics [19,20]. A neutral atom with a magnetic quantum number m experiences a linear Zeeman potential $U(\mathbf{r}) = \mu_B g m |\mathbf{B}(\mathbf{r})|$, where μ_B is the Bohr magneton, g is the Landé g factor, and $\mathbf{B}(\mathbf{r})$ is the magnetic field at $\mathbf{r}$. Atoms with $gm > 0$ are transversely confined near the minimum in field magnitude shown in Fig. 1; however, they are free to move in the z direction, parallel to the wires. Nonadiabatic changes in m near the field minimum can be exponentially suppressed with a holding field B_h applied in the axial direction z [21]. Near the guide center, the potential forms a cylindrically symmetric two-dimensional simple harmonic oscillator with classical oscillation frequency $\omega = [\mu_B g m(2\mu_0 I/\pi S^2)^2/MB_h]^{1/2}$, where μ_0 is the
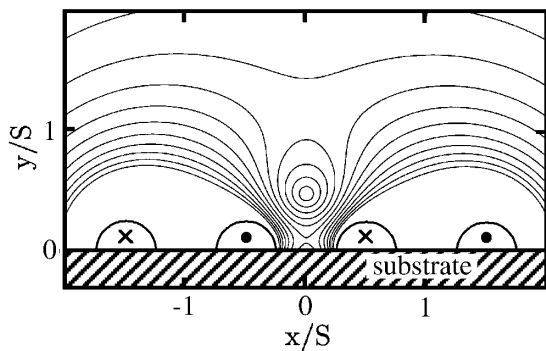
FIG. 1. Magnetic field contours above a microelectromagnet waveguide. Four parallel wires, separated by a distance S and with antiparallel current flow (marked "·" for $+z$ and "×" for $-z$) are mounted on a substrate (crosshatched), which serves both to support the wires mechanically and to dissipate the heat produced. A potential minimum is formed above the wires and can be used to guide atoms in the out-of-plane direction z. Twelve contours, equally spaced by $B_0/4$, are shown, where $B_0 = \mu_0 I/2\pi S$ and $\pm I$ ($\pm 2I$) is the current in the inner (outer) wire pair.
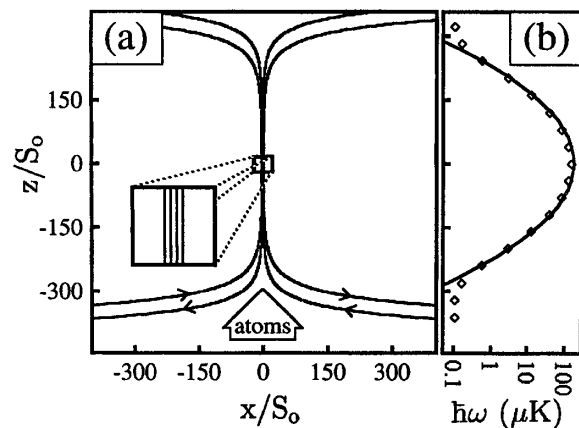

FIG. 2. (a) Top-down view of a waveguide wire geometry which creates a quantum point contact for atoms. The direction of current flow is indicated on the wires (solid lines). A constriction with $\ell = 100S_0$ is shown. (b) Level spacing $\hbar\omega$ (in μK) of transverse oscillator states versus axial distance z. Points ($\diamond$) are based on numerical calculations of the field curvature at each z above the wire configuration shown in (a); the line is based on the parallel-wire scaling $S(z)^{-2}$. Both calculations assume Na atoms in the $|F = 1, m_F = +1\rangle$ state, $S_0 = 1\ \mu$m, $I = 200$ mA, and $B_h = 35$ G.

permeability of free space, I is the inner wire current, $2I$ is the outer wire current, S is the center-to-center wire spacing, and M is the mass of the atoms. Sodium (^{23}Na) in the $|F = 1, m_F = +1\rangle$ state would have a classical oscillation frequency of $\omega = 2\pi \times 3.3$ MHz and a root mean squared (rms) ground state width $b = \sqrt{\hbar/2M\omega} = 8.1$ nm in a waveguide with $S = 1\ \mu$m and $I = 0.1$ A. The fabrication of electromagnet waveguides of this size scale and current capacity has been demonstrated [20].

A constriction in the waveguide potential can be created by contracting the spacing between the wires of the waveguide. The constriction strength can be tuned dynamically by changing the current in the wires. Figure 2a shows a top-down view of a constriction whose wire spacing $S(z)$ is smoothly varied as

$$S(z) = S_0 \exp\left[\frac{z^2}{2\ell^2}\right], \qquad (1)$$

where S_0 is the spacing at $z = 0$ and ℓ is the characteristic channel length. Assuming the wires are nearly parallel, the guide width, depth, oscillation frequency, and curvature scale as $S(z)$, $S(z)^{-1}$, $S(z)^{-2}$, and $S(z)^{-4}$, respectively. For $\ell = 100S_0$, field calculations above this curved-wire geometry show that the parallel-wire approximation is valid for $|z| \lesssim 3\ell$, allowing for a well-defined waveguide potential over a factor of more than 10^3 in level spacing (see Fig. 2b). Our particular choice of $S(z)$ is somewhat arbitrary but prescribes one way in which wires can form a smooth, constricting waveguide as well as run to contact pads (necessary to connect the wires to a power supply) far enough from the channel ($\gg \ell$) that their geometry is unimportant. The total "footprint" of this device (not including contact pads) is approximately

$10\ell \times 10\ell$, or about 1 mm^2, for $S_0 = 1\ \mu$m and $\ell = 100S_0$.

Atoms approach the constriction from the $-z$ direction, as shown in Fig. 2a. We calculate the propagation of the atom waves through the constriction by solving the time-dependent Schrödinger equation in three spatial dimensions. It is important to note that the nature of quantum transport requires fully quantum-mechanical calculations, even for the longitudinal degree of freedom within the waveguide. The Hamiltonian for an atom near the axis of the four-wire waveguide described by Eq. (1) is

$$\hat{H}_{QPC} = \frac{\hat{p}^2}{2M} + \frac{1}{2} M \omega_0^2 e^{-2\hat{z}^2/\ell^2}(\hat{x}^2 + \hat{y}^2), \qquad (2)$$

where $^\wedge$ denotes an operator, ω_0 is the transverse oscillation frequency at $z = 0$, and we have assumed the parallel-wire scaling of field curvature, $S(z)^{-4}$. Since a direct numerical integration approach is computationally prohibitive, we developed a model that neglects nonadiabatic propagation at the entrance and the exit of the channel. The waveguide potential is truncated at $z = \pm z_T$, the planes between which atoms can propagate adiabatically in the waveguide, and the wave function amplitude ψ and its normal derivative $\partial\psi/\partial z$ are matched between plane-wave states ($|z| > z_T$) and the modes of the waveguide ($|z| < z_T$). We found that, for $\ell \gtrsim 10b_0$, a two-dimensional version of the model could reproduce the transmissions and spatial output distributions of a two-dimensional split-operator fast Fourier transform integration of $\hat{H}_{QPC}(\hat{x}, \hat{z})$ with the full waveguide potential. This agreement gave us confidence in our three-dimensional model of atom propagation through the constriction.

The cross section for an incident atomic plane wave to be transmitted through a constriction is dependent on the plane-wave energy E_I and incident angle. However, if the rms angular spread of incident plane waves σ is much greater than the rms acceptance angle $\alpha \sim [\ln(\ell/b_0)b_0^2/\ell^2]^{1/4}$, we can integrate over all solid angles and define a conductance Φ dependent only on parameters of the constriction and the kinetic energy E_I of the incident atoms:

$$\Phi(E_I) = \frac{F}{J_0 f(0,0)}, \qquad (3)$$

where F is the total flux of atoms (in s^{-1}) transmitted through the constriction and $J_0 f(0,0)$ is the incident on-axis brightness (in $\text{cm}^2\,\text{s}^{-1}$). The transverse momentum distribution $f(k_x, k_y)$ is defined as follows: in the plane-wave basis $\{|\mathbf{k}\rangle\}$, we consider a density distribution of atoms on the energy shell $a(\mathbf{k})d\mathbf{k} = (C/k_z^0)\delta[k_z - k_z^0]f(k_x, k_y)d\mathbf{k}$, where $C = \hbar J_0/2\pi E_I$, $\hbar k_z^0 = [2ME_I - \hbar^2(k_x^2 + k_y^2)]^{1/2}$, and $f(k_x, k_y)$ is normalized such that the incident flux density $J_0 = \int d\mathbf{k}\, a(\mathbf{k})\hbar k_z/M$. When applied to the diffusion of an isotropic gas ($f = 1$) through a hole in a thin wall, Φ is equal to the area of the hole; for a channel with a small acceptance angle, $\alpha \ll \sigma$, Φ is the effective area at the narrowest cross section of the channel. We consider a distribution of incident energies $g(E_I)$ with a rms spread ΔE, centered about $\overline{E_I}$. As an example, the ^{23}Na source described in Ref. [11] has a monochromaticity $\overline{E_I}/\Delta E \approx 50$ for atoms traveling at 30 cm/s, or $\lambda_{\text{dB}} = 50$ nm. To meet the constraint $\sigma \gg \alpha$ [22], such a source can be reflected off of a diffuser [23], such as the demagnetized magnetic tape described in Ref. [24], or translated laterally to reduce time-averaged spatial coherence. Assuming the spatial density of atoms is preserved during propagation [25], such a source can have a flux density of $J_0 \approx 2 \times 10^{10}\ \text{cm}^{-2}\,\text{s}^{-1}$.

The quantized conductance for atoms is shown in Fig. 3 and is the central result of this Letter. Conductance $\Phi/(\lambda_{\text{dB}}^2/\pi)$ is shown as a function of mean energy $\overline{E_I}/\hbar\omega_0$ and energy spread $\Delta E/\hbar\omega_0$. In the limits $\hbar\omega_0 \gg \Delta E$ and $\ell \gg b_0$, one can show analytically that the conductance is $\Phi = (\lambda_{\text{dB}}^2/\pi)N$, where N is the number of modes above cutoff at $z = 0$. The "staircase" of Φ versus $\overline{E_I}/\hbar\omega_0$ is a vivid example of quantum transport, as it demonstrates the quantum mechanical nature of the center-of-mass motion. For all of Fig. 3, we have assumed $\ell = 10^3 S_0 \approx 10^5 b_0$; in the particular case of the Na source discussed above, and assuming $\sigma = 25$ mrad, the first step ($\Phi = \lambda_{\text{dB}}^2/\pi$) corresponds to a transmitted flux of ~ 500 atoms s^{-1}, which is a sufficient flux to measure via photoionization.

We can understand several features shown in Fig. 3 by considering the adiabatic motion of atoms within the waveguide. As atom waves propagate through the constricting waveguide, modes with transverse oscillator states (n_x, n_y) such that $\hbar\omega_0(n_x + n_y + 1) - \overline{E_I} \gtrsim 2M\hbar^2/\ell^2$ will con-
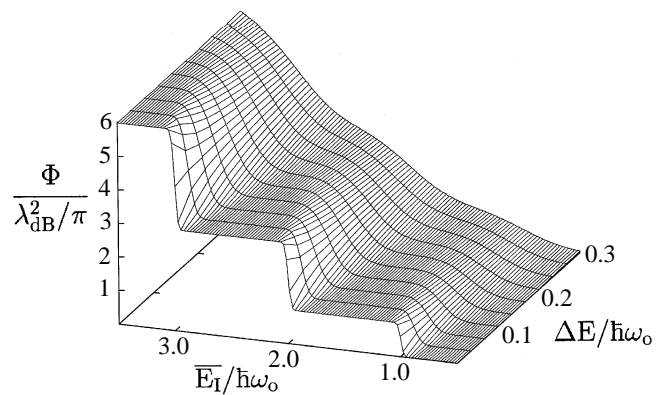

FIG. 3. Conductance Φ through a quantum point contact, as a function of average incident energy $\overline{E_I}$ and energy spread ΔE. Φ is plotted in terms of the quantized unit of conductance, $\lambda_{\text{dB}}^2/\pi$, and $\overline{E_I}$ and $k_B T$ are plotted in terms of $\hbar\omega_0$, the level spacing at the narrowest point of the constriction. The lowest ΔE shown, $0.02\hbar\omega_0$, corresponds to the example for ^{23}Na discussed in the text.

tribute negligible evanescent transmission and adiabatically reflect before $z = 0$. Steps occur when the number of allowed propagating modes changes: the mth step appears at $\hbar\omega_0 = \overline{E_I}/m$. Note that this condition can also be written $b_0 = \sqrt{m}\,\lambda_{\text{dB}}/2\pi$, demonstrating that transverse confinement on the order of $\lambda_{\text{dB}}/2\pi$ is essential to seeing conductance steps in a QPC. Since low-lying modes occupy a circularly symmetric part of the potential, the mth step involves m degenerate modes and is m times as high as the first step. The large aspect ratio of the atom QPC allows for a sufficiently gentle constriction to suppress partial reflection at the entrance to the guide, such that the sharpness of steps and flatness between them is limited only by the spread in incident atom energies.

It is interesting to compare the electron and atom QPC systems. If contact is made between two electron reservoirs whose chemical potentials differ by $e\Delta V < k_B T \ll E_F$, where ΔV is the applied voltage, T is the temperature of the electron gas, and E_F is the Fermi energy, then the current that flows between them will be carried by electrons with an energy spread $k_B T$ and a mean energy E_F. For a cold atom beam, the particle flow is driven by kinetics instead of energetics. The incident kinetic energy $\overline{E_I}$ corresponds to E_F, and the energy spread $\Delta E \ll \overline{E_I}$ corresponds to $k_B T$. The quantum of conductance for both systems can be formulated in terms of particle wavelength: the classical conductance of a point contact of area A connecting two three-dimensional gases of electrons is $G = (e^2 k_F^2 A)/(4\pi^2\hbar)$ [26], such that, if $G = Ne^2/\pi\hbar$, the effective area is $A = N\lambda_F^2/\pi$.

In order to determine the accuracy of conductance quantization, three measurements [F, $J_0 f(0,0)$, and λ_{dB}] are necessary for the atom QPC instead of two measurements (current and ΔV) for the electron QPC. The reduced number of degrees of freedom for electrons results from their

Fermi degeneracy: the net current is carried by electrons whose incident flux density J_0 and wavelength λ_F are functions of E_F and ΔV. As a thought experiment, the simplicity of an externally tuned J_0 and λ_{dB} could also be extended to neutral atoms, if two degenerate ensembles of fermionic atoms were connected by a QPC and given a potential difference ΔU. We can redefine neutral atom conductance as $\Gamma = F/\Delta U$, where F is the transmitted atom flux, just as the electron conductance G is the ratio of electron flux (current) to potential difference (voltage). One can show that

$$\Gamma = \frac{N}{h}, \qquad (4)$$

assuming $\Delta U < k_B T \ll E_F$, where T is the temperature of the Fermi ensembles and N is the number of modes above cutoff.

Two QPC's can form a trap between them, just as a pair of electron QPC's form a quantum dot [13]. For $\hbar\omega_0 > \overline{E_I}$, all modes of the QPC are below cutoff and evanescent transmission is dominated by tunneling of atoms occupying the $(0,0)$ mode. While the quantum dot between them is energetically isolated, atoms can still tunnel into and out of the dot. For cold fermionic atoms, the Pauli exclusion principle would enable a single atom to block transmission through the trap, just as the charging energy of a single electron can block transmission in electron quantum dots; such a blockade might be used to make a single-atom transistor. In such a single-atom blockade regime, quantum dots can also show a suppression of shot noise below the Poissonian level [13]. Note that spectroscopic measurement of neutral atom traps with resolvable energy levels has been suggested previously [17] in analogy to spectroscopic measurement of electron quantum dots. We emphasize that the loading and observation of such a small trap with two or more QPC "leads" is a powerful configuration for atom optics, because loading a small, isolated trap is problematic, and because spectroscopy near the substrate is complicated by light scattering and inaccessibility.

In conclusion, we show how an electromagnet waveguide could be used to create a quantum point contact for cold neutral atoms. This device is an example of a new physical regime, quantum transport within microfabricated atom optics.

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