Volumetric negative-refractive-index metamaterials based upon the shunt-node transmission-line configuration

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A volumetric negative-refractive-index transmission-line (NRI-TL) metamaterial is presented. This structure constitutes a natural extension of the planar NRI-TL metamaterials and maintains the desired features of broad bandwidth and low transmission loss. Unlike their planar counterparts, the proposed volumetric NRI-TL metamaterials can effectively couple incident plane waves from free space. The proposed topology can be readily made by stacking layers that are individually fabricated using standard multilayer printed-circuit board techniques at microwave frequencies. However, the creation of the volumetric structure results in the presence of a parasitic parallel-plate mode. This mode can interfere with the desired backward wave mode of the metamaterial, causing a stop band to appear. To facilitate the rapid analysis of this new design, a multiconductor transmission line model was developed. Through the use of this model and full-wave simulations, it will be demonstrated that this unwanted parallel-plate mode can be eliminated by properly arranging the vertical inductive loading wires. Using this process, it will be shown that a properly designed inductive load can result in a practical NRI metamaterial slab which is matched to free space over a large bandwidth (22%) and with low insertion loss (<1 dB). This approach can also be used to design NRI-TL metamaterials with backward wave dispersion bandwidths of over 140%. © 2007 American Institute of Physics. [DOI: 10.1063/1.2803924]

I. INTRODUCTION

Metamaterials based upon loaded transmission lines offer the potential of large negative refractive index (NRI) bandwidths with low-loss operation. Thus far, the research in this area has focused primarily on two-dimensional (2D) designs. These type of designs have been based upon a 2D “dual” transmission-line (TL) foundation, meaning that the loading of the TL reverses the natural orientation of the TL's capacitance and inductance. This results in a transmission line that has a series capacitive and shunt inductive nature, which gives rise to the backward wave propagation necessary for negative refraction. This concept of loaded TJs has also been recently extended to the optical domain in order to realize negative refraction and left-handed propagation at infrared frequencies. Efforts to adapt these ideas to create a three-dimensional (3D) NRI metamaterial have proven the difficulty of physically realizing these type of materials. In an important development, there have been recent reports of experimental evidence of a 3D isotropic lens based upon loaded TJs. However, this structure was excited by embedded sources and thus did not involve coupling from an incident free-space plane wave. This design is also difficult to realize due to the fact that it relies on connecting vertical printed circuit boards to the main horizontal boards. Moreover, isotropy is limited for electric fields that are vertically oriented with respect to the boards.

An intermediate step before the realization of a complete 3D NRI metamaterial is to extend the 2D dual TL design into the third dimension, so that cylindrical waves can be focused. This type of a 2D “volumetric” metamaterial was introduced by Iyer and Eleftheriades. The transmission-line design involved the stacking of a number of 2D dual TL layers. These layers contained an array of series TL unit cells, which made up the NRI medium. This type of dual TL configuration is created by connecting the transmission lines which run in the two orthogonal directions in the horizontal plane in a series fashion, as illustrated in Fig. 1(a). This is different from the shunt-node configuration, shown in Fig. 1(b), in which the lines are connected in shunt, which was at the heart of the original work in this area. Other approaches for realizing volumetric NRI metamaterials include using arrays of split-ring resonators and thin wires. In addition, the focusing effects of NRI metamaterials have also been observed through the interaction of magnetoinductive waves excited along coupled layers of split-ring resonators. However, since these techniques rely on the use of resonant elements, the transmission losses are often much larger than those that have been observed in TL based NRI metamaterials.

FIG. 1. Series (a) and shunt (b) node topologies for 2D dual TL metamaterials
The design of the volumetric series-node NRI metamaterial followed a similar process as that of the 2D dual TL. However, it was found that the stacking of the layers resulted in the introduction of an added coupling factor. This extra coupling was due to the inductive coupling between the 2D dual TL element and the host parallel-plate waveguide, which was created by stacking the 2D layers. Nevertheless, it was shown that if this coupling was accounted for in the design process a mode with moderately broad bandwidth NRI characteristics could be realized.8

The purpose of this paper is to present the concept and simulated results of a volumetric NRI metamaterial based upon the shunt-node 2D dual TL. The resulting volumetric NRI-TL metamaterial maintains the advantage of broad bandwidth and low insertion loss that was enjoyed by its planar counterpart. Moreover, this new topology can be easily coupled to a plane wave incident from free space, which can eliminate the parasitic forward PPW mode. Last, it will be demonstrated that this type of metamaterial can be easily coupled to a plane wave incident from free space, which has a vertically polarized electric field. This is an important characteristic for the use of such a metamaterial in subdiffraction-limited imaging applications.

II. MULTICONDUCTOR TRANSMISSION-LINE MODEL

In order to create a volumetric metamaterial based on the shunt-node 2D dual TL design, a number of layers must be stacked upon one another. This process results in the addition of another conductor above the standard 2D dual TL unit cell. Through image theory, this is equivalent to having the structure extend to infinity in the vertical direction. This new unit cell is depicted in Fig. 2. A second loading inductance, \( L_s \), has been placed between the microstrip line and the top ground plane. It will be demonstrated later that this inductance is necessary to enable backward wave propagation through a metamaterial based upon this unit cell.

A very useful model for this type of structure can be developed using multiconductor transmission-line (MTL) theory.14 For this model, the unit cell is considered to be made up of two loaded transmission lines over a common ground plane. The equivalent TL circuit is illustrated in Fig. 3. A similar analysis was presented in 2004 by Elek and Eleftheriades for a shielded Sievenpiper structure,15 which consisted of a microstrip patch loaded with a shunt inductance and series capacitors. For the purposes of this work, the model was developed using a microstrip transmission line of arbitrary width loaded by series capacitors, \( C_0 \), and both the shunt inductance, \( L_f \), and the “shorting” inductance, \( L_s \). The primed inductances indicate that the shunt-loading effects of the open-circuited stubs located transverse to the direction of propagation have been accounted for.

To begin the analysis, one can utilize the concept of transfer matrices, which are also commonly referred to as chain, or \( A B C D \), matrices. For a network with \( n \) ports, these matrices relate the output voltages and currents of that network to the input voltages and currents through a \( 2n \times 2n \) matrix. For this unit cell, this can be written as

\[
\begin{bmatrix}
V_{1f} \\
V_{2f} \\
I_{1f} \\
I_{2f}
\end{bmatrix} = T_{\text{UnitCell}} \begin{bmatrix}
V_{1o} \\
V_{2o} \\
I_{1o} \\
I_{2o}
\end{bmatrix},
\]

with \( T_{\text{UnitCell}} \) being the \( 4 \times 4 \) matrix which is calculated from the cascade of the transfer matrices of the different sections of the unit cell

\[
T_{\text{UnitCell}} = [T_{2C}][T_{\text{MTL}}][T_{L}][T_{\text{MTL}}][T_{2C}].
\]

Once the transfer matrix of the unit cell has been determined, the dispersion properties of this metamaterial can be calculated. This process, which is described in Ref.15, relies on Bloch’s theorem and the evaluation of the eigenvalues of \( T_{\text{UnitCell}} \).

In order to find the overall transfer matrix for the unit cell, the individual matrices must first be found. The effect of the series capacitive loading at either end of the unit cell can be represented by the transfer matrix, \( T_{2C} \).
with \([I]\) being the \(2 \times 2\) identity matrix and \([0]\) being the \(2 \times 2\) null matrix. This matrix illustrates that only the voltage along the first line is affected by the presence of the series capacitance. The matrix for the lumped inductive loading is similar and can be written as

\[
T_L = \begin{bmatrix}
[I] & 0 \\
0 & [I]
\end{bmatrix},
\]

where \(L'\) represents the parallel combination of \(L_S'\) and \(L_0'\).

The transfer matrix for the section of the coupled lines, \(T_{\text{MTL}}\), is a function of the modal wave impedance matrix, \(Z_w\), and the propagation constant matrix, \(\Gamma\). Each of these is a \(2 \times 2\) matrix, with the off diagonal elements representing the line-to-line coupling which can occur in multiconductor transmission lines. The details of how this matrix is calculated can be found in Ref. 14, but the modal characteristics of the MTL configuration are determined by finding the per-unit-length capacitance and inductance matrices. For an arbitrary MTL layout, a two-dimensional FEM solver, such as the commercially available COMSOL Multiphysics, can be used to find these quantities. In general this results in the matrix

\[
C = \begin{bmatrix}
C_1 & C_m \\
C_m & C_2
\end{bmatrix},
\]

with the inductance matrix calculated from running the same simulation with an all-air version of the MTL. This gives

\[
L = \frac{\mu_0 \varepsilon_0}{C} \begin{bmatrix}
L_1 & L_m \\
L_m & L_2
\end{bmatrix}.
\]

From these matrices, the propagation constant matrix is \(\Gamma = \sqrt{LC}\) and the wave impedance matrix is \(Z_w = j \omega \Gamma^{-1} L\). Since \(\Gamma\) includes the effects of coupling between the modes, a diagonalization procedure is needed to solve for the modal propagation characteristics of the modes supported by the MTL structure.14 With the combined use of COMSOL and MATLAB, the dispersion properties can be quickly and accurately evaluated.

### III. SHIELDED SHUNT-NODE NRI-TL VOLUMETRIC METAMATERIALS

In order to understand the reasoning behind the addition of the second inductive loading, \(L_S\) (see Fig. 2), it is important to first examine the case in which this loading is left out. This topology is essentially the ordinary shunt-node 2D dual TL metamaterial with an additional ground plane above it. This extra ground plane is due to the stacking of multiple layers to create the volumetric structure. Therefore, this can be thought of as a “shielded” version of the ordinary 2D dual TL metamaterial.

### IV. SYMMETRIC INDUCTIVE LOADING

One method for prohibiting this low-frequency FW mode propagation is to place a second shunt inductive load, \(L_3\), in line with the main shunt inductance, \(L_0\), between the middle line and the top plate as shown in Fig. 2. As an initial test of this topology, the loading inductances were made to be equal and the middle line was placed exactly at the midpoint between the top and bottom plates (\(h_1 = h_2 = 5\) mm).

The resulting dispersion characteristics for this type of metamaterial are presented in Fig. 4. Also shown in this figure are the results from an eigenmode simulation carried out in Ansoft’s HFSS, as well as the ordinary TL theory results for an open, or unshielded 2D dual TL.16 Both the MTL theory and the FEM simulations show the presence of a low-frequency forward-wave (FW) mode that closely follows the light line. This is the transverse electromagnetic (TEM) mode which is characteristic of a PPW waveguide. A backward-wave (BW) mode, which is due to the 2D dual TL, appears below 2 GHz. The upper cutoff frequency of the BW mode in the shielded structure is quite a bit different than that of the open 2D dual TL (1.9 GHz compared with 1.4 GHz). Since this cutoff frequency is primarily controlled by the series capacitive loading, \(C_0\), the shielding reduces the effectiveness of this series capacitance. This is an indication that the parallel-plate waveguide introduces an additional series inductance to the equivalent circuit of the 2D dual TL unit cell. This mutual coupling between the two modes is also evident by the fact that a stopband occurs (between 1.4 and 1.9 GHz) in the region where the two modes overlap. This modal coupling is a well-known phenomenon in NRI-TL metamaterials. It has been observed in a microstrip-based NRI material where the BW mode coupled to a forward surface-wave mode.3 This concept has also been used to realize very compact, highly directive couplers at microwave frequencies.17 To isolate the desired BW mode, it is clear that the low-frequency FW mode must be inhibited in some fashion.
The resulting dispersion diagram is shown in Fig. 5. It is clear that the FW mode has been effectively cutoff and a BW mode with a large bandwidth (100%) is achieved. The electric field magnitude and direction for this structure at 2 GHz is shown in Fig. 6. The field is equally distributed between the top and bottom halves, which is indicative of a stripline-like NRI medium. However, this type of a field pattern makes it very difficult to excite this mode with an incident plane wave.

V. ASYMMETRIC INDUCTIVE LOADING

A. Dispersion analysis

To enable the coupling of this NRI metamaterial structure to an external plane wave, the modal electric field must become completely polarized in the same direction as the incident field. This means that the field should become completely confined to the space either above or below the middle transmission line (see Fig. 2). In order to achieve this type of a field distribution, an asymmetry in the inductive loading is required.

The dispersion diagram for an example of this type of asymmetric loading is shown in Fig. 7. In this case, the main loading is still $L_0=15$ nH while the second inductive load is changed to $L_S=1$ nH and the height of the lower region has been reduced to $h_1=2$ mm. This height reduction was necessary to obtain a good match to a free-space plane wave excitation, as will be discussed later. For this design, the forward PPW mode is still suppressed leaving only the backward propagation between 1.08 and 1.94 GHz. The field plot shown in Fig. 8 demonstrates the ability of this configuration...
to confine the fields to the lower portion of the three conductor transmission line system. This enables a slab built up from this unit cell design to be well coupled to a free-space plane wave.

The dispersion properties of the unshielded ordinary 2D dual TL unit cell are also presented in Fig. 7. The obvious discrepancy between the characteristics of the BW mode for the shielded and unshielded cases shows that the shielding has a significant effect on the NRI metamaterial design. Even though the FW mode is suppressed and the corresponding modal coupling stopband is eliminated, the underlying BW of the 2D dual TL is not just simply “uncovered.” Therefore, a more complex design procedure such as that based on MTL theory is required for these types of NRI metamaterials. Overall, the MTL theory is fairly good at matching the full-wave results. The difference around the $\beta d=0^\circ$ point is likely due to the inaccurate modeling of the host MTL characteristics. It is important to observe that the upper FW modes are also well predicted by the MTL model. This is critical because these modes can interfere with the BW mode characteristics. It is important to observe that the upper FW mode can be seen to exist between 0.35 and 2 GHz (140% bandwidth), and the FW mode starts right at 2 GHz. This type of closed stop band is useful for designing metamaterials since it enables the realization of an index of refraction close to zero ($n=0$), as well as positive and negative values of $n$ around that frequency. For the topology presented in this paper, the dispersion characteristics indicate that as the frequency increases, the FW mode tends toward the same propagation properties as the standard PPW TEM mode.

B. Plane-wave transmission analysis

To verify that this structure could be coupled to external plane waves, a four cell slab was simulated within a parallel-plate waveguide. This slab was based upon a practical unit cell design, which incorporates a Rogers 5880 dielectric base upon which the microstrip transmission line is printed. The upper loading inductance is realized with a 6.83 mm long via with a 0.5 mm radius. Through HFSS simulations, it was found that the equivalent inductance of this via is on the order of 2 nH. As well, the losses of the copper conductors ($\sigma=5.8 \times 10^7 \, S/m$) and those of the finite $Q$ lumped capacitors and inductors were included in the HFSS and MTL simulations. The resulting dispersion diagram for this new design, along with the design parameters, are shown in Fig. 10. It can be seen that a backward wave exists between 1.3 and 1.88 GHz (36% bandwidth). Through further HFSS simulations, it was also observed that the dispersion characteristics of this design did not change at all for component $Q$ values ranging from 50 to 1000.

To test the ability of this unit cell design to be excited by a plane wave, a two-layer, four cell slab was simulated in HFSS. Perfect electric conductors (PECs) were used for the top and bottom planes and perfect magnetic conductors (PMCs) were used for the side planes. Through image theory, this models the case in which an infinitely tall and wide metamaterial slab, which is four cells thick, is illuminated with a plane wave at normal incidence. A 0.5 mm air gap was placed between the layers so that the two layers were electrically isolated from each other, see Fig. 12. The simulated transmission ($|S_{21}|$) and reflection ($|S_{11}|$) scattering parameters of this slab for various component $Q$ values, are shown in Fig. 11. For the $Q=100$ case, it can be seen that a
wide −10 dB matching bandwidth of 22% (1.62–2.03 GHz) is achieved. Field animations within the lower half of this band (1.62–1.85 GHz) are indicative of backward wave propagation, which demonstrate that the proper mode is being excited. This can be seen from the sideview field plots presented in Fig. 12. These plots indicate a very good excitation in both layers of the backward wave mode by the incident plane wave. In addition, the upper and lower frequency limits of the transmission through this four cell slab coincide very well with the dispersion characteristics for unit cell, see Fig. 10. It should also be noted that even with a conservative estimate of $Q_C=Q_L=100$, the insertion loss of the slab is less than −1 dB. If higher quality components are used (i.e., $Q$’s of 500 or more), this loss is reduced to less than −0.5 dB. The fact that the layers are isolated from each other demonstrates that this NRI metamaterial slab could be created by simply stacking the individual layers on top of one another. Each layer can be realized using standard printed-circuit board fabrication techniques, which simplifies the overall construction of a NRI metamaterial slab based on this shunt-node loaded-TL approach.

VI. CONCLUSIONS

This paper has demonstrated the possibility of using the 2D dual shunt-node TL configuration to create a volumetric NRI metamaterial. It has been shown that an additional inductive load is required to inhibit the PPM mode with interfering with the desired backward wave mode. As well, an asymmetry in the inductive loading is required to enable coupling to an incident plane wave. Through an analysis of the reflection and transmission properties of a metamaterial slab made from this unit cell design, excellent matching to a free-space plane wave has been achieved for a very practical design. This approach also maintains the broad bandwidth and low loss characteristics of the backward wave mode, which is a key advantage of loaded TL metamaterials. Indeed, a backward wave dispersion bandwidth of 100% was demonstrated for the symmetric inductive load topology and a 140% bandwidth was achieved in the asymmetric loading case with a closed stopband (enabling access to a refractive index $n=0$). As well, the practical four-cell metamaterial slab based upon the asymmetric loading concept was well matched to a free-space plane wave over a 22% bandwidth. These large bandwidths are a result of the tight coupling between the resonant elements of the unit cell. This point-of-view is further supported by a recent design of a large bandwidth metamaterial based upon the split-ring-resonator/wire approach in which the split-ring resonators are electrically connected. Moreover, the resulting insertion loss is only moderate (e.g., on the order of −0.25 dB per unit cell for $Q=100$ component value). It was also evident from this work that the shielding and added inductive loading resulted in dispersion characteristics which were much different from those predicted by ordinary 2D dual TL theory. Therefore, a more complex analysis technique, such as the MTI modeling method, is required for accurate design of this topology of TL metamaterials. This proposed NRI-TL metamaterial is compatible with standard printed-circuit-board fabrication techniques at microwave frequencies and a volumetric slab can be easily made by stacking multiple layers of this metamaterial type. It is believed that this volumetric shunt-node TL metamaterial will lead to the further development of super resolution lenses, which are possible with NRI slabs of this type.