Online Appendix to

Asymmetric Attention and Stock Returns

Any investor $i \in [0, 0.5]$ belongs to the local region, has an ex-ante information set $\Omega_i = \{U_L\}$, and has an ex-post information set $I_i = \{\tilde{U}_L, \tilde{Y}_{iL}, \tilde{Y}_{iN}\}$. Any $i \in (0.5, 1]$ belongs to the nonlocal region, has an ex-ante information set $\Omega_i = \emptyset$, and has an ex-post information set $I_i = \{\tilde{Y}_{iL}, \tilde{Y}_{iN}\}$. Any investor i, with a constant risk aversion parameter equal to one, maximizes the utility function

$$EU_i = E\left[E(W_i' \mid I_i) - \frac{1}{2}V(W_i' \mid I_i) \mid \Omega_i\right], \tag{1}$$

where W_i' is the wealth of the investor in the last period, subject to the budget constraint

$$W_i' = W_0\bar{R} + q_{iL}(\tilde{R}_L - \bar{R}P_L) + q_{iN}(\tilde{R}_N - \bar{R}P_N), \tag{2}$$

where q_{iL} and q_{iN} are the asset holdings of the investor, and P_L and P_N are the asset prices of the local and nonlocal asset, which are taken as given. The attention allocation choice of investor i will lead to a signal $\tilde{Y}_{ij}$ about each risky asset $j = L, N$ given by

$$\tilde{Y}_{ij} = \tilde{R}_j + \tilde{\eta}_{ij},$$

where $\tilde{\eta}_{ij} \sim N(0, \sigma^2_{\eta ij})$.

We solve the model using backward induction. First, given an arbitrary information choice, each investor decides her optimal asset holdings. Second, given the optimal risky asset demand for each signal, each investor decides her optimal information choice.

1 Portfolio Choice

First, the investor chooses the optimal risky asset demand taking the signals as given. After observing the signals, the investor derives her posterior beliefs for each asset $j = L, N$ and maximizes the following utility function

$$E(W_i' \mid I_i) - \frac{1}{2}V(W_i' \mid I_i).$$

Substituting in the budget constraint (2), we obtain

$$\left[W_0\bar{R} + q_{iL}(E[\tilde{R}_L \mid I_i] - \bar{R}P_L) + q_{iN}(E[\tilde{R}_N \mid I_i] - \bar{R}P_N)\right] - \frac{1}{2}\left[q_{iL}^2 V[\tilde{R}_L \mid I_i] + q_{iN}^2 V[\tilde{R}_N \mid I_i]\right].$$

Taking the first-order condition with respect to q_{ij} for $j = L, N$, we obtain

$$q_{ij} = \frac{E[\tilde{R}_j \mid I_i] - \bar{R}P_j}{V[\tilde{R}_j \mid I_i]}. \tag{3}$$

This equation tells us that the investor will buy more of assets that have high expected payoffs and low conditional volatility. Note that mean-variance preferences imply a demand for risky assets that does not depend on wealth.

2 Information Choice

Second, the investor chooses the optimal allocation of information resources, κ_{iL} and κ_{iN}. Taking into account the optimal asset demand given by equation (3), investors maximize their objective function given by equation (1) subject to the information constraint.

Substituting q_{ij} back into the utility function (1), we obtain:

$$EU_i = W_0\bar{R} + \frac{1}{2}E\left[\frac{\left(E[\tilde{R}_L \mid I_i] - \bar{R}P_L\right)^2}{V[\tilde{R}_L \mid I_i]} + \frac{\left(E[\tilde{R}_N \mid I_i] - \bar{R}P_N\right)^2}{V[\tilde{R}_N \mid I_i]} \mid \Omega_i\right] \tag{4}$$

We have to take the expectation of a squared random variable. Recall that for any random variable x, we can calculate $E[x^2] = V(x) + [E(x)]^2$. In this particular case, for any investor i and asset j,

$$x = \frac{E[\tilde{R}_j \mid I_i] - \bar{R}P_j}{(V[\tilde{R}_j \mid I_i])^{1/2}},$$

where the expectation is given by

$$E(x) = \frac{E[\tilde{R}_j \mid \Omega_i] - \bar{R}P_j}{(V[\tilde{R}_j \mid I_i])^{1/2}},$$

and the variance is given by

$$V(x) = \frac{V\left[E[\tilde{R}_j \mid I_i] \mid \Omega_i\right]}{V[\tilde{R}_j \mid I_i]} = \frac{V[\tilde{R}_j \mid \Omega_i] - V[\tilde{R}_j \mid I_i]}{V[\tilde{R}_j \mid I_i]}.$$

We apply the law of total variance $V[E(X \mid Y)] = V(X) - E[V(X \mid Y)]$ is applied in the second equality. Therefore, for $j = L, N$, we obtain

$$E[x^2] = E\left[\frac{\left(E[\tilde{R}_j \mid I_i] - \bar{R}P_j\right)^2}{V[\tilde{R}_j \mid I_i]}\right] = \frac{V[\tilde{R}_j \mid \Omega_i] - V[\tilde{R}_j \mid I_i]}{V[\tilde{R}_j \mid I_i]} + \frac{\left(E[\tilde{R}_j \mid \Omega_i] - \bar{R}P_j\right)^2}{V[\tilde{R}_j \mid I_i]}.$$

Applying this result to the investor's expected utility for any posterior belief in equation (4), we obtain

$$EU_i = W_0\bar{R} - 1 + \frac{1}{2}\frac{V[\tilde{R}_L \mid \Omega_i]}{V[\tilde{R}_L \mid I_i]}(1 + \theta_{iL}^2) + \frac{1}{2}\frac{V[\tilde{R}_N \mid \Omega_i]}{V[\tilde{R}_N \mid I_i]}(1 + \theta_{iN}^2), \tag{5}$$

where $\theta_{ij}^2 = \frac{\left(E[\tilde{R}_j \mid \Omega_i] - \bar{R}P_j\right)^2}{V[\tilde{R}_j \mid \Omega_i]}$ is the squared Sharpe ratio of asset $j = L, N$ for investor i. The information constraint is given by

$$\kappa = \log V[\tilde{R}_L \mid \Omega_i] - \log V[\tilde{R}_L \mid I_i] + \log V[\tilde{R}_N] - \log V[\tilde{R}_N \mid I_i],$$

which can be rewritten as

$$\frac{e^\kappa}{V[\tilde{R}_L \mid \Omega_i]V[\tilde{R}_N \mid \Omega_i]} = \frac{1}{V[\tilde{R}_L \mid I_i]}\frac{1}{V[\tilde{R}_N \mid I_i]}. \tag{6}$$

Hence, investors maximize their objective function given by equation (5) subject to the information constraint (6). Since every signal variance $\sigma_{\eta iL}^2$ and $\sigma_{\eta iN}^2$ has a unique posterior belief variance $V[\tilde{R}_L \mid I_i]$ and $V[\tilde{R}_N \mid I_i]$ associated with it, we can economize on notation and optimize over the inverse of posterior variances. Thus, the problem simplifies to maximizing a weighted sum subject to a product constraint. Note that a posterior variance can never exceed a prior variance:

$$V[\tilde{R}_j \mid I_i] \leq V[\tilde{R}_j \mid \Omega_i].$$

We can write our optimization problem as

$$\max_{x_1, x_2} a_1 x_1 + a_2 x_2$$

subject to

$$a_3 = x_1 x_2,$$

where a_1, a_2 and a_3 are positive constants and $x_1 \geq 0$ and $x_2 \geq 0$. If we substitute the constraint into the objective function, then we get the following unconstrained optimization problem

$$\max_{x_1} a_1 x_1 + a_2 \frac{a_3}{x_1}.$$

The objective function is a convex function since the second-order condition is given by $2a_2 a_3 x_1^{-3} \geq 0$ as long as $x_1 \geq 0$, hence the solution to the optimization problem is a corner solution. There are two corner solutions to the optimization problem. The first solution is to use all information resources to learn about the local asset such that conditional variances are given by $V[\tilde{R}_L \mid I_i] = \frac{V[\tilde{R}_L \mid \Omega_i]}{e^{\kappa}}$ and $V[\tilde{R}_N \mid I_i] = \sigma_R^2$, and expected utility equals $EU_i = W_0 \bar{R} - 1 + e^{\kappa}(1 + \theta_{iL}^2) + (1 + \theta_{iN}^2)$. The second solution is to use all information resources to learn about the nonlocal asset such that conditional variances are given by $V[\tilde{R}_L \mid I_i] = V[\tilde{R}_L \mid \Omega_i]$ and $V[\tilde{R}_N \mid I_i] = \frac{\sigma_R^2}{e^{\kappa}}$, and expected utility equals $EU_i = W_0 \bar{R} - 1 + (1 + \theta_{iL}^2) + e^{\kappa}(1 + \theta_{iN}^2)$. The optimal information choice by investor i is to allocate all information resources to learn about the asset j with the highest squared Sharpe ratio θ_{ij}^2.