The Asset Pricing and Real Implications of Relationship Disclosure

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Abstract

In many scenarios, investors in financial markets are uncertain about the relationship between two firms and have to rely on firms' disclosure of such relationship. We develop a theory to study the asset pricing implications of this relationship uncertainty and how such relationship uncertainty affects firms' incentives to form and disclose their relationships to the public in the first place (i.e., the real implications). We find that while disclosing relationships has a positive price impact by increasing the expected cash flow, it also has a negative price impact by reducing the diversification benefit (or, equivalently, increasing the diversification cost) of investing in multiple firms that have more correlated cash flows. The price impact upon relationship disclosure is therefore not monotone: it increases with the expected benefit of relationship and decreases with the risk of the underlying relationship.

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One main policy implication of our analysis is that mandatory disclosure of firm relationships may both destroy relationship development and reduce investor welfare. In other words, disclosing relationship information can have real consequences on cash flows through affecting firm relationships at both the intensive and the extensive margins. The results are robust to a battery of extensions.

Keywords: firm relationships, asset prices, disclosure, matching intensity, collaboration intensity

JEL: D82, G14, G18, M41, M45

1 Introduction

In this paper we propose a model that studies the asset pricing and real implications of disclosure about relationship between firms. Firms do not operate in isolation and financial markets consist of a network of companies that are linked to and interact with each other. These companies are linked through contractual relationships such as customer-supplier agreements or implicit relationships such as strategic alliances, common production or labor exposures, and similar regulation requirements or litigation risks. Not surprisingly, regulators require many disclosures of such linkages. For example, SFAS No. 131 requires firms to disclose the existence of and the total amount of revenues from each major customer that represents more than 10 percent of a firm's sales revenue. As another example, Regulation S-K requires firms to disclose all material contracts or agreements into which a firm enters. Such relationship disclosure will clearly affect investors' perception of the firms' underlying cash flows and thus their capital market prices. Concerns about capital market prices will in turn have real effects on firms' investment in such relationship in the first place (Kanodia and Sapra (2016)). In other words, disclosure of relationship affects the correlation of firms' cash flows endogenously.

Traditional theories on asset prices and disclosure assume that the correlation structure of asset payoffs is exogenously given and known to investors through links among firms. However, knowledge of such payoffs is often obtained through firms' endogenous disclosure choices. In addition, disclosure of such relationship is often incomplete and differs among firms. For instance, while regulation SFAS No. 131 requires firms to disclose the existence of and the percentage of revenues from each major customer that represents more than 10 percent of a firm's sales revenue, firms can choose whether to disclose the identity of the major customer. For example, Pepsi disclosed in its 2020 annual report that "Walmart and its affiliates (including Sam's) represented approximately 14% of our consolidated net revenue," whereas Nike only disclosed in its 2020 annual report that "our three largest customers accounted for approximately 24% of sales in the United States" without disclosing the names of those customers. In addition, as discussed in Ellis et al. (2012), firms can choose to voluntarily disclose non-major customers. The possibility of disclosing some but not all relationships suggests that investors are uncertain about relationships and exposure between firms. Uncertainty about such relationships and exposures implies that investors have imperfect knowledge about the correlation structure of asset payoffs. In this paper, we study how investors value assets when there is such uncertainty about relationships, and the incentives of firms to disclose and form such relationships. While previous studies on firms' disclosure of relationships (e.g., Ellis et al. (2012) and Verrecchia and Weber (2006)) focus mostly on how proprietary cost affects firms' disclosure choices, we abstract away from such proprietary cost and instead focus on how the residual uncertainty about relationships affects firms' disclosure choices and, perhaps more importantly, firms' choices of whether to establish a relationship in the first place, through affecting firms' stock prices.

In our setting, there are two firms with an uncertain exposure to a common relationship risk factor, which we label as "matching intensity." [1] We assume that if two firms form a high (low) intensity match, their collaboration will generally lead to more (less) cash flows (in expectation). The valuation of both firms is determined by a representative investor who forms her portfolio in a competitive asset market. Having a relationship has two effects on the firms' valuation. First, the cash flows have an additional payoff component with a positive mean (i.e., firms on average collaborate on positive NPV projects). This increase in the mean of asset payoffs increases asset prices via boosting the investor's perceived returns on investing in those risky assets. Second, the cash flows of the two firms are more risky and become more correlated, which harms the investor's ability to diversify her portfolio, thereby lowering her demand for those risky assets and their prices.

The effect of the relationship on firms' prices in turn affects the optimal disclosure policy about matching intensity. Unlike most of the existing studies on disclosure, disclosure in our setting is about disclosing the exposure to a common component in the asset payoffs of the two firms, which generates the correlation structure between the two firms. It is not about disclosing

[1] Therefore our setting can be literally interpreted as two big firms in a large economy, which may not be unreasonable in the example of Walmart being a major customer of Pepsi, in addition to making the model tractable. We discuss how our results can be extended in settings with N-firms in Section 6.

the realization of the fundamentals as in most research on disclosure (see, e.g., Dye (2001),Verrecchia (2001), Beyer et al. (2010), Stocken (2013), Kanodia and Sapra (2016), and Goldstein and Yang (2017) for excellent reviews) and in particular, the literature on proprietary cost (Verrecchia (1983)). Even when abstracting away from such proprietary cost, we find that firms may not always want to make relationship disclosures. Specifically, firms in relationships with very large or very small collaboration intensity relative to the collaboration risk of forming such relationships will choose to fully disclose their relationships, while firms with collaboration intensity at an intermediate level relative to the collaboration risk will choose not to disclose any information about their relationship. Intuitively, firms with very large expected collaboration intensity relative to collaboration risk have an incentive to fully disclose to let the investor be aware that the increase in cash flow relative to increase in risk is large. When the collaboration intensity is sufficiently small relative to collaboration risk, the investors are particularly concerned about the diversification cost and therefore put more weight on the high uncertainty scenario in the absence of disclosure. The firm then has an incentive to commit to full disclosure to clarify when such uncertainty is low. For firms with intermediate collaboration intensity relative to collaboration risk, investors are not much concerned about the diversification cost and therefore put more weight on the low uncertainty scenario in the absence of disclosure. The firm then has an incentive to not disclose to make such clarification. Algebraically, the results are driven by the effect of more precise disclosure on higher moments of cash flow distribution, specifically, skewness and kurtosis, which relates our finding to that in Heinle et al. (2018). However, as discussed in more detail in Section 2, our binary distribution of relationship intensity implies that more precise disclosure does not unambiguously increase skewness and kurtosis.

We then use our framework to investigate the choice of firms to form relationships and how the choice of forming relationships interacts with the choice of disclosing such relationship, that is, the real effects of such disclosure. Unlike previous studies, we do not take the relationship as given but we treat the decision to form a relationship as endogenous. Firms will choose to form a relationship when the expected price of forming a relationship is higher than the expected price when there is no relationship. We assume the decision is made before the firm is aware whether the relationship will be a high intensity or low intensity match. We find that when the expected benefits of forming a relationship are very large relative to the costs, firms choose to form a relationship with disclosure. Instead, when the expected benefits are at intermediate levels relative to the costs, firms form a relationship without disclosure. Intuitively, when the expected benefit is sufficiently high, forming a relationship increases the perceived firm value and thus expected price. Conditional on forming a relationship, whether or not firms would want to disclose depends on the relative magnitude of the expected benefit, as discussed above. Finally, when the relative expected benefits are low, firms choose not to form a relationship.

This new setting allows us to analyze the implications of policies that mandate disclosure of relationships. We find that even absent proprietary cost concerns, mandatory disclosure may prevent relationship formation. Specifically, we find that when firms are forced to disclose their relationships, some relationships that would have been formed, specifically, when the expected benefits of forming a relationship relative to the costs are at an intermediate range, will not be formed at all. The reason is that, as discussed above, firms would voluntarily choose not to disclose in this case. Therefore, mandatory disclosure, through reducing expected prices, results in firms not forming relationships in the first place. Thus, mandatory disclosure will have an effect on the extensive margin of relationships. In addition, we show that such real effects have an unambiguously negative effect on investor welfare, as the reduced expected cash flow more than compensates the reduced risk from not forming a (risky) relationship in the first place. In contrast, mandatory disclosure have an unambiguously positive effect on investor welfare when relationship formation is not destroyed, by lowering the price that investors pay for firms. Our paper therefore provides a novel trade-off regarding regulation aimed at relationship disclosure such as SFAS 131.

Finally, we analyze a couple of extensions of the model. We first extend our basic framework to settings of N-firms. We consider two settings: in the first setting, there are N-firms and either N-firms can choose to form relationship with each other or not, sharing a similar spirit to prior work on strategic network formation (e.g.,Goyal and Vega-Redondo (2007)); in the second setting, N-1 firms have already formed relationship with each other and the Nth firm is considering whether to join the relationship network. In both settings, we find that our results in the basic framework remain qualitatively unchanged: firms would voluntarily disclose their relationship if the collaboration intensity relative to collaboration risk is either sufficiently large or small, and mandatory relationship destroys such relationship if firms' collaboration intensity relative to the collaboration risk is in the intermediate range.

We also consider a setting where there is correlation between firm-specific cash flow and cash flow from relationship formation. We find that firms are more likely to disclose relationship when the correlation is negative and less likely when the correlation is positive. Intuitively, negative (positive) correlation reduces (increases) the diversification cost, making firms more likely to disclose. Nevertheless, our results in the basic framework remain qualitatively unchanged so long as the correlation is not too negative, as firms will always disclose their relationships when the correlation becomes too negative.

We finally consider a setting when firms observe the realization of relationship matching intensity and chooses whether to disclose ex-post. Without adding frictions full disclosure is optimal, in line with the previous literature. When adding frictions such as disclosure cost, whether firms will disclose high matching intensity or low matching intensity depends on whether the expected benefit from relationship formation is larger than the increase in diversification cost, with firms disclosing high (low) matching intensity when the expected benefit is large (small).

The rest of this paper is organized as follows. Section 2 discusses related literature. Section 3 introduces our framework to study the asset pricing implications regarding uncertainty of a relationship between two firms. Section 4 addresses firms' voluntary disclosure strategies of relationships. Section 5 discusses how relationship disclosure affects formation of relationship in the first place, that is, the real implications of relationship disclosure. Section 6 discusses extensions to N firms. Section 7 introduces correlation between firm-specific cash flow and cash flow from relationship formation. Section 8 explores ex-post relationship. Section 9 examines the effect of disclosure mandate on investors' welfare. Section 10 discusses implications of our analysis. Finally, Section 11 concludes. All proofs are relegated to the Appendix. More technical details and variations of the model are included in the Online Appendix.

2 Related Literature

Our paper is related to the extensive literature that has examined the implications of disclosure for market quality. Recent studies analyze the implications of financial disclosure and argues that both the amount (e.g.,Tang (2014)) and the type of the information disclosed (e.g., Goldstein and Yang (2019)) are crucial determinants of market quality (see Verrecchia (2001), Dye (2001), and, more recently, Goldstein and Yang (2017) for surveys on this topic). The main reason to disclose information voluntarily is to reduce information asymmetries between investor and firms (e.g., Diamond and Verrecchia (1991), Easley and O'hara (2004), Lambert et al. (2007)) or reducing uncertainty about future payoffs (e.g., Barry and Brown (1985), Coles and Loewenstein (1988), Cheynel (2013)). The implications of mandatory disclosure are studied by Fishman and Hagerty (2003) who show that customer's sophistication level is of great importance to determine the benefit of mandatory disclosure, and by the real effects literature (Kanodia and Sapra (2016)) showing that mandatory disclosure may have unintended consequences of distorting firms' real decisions. Unlike these prior studies, where disclosure is related to the realization of private information, our paper focuses on disclosure of firm relationships and its implications for asset prices.

Our paper is also related to the vast literature on diversification discount (e.g., Lang and Stulz (1994), Berger and Ofek (1995)), which focuses on the cost of having multiple segments inside a firm instead of forming a relationship between firms. While this literature in general documents a discount for diversified firms relative to the sum of the estimated value of their segments, Campa and Kedia (2002) show that after controlling for the endogenous decision of firms to diversify, diversification actually improves firm value, which is similar in spirit to our assumption that forming relationship improves firm value. However, the diversification literature finds that diversification within a firm reduces risk, whereas we find that relationship formation increases the systematic risk. This result is consistent with the empirical findings in Herskovic (2018), who documents that networks in production (a source of inter-firm relationship) are sources of systematic risk reflected in equilibrium asset prices.

Another line of research studies capital market response to firms' disclosure choice about certain relationships, and how the existence of such relationships affect firms' public disclosures. Examples include disclosure about consumers (Ellis et al. (2012)), material contracts (Verrecchia and Weber (2006), Samuels (2021)), and strategic alliances (Ma (2019), Kepler (2021)). Analytically, Darrough and Stoughton (1990) and Wagenhofer (1990) model the decision to disclose information on customer-supplier relationships when there is a benefit of reducing information asymmetries but also a cost of revealing proprietary information to competitors. Correspondingly, most if not all of the papers are couched in a proprietary cost framework, suggesting that firms should always disclose such relationships in the absence of proprietary cost. In contrast, our paper shows that even without proprietary cost, firms may not want to make relationship disclosure because such disclosure results in a diversification cost. In addition, our paper also studies the effect of disclosure on the firms' decision to form a relationship, whereas relationships are assumed to be exogenous in those papers.

In a closely related paper, Heinle et al. (2018) analyze the effects of factor-exposure uncertainty on asset prices and the implications of disclosure about exposure. Similarly, our paper models uncertainty about a relationship as uncertainty about exposure to a common factor between two firms. Since there are two firms in our model, we are able to capture a diversification cost of having a relationship that is not explored by Heinle et al. (2018). Also, the way we model exposure to the common factor is different because we are pursuing different research questions. Our model is able to capture a situation in which an investor does not know whether two firms have a relationship, while this is not possible in the previous literature on disclosure. In addition, as stated previously, our paper endogenizes the firms' decision to form a relationship instead of taking it as exogenously given. Of course, our paper also shares a lot of similarities, e.g., prices are affected by higher order moments of cash flow distribution and more disclosure does not necessarily increase the cost of capital. However, our paper also differs from Heinle et al. (2018) in this aspect, by showing that more precise disclosure does not unambiguously increase skewness and decrease kurtosis. We believe this is due to our binary distribution of factor exposure being not a symmetric distribution, as the normal distribution in Heinle et al. (2018). In addition, our binary distribution provides us with a tractable framework to characterize the necessary and sufficient conditions for more precise disclosure to decrease the cost of capital, thus allowing us to answer the question of the real effects of more precise disclosure on relationship formation.[2]

A growing literature studies endogenous network formation, in which the aim of forming such network could be acquiring information (Herskovic and Ramos (2018), Galeotti and Goyal (2010)), or forming input-ouput relationships (Acemoglu and Azar (2017), Taschereau-Dumouchel (2017), Lim (2018), Oberfield (2018), Tintelnot et al. (2018)).[3] In contrast to these papers, disclosure policy is at the core of our analysis. Furthermore, our paper contributes to this literature by showing that disclosure policies can affect customer-supplier relationships. More specifically, mandatory disclosure may prevent such relationship formation in the first place.

Finally, our paper features relationship uncertainty, which connects our framework to a few theoretical studies in the literature that have considered uncertain factor loadings in various forms (e.g., Armstrong et al., 2013; Beyer and Smith, 2021; Huang et al., 2021) and a more recent literature on disclosing algorithms (e.g., Brunnermeier et al. (2020), Sun (2021)). In these papers, the random factor loadings or the statistical properties of algorithms are exogenous, while our framework makes an effort to endogenize these random loadings via relationship formation.

[2] See Section 4 for more detailed discussions.

[3] See also Bernard and Moxnes (2018) and Carvalho and Tahbaz-Salehi (2018) for recent literature reviews on production networks in international trade and macroeconomics, respectively.

3 Conceptual Framework

In this section, we build a framework to study the effects of uncertainty of a relationship between two firms on asset prices.

3.1 Setup

Consider two symmetric firms: A and B. We normalize the number of shares of each firm to 1. The cash flows of each firm have two components and are given by $\tilde{F}_A = \tilde{V}_A + \tilde{\Delta}$ and $\tilde{F}_B = \tilde{V}_B + \tilde{\Delta}$. The components $\tilde{V}_A$ and $\tilde{V}_B$ are firm-specific (and so mutually independent) and normally distributed with mean $\bar{V}$ and variance σ_V^2: $\tilde{V}_A \sim N(\bar{V}, \sigma_V^2)$ and $\tilde{V}_B \sim N(\bar{V}, \sigma_V^2)$. The component $\tilde{\Delta}$ is common between the two firms and reflects the cash flow correlation driven by the relationship between both firms. This second component is given by

$$\tilde{\Delta} = \tilde{\rho}\tilde{\delta}, \text{ with } \tilde{\delta} \sim N\left(\bar{\delta}, \sigma_{\delta}^2\right) \text{ and } \tilde{\rho} = \begin{cases} \rho_h & \text{with probability } \pi, \\ \rho_l & \text{with probability } 1 - \pi, \end{cases} \tag{1}$$

where $\rho_h > \rho_l \geq 0$ and $\bar{\delta} \geq 0$. The random variables $\tilde{\rho}, \tilde{\delta}, \tilde{V}_A$, and $\tilde{V}_B$ are mutually independent.

In this setup, a relationship between the two firms determines their payoff correlation through two elements, $\tilde{\rho}$ and $\tilde{\delta}$. First, a relationship may turn out to be a high intensity match with ρ_h or a low intensity match with ρ_l. We refer to $\tilde{\rho}$ as "matching intensity." Second, given matching intensity, if the two firms engage in intensive collaboration, say, collaborating across multiple product lines, their payoff correlation tends to be strong. We interpret $\tilde{\delta}$, or more specifically, its mean $\bar{\delta}$, as "collaboration intensity."

A special case of specification (1) is $\pi = \rho_l = 0$, which implies that $\tilde{\Delta} = 0$. This degenerate setting corresponds to a benchmark setting with independent cash flows (no relationship between the two firms). Another interesting but less specialized case is $\rho_l = 0$ but $\pi > 0$. In this case, the uncertainty about matching intensity $\tilde{\rho}$ can be reinterpreted as uncertainty about the existence of a relationship between firms: the market is ex ante uncertain about whether two firms are related, but once the market becomes certain that there is a relationship between the two firms, then it is known that the matching intensity of the two firms is ρ_h.

There exists a representative investor, whose preference is $-e^{-\gamma\tilde{W}}$, where $\gamma > 0$ is the absolute risk aversion coefficient and $\tilde{W}$ is the final wealth. The investor is initially endowed with W_0 wealth and chooses the asset holdings that maximize her preference subject to the following budget constraint

$$\tilde{W} = W_0 R + q_A(\tilde{F}_A - RP_A) + q_B(\tilde{F}_B - RP_B), \tag{2}$$

where R is the exogenous risk-free interest rate, q_A and q_B are the asset holdings of the risky assets, and P_A and P_B are asset prices. In the following two subsections, we compute asset prices when the matching intensity is known or random respectively.

As a final remark, we consider a two-firm setting as the main model, which can be literally interpreted as two big firms in their respective industries. A two-firm setting is the simplest way to model relationship among firms and is consistent with the Pepsi-Walmart example discussed in the introduction. While it certainly raises the issue of whether the risk exposure due to the relationship among two firms can be diversified away in a multiple-firm setting, we later in Section 6 extend our analysis to N-firm settings and show that our results remain largely the same, when relationship formation among N firms are modelled as a natural extension to that of the two-firm setting. For now we focus on the two-firm setting to better illustrate the driving forces and the underlying intuition of our results.

3.2 Asset Prices under Perfect Matching Information

We use superscript "PI" to denote the case in which the investor has perfect information about the firms' matching intensity $\tilde{\rho}$. That is, the representative investor knows the realization of $\tilde{\rho}$. In this case, the common component of cash flows $\tilde{\Delta}$ follows a Normal distribution. The expectation and variance of final wealth in equation (2) for any realization of $\tilde{\rho}$ are given by

$$E[\tilde{W} \mid \tilde{\rho}] = W_0 R + q_A(\bar{V} + \tilde{\rho}\bar{\delta} - RP_A^{PI}) + q_B(\bar{V} + \tilde{\rho}\bar{\delta} - RP_B^{PI}),$$

and

$$V[\tilde{W} \mid \tilde{\rho}] = q_A^2(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2) + q_B^2(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2) + 2q_A q_B \tilde{\rho}^2\sigma_\delta^2.$$

When the investor maximizes her expected utility subject to the budget constraint, her demand for asset A is given by

$$q_A^{PI} = \frac{(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2)(\bar{V} + \tilde{\rho}\bar{\delta} - RP_A^{PI}) - \tilde{\rho}^2\sigma_\delta^2(\bar{V} + \tilde{\rho}\bar{\delta} - RP_B^{PI})}{\gamma[(\sigma_V^2 + \tilde{\rho}^2\sigma_\delta^2)^2 - (\tilde{\rho}^2\sigma_\delta^2)^2]}. \tag{3}$$

Similarly, one can find the demand for asset B. The demand of one asset is affected by the demand of the other asset. We can find the asset prices under perfect information by imposing the market-clearing conditions $q_A = 1$ and $q_B = 1$. The price for asset $j \in \{A, B\}$ under perfect information is given by

$$P_j^{PI}(\tilde{\rho}) = \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}\tilde{\rho} - \frac{2\gamma\sigma_\delta^2}{R}\tilde{\rho}^2, \text{ for } j \in \{A, B\}. \tag{4}$$

The first term of the price would be the price when $\tilde{\rho} = \tilde{\Delta} = 0$, that is, when firms have no relationship. The second and third terms convey the additional effects of relationships in prices. Specifically, the second term captures the benefit of having a relationship, which is an additional payoff factor. The third term, instead, captures the cost of forming a relationship: payoffs of both assets are more risky and become correlated, making it more difficult for the investor to diversify risk. The risk aversion parameter γ affects prices in two ways. First, the weight of the variance in the price increases with risk aversion. Second, the risk aversion parameter also affects the weight associated with the decrease in the investor's ability to diversify when firms form a relationship. Under both effects, prices decrease with risk aversion.

The next corollary characterizes how the features of a relationship influence asset prices under perfect matching information. The proof is straightfoward and thus omitted.

Corollary 1. *Asset prices under perfect matching information (i) increase in $\bar{\delta}$, the level of collaboration intensity; (ii) decrease in σ_δ^2, the risk of the relationship; and (iii) increase in $\tilde{\rho}$, the level of matching intensity, if and only if $\tilde{\rho} < \frac{\bar{\delta}}{4\gamma\sigma_\delta^2}$.*

Corollary 1 is intuitive. Prices will increase with the expected benefit of the relationship due to its positive impact on the cash flow, decrease with the underlying risk of the relationship due to its negative impact on the variance of the cash flow. Higher matching intensity increases both the mean and the variance of the underlying cash flow and so it increases asset prices only when the mean effect is sufficiently large. Since the mean effect increases linearly with $\tilde{\rho}$ but the variance effect increases quadratically with $\tilde{\rho}$ (and thus increases faster), higher matching intensity increases asset prices only when $\tilde{\rho}$ is sufficiently small.

3.3 Asset Prices under Matching Uncertainty

Facing uncertainty about matching intensity, the representative investor has to compute her expected utility without knowing $\tilde{\rho}$. For a given realization of $\tilde{\rho}$, final wealth follows a normal distribution. If $\tilde{\rho} = \rho_h$, then $\tilde{W} \sim N(\mu_W(h), \sigma_W^2(h))$, while if $\tilde{\rho} = \rho_l$, then $\tilde{W} \sim N(\mu_W(l), \sigma_W^2(l))$, where $\mu_W(s)$ and $\sigma_W^2(s)$ for $s \in \{l, h\}$ are the expectation and variance of final wealth conditional on $\tilde{\rho} = \rho_s$, and are given by

$$\mu_W(s) = E[\tilde{W} \mid \tilde{\rho} = \rho_s] = W_0 R + q_A(\bar{V} + \rho_s \bar{\delta} - RP_A) + q_B(\bar{V} + \rho_s \bar{\delta} - RP_B),$$

and

$$\sigma_W^2(s) = V[\tilde{W} \mid \tilde{\rho} = \rho_s] = q_A^2(\sigma_V^2 + \rho_s^2 \sigma_\delta^2) + q_B^2(\sigma_V^2 + \rho_s^2 \sigma_\delta^2) + 2 q_A q_B \rho_s^2 \sigma_\delta^2.$$

However, before $\tilde{\rho}$ is realized, final wealth does not follow a normal distribution and we cannot apply standard results of the CARA-Normal framework. The distribution of final wealth is a mixture of two normal distributions. To solve the portfolio choice, we need to calculate the expected utility of final wealth by using the Law of Iterated Expectations, resulting in:

$$\begin{aligned} EU &= E\left[E(-\exp(-\gamma \tilde{W}) \mid \tilde{\rho})\right] \\ &= -\pi \exp\left[-\gamma\left(\mu_W(h) - \frac{\gamma \sigma_W^2(h)}{2}\right)\right] - (1-\pi)\exp\left[-\gamma\left(\mu_W(l) - \frac{\gamma \sigma_W^2(l)}{2}\right)\right]. \end{aligned} \tag{5}$$

If we take the first-order conditions with respect to q_A and q_B and plug the market-clearing conditions $q_A = 1$ and $q_B = 1$, we can compute prices for asset $j \in \{A, B\}$ as specified in the following proposition:

Proposition 1 (Asset Prices under Matching Uncertainty). *The price of asset $j \in \{A, B\}$ is given by*

$$\begin{aligned} P_j &= \frac{\bar{V} - \gamma\sigma_V^2}{R} \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right] \\ &- \frac{2\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right]. \end{aligned} \tag{6}$$

The asset price in this economy has three distinctive terms. The first term of the price would be the price of the asset if there was no common element $\tilde{\Delta}$ in the cash flows, that is, firms A and B would be independent of each other. The second term captures the benefit of having a relationship, while the third term, instead, captures the cost of forming a relationship. Note that both the benefit and the cost term come from expectations of utilities taken with respect to the prior distribution of $\tilde{\rho}$, hence containing the exponential terms. In other words, we can rewrite equation (6) as

$$P_j = \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h\pi^{ra} + \rho_l(1-\pi^{ra})] - \frac{2\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{ra} + \rho_l^2(1-\pi^{ra})], \tag{7}$$

where

$$\pi^{ra} = \frac{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}. \tag{8}$$

can be viewed as the risk-adjusted prior probability that $\tilde{\rho} = \rho_h$, for an investor with CARA utility and a portfolio of firms with correlated cash flows.

Note that when $\rho_h = \rho_l = \rho$, i.e., when there is no relationship uncertainty, we have the standard result in a CARA-Normal setup that the price of the relationship part is equal to the mean of cash flow minus the product of twice the risk-aversion coefficient (as there are two firms) and the variance of cash flow (discounted by the risk-free rate), i.e., the price is only determined by the first and second moment of cash flow,

$$
\begin{aligned}
P_j &= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R} - \frac{2\gamma\sigma_\delta^2}{R} \\
&= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{E[\tilde{\delta}]}{R} - \frac{2\gamma var[\tilde{\delta}]}{R}.
\end{aligned}
$$

When $\rho_h \neq \rho_l$, i.e., in the presence of relationship uncertainty, the price of the relationship part is determined by the higher order moments of cash flow, as in Heinle et al. (2018). We can write the price expression as

$$
\begin{aligned}
P_j &= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{E[\tilde{\delta}\tilde{\rho}]}{R} - \frac{2\gamma var[\tilde{\delta}\tilde{\rho}]}{R} \\
&\quad + \frac{\bar{\delta}(\pi^{ra} - \pi)(\rho_h - \rho_l)}{R} - \frac{2\gamma}{R}\{\sigma_\delta^2(\pi^{ra} - \pi)(\rho_h^2 - \rho_l^2) - \bar{\delta}^2\pi(1-\pi)(\rho_h - \rho_l)^2\} \\
&= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\phi E[\tilde{\delta}\tilde{\rho}]}{R} - \frac{2\gamma}{R}\zeta, \qquad (9)
\end{aligned}
$$

where

$$E[\widetilde{\delta\rho}] = \bar{\delta}[\rho_h\pi + \rho_l(1-\pi)],$$

$$var[\tilde{\delta}\tilde{\rho}] = \sigma_\delta^2[\pi\rho_h^2 + (1-\pi)\rho_l^2] + \bar{\delta}^2\pi(1-\pi)(\rho_h - \rho_l)^2,$$

$$\phi = 1 + \frac{(\pi^{ra} - \pi)(\rho_h - \rho_l)}{\rho_h\pi + \rho_l(1-\pi)},$$

and

$$\zeta = \sigma_\delta^2[\pi^{ra}\rho_h^2 + (1-\pi^{ra})\rho_l^2].$$

As one can clearly see in the second line of equation (9), in addition to the usual mean and variance term, P_j is determined by the difference between π^{ra} and π, which clearly depends on higher order moments, based on a Taylor series expansion argument of the exponential terms. In the third line of equation (9), we compare our price expression to that of equation (3) in Lemma

1 of Heinle et al. (2018). Note that different from that in Heinle et al. (2018), ϕ is not necessarily always bigger than 1 as it depends on the relative magnitude of π^{ra} and π. We are able to show that $\pi^{ra} > \pi$, and thus $\phi > 1$ if and only if

$$\frac{\bar{\delta}}{\gamma\sigma_{\delta}^{2}} < \rho_h + \rho_l.$$

In other words, one need $\frac{\bar{\delta}}{\gamma\sigma_{\delta}^{2}}$ to be sufficiently low relative to ρ for the prior mean to have an amplifying effect on prices. Intuitively, when $\bar{\delta}$ is sufficiently low or σ_{δ}^{2} is sufficiently high, investors face more downside risk of the factor relative to the factor exposure. Uncertainty about factor exposure therefore preserves the upside potential and amplifies the effect of prior mean on prices. Note that this difference stems from the fact that the distribution of $\tilde{\Delta}$ in our setting is not necessarily positively skewed but is positively skewed in Heinle et al. (2018) when the mean of both components of the cash flow is positive.

Also different from Heinle et al. (2018), we can also show that it is not necessarily true that $\zeta > var[\widetilde{\delta}\widetilde{\rho}]$, that is, uncertainty about factor exposure also does not necessarily increase kurtosis, which we conjecture is because our factor exposure distribution is not symmetric around the mean, as in Heinle et al. (2018). When the factor exposure distribution is symmetric around the mean, more uncertainty increases the probability of tail events occurring in both tails thus amplifying kurtosis, which is not necessarily true if factor exposure distribution is highly skewed to one tail. Please see the Appendix for more details of the statistical properties of $\tilde{\Delta}$.

In contrast to the case with perfect matching information, asset prices under matching uncertainty is generally ambiguous in $\bar{\delta}$ and σ_{δ}^{2} due to their ambiguous effects on higher order moments of the cash flow distribution so we cannot make any general statements. We now examine the firms' optimal disclosure strategies of relationships.

4 Voluntary Disclosure of Relationships

In this section, we study the optimal disclosure policy about matching intensity $\tilde{\rho}$. To capture relationship disclosure, we assume that firms can commit to disclosing information about their matching intensity $\tilde{\rho}$. Such assumption of ex-ante commitment of voluntarily disclosing information is quite common in the literature (e.g., Diamond (1985)) and is consistent with the findings of a large stream of empirical literature showing that firms can ex-ante commit to disclosing with various information quality by, e.g., providing forecasts with different frequency and quality (e.g.,, Ajinkya et al. (2005), Karamanou and Vafeas (2005) and Baginski and Rakow (2012)).[4] In principle, we can also model relationship disclosure as disclosing information about $\tilde{\delta}$ or about $\tilde{\Delta}$. We do not explore these alternatives in the current paper.

Before the realization of $\tilde{\rho}$, firms commit to a joint disclosure policy that will take place once firms observe the realization of $\tilde{\rho}$. They will send a message $\tilde{m} \in \{h, l\}$ to indicate if they are in the state with ρ_h or ρ_l. Specifically, firms can choose the following probability $\alpha \in [\frac{1}{2}, 1]$ at zero cost:

$$\begin{aligned} Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_l) = \alpha, \\ Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_l) = 1 - \alpha. \end{aligned} \tag{10}$$

In the limit, when $\alpha = 1$, firms provide perfect disclosure of the realization of $\tilde{\rho}$. Instead, when $\alpha = 1/2$, firms provide no information about the realization of $\tilde{\rho}$. Any α in between will generate only partial disclosure about the realization of $\tilde{\rho}$.

We augment the main conceptual framework to a setup with three stages. The timeline of the augmented setup is described by Figure 1. At $t = 1$, before the realization of $\tilde{\rho}$, firms can commit to a disclosure policy α. At $t = 2$, firms observe $\tilde{\rho}$ and provide a joint message $\tilde{m} \in \{h, l\}$ based on the disclosure policy α. At $t = 3$, the investor updates her beliefs about the realization of $\tilde{\rho}$ based on the message received and the disclosure policy, decides her portfolio choice, and asset

[4] Section 8 discusses results when firms can choose to disclose relationships ex post.

prices are determined in equilibrium.

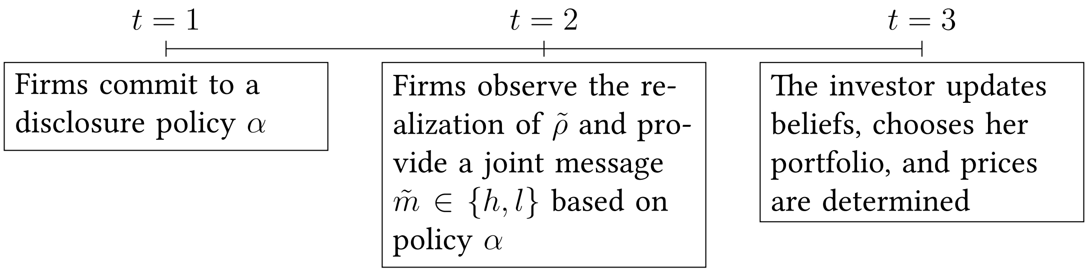


Figure 1: Timeline with Voluntary Disclosure of Relationships

Firms will choose the disclosure policy α that minimizes their costs of capital (CoC henceforth) given by $CoC_j \equiv E[\tilde{F}_j - P_j]$ for $j = \{A, B\}$. Since the decision on disclosure is made before firms observe the realization of $\tilde{\rho}$, minimizing the costs of capital is equivalent to maximizing the expected price. Specifically, since $E[\tilde{F}_j]$ is a constant independent of α, then $\min_\alpha E[\tilde{F}_j - P_j]$ is equivalent to $\max_\alpha E[P_j]$. The model is solved using backward induction. First, we solve for the investor's demands and asset prices. Second, we solve for the firms' committed disclosure policies that maximize the firms' expected prices.

The investor's problem is the same as that derived in Section 3 for the conceptual framework with probabilities π and $1-\pi$ being updated based on the message received at $t = 2$. Let us denote $\pi^h \equiv Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = h)$ and $\pi^l \equiv Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = l)$ as the probabilities of $\tilde{\rho} = \rho_h$ that are assigned by the investor after receiving the message $\tilde{m} = h$ and $\tilde{m} = l$, respectively. Accordingly, $1 - \pi^h$ and $1 - \pi^l$ are the posterior probabilities of $\tilde{\rho} = \rho_l$ assigned by the investor after receiving the message $m = h$ and $m = l$, respectively. Following Bayes' rule, these probabilities are given by

$$\begin{aligned} \pi^h = Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = h) &= \frac{Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) Pr(\tilde{\rho} = \rho_h)}{Pr(\tilde{m} = h)} = \frac{\alpha\pi}{\alpha\pi + (1-\alpha)(1-\pi)}, \\ \pi^l = Pr(\tilde{\rho} = \rho_h \mid \tilde{m} = l) &= \frac{Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) Pr(\tilde{\rho} = \rho_h)}{Pr(\tilde{m} = l)} = \frac{(1-\alpha)\pi}{(1-\alpha)\pi + \alpha(1-\pi)}. \end{aligned} \tag{11}$$

There will be one asset price for $\tilde{m} = h$ and another one for $\tilde{m} = l$. The expression of the prices will be similar to the one in the conceptual framework in equation (6), but with adjusted probabilities π^m for $\tilde{m} \in \{h, l\}$. Hence, for any message $\tilde{m} \in \{h, l\}$, the price $P_j(\alpha; \tilde{m})$ for $j \in \{A, B\}$ is given by

$$\begin{aligned} P_j(\alpha; \tilde{m}) &= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi^m \rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi^m)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi^m e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi^m) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right] \\ &- \frac{2\gamma\sigma_\delta^2}{R}\left[\frac{\pi^m \rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi^m)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi^m e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi^m) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right], \end{aligned} \tag{12}$$

where π^m is the posterior probability after receiving message m given by equation (11). Using the similar risk-adjusted probability formulation as in equation (7), we can rewrite the price expressions given disclosure as

$$P_j(\alpha; \widetilde{m}) = \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h \pi^{ram} + \rho_l(1 - \pi^{ram})] - \frac{2\gamma\sigma_\delta^2}{R}[\rho_h^2 \pi^{ram} + \rho_l^2(1 - \pi^{ram})], \tag{13}$$

where

$$\pi^{ram} = \frac{\pi^m e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}}}{\pi^m e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi^m) e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}. \tag{14}$$

is the risk-adjusted probability that $\widetilde{\rho} = \rho_h$ given disclose of m.

Given the asset prices for any α, firms will choose the joint optimal disclosure policy α^* that maximizes their expected asset prices. Since both firms are symmetric, we can focus on the problem of one firm. The expected asset prices are given by

$$E[P_j(\alpha; \tilde{m})] = P_j(\alpha; \tilde{m} = h)Pr(\tilde{m} = h) + P_j(\alpha; \tilde{m} = l)Pr(\tilde{m} = l). \tag{15}$$

The next corollary shows that an ex-post disclosure of higher relationship intensity is not necessarily good news to the market, relative to the ex-ante price. Again, the proof is straightforward and thus omitted.

Corollary 2. *Disclosure of $\widetilde{m} = h$ ($\widetilde{m} = l$) increases (decreases) prices if and only if* $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > 2(\rho_h +$

ρ_l).

Corollary 2 is similar to Proposition 1 of Heinle et al. (2018) in the sense that disclosure of high relationship intensity is not necessarily good news to the market as the price may drop. High relationship intensity increases the expected cash flow but also increases the underlying risk of the expected cash flow. Therefore, high relationship intensity is good news if and only if the benefit of the increase in expected cash flow dominates the underlying risk. Since $\widetilde{m}$ is binary, the Bayesian updating is not linear. Therefore we are not able to follow Heinle et al. (2018) to express the price as a simple expression. Due to this complication, we are not able to provide analytical results on the comparative statics of both the ex-ante (i.e., before any disclosure) and ex-post (i.e., after disclosure) prices. We provide some graphical illustrations instead. Figure 2 shows how the ex-ante price and the two ex-post prices (i.e., when $\widetilde{m} = h$ and $\widetilde{m} = l$) change with respect to $\bar{\delta}$ and σ_δ^2, respectively, when disclosure quality (i.e., α) varies. While the general trend is ambiguous, price increases with $\bar{\delta}$ for large values of $\bar{\delta}$ and decreases with σ_δ^2 for large values of σ_δ^2. The parameter values are reported on top of the panels.

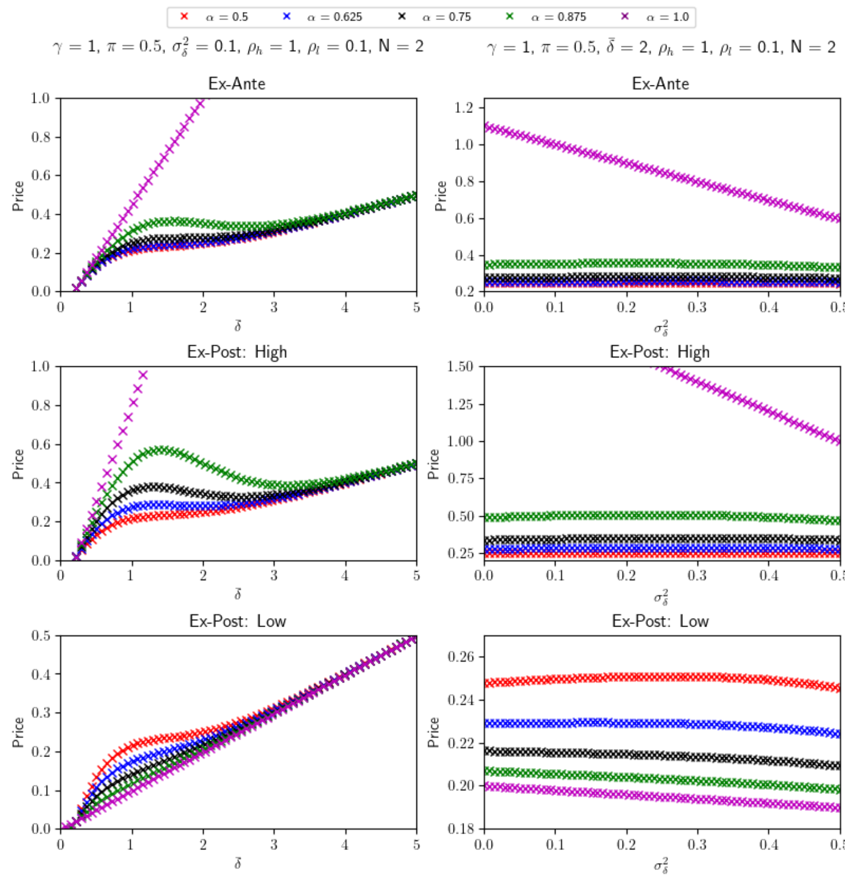
Given our focus is on the optimal disclosure policy, we now proceed to characterize it. Without loss of generality, we assume that if a firm is indifferent between any policy α, it will choose full disclosure ($\alpha = 1$). The following proposition shows under which conditions firms will choose to opt for a full disclosure policy or a non-disclosure policy.

Proposition 2. *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \textit{if } \frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l, \\ 1, & \textit{otherwise} . \end{cases} \tag{16}$$

Firms choose to commit to a non-disclosure policy $\alpha^* = \frac{1}{2}$ if and only if $\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h+\rho_l$. Otherwise, they commit to a full disclosure policy $\alpha^* = 1$. While the optimal disclosure policy is a corner solution, it is not robust to slight perturbations of the model. For example, we can add a disclosure cost that increases with α. In this case, the optimal solution will still be no disclosure

Figure 2: Ex-Ante and Ex-Post Prices



if and only if $\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l$ but may be interior otherwise. We choose not to include a disclosure cost in our main setting as essentially we are comparing $\alpha^* = 1$ with $\alpha^* = \frac{1}{2}$, which can be interpreted as more or less disclosure, while abstracting away from other factors that may affect disclosure precision.[5] The conditions on $\alpha^* = 1$ versus $\alpha^* = \frac{1}{2}$ being the equilibrium can be used to derive predictions for when more disclosure is more likely, which we will discuss in more detail in Section 10 regarding empirical implications.

To further understand the conditions in Proposition 2, we rewrite equation (15) using the

[5] Section 8.2 considers the case of a disclosure cost when firms can disclose ex post.

risk-adjusted probabilities as follows:

$$\begin{aligned}
& E[P_j(\alpha;\widetilde{m})] \\
= \; & P_j(\alpha;\widetilde{m}=h)\Pr(\widetilde{m}=h) + P_j(\alpha;\widetilde{m}=l)\Pr(\widetilde{m}=l) \\
= \; & \{\frac{\bar{V}-\gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h\pi^{rah} + \rho_l(1-\pi^{rah})] - \frac{2\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{rah} + \rho_l^2(1-\pi^{rah})]\}\Pr(h) \\
& +\{\frac{\bar{V}-\gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}[\rho_h\pi^{ral} + \rho_l(1-\pi^{ral})] - \frac{2\gamma\sigma_\delta^2}{R}[\rho_h^2\pi^{ral} + \rho_l^2(1-\pi^{ral})]\}\Pr(l) \\
= \; & \frac{\bar{V}-\gamma\sigma_V^2}{R} + \frac{\phi E[\widetilde{\delta}\widetilde{\rho}]}{R} - \frac{2\gamma}{R}\zeta, \qquad (17)
\end{aligned}$$

where

$$\phi = 1 + \frac{(\pi^{rad}-\pi)(\rho_h-\rho_l)}{\rho_h\pi + \rho_l(1-\pi)},$$

and

$$\zeta = \sigma_\delta^2[\pi^{rad}\rho_h^2 + (1-\pi^{rad})\rho_l^2],$$

where

$$\pi^{rad} = \pi^{rah}\Pr(h) + \pi^{ral}\Pr(l),$$

and we replace $\Pr(\widetilde{m}=i)$ with $\Pr(i)$ for $i \in \{h,l\}$ to shorten the notation.

As in the analysis in Heinle et al. (2018), the marginal impact of a change in disclosure precision, α, on price is

$$\begin{aligned}
\frac{\partial E[P_j(\alpha;\widetilde{m}]}{\partial\alpha} &= \frac{1}{R}(E[\widetilde{\delta}\widetilde{\rho}]\frac{\partial\phi}{\partial\alpha} - 2\gamma\frac{\partial\xi}{\partial\alpha}) \\
&= \frac{1}{R}[\underbrace{\bar{\delta}(\rho_h-\rho_l)\frac{\partial\pi^{rad}}{\partial\alpha}}_{\text{Skewness term}} - \underbrace{2\gamma\sigma_\delta^2(\rho_h^2-\rho_l^2)\frac{\partial\pi^{rad}}{\partial\alpha}}_{\text{kurtosis term}}]. \qquad (18)
\end{aligned}$$

We find that the sign of $\frac{\partial\pi^{rad}}{\partial\alpha}$ is ambiguous. So, unlike Heinle et al. (2018) case, in our setting more precise disclosure does not unambiguously increase the skewness or kurtosis terms. However, more precise disclosure always change the skewness or kurtosis term in the same direction, therefore resulting in a tradeoff. We conjecture that the difference is due to that in Heinle et al.

(2018), the distribution of factor exposure is normal and therefore symmetric with respect to the mean whereas in our setting, the factor exposure is binary and can only be positive, resulting in a truncated and therefore asymmetrical distribution with respect to the mean. In fact, we can show that in the hypothetical case of $\pi = \frac{1}{2}$ and $\rho_h = \rho = -\rho_l > 0$ (i.e., symmetric with respect to the mean), we can sign $\frac{\partial \pi^{rnd}}{\partial \alpha}$, more specifically, $\frac{\partial \pi^{rnd}}{\partial \alpha} \geq 0$ with the equality reached when $\alpha = \frac{1}{2}$, which will be consistent with the result in Heinle et al. (2018) that more precise disclosure increases skewness but decreases kurtosis.

When $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$, more precise disclosure increases the skewness and the marginal impact increases with $\bar{\delta}$ and $\rho_h - \rho_l$. Intuitively, the higher $\bar{\delta}$ and the larger the difference between $\rho_h - \rho_l$, the more skewed the distribution will be, and thus the larger the impact of the skewness term. When $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$, more precise disclosure also increases the kurtosis term and the marginal impact increases with σ_δ^2 and $\rho_h^2 - \rho_l^2$. Intuitively, the larger the variation in $\tilde{\delta}$ and $\tilde{\rho}$, the larger the probability of tail risk, and thus the larger the impact of the kurtosis term. Therefore, when $\bar{\delta}(\rho_h - \rho_l) > 2\gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2)$, or, equivalently, $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > 2(\rho_h + \rho_l)$, the impact of the skewness term dominates that of the kurtosis term.

We can show that $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$ if and only if $e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}} > e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}}$, or, equivalently, $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \rho_h + \rho_l$. Note that π^{rad} can be viewed as the risk-adjusted probability that $\rho = \rho_h$ occurs. When $e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}} > e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}}$, the risk-adjusted probability puts more weight (relative to the Bayesian weight) on the probability that $\rho = \rho_h$ occurs, resulting in full disclosure maximizing the risk-adjusted probability that $\rho = \rho_h$ occurs. Intuitively, when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > \rho_h + \rho_l$, the expected factor value (relative to the risk) is so high that the investor faces less downside risk of the factor relative to the factor exposure. Risk-adjusted probability then puts more weight on $\rho = \rho_h$.

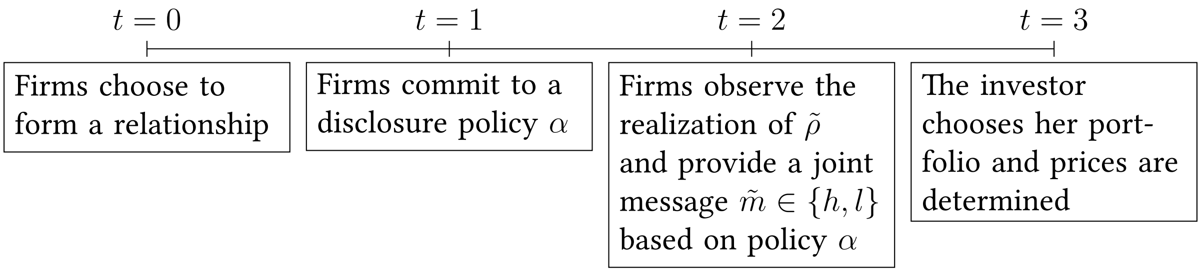
In summary, when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > 2(\rho_h + \rho_l)$, $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$ and more disclosure increases both the skewness term and the kurtosis term but the effect on the skewness term dominates, resulting in perfect disclosure maximizing the price; when $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} < (\rho_h + \rho_l)$, $\frac{\partial \pi^{rad}}{\partial \alpha} < 0$ and more disclosure decreases both the skewness term and the kurtosis term but the effect on the kurtosis term dominates (is more negative), again resulting in perfect disclosure maximizing the price; when

$(\rho_h+\rho_l) < \frac{\bar{\delta}}{\gamma\sigma_{\delta}^2} < 2(\rho_h+\rho_l)$, $\frac{\partial \pi^{rad}}{\partial \alpha} > 0$ and more disclosure increases both the skewness term and the kurtosis term but the effect on the kurtosis term dominates, resulting in no disclosure maximizing the price. Similar to Heinle et al. (2018), more disclosure does not necessarily decrease the cost of capital because its effect on the higher order moments terms are not unambiguously price-increasing. Different from Heinle et al. (2018), the asymmetric binary distribution in our setting results in more disclosure not having an unambigously positive effect on skewness and negative effect on kurtosis. Perhaps more importantly, the binary structure provides us the tractability to be able to characterize the necessary and sufficient conditions for disclosure to increase the cost of capital, which is useful in our subsequent analysis of when firms choose to form relationship in the first place.

From a general intuition perspective, note that when the collaboration intensity $\bar{\delta}$ of having a relationship is very high relative to the collaboration risk σ_{δ}^2, the firm is more likely to get a high cash flow from having the relationship. Firms therefore want to disclose to let the investor be aware of the high expected benefit of the relationship. When the collaboration intensity is sufficiently low, the investor is more concerned about the increased expected diversification cost, in particular when the matching intensity is high, i.e., $\rho = \rho_h$, therefore putting more weight on this scenario in the absence of disclosure (in other words, the risk-neutral probability of $\rho = \rho_h$ becomes smaller). Firms then have an incentive to disclose to let the investor be aware when ρ is low. When the collaboration intensity is at intermediate levels relative to the cost of forming a relationship, the investor is not much concerned about the increased diversification cost, in particular when the matching intensity is high, i.e., $\rho = \rho_h$, therefore putting less weight on this scenario in the absence of disclosure. Firms then have an incentive to not disclose to prevent the investor from being aware of the possible scenario that ρ is high. Thus, full disclosure of a relationship is not always optimal when the objective is to maximize the expected asset price of the firm (or equivalently, to minimize the cost of capital). The following corollary provides the asset prices under non-disclosure and full disclosure.

Corollary 3. *The price of asset $j \in \{A, B\}$ when the optimal policy is non-disclosure ($\alpha^* = \frac{1}{2}$) is*

Figure 3: Timeline with Relationship Formation and Voluntary Relation Disclosure



given by equation (6). Instead, when the optimal policy is full disclosure ($\alpha^* = 1$), asset prices are given by equation (4) and the expected asset price is given by

$$E[P_j(\alpha^* = 1; \tilde{m})] = \frac{\bar{V} - \gamma\sigma_V^2}{R} + \frac{\bar{\delta}}{R}(\pi\rho_h + (1-\pi)\rho_l) - \frac{2\gamma\sigma_\delta^2}{R}(\pi\rho_h^2 + (1-\pi)\rho_l^2). \tag{19}$$

5 Relationship Formation

In this section, we study the real effects of relationship disclosure, that is, how disclosing relationship affects firms' choices to form relationships in the first place. We augment the conceptual framework with disclosure from the previous section to a setting with four stages. The timeline of the economy is now given by Figure 3. At $t = 0$, firms choose whether to form a relationship. At $t = 1$, before the realization of $\tilde{\rho}$, firms can commit to a disclosure policy α. At $t = 2$, firms observe $\tilde{\rho}$ and provide a joint message $\tilde{m} \in \{h, l\}$ based on the disclosure policy α. At $t = 3$, the investor updates her beliefs about the realization of $\tilde{\rho}$ based on the message received and the disclosure policy, and decides her portfolio choice, and prices are determined in equilibrium.

As in the previous section, firms' objectives are to maximize expected prices. That is, a firm will choose to form a relationship if and only if the expected price of the firm when forming a relationship is higher than the expected price of the firm under no relationship, taking into account the strategic actions of the other firm. Without loss of generality, we assume that if a firm is indifferent between forming or not forming a relationship, the firm will choose not to form a relationship.

If firms choose not to have a relationship, then the cash flows of each firm have only one component and are given by $\tilde{F}_A = \tilde{V}_A$ and $\tilde{F}_B = \tilde{V}_B$, which corresponds to the degenerate case of $\pi = \rho_l = 0$ in Section 3. We use superscript "N" to denote the case in which the two firms have not formed a relationship. Taking prices as given, demand for asset $j \in \{A, B\}$ is standard in a CARA-Normal framework and given by

$$q_j^N = \frac{\bar{V} - RP_j^N}{\gamma \sigma_V^2}. \tag{20}$$

The demand of the investor for asset j is independent of the demand for the other asset in the economy. The demand depends positively on the expected excess returns and negatively on the variance of the asset and the risk aversion of the investor. We can compute prices using the market-clearing conditions: $q_A = 1$ and $q_B = 1$. The prices of the two firms are the same and given by

$$P_j^N = \frac{\bar{V} - \gamma \sigma_V^2}{R}, \tag{21}$$

for $j \in \{A, B\}$. The price is the present discounted value of expected payoffs adjusted for the risk associated with holding the asset.

If firms choose to have a relationship, then cash flows have two components $\tilde{F}_A = \tilde{V}_A + \tilde{\Delta}$ and $\tilde{F}_B = \tilde{V}_B + \tilde{\Delta}$ as in the conceptual framework in Section 3. The second component $\tilde{\Delta}$ is given by equation (1) and the price for firm $j \in \{A, B\}$ is given by (12). Firms will compare the expected price (15) of forming a relationship under the optimal disclosure policy α^* from equation (16) with the price (21) under no relationship, taking into account the choice of the other firm. The payoff matrix is given by Figure 4 and the definition of the equilibrium of this game and the definitions of a relationship equilibrium, a no-relationship equilibrium, and a Pareto-dominant equilibrium are presented below.

Definition 1. *An equilibrium consists of (i) prices when there is no relationship P_A^N and P_B^N that satisfy the market-clearing conditions and are given by (21); (ii) prices when there is a relationship $P_A(\alpha; m)$ and $P_B(\alpha, m)$ that satisfy the market-clearing conditions and are given by (12); (iii) a*

Figure 4: Payoffs under Relationship Formation

		Firm B Relation	Firm B No Relation
Firm A	Relation	$E[P_A(\alpha^*;\tilde{m})], E[P_B(\alpha^*;\tilde{m})]$	P_A^N, P_B^N
	No Relation	P_A^N, P_B^N	P_A^N, P_B^N

disclosure policy α that maximizes (15) and is given by (16); and (iv) the firms' decisions to form a relationship that form a Nash equilibrium in Pure Strategies of the game in Figure 4.

Definition 2. *(i) A relationship equilibrium is an equilibrium in which both firms A and B choose to form a relationship. (ii) A no-relationship equilibrium is an equilibrium in which at least one of the firms A or B chooses not to form a relationship. (iii) An equilibrium is Pareto-dominant if it has the highest expected price across all equilibria.*

5.1 Equilibrium Characterization

The following Lemma shows that there always exists an equilibrium with no relationship.

Lemma 1. *A no-relationship equilibrium always exists.*

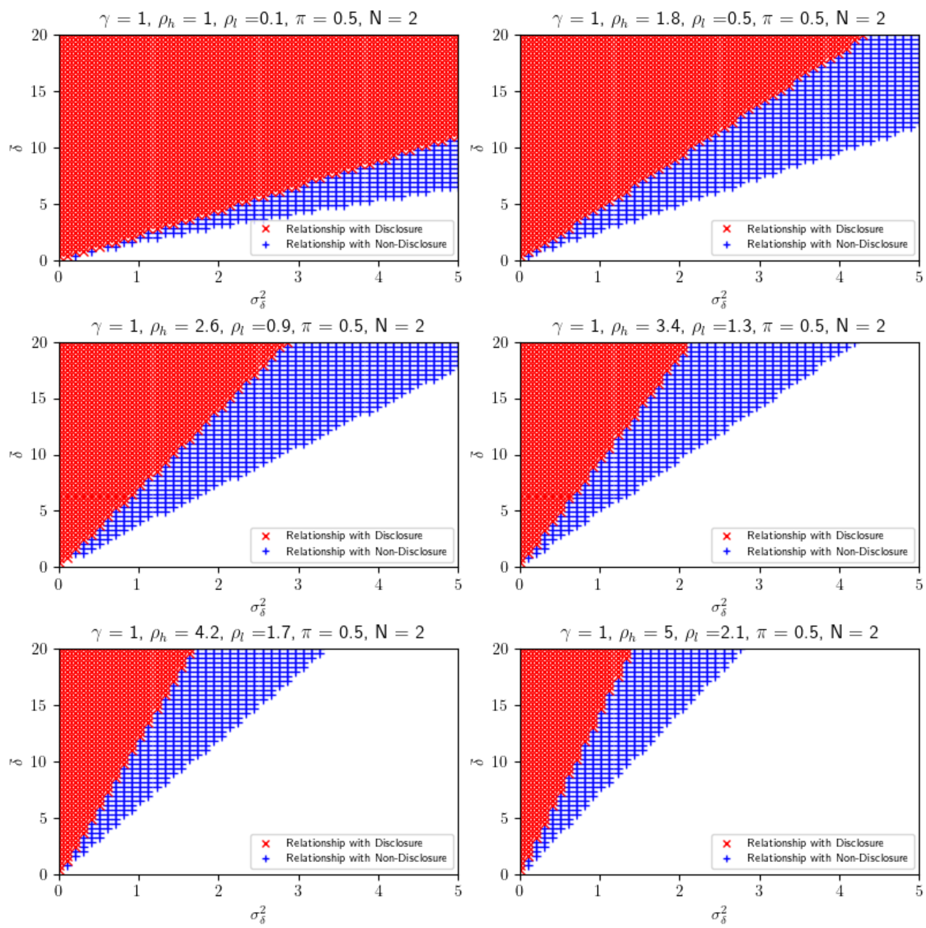
Intuitively, since a relationship is formed only when both firms choose to do so, no firm has any incentive to deviate from a no-relationship equilibrium as there is no benefit from a unilateral deviation. For the purpose of this paper, we will focus on the Pareto-dominant equilibrium. As mentioned above, a Pareto-dominant equilibrium is the equilibrium with the highest expected price.[6] There will be a Pareto-dominant equilibrium with a relationship when $E[P_j(\alpha^*;\tilde{m})] > P_j^N$. The next proposition characterizes the conditions for the existence of a Pareto-dominant relationship equilibrium.

Proposition 3. *There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with full-disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > (\rho_l+\rho_h)$ or $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_l+\rho_h}{2})$, with the latter possible if and only if $\pi\rho_h < (1-\pi)\rho_l$. There exists a Pareto-dominant relationship*

[6] In Section 9 we discuss the relation between Pareto-dominance and social welfare.

equilibrium in which firms will form a relationship with non-disclosure ($\alpha^* = \frac{1}{2}$) *if and only if* $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in \left(\max\left(\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}, \frac{\rho_l+\rho_h}{2}\right), \rho_l+\rho_h\right)$*, with* $\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}} < \frac{\rho_l+\rho_h}{2}$ *if and only if* $\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} < (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}$.

Figure 5: Pareto-Dominant Equilibrium with Relationship Formation



Proposition 3 states that firms will form relationship if and only if $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2}$ is sufficiently large, that is, when the benefit of increased collaboration intensity from relationship formation is sufficiently larger than the cost of increased colloboration risk. The rest of the conditions comes from that in Proposition 2 regarding when the optimal disclosure policy is full disclosure versus non-disclosure. Figure 5 shows the parameter range for which a Pareto-dominant relationship equilibrium exists. There are six parameters involved, that is, π, γ, ρ_h, ρ_l, $\bar{\delta}$, and σ_δ^2. In the figure, we assume $\gamma = 1$ and $\pi = 0.5$, choose several values for ρ_h and ρ_l, and show equilibria with relationship formation when we vary $\bar{\delta}$ and σ_δ^2. We observe that equilibria with relationship formation can be Pareto-dominant under both a disclosure and a non-disclosure policy. Intuitively, when the collaboration intensity $\bar{\delta}$ of forming a relationship is high relative to the collaboration risk σ_δ^2, a Pareto-dominant relationship equilibrium exists. When the collaboration intensity is very high relative to the collaboration risk of forming a relationship, firms choose to commit to disclosing the nature of their relationship, as the benefit of disclosure exceeds the cost. When the expected collaboration intensity is at intermediate levels relative to the collaboration risk of forming a relationship, firms prefer to commit to a non-disclosure regime, as investors are (relatively) less concerned about the increase in diversification cost (relative to when the collaboration intensity is sufficiently low relative to the collaboration risk) and put more weight on the scenario when ρ is low in the absence of disclosure. Therefore, firms choose not to disclose. However, firms would still prefer to form a relationship as the expected benefit of forming a relationship still dominates the cost of increased diversification cost. Finally, we observe that when the expected collaboration intensity of forming relationship is low relative to the collaboration risk of increased diversification cost, there is a unique equilibrium where no relationship is formed.

5.2 Mandatory Disclosure

In this subsection, we study the implications of regulations mandating relationship disclosure, such as Regulation SFAS No. 131 under which firms must disclose their major operating segments and the existence of major customers. The next proposition shows that the introduction of mandatory disclosure ($\alpha = 1$) may lead to destruction of previously formed relationship under non-disclosure ($\alpha = \frac{1}{2}$).

Proposition 4. *Mandatory disclosure may destroy relationship formation. Specifically, for any firm $j \in \{A, B\}$, $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^N > E[P_j(\alpha = 1; \tilde{m})]$ if and only if*

$$\max\left\{\frac{\rho_h + \rho_l}{2}, \frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right\} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l},$$

which requires $\pi\rho_h > (1-\pi)\rho_l$. Mandatory disclosure will not destroy relationship formation when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}$. In particular, when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \rho_h+\rho_l)$, mandatory disclosure will force firms to disclose but will keep relationship formation intact.

When the parameter condition in the above proposition is satisfied, moving from a non-disclosure regime ($\alpha = \frac{1}{2}$) to a disclosure regime ($\alpha = 1$) would break relationships previously formed. Intuitively, when the benefit of disclosure relative to the cost is in the intermediate region, firms would optimally form a relationship but not disclose, because $E[P_j(\alpha = \frac{1}{2}; \tilde{m})] > P_j^N$, as the benefit of forming a relationship is still sufficiently large. However, once regulation forcing disclosure is introduced, then forming relationships would be suboptimal because $P_j^N > E[P_j(\alpha = 1; \tilde{m})]$, that is, forcing firms to disclose results in firms not establishing relationship in the first place.

Figure 6: Mandatory Disclosure and Destruction of Relationships

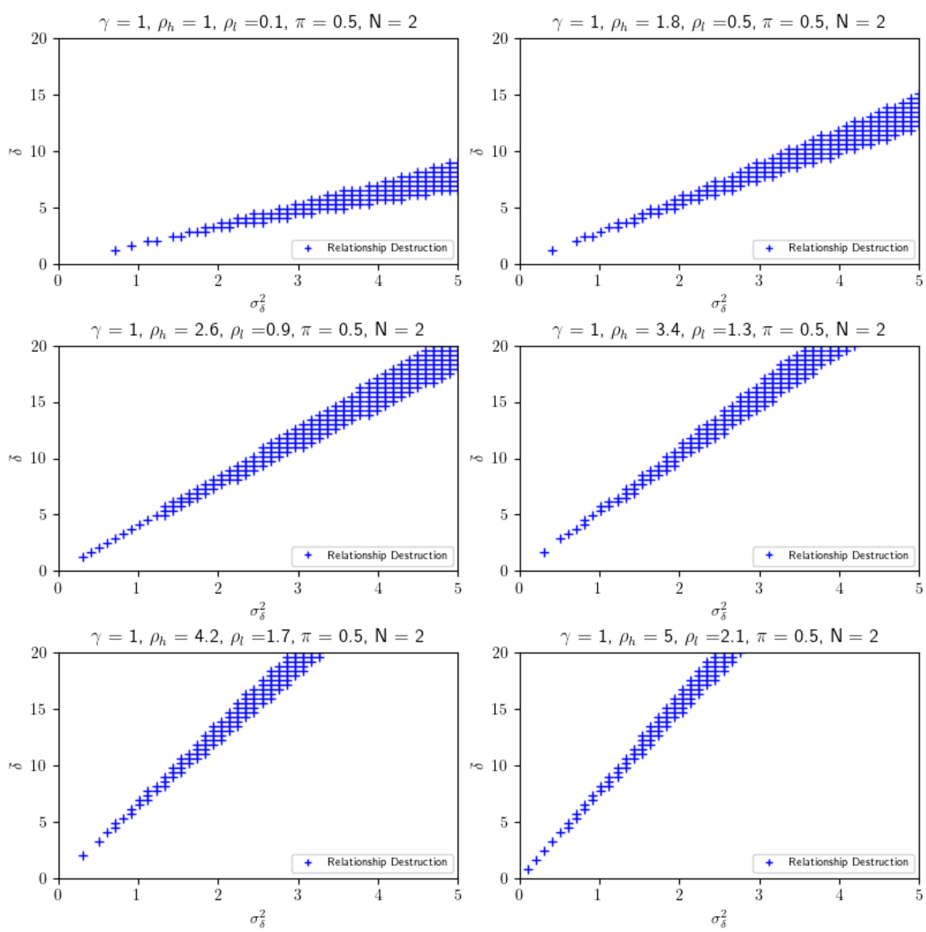


Figure 6 shows the parameter range for which forcing disclosure destroys the formation of relationships. In the figure, we assume $\gamma = 1$ and $\pi = 0.5$, choose several values for ρ_h and ρ_l, and show relationship formation destruction when we vary parameters $\bar{\delta}$ and σ_δ^2. For most combinations of ρ_h and ρ_l, there exists a region for which relationships are formed only under the non-disclosure policy when $\bar{\delta}$ relative to σ_δ^2 is in an intermediate range. Thus, when the collaboration intensity of forming a relationship is at some intermediate level relative to the collaboration risk, mandatory disclosure may destroy relationship formation and will affect the extensive margin of a relationship between two firms.

6 N-Firm Extensions

In the main setting we focus on disclosure of the relationship in a two-firm setting. This naturally raises the concern regarding whether the results hold more generally in a N-firm setting. In this section we extend our analysis to N firms and show that our results hold qualitatively, when the relationship formation among N firms are properly modelled.

Clearly, when N gets large, if relationship formation is only among a subset of firms, the diversification cost would approach zero as N goes to infinity, unless we assume that for some reason the idiosyncratic risk is priced (e.g., segmented markets). Therefore, we need to model relationship formation as firms forming a relationship network across many firms, and there are numerous ways to model such a systematic relationship network. To preserve tractability we consider two approaches to modelling systematic relationship formation. Under the first approach, we compare the case when none of the firms forms relationship versus the case when all N firms move together and form relationship with each other. This approach shares a similar spirit to the prior literature on strategic network formation (e.g., Goyal and Vega-Redondo (2007)). Under the second approach, we assume that N-1 firms have already formed relationship with each other (i.e., the N-1 firms have a common component with each other) and examine whether the Nth firm (the new joiner) would want to form relationship with the existing N-1 firms or not (i.e., forming bilateral relationships). In both approaches, the relationship modelled in the N-firm setting can be understood as the notion of "coalition" in economic theory, which requires mutual benefit for all parties for equilibrium coalition formation, i.e., if any party does not agree then relationship cannot be formed (see,e.g., page 247 of the survey paper by Ray and Vohra (2015)). For simplicity we only list the main results and discuss the main intuitions and relegate the rest of the algebraic details to a separate online appendix.

6.1 All N Firms Moving Together

There are N firms, indexed by $i \in \{1, 2, ..., N\}$. Each firm's cash flow is characterized by $\tilde{F}_i = \tilde{V}_i + \tilde{\Delta}$, where $V_i \sim N(\bar{V}, \sigma_V^2)$ is the firm-specific cash flow component, and $\tilde{\Delta} = \tilde{\rho}\tilde{\delta}$ is the common component if all firms end up with a relationship. As in the two-firm case, $\tilde{\delta} \sim N(\bar{\delta}, \sigma_\delta^2)$ and $\tilde{\rho} = \rho_h$ with probability π and ρ_l with probability $(1-\pi)$. We again assume that $\tilde{V}_1, \ldots, \tilde{V}_N, \tilde{\delta}$ are jointly normal and independent of $\tilde{\rho}$.

6.1.1 Relationship Disclosure

We first show that the conditions for firms to disclose relationship is qualitatively the same as Proposition 2, i.e., firms will not disclose if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2}$ is in the middle.

Proposition 5. *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and is given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \textit{if } \frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{N\gamma\sigma_\delta^2} < \rho_h + \rho_l, \\ 1, & \textit{otherwise}. \end{cases} \tag{22}$$

Note that Proposition 2 can be seen as a special case of Proposition 5 with $N = 2$. Intuitively, while the presence of more firms increases the expected cash flow, it also increases the diversification cost. Since diversification cost increases at a faster rate ($\sim N^2$) than that of the increase in expected cash flow ($\sim N$), firms find it more costly to not disclose as N increases, resulting in no disclosure less likely to be an equilibrium.

6.1.2 Relationship Formation

We now consider the conditions when firms would form relationships, given their subsequent disclosure strategies. The next Proposition shows that, in this case, the conditions for forming relationship are again qualitatively the same as that in the two-firm case.

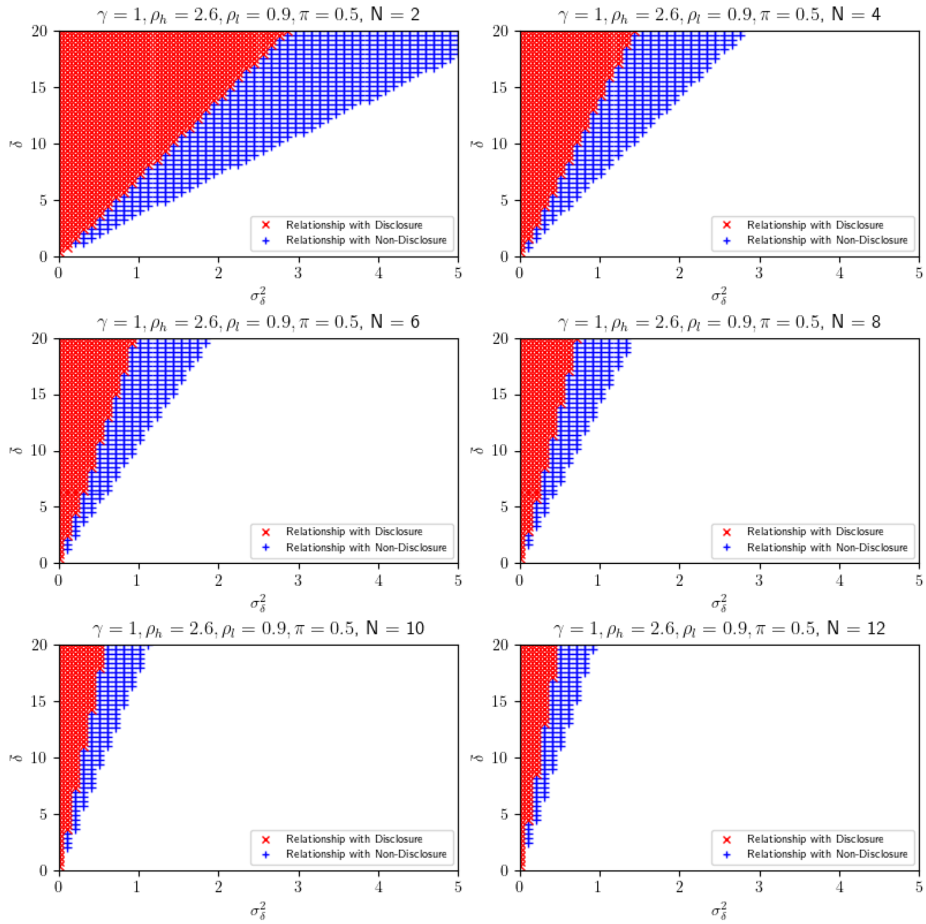
Proposition 6. *There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with full disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} > (\rho_l + \rho_h)$ or $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_l+\rho_h}{2})$, with the latter possible if and only if $\pi\rho_h < (1-\pi)\rho_l$. There exists a Pareto-dominant relationship equilibrium in which firms will form a relationship with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} \in (\max(\frac{\pi\rho_h^2 e^{\frac{\gamma^2}{2}\rho_h^2 N^2\sigma_\delta^2+\gamma N\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{\frac{\gamma^2}{2}\rho_l^2 N^2\sigma_\delta^2+\gamma N\rho_h\bar{\delta}}}{\pi\rho_h e^{\frac{\gamma^2}{2}\rho_h^2 N^2\sigma_\delta^2+\gamma N\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{\frac{\gamma^2}{2}\rho_l^2 N^2\sigma_\delta^2+\gamma N\rho_h\bar{\delta}}}, \frac{\rho_l+\rho_h}{2}), \rho_l+\rho_h)$, with $\frac{\pi\rho_h^2 e^{\frac{\gamma^2}{2}\rho_h^2 N^2\sigma_\delta^2+\gamma N\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{\frac{\gamma^2}{2}\rho_l^2 N^2\sigma_\delta^2+\gamma N\rho_h\bar{\delta}}}{\pi\rho_h e^{\frac{\gamma^2}{2}\rho_h^2 N^2\sigma_\delta^2+\gamma N\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{\frac{\gamma^2}{2}\rho_l^2 N^2\sigma_\delta^2+\gamma N\rho_h\bar{\delta}}} < \frac{\rho_l+\rho_h}{2}$ if and only if $\pi\rho_h e^{\frac{\gamma^2}{2}\rho_h^2 N^2\sigma_\delta^2+\gamma N\rho_l\bar{\delta}} < (1-\pi)\rho_l e^{\frac{\gamma^2}{2}\rho_l^2 N^2\sigma_\delta^2+\gamma N\rho_h\bar{\delta}}$.*

Again, Proposition 3 can be seen as a special case of Proposition 6 when $N = 2$ and our general intuition is retained. In particular, it is straightforward from Proposition 3 that mandatory relationship disclosure will destroy relationship formation in the first place when the collaboration intensity of the relationship relative to the collaboration risk is in the intermediate region, i.e., if

$$\max(\frac{\rho_h+\rho_l}{2}, \frac{\pi\rho_h^2 e^{\gamma\bar{\delta}N\rho_l+\frac{\gamma^2}{2}\rho_h^2N^2\sigma_\delta^2}+(1-\pi)\rho_l^2 e^{\gamma\bar{\delta}N\rho_h+\frac{\gamma^2}{2}\rho_l^2N^2\sigma_\delta^2}}{\pi\rho_h e^{\gamma\bar{\delta}N\rho_l+\frac{\gamma^2}{2}\rho_h^2N^2\sigma_\delta^2}+(1-\pi)\rho_l e^{\gamma\bar{\delta}N\rho_h+\frac{\gamma^2}{2}\rho_l^2N^2\sigma_\delta^2}})$$
$$< \quad \frac{\bar{\delta}}{\gamma N\sigma_\delta^2} < \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}.$$

As in the two-firm case, mandatory disclosure will not destroy relationship formation when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}$. In particular, when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \rho_h+\rho_l)$, mandatory disclosure will force firms to disclose but will keep relationship formation intact. Figures 7 and 8, corresponding to Figures 5 and 6 in the two-firm setting, illustrate the equilibrium relationship formation and when mandatory disclosure destroys relationship formation in the N-firm network setting.

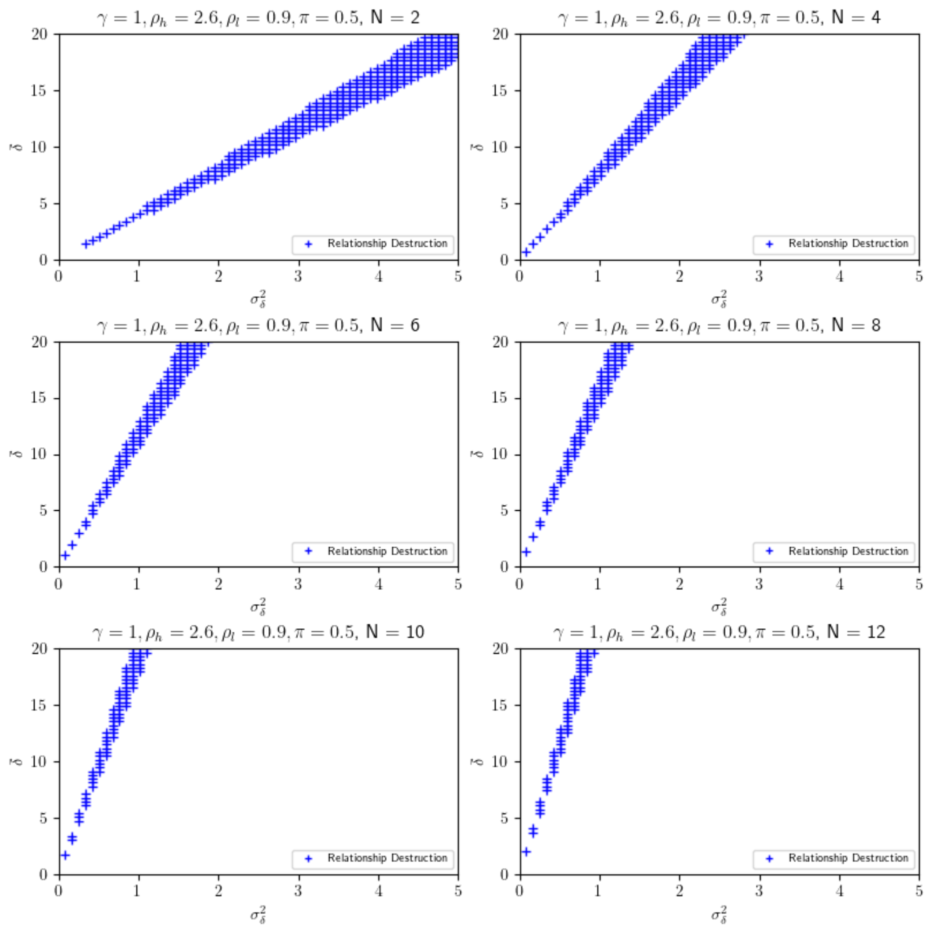
Figure 7: Pareto-Dominant Equilibrium with Relationship Formation in the N-Firm Network



6.2 N-1 Firms already Forming Relationship

We now consider the second approach to modeling a systematic relationship network, under which N-1 firms have already formed relationship with each other and the Nth firm is considering whether to form relationships with the N-1 firms. For the N-1 existing firms, the cash flows are $\tilde{F}_i = \tilde{V} + \sum_{j\in -I} \tilde{\rho}\tilde{\delta}_{ij}$, where $-I$ denotes $\{1, \dots, N\} \setminus \{i\}$, $\tilde{V}_i \sim (\bar{V}, \sigma_V^2)$, $i = 1, \dots, N$, and $\tilde{\delta}_{ij} \sim (\bar{\delta}, \sigma_\delta^2)$, $1 \leq i < j \leq N$. Again, we first establish the conditions for firms to disclose relationship when such relationship exists, followed by the conditions for firms to form such relationships, with or without disclosure.

Figure 8: Mandatory Disclosure and Destruction of Relationships in the N-Firm Network



6.2.1 Relationship Disclosure

We first show that the conditions for firms to disclose relationship are identical to Proposition 2.

Proposition 7. *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and is given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \text{if } \rho_h + \rho_l < \frac{\bar{\delta}}{\gamma \sigma_\delta^2} < 2(\rho_h + \rho_l), \\ 1, & \text{otherwise} . \end{cases} \tag{23}$$

To understand the intuition regarding why the conditions in Proposition 7 is the same as that in Proposition 2, note that from either the perspective of the new joiner (the Nth firm) or any of the existing firms (the rest N-1 firms), they are choosing between the price when two firms collaborate and the no-collaboration price, which is essentially a bilateral problem as in the situation characterized by Proposition 2.

6.2.2 Relationship Formation

The conditions regarding when the relationship would be formed differ depending on whether we are looking at the new joiner or the existing firms, as the two types of firms are asymmetric before relationship formation. The conditions when the relationship would be formed would thus be the intersection of the two conditions. We summarize the main conclusions in the next Proposition and relegate all the technical details to the appendix.

Proposition 8. *There exists a Pareto-dominant relationship equilibrium in which relationships are formed between the existing firms and the new joiner with full disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_l + \rho_h$ or $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_l+\rho_h}{2})$, with the latter possible if and only if $\pi\rho_h < (1-\pi)\rho_l$. There exists a Pareto-dominant relationship equilibrium in which relationships are formed between the existing firms and the new joiner with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if*

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in \left(\max\left(\frac{[(N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)]}{[(N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]}, \frac{\rho_l+\rho_h}{2}\right), \rho_l+\rho_h\right).$$

A necessary condition for

$$\frac{[(N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)]}{[(N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]} < \frac{\rho_l+\rho_h}{2}$$

is that $\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} < (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}$.

Again, we see that the insights from the two-firm setting extends qualitatively here. In particular, mandatory relationship disclosure destroys relationship formation in the first place between the existing firms and the new joiner when the collaboration intensity of the relationship relative to the collaboration risk is in the intermediate region, i.e., when

$$\max\left(\frac{\rho_h+\rho_l}{2}, \frac{\pi\rho_h^2 e^{\gamma\bar{\delta}N(N-1)\rho_l+\frac{\gamma^2}{2}\rho_h^2 N(N-1)\sigma_\delta^2}+(1-\pi)\rho_l^2 e^{\gamma\bar{\delta}N(N-1)\rho_h+\frac{\gamma^2}{2}\rho_l^2 N(N-1)\sigma_\delta^2}}{\pi\rho_h e^{\gamma\bar{\delta}N(N-1)\rho_l+\frac{\gamma^2}{2}\rho_h^2 N(N-1)\sigma_\delta^2}+(1-\pi)\rho_l e^{\gamma\bar{\delta}N(N-1)\rho_h+\frac{\gamma^2}{2}\rho_l^2 N(N-1)\sigma_\delta^2}}\right)$$
$$< \quad \frac{\bar{\delta}}{\gamma N\sigma_\delta^2} < \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}.$$

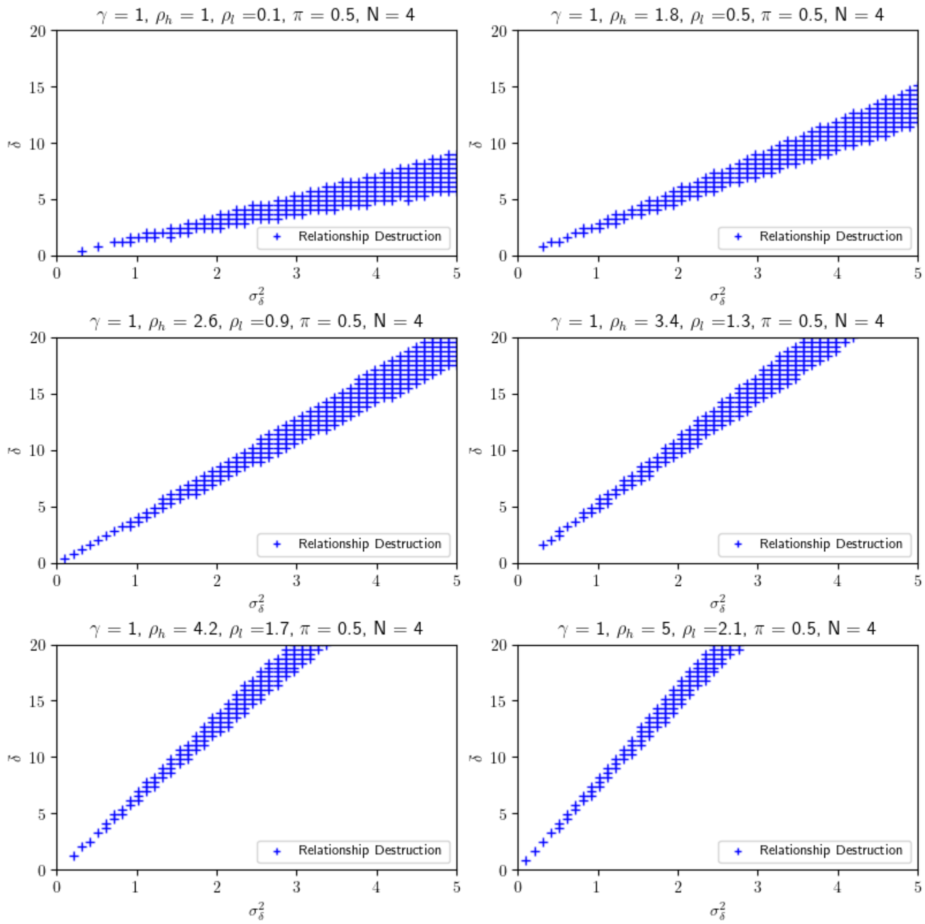
Note that the condition for mandatory relationship formation to destroy relationship formation in the first place is less likely to be satisfied when N gets larger. Intuitively, for the existing firms who already form relationship with N-1 firms, forming relationship with another firm with independent cash flows increases diversification and thus reduces the diversification cost, as N gets larger. Therefore, the firm is more likely to voluntarily disclose relationship formation and mandatory disclosure is less likely to destroy such formation.

Figure 9 illustrate how mandatory disclosure destroys relationship, with the parameter values reported on top of the panels. Again, such relationship destruction happens when the collaboration intensity relative to the collaboration risk is in the intermediate region. As in the two-firm case, mandatory disclosure will not destroy relationship formation when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}$. In particular, when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \rho_h+\rho_l)$, mandatory disclosure will force firms to disclose but will keep relationship formation intact.

7 Introducing Correlation between Firm-Specific Cash Flows and Cash Flows from Relationship Formation

So far in our analysis we assume that the firm-specific cash flow component before forming a particular relationship, $\widetilde{V}_i$, and the cash flow component coming from that particular relationship formation, $\widetilde{\Delta}$, are mutually independent. In reality, it is quite plausible that those cash flow components can be correlated. For example, suppose a cloud service provider has several big retailers as customers. Then, securing another retailer as an additional customer may result in the existing cash flow from the cloud service provider being even more positively correlated with the

Figure 9: Mandatory Disclosure and Destruction of Relationships For the New Joiner and Existing Firms



cash flow from the additional retailer customer and thus the diversification cost becoming even bigger, whereas securing a luxury goods retailer may result in the cash flows being negatively correlated, to the extent that luxury goods consumption is relatively recession-proof.

In this section we build on the N-firm setting from Section 6 and introduce correlation between $\tilde{V}_i$ and a component of $\tilde{\Delta}$, $\tilde{\delta}$. Denote Cov($\tilde{V}_i, \tilde{\delta}$) = $\sigma_{\delta i}$, and let C = $\sum_{i=1}^{N} \sigma_{\delta i}$. Again we first look at optimal relationship disclosure, when such relationship exists, and then examine optimal relationship formation.

7.1 Relationship Disclosure

The results on releationship disclosure are summarized in the following Proposition:

Proposition 9. *The optimal disclosure policy is*

$$\alpha^* = \begin{cases} \frac{1}{2}, \textit{ if } \bar{\delta} \textit{ is between } C\gamma + N\gamma\sigma_\delta^2(\rho_h + \rho_l) + N\gamma\sigma_{\delta i} \textit{ and } C\gamma + \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l), \\ 1, \textit{ otherwise.} \end{cases}$$

Proposition 9 shows that the results are largely in line with that in the main setting: the relationship increases the expected cash flow but also increases the diversification cost due to the common component of the cash flow. The introduction of correlation between $\widetilde{V}_i$ and $\widetilde{\delta}$, however, also generates some subtle differences in our results.

If the correlation is positive, then $\sigma_{\delta j} > 0$ and $C > 0$, we have $\frac{C}{N\sigma_\delta^2} + \frac{\sigma_{\delta j}}{\sigma_\delta^2} + \rho_h + \rho_l > \frac{C}{N\sigma_\delta^2} + \frac{\rho_h + \rho_l}{2}$ and so our results are largely consistent with the previous section, namely that a firm will choose to make relationship disclosure if and only if $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2}$ is sufficiently large or small. Intuitively, the driving forces with positive correlation are similar to that in our main setting: relationship disclosure increases the expected cash flow due to the collaboration intensity but also exacerbates the diversification cost due to the collaboration risk, even more so due to the positive correlation. Therefore, the range of no disclosure actually increases.

If the correlation is negative, then $\sigma_{\delta j} < 0$ and $C < 0$. In this case relationship disclosure actually reduces the diversification cost due to a reduction of the collaboration risk, resulting in the firm more likely to disclose. In the extreme case when $\frac{C}{N\sigma_\delta^2}$ is sufficiently negative relative to $\rho_h + \rho_l$ so that both $\frac{C}{N\sigma_\delta^2} + \frac{\rho_h + \rho_l}{2}$ and $\frac{C}{N\sigma_\delta^2} + \frac{\sigma_{\delta j}}{\sigma_\delta^2} + \rho_h + \rho_l$ are negative, we will have α^* always equal to 1. In this case, the negative correlation is so large as to dominate the increase in diversification cost due to the common component $\widetilde{\Delta}$ (which is determined by ρ_h and ρ_l). Therefore relationship disclosure only has benefits and the firm will therefore always disclose.

7.2 Relationship Formation

In the Online Appendix we show that mandatory disclosure destroys relationship if and only if

$$\frac{(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(\pi\rho_h+(1-\pi)\rho_l)} > \frac{\left(\bar{\delta}-C\gamma-N\gamma\sigma_{\delta j}\right)}{N\gamma\sigma_\delta^2} > \frac{\left(\pi\rho_h^2 e^{N\bar{\delta}\gamma\rho_l+\frac{\gamma^2\cdot\left(2CN\rho_h+N^2\rho_h^2\sigma_\delta^2\right)}{2}}+\rho_l^2\cdot(1-\pi)\,e^{N\bar{\delta}\gamma\rho_h+\frac{\gamma^2\cdot\left(2CN\rho_l+N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\left(\pi\rho_h e^{N\bar{\delta}\gamma\rho_l+\frac{\gamma^2\cdot\left(2CN\rho_h+N^2\rho_h^2\sigma_\delta^2\right)}{2}}+\rho_l\,(1-\pi)\,e^{N\bar{\delta}\gamma\rho_h+\frac{\gamma^2\cdot\left(2CN\rho_l+N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}.$$

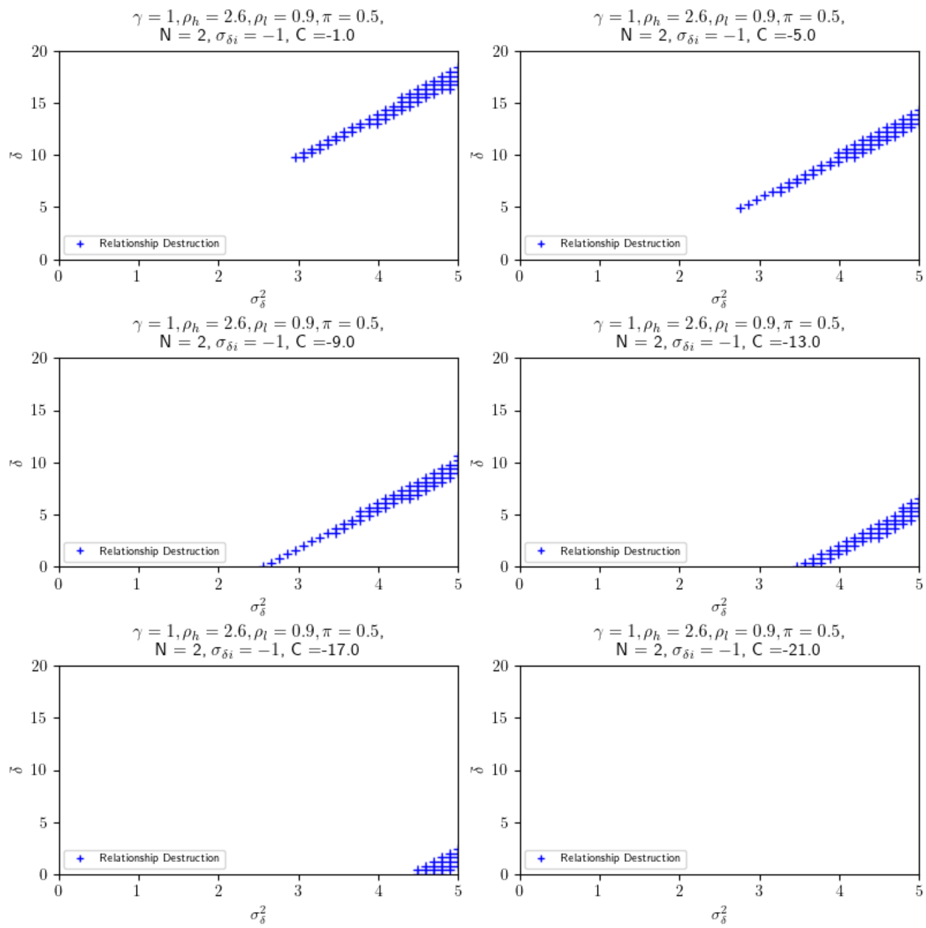
Again, the results are largely in line with that in the main setting. Positive correlation between $\tilde{V}_i$ and $\tilde{\delta}$ results in the firm less likely to form a relationship as diversification cost from the common relationship increases even more, whereas negative correlation results in the firm more likely to form a relationship, and there will be circumstances when mandatory relationship destroys disclosure formation in the first place, so long as C and $\sigma_{\delta i}$ is not too negative.

Due to the complexities of the expressions, we resort to numerical examples to illustrate our results. Figure 10 shows the equilibrium relationship formation and when mandatory disclosure destroys relationship formation. For a broad range of parameter values, we can see that mandatory disclosure destroys relationship formation when the collaboration intensity of relationship formation relative to the collaboration risk (after properly accounting for the correlation between $\tilde{V}_i$ and $\tilde{\delta}$) is in the intermediate region, in line with our previous findings. We also see that when C becomes more negative, the region that mandatory disclosure destroys relationship formation shrinks and does not exist when C is very negative, consistent with our discussions.

8 Ex-Post Disclosure

In our main setting we focus on firms committing ex-ante to a disclosure strategy. While this is in line with some of the literature (e.g., Diamond (1985)) and can be justified on the ground that voluntary disclosure ex post is not credible, it is hard to argue that voluntary relationship disclosures are not credible, as it is hard to imagine that a firm could blatantly lie about whether

Figure 10: Mandatory Disclosure and Destruction of Relationships in the Correlation Model



it has a contractual relationship with another firm. Therefore in this section we study a setting in which firms can choose whether to disclose ex-post, i.e., after observing the realization of $\tilde{\rho}$. We first show, not surprisingly, that without any disclosure friction, full disclosure is always optimal. We then add a disclosure cost friction and illustrate when there is an interior optimal disclosure precision α.

8.1 No Disclosure Cost

We first show that full disclosure is always fully revealing without any disclosure friction.

Proposition 10. *The optimal ex-post disclosure is fully revealing. Specifically, firms observing $\tilde{\rho} = \rho_h$ wants to disclose and firms observing $\tilde{\rho} = \rho_l$ is indifferent between disclosing and not disclosing* *if and only if* $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} > \rho_h + \rho_l$*; firms observing* $\tilde{\rho} = \rho_l$ *wants to disclose and firms observing* $\tilde{\rho} = \rho_h$ *is indifferent between disclosing and not disclosing if and only if* $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} < \rho_h + \rho_l$.

Proposition 10 follows from the literature on ex-post disclosure (Grossman (1981), Milgrom (1981)): without any disclosure frictions, those with good news will disclose and those with bad news will not but will be inferred as having bad news. To obtain an interior optimal disclosure precision, we need to introduce some disclosure friction, which we do next.

8.2 Costly Ex-Post Disclosure

We now introduce disclosure cost in the spirit of Jovanovic (1982) and Verrecchia (1983). Different from the assumption of a constant disclosure cost, we assume that firms can disclose with precision α with a cost that is increasing and convex in the precision. Specifically, we assume a cost form of $C(\alpha) = K(2\alpha - 1)^n \times R\mathbb{E}[P_j(\alpha; \tilde{m})]$, $K > 0, n \geq 2$ so firms choose $\alpha \in [\frac{1}{2}, 1]$ to maximize $P(\alpha) = (1 - K(2\alpha - 1)^n) \times R\mathbb{E}[P_j(\alpha; \tilde{m})]$. We focus on the case of $\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} > \rho_h + \rho_l$, i.e, firms observing $\tilde{\rho} = \rho_l$ will not disclose and firms observing $\tilde{\rho} = \rho_h$ may choose to disclose or not, as the reverse case is qualitatively the same. We are able to characterize conditions regarding the price function for an interior maximum, as illustrated in the next proposition and corollary. However, given the complexity of the price expression, we are not able to solve the problem in closed form and so rely on numerical examples to illustrate.

Proposition 11. *The following are sufficient conditions for an interior optimal* α*:* $Price''(\alpha = \frac{1}{2}) > 0$ *and* $Price(1) \leq Price(\frac{1}{2})$*, where* $Price''(.)$ *is the second order derviative of the price function.*

Corollary 4. *Given* $\mathbb{E}[P_j(\alpha; \tilde{m})]$*, then the costly disclosure price* $Price(\alpha)$ *will have an interior maximum if:*

$$\begin{cases} 1 - \frac{\mathbb{E}[P_j(\alpha=\frac{1}{2};\tilde{m})]}{\mathbb{E}[P_j(\alpha=1;\tilde{m})]} \leq K < \frac{\frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2}\Big|_{\alpha=\frac{1}{2}}}{8\mathbb{E}[P_j(\alpha=\frac{1}{2};\tilde{m})]} \text{ and } n = 2, \\ 1 - \frac{\mathbb{E}[P_j(\alpha=\frac{1}{2};\tilde{m})]}{\mathbb{E}[P_j(\alpha=1;\tilde{m})]} \leq K \text{ and } n \geq 3. \end{cases}$$

If we assume $\bar{\delta} > \gamma N \sigma_\delta^2(\rho_h + \rho_l)$, and $n \geq 3$, then K =1 will always satisfy the conditions in Corollary 4 and ensure an interior maximum, as we now graphically illustrate. Figure 11 graphically illustrates the price as a function of disclosure quality for various parameter combinations. As can be seen from the figure, when there is no disclosure cost ($K = 0$) or when marginal disclosure cost is sufficiently small, full disclosure is optimal as price increases with disclosure quality. When K becomes larger, then there will be an interior optimal degree of disclosure.

Figure 11: Price As a Function of Disclosure Quality in the Costly Ex Post Disclosure Setting

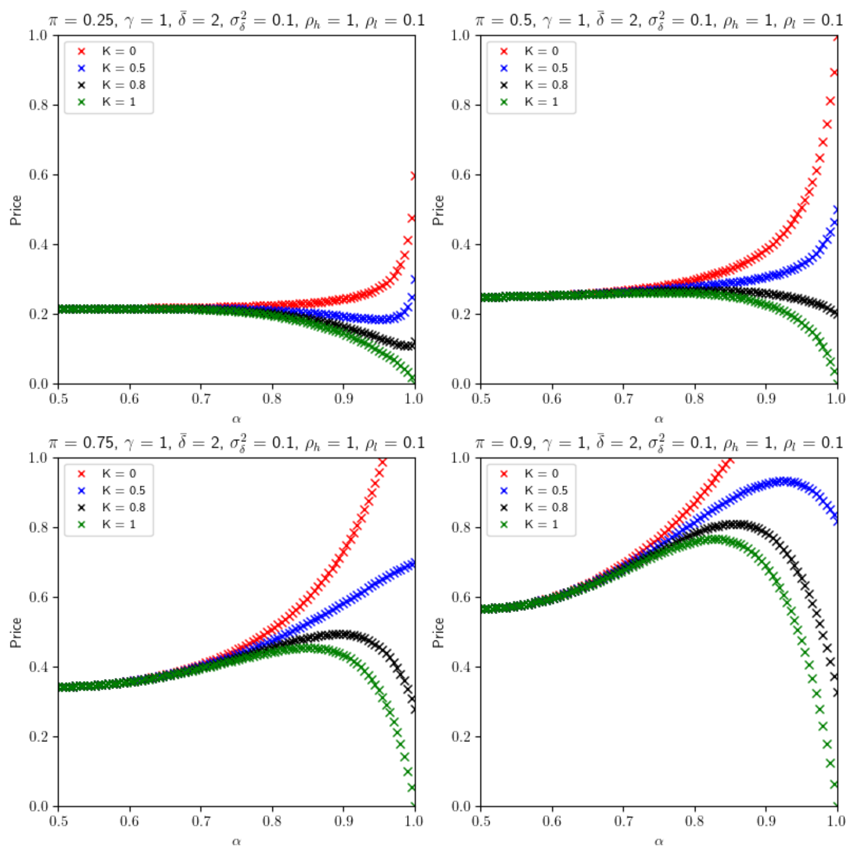
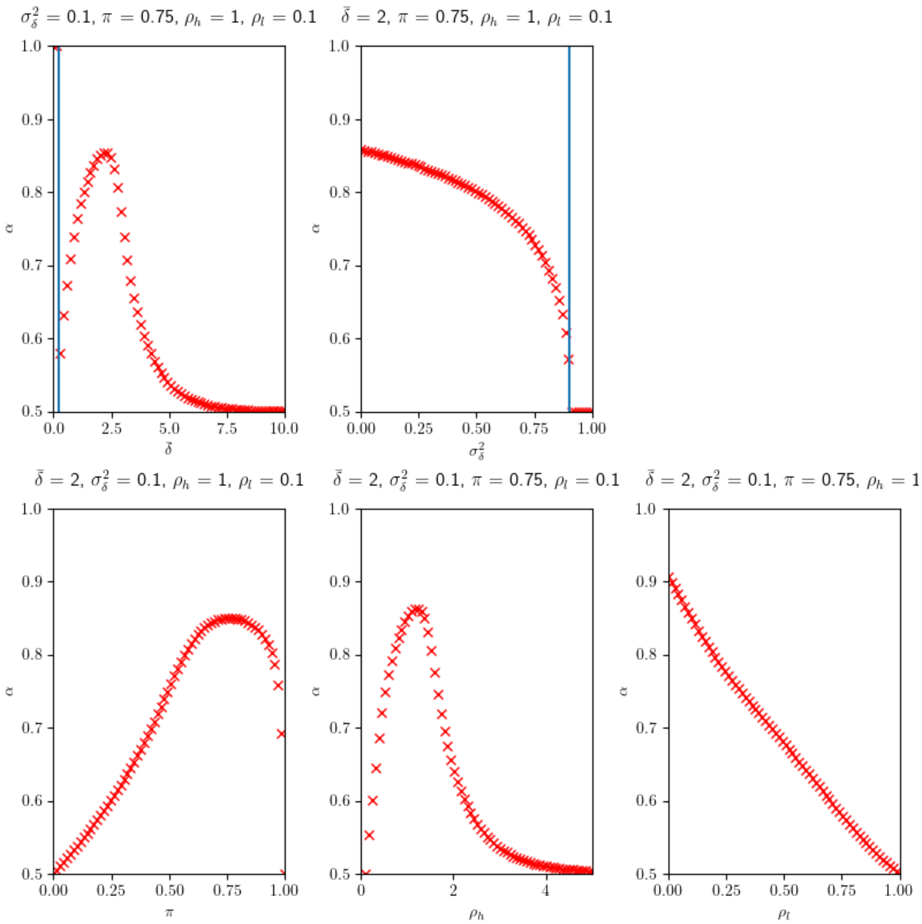


Figure 12 provides numerical examples for the optimal ex-post disclosure quality as a function of various exogenous parameters. One can see that the optimal disclosure quality generally decreases with σ_δ^2, i.e. the firm is less likely to disclose if the relationship becomes riskier.

9 Welfare Analysis of Mandatory Relationship Disclosure

So far we have been focusing on how relationship disclosure affects the cost of capital or asset prices. However, in a setting with real decisions that affect cash flows, asset prices are not

Figure 12: Comparative Statics of Optimal Disclosure Quality



equivalent to measures of investors' welfare (e.g., (Gao, 2010)). In addition, in the main setting we focus on the cost of relationship disclosure in affecting firm price, without considering the potential benefit of relationship disclosure in reducing uncertainty faced by risk-averse investors and a potential increase in investors' welfare. In this section we use the setting in Section 6.1 to study whether mandatory disclosure regulation increases or decreases investor welfare, i.e., the expected utility, as this setting is a generalization of our main setting of two firms. When firms choose to voluntarily disclose relationship, such regulation clearly does not affect welfare, so the interesting question is how such regulation affects welfare when firms voluntarily choose to not disclose relationship. The following proposition shows that mandatory disclosure regulation increases investor welfare when such disclosure does not result in destruction of relationship formation but decreases investor welfare when such disclosure results in destruction of relationship formation.

Proposition 12. *Mandatory disclosure makes the investor strictly better off when firms are forced to disclose relationship without destroying the underlying relationship formation and makes the investor strictly worse off when relationship formation is destroyed.*

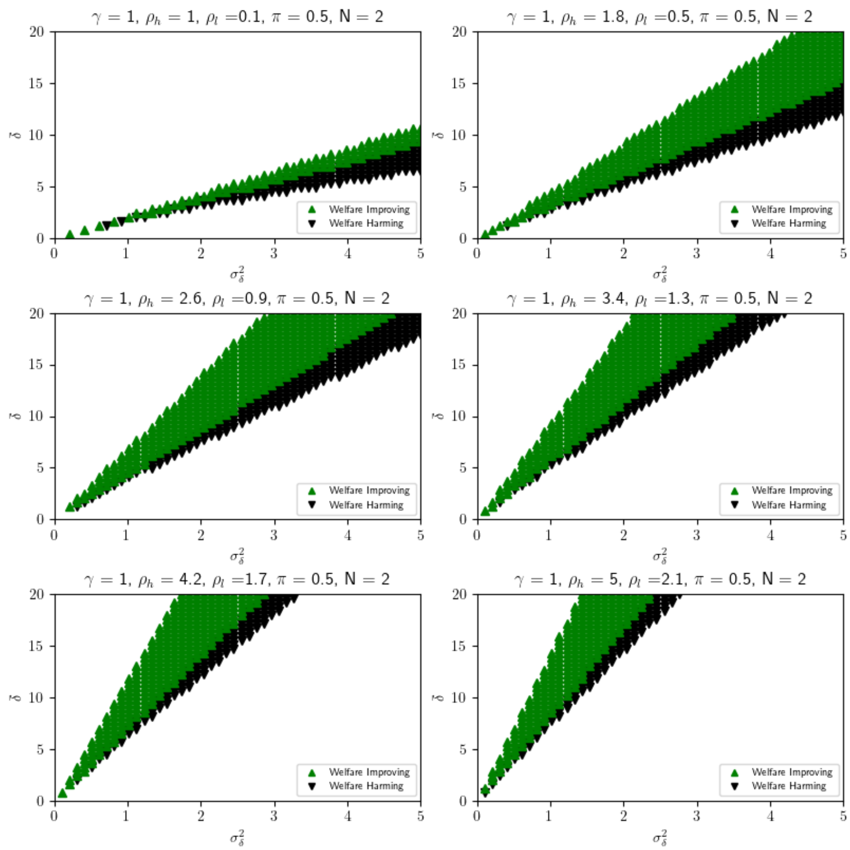
Intuitively, Proposition 12 states that mandatory relationship disclosure has welfare benefits if firms are still forming the relationship, as such disclosure reduces the prices that the investor pays for (which is why firms choose not to disclose in the first place). However, when mandatory relationship disclosure destroys relationship formation, the reduction in expected cash flow due to firms not forming relationship more than offsets the price reduction benefit to the investor. In other words, mandating relationship disclosure is beneficial if such disclosure does not have adverse real effects.

Figure 13 graphically illustrates when mandatory disclosure increases and decreases investor welfare, for various parameter combinations. When $\frac{\bar{\delta}}{N\gamma\sigma_{\delta}^2}$ is in the intermediate range, mandatory disclosure reduces investor welfare by destroying relationship formation. When $\frac{\bar{\delta}}{N\gamma\sigma_{\delta}^2}$ becomes higher, mandatory disclosure increases investor welfare by forcing firms to disclose but does not destroy relationship formation. When $\frac{\bar{\delta}}{N\gamma\sigma_{\delta}^2}$ becomes even higher, firms will voluntarily disclose relationship so mandatory disclosure has no effect.

10 Empirical and Regulatory Implications

Our results provide several implications related to disclosure of relationships among firms. First, Corollaries 1 and 2 and the numerical examples offer some predictions on price responses to relationship disclosure. We show that price response to relationship disclosure will be lower if the relationship is riskier, and will be higher if the expected levels of matching intensity and collaboration intensity are higher. To the extent that expected levels of matching intensity and collaboration intensity are more important for small firms' cash flows, the price response will be higher in magnitude for smaller firms relative to larger firms in the relationship. Interestingly,

Figure 13: Effect of Mandatory Disclosure on Investor Welfare



Corollary 2 also suggests that disclosure of a more intensive relationship (than expected) may not necessarily generate a positive price response, if the relationship turns out to be very risky relative to the expected benefits.

Second, we show that relationship disclosure increases the perceived risk of firms due to a decrease in diversification benefit, which may result in lower ex-ante prices. Therefore, we predict that firms making relationship disclosure are more risky than firms that do not make such disclosure, which can be empirically proxied by, e.g., return volatility or beta.

Third, we show that firms that choose to voluntarily disclose relationship have either high or low expected benefits from such relationship. To the extent that the benefits from relationship translate into higher future earnings and cash flows, our results imply that in the pre-SFAS 131 regime when relationship disclosures are voluntary, firms that choose to disclose relationship would have either high or low future expected earnings and cash flows from operations whereas firms that choose not to disclose such relationship will have intermediate future expected earnings and cash flows.

Fourth, our analysis of N-firm settings in Figure 7 shows that the more firms form relationship with each other (i.e., a strategic alliance network), the less likely firms will voluntarily disclose about such relationship, as the range when no disclosure is optimal is increasing in N. While this result seems counterintuitive, it also suggests that conditional on firms disclosing about a strategic alliance network, the average benefit to each firm would be larger, relative to the increase in risk, the larger the network is.

Fifth, our analysis of the setting with correlation between $\tilde{V}_i$ and $\tilde{\delta}$ in Section 7 suggests that the firm is more likely to make relationship disclosure if the correlation of the cash flow generated between the new relationship and existing relationships is more negative (or less likely to be positive). Therefore, conditional on firm disclosing a new customer relationship, such relationship should generate higher expected future cash flows if the customer is more correlated with the firms' existing customers, which can be proxied by, e.g., correlation of stock returns, correlation of past earnings.

Sixth, our analysis of the setting of ex post disclosure in Section 8 suggest that firms are more likely to disclose that matching intensity is high if the expected future benefit is sufficiently high and are more likely to disclose that matching intensity is low if the expected future benefit is sufficiently low. However, adding a cost of disclosure (e.g., proprietary cost) then suggests that firms will choose to optimally disclose less if the underlying relationship is riskier.

Finally, we show that mandating relationship disclosure has the unintended consequences of discouraging firms to form beneficial relationships in the first place, which should be of interests to regulators contemplating more relationship disclosures. The results also imply that firms who voluntarily disclose in the pre-SFAS 131 regime will make less relationship disclosure post-SFAS 131, when such disclosure becomes mandatory. However, those firms will also have higher profitability post SFAS-131 as relationships that only have high benefits will be formed. In addition, results from Section 9 suggest that such mandatory disclosure will result in a lower investor welfare if and only if it destroys relationship formation, which may have macroeconomic implications for, e.g., consumption growth and GDP. When such disclosure does not result in destroying relationship formation, mandatory disclosure cannot decrease investor welfare.

11 Conclusion

In this paper, we examine how the payoff correlation structure is endogenously generated and study the incentives and implications of firms to form and disclose such correlations. Since asset payoffs are cash flows generated by production, in principle, any payoff correlation structure must trace back to the production process. In this paper, we open the black box of the production process from a particular perspective, namely, firm relationships.

This paper develops a new conceptual framework to analyze the incentives of firms to form and disclose relationships and its implications for asset prices, abstracting away from the proprietary cost framework on which most prior literature focuses. Forming a relationship generates synergies between firms. But relationship formations also incur a cost. Relationship formation makes the performance of the firms correlated, reducing the ability to hedge the risk of investing in the firms and generating additional risk in financial markets. We first study the trade-offs on asset prices when there is uncertainty about the relationship between two firms. Having a relationship generates two effects on the cash flows of the firms. First, the cash flows of the two firms have an additional payoff component with a positive mean. Second, the cash flows also become more correlated. On the one hand, the increase in the mean of asset payoffs raises the investor's perceived returns on investing in risky assets and thus increases her demand for the assets and their prices. On the other hand, the increase in the variance of the asset payoffs decreases the investor's ability to diversify her portfolio as cash flows are now correlated, which decreases the investor's demand for the assets and their prices.

We further analyze the optimal voluntary disclosure policies about a relationship. Unlike previous literature, disclosing a relationship in our framework is about disclosing the existence of a common component in the asset payoffs of the two firms, generating a correlation structure between the two firms. It is not about disclosing the realization of the fundamentals as in most research on disclosure. We finally examine under which conditions firms choose to form relationships and their collaboration intensity. Our analysis provides insights for regulations mandating relationship disclosure, by highlighting the consequences of such regulations for asset prices and relationship formation, as well as proposing an endogenous and non-proprietary cost of such disclosure.

As a final remark, in this paper we consider a two-firm setting for the sake of tractability. This setting can be literally interpreted as two big representative firms. For example, Walmart and Pepsi can be considered as two big representative firms in their respective industries. In a large economy with N firms, the number of relationships among firms is large and modelling relationship between any two of the N firms becomes quite challenging. While we make some attempts in Section 6 to show that our results largely hold with appropriate extensions of relationship formation to N-firm settings, there is always the issue that whether the extension to the N-firm setting correctly captures the interactions of N firms in reality. We leave that for future work but conjecture that our results will hold so long as the modelling of relationship among firms results in relationship being systematic, which is supported by the empirical evidence in Herskovic (2018).

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Appendix

Statistical Properties of Cash Flows

The common component of the cash flows $\tilde{\Delta}$ follows a mixture of two Normal distributions. Hence, $\tilde{\Delta}$ does not follow a Normal distribution. Since $\tilde{\rho}$ affects both the mean and the variance of $\tilde{\Delta}$, uncertainty about $\tilde{\rho}$ generates uncertainty about both the mean and the variance of the common component in the cash flows. Hence, states of the world with high mean will also have high variance, generating skewness in the distribution of the common component.

The sign of skewness of the common component of the cash flows is going to be given by the sign of the following expression:

$$E\left[(\tilde{\Delta} - E[\tilde{\Delta}])^3\right] = \pi(1-\pi)(\rho_h - \rho_l)\bar{\delta}[(1-2\pi)(\rho_h - \rho_l)^2\bar{\delta} + 3(\rho_h^2 - \rho_l^2)\sigma_\delta^2].$$

Skewness is a measure of asymmetry of the distribution of cash flows. For $\pi = 0.5$, the skewness will always be strictly positive. Risk averse investors have preferences over higher moments of the distribution. Specifically, risk averse investors value more assets with positive skewness, and it will be reflected positively in asset prices. There will only be negative skewness when both $\pi > 0.5$ and $\bar{\delta}$ is large. The uncertainty about the variance of $\tilde{\Delta}$ will also generate kurtosis in the distribution of the common component. Kurtosis increases the probability of extreme outcomes and will lead to a decrease in asset prices as a result.

In addition, the distribution of the common component $\tilde{\Delta}$ may be unimodal or bimodal depending on the parameters of the distribution. Define $d \equiv \frac{(\rho_h - \rho_l)\bar{\delta}}{2\sigma_\delta^2\sqrt{\rho_h\rho_l}}$. The distribution of the common component $\tilde{\Delta}$ is unimodal if and only if $d \leq 1$ or

$$|\ln(1-\pi) - \ln(\pi)| \geq 2\log\left(d - \sqrt{d^2 - 1}\right) + 2d\sqrt{d^2 - 1}.$$

For sufficiently low $\bar{\delta}$, the distribution of the common component $\tilde{\Delta}$ will be unimodal and the distribution will have positive skewness as described above. For sufficiently high $\bar{\delta}$ and if $\pi > 0.5$, the distribution of the common component will be bimodal and have negative skewness. For $\pi \leq 0.5$ and large $\bar{\delta}$, the distribution of the common component will be bimodal and have positive skewness.

Proof of Proposition 1

The representative investor maximizes equation (5) with respect to q_A and q_B. The first-order condition (FOC) with respect to q_A is given by

$$\pi \exp\left[-\gamma\left(\mu_W(h) - \frac{\gamma\sigma_W^2(h)}{2}\right)\right](\bar{V} + \rho_s\bar{\delta} - RP_A - \gamma q_A(\sigma_V^2 + \rho_h^2\sigma_\delta^2) - \gamma q_B\rho_h^2\sigma_\delta^2 +$$
$$(1-\pi)\exp\left[-\gamma\left(\mu_W(l) - \frac{\gamma\sigma_W^2(l)}{2}\right)\right](\bar{V} + \rho_l\bar{\delta} - RP_A - \gamma q_A(\sigma_V^2 + \rho_l^2\sigma_\delta^2) - \gamma q_B\rho_l^2\sigma_\delta^2 = 0.$$

We can get the FOC with respect to q_B in the same way. When we plug the market clearing conditions $q_A = 1$ and $q_B = 1$ into both conditions, we get prices for asset $j \in \{A, B\}$:

$$\begin{aligned} P_j &= \frac{\bar{V} - \gamma\sigma_V^2}{R} + \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right] \\ &- \frac{2\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} + (1-\pi)e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}}\right]. \end{aligned}$$

Proof of Proposition 2

We know that $\pi^h \geq \pi^l$ as $\alpha \geq \frac{1}{2}$. In addition, from equation 14 π^{ram} is increasing in π^m as its inverse is decreasing in π^m. Therefore, from equation 15 $\widetilde{m} = h$ increases prices (and $\widetilde{m} = l$ decreases prices) if and only if $P_j(\alpha; \widetilde{m})$ is increasing in π^{ram}. From equation 13, the derivative of $P_j(\alpha; \widetilde{m})$ with respect to π^{ram} is

$$\begin{aligned} & \frac{\bar{\delta}}{R}(\rho_h - \rho_l) - \frac{2\gamma\sigma_\delta^2}{R}(\rho_h^2 - \rho_l^2) \\ = & \frac{\rho_h - \rho_l}{R\gamma\sigma_\delta^2}[\frac{\bar{\delta}}{\gamma\sigma_\delta^2} - 2(\rho_h + \rho_l)] \\ > & 0, \end{aligned}$$

if and only if $\frac{\bar{\delta}}{\gamma\sigma_\delta^2} > 2(\rho_h + \rho_l)$.

Proof of Proposition 2

The FOC of the expected asset price (15) with respect to α equals zero at $\alpha = \frac{1}{2}$ is:

$$\left.\frac{\partial E[P_j(\alpha;\tilde{m})]}{\partial \alpha}\right|_{\alpha=\frac{1}{2}} = 0.$$

The relevant expression for the sign of the second-order condition (SOC) $\frac{\partial^2 E[P_j(\alpha;\tilde{m})]}{\partial \alpha^2}$ for $\alpha = \frac{1}{2}$ to be a maximum is given by

$$\left[2\gamma\sigma_\delta^2(\rho_h + \rho_l) - \bar{\delta}\right]\left(e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} - e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}\right) < 0;$$

otherwise $\alpha = \frac{1}{2}$ is a minimum. There are two ways under which the inequality above is satisfied for $\alpha = \frac{1}{2}$ to be a maximum:

1. If both of these conditions are satisfied: $\bar{\delta} > 2\gamma\sigma_\delta^2(\rho_h+\rho_l)$ and $e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} > e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}$. These two conditions are satisfied if and only if $\gamma(\rho_l + \rho_h)\sigma_\delta^2 > \bar{\delta} > 2\gamma(\rho_l + \rho_h)\sigma_\delta^2$. This is not a feasible condition.

2. If both of these conditions are satisfied: $\bar{\delta} < 2\gamma\sigma_\delta^2(\rho_h+\rho_l)$ and $e^{2\gamma^2\rho_h^2\sigma_\delta^2 + 2\gamma\rho_l\bar{\delta}} < e^{2\gamma^2\rho_l^2\sigma_\delta^2 + 2\gamma\rho_h\bar{\delta}}$. These two conditions are satisfied if and only if $\gamma(\rho_l + \rho_h)\sigma_\delta^2 < \bar{\delta} < 2\gamma(\rho_l + \rho_h)\sigma_\delta^2$. This is a feasible condition.

Hence, we can conclude that $\alpha = \frac{1}{2}$ is a maximum if and only if $\gamma(\rho_l + \rho_h)\sigma_\delta^2 < \bar{\delta} < 2\gamma(\rho_l + \rho_h)\sigma_\delta^2$. Otherwise, $\alpha = 1$ is a maximum since $\alpha = \frac{1}{2}$ is a minimum and $\frac{\partial E[P_j(\alpha;\tilde{m})]}{\partial \alpha} > 0$ for any $\alpha \in (\frac{1}{2}, 1]$.

Proof of Corollary 3

When the optimal policy is non-disclosure ($\alpha^* = \frac{1}{2}$), the message received by the representative investor contains no information and we are in the scenario of Section 3.3, where the asset price is given by equation (6). In this case, the price is independent of the message, and thus $E[P_j(\alpha^* = \frac{1}{2};\tilde{m})]$ is also given by equation (6). Instead, when the optimal policy is full disclosure ($\alpha^* = 1$), the message received by the representative investor contains full information and asset prices are given by equation (4) depending on the realization of $\tilde{\rho}$. Using equation (15), we can calculate the expected asset price as

$$E[P_j(\alpha^* = 1;\tilde{m})] = \pi P_j^{PI}(\tilde{\rho} = \rho_h) + (1-\pi)P_j^{PI}(\tilde{\rho} = \rho_l),$$

where we have used that $Pr(\tilde{m} = h) = \pi$, $Pr(\tilde{m} = l) = 1 - \pi$, $P_j(\alpha^* = 1; \tilde{m} = h) = P_j^{PI}(\tilde{\rho} = \rho_h)$, and $P_j(\alpha^* = 1; \tilde{m} = l) = P_j^{PI}(\tilde{\rho} = \rho_l)$.

Proof of Lemma 1

From Figure 4, we can see that no firm has any incentive to deviate from a no-relationship equilibrium as there is no benefit from an individual deviation.

Proof of Proposition 3

Under the optimal disclosure policy $\alpha^* = 1$, using equations (19) and (21), for any firm $j \in \{A, B\}$, we get that $P_j^N < E[P_j(\alpha^* = 1; \tilde{m})]$ if and only if

$$(\pi\rho_h + (1-\pi)\rho_l)\bar{\delta} > 2\gamma\sigma_\delta^2(\pi\rho_H^2 + (1-\pi)\rho_L^2).$$

In addition, from Proposition 2 we know that $\alpha^* = 1$ is optimal if and only if either $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_h + \rho_l$ or $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \frac{\rho_h+\rho_l}{2}$. We can show that $\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l} < \rho_h + \rho_l$ as it is equivalent to

$$\pi(1-\pi)\rho_h\rho_l > 0.$$

We can also show that $\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l} < \frac{\rho_h+\rho_l}{2}$ if and only if

$$\pi\rho_h(\rho_h - \rho_l) < (1-\pi)\rho_l(\rho_h - \rho_l),$$

which is equivalent to

$$\pi\rho_h < (1-\pi)\rho_l.$$

Under the optimal disclosure policy $\alpha^* = \frac{1}{2}$, using equations (6) and (21), for any firm $j \in \{A, B\}$, we get that $P_j^N < E[P_j(\alpha^* = \frac{1}{2}; \tilde{m})]$ if and only if

$$(\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}})\bar{\delta} > 2\gamma\sigma_\delta^2(\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}).$$

In addition, from Proposition 2 we know that $\alpha^* = \frac{1}{2}$ is optimal if and only if $\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l$. $\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}} < \rho_h + \rho_l$ as it is equivalent to

$$\rho_h\rho_l[\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} + (1-\pi)e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}] > 0.$$

We can also show that $\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}} < \frac{\rho_h+\rho_l}{2}$ if and only if

$$\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}(\rho_h-\rho_l) < (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}(\rho_h-\rho_l),$$

which is equivalent to

$$\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}} < (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}.$$

Proof of Proposition 4

There are three conditions that need to be satisfied so that $E[P_j(\alpha=\frac{1}{2};\tilde{m})] > P_j^N > E[P_j(\alpha=1;\tilde{m})]$ for any firm $j\in\{A,B\}$.

1. For both conditions $E[P_j(\alpha=\frac{1}{2};\tilde{m})] > P_j^N$ and $P_j^N > E[P_j(\alpha=1;\tilde{m})]$ to be jointly satisfied, the following inequality must be satisfied

$$\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l} > \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}+(1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}}}.$$

   The first inequality arises from $P_j^N > E[P_j(\alpha=1;\tilde{m})]$ and the second inequality arises from $E[P_j(\alpha=\frac{1}{2};\tilde{m})] > P_j^N$.

2. The sufficient conditions stated in Proposition 2 need to be violated. Otherwise, there exists an equilibrium with relationship formation under both a disclosure and non-disclosure policy or alternatively there is a unique equilibrium with no relationships in both economy. Hence, the following condition must be satisfied

$$\rho_h > \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_l.$$

3. For the upper bound of condition 1 to be larger than the lower bound, we need

$$(\rho_h-\rho_l)(e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\bar{\delta}} - e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\bar{\delta}}) > 0,$$

   which is only satisfied when

$$(\rho_l+\rho_h) < \frac{\bar{\delta}}{\gamma\sigma_\delta^2}.$$

Conditions 3 implies that the upper bound of condition 1 is always strictly higher than the lower bound of that condition. Also conditions 2 and 3 are weaker than condition 1. Hence, condition 1 is a necessary and sufficient condition.

Note that for condition 1 to be satisfied, we need

$$\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\overline{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\overline{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\overline{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\overline{\delta}}} < \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l},$$

which is equivalent to

$$e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\overline{\delta}} > e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\overline{\delta}},$$

that is,

$$\frac{\overline{\delta}}{2\gamma\sigma_\delta^2} > \frac{\rho_h+\rho_l}{2}.$$

Note that we also need

$$\frac{\rho_h+\rho_l}{2} < \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l},$$

which is equivalent to

$$\pi\rho_h > (1-\pi)\rho_l.$$

Therefore , the condition can be finally reduced to

$$\max\Big(\frac{\rho_h+\rho_l}{2}, \frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\overline{\delta}} + (1-\pi)\rho_l^2 e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\overline{\delta}}}{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma\rho_l\overline{\delta}} + (1-\pi)\rho_l e^{2\gamma^2\rho_l^2\sigma_\delta^2+2\gamma\rho_h\overline{\delta}}}\Big) < \frac{\overline{\delta}}{2\gamma\sigma_\delta^2} < \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l},$$

which requires $\pi\rho_h > (1-\pi)\rho_l$.

Proof of Proposition 5

For simplicity we use the expression of RP_j as R is a constant so maximizing P_j is equivalent to maximizing RP_j.

We have

$$\begin{aligned} RP_j(\alpha;\tilde{m}) = \bar{V} - \gamma\sigma_V^2 + \bar{\delta}&\left[\frac{\pi^m\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi^m)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi^m e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi^m) e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\ - \gamma N\sigma_\delta^2&\left[\frac{\pi^m\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi^m)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi^m e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi^m) e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right], \end{aligned}$$

$$E[RP_j(\alpha;\tilde{m})] = RP_j(\alpha;\tilde{m}=h)Pr(\tilde{m}=h) + RP_j(\alpha;\tilde{m}=l)Pr(\tilde{m}=l).$$

Relevant expression for FOC:

$$-\pi^2 \cdot (2\alpha-1)(\pi-1)^2(\rho_h-\rho_l)(2\alpha\pi-\alpha-\pi)^3\left(N\gamma\rho_h\sigma_\delta^2+N\gamma\rho_l\sigma_\delta^2-\bar{\delta}\right)(2\alpha\pi-\alpha-\pi+1)^2 e^{N\bar{\delta}\gamma\rho_h}e^{N\bar{\delta}\gamma\rho_l}$$
$$\times\Big(-\pi e^{2N\bar{\delta}\gamma\rho_h+\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}+2\pi e^{N\bar{\delta}\gamma\rho_h+N\bar{\delta}\gamma\rho_l+N^2\gamma^2\rho_h^2\sigma_\delta^2+N^2\gamma^2\rho_l^2\sigma_\delta^2}-\pi e^{2N\bar{\delta}\gamma\rho_l+\frac{3N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}$$
$$+e^{2N\bar{\delta}\gamma\rho_h+\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}-e^{N\bar{\delta}\gamma\rho_h+N\bar{\delta}\gamma\rho_l+N^2\gamma^2\rho_h^2\sigma_\delta^2+N^2\gamma^2\rho_l^2\sigma_\delta^2}\Big).$$

Relevant expression for second order condition (SOC):

$$\left(e^{N\bar{\delta}\gamma\rho_l+N^2\gamma^2\rho_h^2\sigma_\delta^2+\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}-e^{N\bar{\delta}\gamma\rho_h+\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+N^2\gamma^2\rho_l^2\sigma_\delta^2}\right)\left(N\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta}\right)<0.$$

To find the optimal α^*, we start by considering $\frac{\partial\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}$:
First, $\frac{\partial\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}\Big|_{\alpha=\frac{1}{2}}=0$.
Second, looking at the SOC $\frac{\partial^2\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha^2}$ for $\alpha=\frac{1}{2}$ to be a maximum is given by:

$$(\frac{N}{2}\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta})\left(N\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta}\right)<0.$$

Finally, breaking down the FOC:
First we notice: for all $\alpha\in(\frac{1}{2},1)$ and $\pi\in(0,1)$:

$$\pi^2\cdot(2\alpha-1)(\pi-1)^2(\rho_h-\rho_l)(2\alpha\pi-\alpha-\pi+1)^2e^{N\bar{\delta}\gamma\rho_h}e^{N\bar{\delta}\gamma\rho_l}>0$$
$$\text{and: }-(2\alpha\pi-\alpha-\pi)^3>0.$$

Looking at the exponential term:

$$e^{2N\bar{\delta}\gamma\rho_h+\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}-e^{N\bar{\delta}\gamma\rho_h+N\bar{\delta}\gamma\rho_l+N^2\gamma^2\rho_h^2\sigma_\delta^2+N^2\gamma^2\rho_l^2\sigma_\delta^2}$$
$$+\pi\Big(2e^{N\bar{\delta}\gamma\rho_h+N\bar{\delta}\gamma\rho_l+N^2\gamma^2\rho_h^2\sigma_\delta^2+N^2\gamma^2\rho_l^2\sigma_\delta^2}-e^{2N\bar{\delta}\gamma\rho_h+\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}-e^{2N\bar{\delta}\gamma\rho_l+\frac{3N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}+\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}\Big)$$

is determined by :$(\frac{N}{2}\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta})$.

As a result, the sign of the FOC is determined by:

$$(\frac{N}{2}\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta})\left(N\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta}\right),\ \text{the SOC expression.}$$

This allows us to conclude that when $\alpha^*=\frac{1}{2}$ is a minimum, the derivative is increasing over

$(\frac{1}{2}, 1)$ and so the maximum occurs at $\alpha^* = 1$.

Proof of Proposition 6

When $\alpha = 1$:

$$RP_j(\alpha^* = 1; \tilde{m}) = \bar{V} - \gamma\sigma_V^2 + \bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) - N\gamma\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2),$$
$$RP_j^N = \bar{V} - \gamma\sigma_V^2.$$

So form relationship if: $\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) > N\gamma\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2)$, which is equivalent to

$$\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}.$$

From Proposition 5 we know that firms disclose if and only if $\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} < \frac{\rho_l+\rho_h}{2}$ or $\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} > \rho_l + \rho_h$. We know that

$$\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} \leq \rho_h + \rho_l,$$

as it is equivalent to

$$\rho_h\rho_l \geq 0.$$

Therefore a relationship will be formed with full-disclosure if $\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} > \rho_l + \rho_h$, or when $\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_l+\rho_h}{2})$. The latter case is possible only if

$$\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} < \frac{\rho_l + \rho_h}{2},$$

which is equivalent to

$$\pi\rho_h(\rho_h - \rho_l) < (1-\pi)\rho_l(\rho_h - \rho_l),$$

that is,

$$\pi\rho_h < (1-\pi)\rho_l.$$

When $\alpha = \frac{1}{2}$, price given by the standard price equation in Section 1:

So form a relationship if

$$\begin{aligned} &\overline{\delta}[\pi\rho_h e^{\gamma(\overline{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\overline{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}] \\ > \; &\gamma N\sigma_\delta^2[\pi\rho_h^2 e^{\gamma(\overline{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\overline{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}], \end{aligned}$$

which is equivalent to

$$\frac{\overline{\delta}}{\gamma N\sigma_{\delta}^{2}} > \frac{\pi\rho_{h}^{2}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}^{2}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}{\pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}.$$

From Proposition 5 we know that firms will not disclose if and only if $\frac{\rho_l+\rho_h}{2} < \frac{\overline{\delta}}{N\gamma\sigma_{\delta}^{2}} < \rho_l + \rho_h$. We know that

$$\frac{\pi\rho_{h}^{2}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}^{2}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}{\pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}} \leq \rho_l + \rho_h,$$

as it is equivalent to

$$[\pi e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}]\rho_h\rho_l \geq 0.$$

Therefore a relationship will be formed with no disclosure if

$$\begin{aligned} & \max\Big(\frac{\rho_l+\rho_h}{2}, \frac{\pi\rho_{h}^{2}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}^{2}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}{\pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}\Big) \\ < \; & \frac{\overline{\delta}}{N\gamma\sigma_{\delta}^{2}} < \rho_l + \rho_h. \end{aligned}$$

In addition,

$$\frac{\pi\rho_{h}^{2}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}^{2}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}}{\pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}+(1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}} < \frac{\rho_l+\rho_h}{2}$$

if and only if

$$\begin{aligned} & \pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})}(\rho_h-\rho_l) \\ < \; & (1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}(\rho_h-\rho_l), \end{aligned}$$

which is equivalent to

$$\pi\rho_{h}e^{\gamma(\overline{\delta}N\rho_{l})+\frac{\gamma^{2}}{2}(\rho_{h}^{2}N^{2}\sigma_{\delta}^{2})} < (1-\pi)\rho_{l}e^{\gamma(\overline{\delta}N\rho_{h})+\frac{\gamma^{2}}{2}(\rho_{l}^{2}N^{2}\sigma_{\delta}^{2})}.$$

Proof of Proposition 7

Price with Posterior Probability π^m after sending message $\tilde{m}$

$$RP_j(\alpha;\tilde{m}) = \bar{V} - \gamma\sigma_\delta^2 + \bar{\delta}\,(N-1)\,\frac{\left(\pi^m \rho_h e^{N\bar{\delta}\gamma\rho_l(N-1)+N\gamma^2\rho_h^2\sigma_\delta^2(N-1)} + \rho_l\,(1-\pi^m)\,e^{N\bar{\delta}\gamma\rho_h(N-1)+N\gamma^2\rho_l^2\sigma_\delta^2(N-1)}\right)}{\pi^m e^{N\bar{\delta}\gamma\rho_l(N-1)+N\gamma^2\rho_h^2\sigma_\delta^2(N-1)} + (1-\pi^m)\,e^{N\bar{\delta}\gamma\rho_h(N-1)+N\gamma^2\rho_l^2\sigma_\delta^2(N-1)}}$$
$$-\,2\gamma\sigma_\delta^2\,(N-1)\,\frac{\left(\pi^m \rho_h^2 e^{N\bar{\delta}\gamma\rho_l(N-1)+N\gamma^2\rho_h^2\sigma_\delta^2(N-1)} + \rho_l^2\cdot(1-\pi^m)\,e^{N\bar{\delta}\gamma\rho_h(N-1)+N\gamma^2\rho_l^2\sigma_\delta^2(N-1)}\right)}{\pi^m e^{N\bar{\delta}\gamma\rho_l(N-1)+N\gamma^2\rho_h^2\sigma_\delta^2(N-1)} + (1-\pi^m)\,e^{N\bar{\delta}\gamma\rho_h(N-1)+N\gamma^2\rho_l^2\sigma_\delta^2(N-1)}}.$$

Expected Price with disclosure policy α:

$$R\mathbb{E}[P_j(\alpha;\tilde{m})] = RP_j(\alpha;\tilde{m}=h)Pr(\tilde{m}=h) + RP_j(\alpha;\tilde{m}=l)Pr(\tilde{m}=l).$$

Relevant expression for FOC:

$$-\,\pi^2\,(N-1)\,(2\alpha-1)\,(\pi-1)^2\,(\rho_h-\rho_l)\left(\pi e^{N\gamma\left(N\bar{\delta}\rho_l+N\gamma\rho_h^2\sigma_\delta^2+\bar{\delta}\rho_h+\gamma\rho_l^2\sigma_\delta^2\right)} + (1-\pi)\,e^{N\gamma\left(N\left(\bar{\delta}\rho_h+\gamma\rho_l^2\sigma_\delta^2\right)+\bar{\delta}\rho_l+\gamma\rho_h^2\sigma_\delta^2\right)}\right.$$
$$\times\, e^{N\gamma\left(N\bar{\delta}\rho_h+N\bar{\delta}\rho_l+N\gamma\rho_h^2\sigma_\delta^2+N\gamma\rho_l^2\sigma_\delta^2+\bar{\delta}\rho_h+\bar{\delta}\rho_l+\gamma\rho_h^2\sigma_\delta^2+\gamma\rho_l^2\sigma_\delta^2\right)}$$
$$\times\left(e^{N\gamma\left(N\bar{\delta}\rho_l+N\gamma\rho_h^2\sigma_\delta^2+\bar{\delta}\rho_h+\gamma\rho_l^2\sigma_\delta^2\right)} - e^{N\gamma\left(N\bar{\delta}\rho_h+N\gamma\rho_l^2\sigma_\delta^2+\bar{\delta}\rho_l+\gamma\rho_h^2\sigma_\delta^2\right)}\right)\left(\bar{\delta}-2\gamma\sigma_\delta^2(\rho_h+\rho_l)\right).$$

Relevant expression for SOC:

$$-\,32\pi^2\,(N-1)\,(\pi-1)^2\cdot(\rho_h-\rho_l)\,e^{-N\bar{\delta}\gamma\rho_h}e^{-N\bar{\delta}\gamma\rho_l}e^{-N\gamma^2\rho_h^2\sigma_\delta^2}e^{-N\gamma^2\rho_l^2\sigma_\delta^2}e^{N^2\bar{\delta}\gamma\rho_h}e^{N^2\bar{\delta}\gamma\rho_l}e^{N^2\gamma^2\rho_h^2\sigma_\delta^2}e^{N^2\gamma^2\rho_l^2\sigma_\delta^2}$$
$$\times\left(e^{-N\bar{\delta}\gamma\rho_l}e^{-N\gamma^2\rho_h^2\sigma_\delta^2}e^{N^2\bar{\delta}\gamma\rho_l}e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} - e^{-N\bar{\delta}\gamma\rho_h}e^{-N\gamma^2\rho_l^2\sigma_\delta^2}e^{N^2\bar{\delta}\gamma\rho_h}e^{N^2\gamma^2\rho_l^2\sigma_\delta^2}\right)\left(\bar{\delta}-2\gamma\sigma_\delta^2(\rho_h+\rho_l)\right).$$

To find the optimal α^*, we start by considering $\frac{\partial\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}$:
First, $\left.\frac{\partial\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}\right|_{\alpha=\frac{1}{2}} = 0$.

Second, looking at the SOC $\frac{\partial^2\mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha^2}$ for $\alpha = \frac{1}{2}$ to be a maximum is given by: First we notice: for all $\alpha\in(\frac{1}{2},1)$ and $\pi\in(0,1)$:

$$32\pi^2\,(N-1)\,(\pi-1)^2\cdot(\rho_h-\rho_l)\,e^{-N\bar{\delta}\gamma\rho_h}e^{-N\bar{\delta}\gamma\rho_l}e^{-N\gamma^2\rho_h^2\sigma_\delta^2}e^{-N\gamma^2\rho_l^2\sigma_\delta^2}e^{N^2\bar{\delta}\gamma\rho_h}e^{N^2\bar{\delta}\gamma\rho_l}e^{N^2\gamma^2\rho_h^2\sigma_\delta^2}e^{N^2\gamma^2\rho_l^2\sigma_\delta^2} > 0.$$

Looking at the exponential term, it is determined by:

$$(\gamma\sigma_\delta^2(\rho_h+\rho_l)-\bar{\delta}).$$

As a result, the sign of the SOC is determined by:

$$\left(\bar{\delta} - \gamma\sigma_\delta^2(\rho_h + \rho_l)\right)\left(\bar{\delta} - 2\gamma\sigma_\delta^2(\rho_h + \rho_l)\right).$$

So the SOC max condition becomes: $\left(\bar{\delta} - \gamma\sigma_\delta^2(\rho_h + \rho_l)\right)\left(\bar{\delta} - 2\gamma\sigma_\delta^2(\rho_h + \rho_l)\right) < 0$.

Finally, breaking down the FOC:

First we notice: for all $\alpha \in (\frac{1}{2}, 1)$ and $\pi \in (0, 1)$:

$$\pi^2 (N-1)(2\alpha - 1)(\pi - 1)^2 (\rho_h - \rho_l)\left(\pi e^{N\gamma\left(N\bar{\delta}\rho_l + N\gamma\rho_h^2\sigma_\delta^2 + \bar{\delta}\rho_h + \gamma\rho_l^2\sigma_\delta^2\right)} + (1-\pi)e^{N\gamma\left(N\left(\bar{\delta}\rho_h + \gamma\rho_l^2\sigma_\delta^2\right) + \bar{\delta}\rho_l + \gamma\rho_h^2\sigma_\delta^2\right)}\right)$$
$$\times e^{N\gamma\left(N\bar{\delta}\rho_h + N\bar{\delta}\rho_l + N\gamma\rho_h^2\sigma_\delta^2 + N\gamma\rho_l^2\sigma_\delta^2 + \bar{\delta}\rho_h + \bar{\delta}\rho_l + \gamma\rho_h^2\sigma_\delta^2 + \gamma\rho_l^2\sigma_\delta^2\right)} > 0.$$

Looking at the exponential term, it is determined by:

$$(\gamma\sigma_\delta^2(\rho_h + \rho_l) - \bar{\delta}).$$

As a result, the sign of the FOC is determined by:

$$\left(\bar{\delta} - \gamma\sigma_\delta^2(\rho_h + \rho_l)\right)\left(\bar{\delta} - 2\gamma\sigma_\delta^2(\rho_h + \rho_l)\right), \text{ the SOC expression.}$$

This allows us to conclude that when $\alpha^* = \frac{1}{2}$ is a minimum, the derivative is increasing over $(\frac{1}{2}, 1)$ and so the maximum occurs at $\alpha^* = 1$.

Proof of Proposition 8

We need to find the intersection of the conditions stated in Proposition OA1 and OA2. The conditions for forming relationship with full disclosure are the same for both the existing firms and the joiner so the intersection is the same, i.e., $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_l + \rho_h$ or $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}, \frac{\rho_l + \rho_h}{2})$. The conditions for forming relationship with non-disclosure are different. We show now that the conditions for the existing firms are more stringent so the intersection of the conditions is the same as those for the existing firms.

We need to show that

$$\frac{\left[(N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] - (N-2)(\pi\rho_h^2 + (1-\pi)\rho_l^2)\right]}{\left[(N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] - (N-2)(\pi\rho_h + (1-\pi)\rho_l)\right]}$$
$$> \frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l) + \gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h) + \gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}. \quad (24)$$

With a slight abuse of notation, denote

$$\begin{aligned}
A &= \pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}, \\
B &= (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}, \\
C &= \pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}, \text{ and} \\
D &= (1-\pi) e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}.
\end{aligned}$$

Then inequality (24) can be written as

$$\frac{\frac{(N-1)A}{C+D} - (N-2)[\pi\rho_h^2 + (1-\pi)\rho_l^2)]}{\frac{(N-1)B}{C+D} - (N-2)[\pi\rho_h + (1-\pi)\rho_l]} > \frac{A}{B},$$

which is equivalent to

$$\frac{(N-1)A - (N-2)[\pi\rho_h^2 + (1-\pi)\rho_l^2)](C+D)}{(N-1)B - (N-2)[\pi\rho_h + (1-\pi)\rho_l](C+D)} > \frac{A}{B},$$

and can be further reduced to

$$\frac{(N-2)\{A - [\pi\rho_h^2 + (1-\pi)\rho_l^2)](C+D)\} + A}{(N-2)\{B - [\pi\rho_h + (1-\pi)\rho_l](C+D)\} + B} > \frac{A}{B}.$$

Rearranging terms result in the condition equivalent to

$$\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} < \frac{A}{B}. \tag{25}$$

Now denote

$$\begin{aligned}
E &= e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}, \text{ and} \\
F &= e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}.
\end{aligned}$$

Then inequality (25) can be written as

$$\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} < \frac{\pi\rho_h^2 E + (1-\pi)\rho_l^2 F}{\pi\rho_h E + (1-\pi)\rho_l F}.$$

Rearranging terms result in the condition equivalent to

$$\rho_h\rho_l(\rho_h - \rho_l)F < \rho_h\rho_l(\rho_h - \rho_l)E,$$

i.e.,

$$F < E.$$

Insert into the expressions of F and E and after rearranging terms result in

$$\bar{\delta}(\rho_h - \rho_l) < \gamma\sigma_\delta^2(\rho_h^2 - \rho_l^2),$$

which is equivalent to

$$\frac{\bar{\delta}}{\gamma\sigma_\delta^2} < \rho_h + \rho_l,$$

which is clearly satisfied. The proof is therefore complete.

Proof of Proposition 9

Relevant expression for FOC:

$$\pi^2 \cdot (2\alpha - 1)(\pi - 1)^2 (\rho_h - \rho_l)(2\alpha\pi - \alpha - \pi)^3 (2\alpha\pi - \alpha - \pi + 1)^2 \left(C\gamma + N\gamma\rho_h\sigma_\delta^2 + N\gamma\rho_l\sigma_\delta^2 + N\gamma\sigma_{\delta j} - \bar{\delta}\right)$$
$$\times \left(\pi e^{2CN\gamma^2\rho_h} e^{2N\bar{\delta}\gamma\rho_l} e^{\frac{3N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} - 2\pi e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l} e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} e^{N^2\gamma^2\rho_l^2\sigma_\delta^2} + \right.$$
$$\pi e^{2CN\gamma^2\rho_l} e^{2N\bar{\delta}\gamma\rho_h} e^{\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} + e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l} e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} e^{N^2\gamma^2\rho_l^2\sigma_\delta^2}$$
$$\left. - e^{2CN\gamma^2\rho_l} e^{2N\bar{\delta}\gamma\rho_h} e^{\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} \right)$$
$$\times e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l}.$$

Relevant expression for SOC:

$$\begin{aligned} & (e^{CN\gamma^2\rho_h} e^{N\bar{\delta}\gamma\rho_l} e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} e^{\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} - e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N^2\gamma^2\rho_l^2\sigma_\delta^2} e^{\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}}) \\ & \times(C\gamma + N\gamma\rho_h\sigma_\delta^2 + N\gamma\rho_l\sigma_\delta^2 + N\gamma\sigma_{\delta j} - \bar{\delta}) \\ < \ & 0. \end{aligned}$$

To find the optimal α^*, we start by considering $\frac{\partial \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}$:
First, $\left.\frac{\partial \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha}\right|_{\alpha=\frac{1}{2}} = 0.$
Second, looking at the SOC $\frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial\alpha^2}$ for $\alpha = \frac{1}{2}$ to be a maximum is given by:

$$\left(C\gamma + N\gamma\sigma_\delta^2(\rho_h + \rho_l) + N\gamma\sigma_{\delta j} - \bar{\delta}\right)\left(C\gamma + \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l) - \bar{\delta}\right) < 0.$$

Finally, breaking down the FOC:

First we notice: for all $\alpha \in (\frac{1}{2}, 1)$ and $\pi \in (0, 1)$:

$$\pi^2 \cdot (2\alpha - 1)(\pi - 1)^2 (\rho_h - \rho_l)(2\alpha\pi - \alpha - \pi + 1)^2 e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l} > 0$$
$$\text{and: } (2\alpha\pi - \alpha - \pi)^3 < 0.$$

Looking at the exponential term:

$$(\pi e^{2CN\gamma^2\rho_h} e^{2N\bar{\delta}\gamma\rho_l} e^{\frac{3N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} - 2\pi e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l} e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} e^{N^2\gamma^2\rho_l^2\sigma_\delta^2} +$$
$$\pi e^{2CN\gamma^2\rho_l} e^{2N\bar{\delta}\gamma\rho_h} e^{\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}} + e^{CN\gamma^2\rho_h} e^{CN\gamma^2\rho_l} e^{N\bar{\delta}\gamma\rho_h} e^{N\bar{\delta}\gamma\rho_l} e^{N^2\gamma^2\rho_h^2\sigma_\delta^2} e^{N^2\gamma^2\rho_l^2\sigma_\delta^2}$$
$$- e^{2CN\gamma^2\rho_l} e^{2N\bar{\delta}\gamma\rho_h} e^{\frac{N^2\gamma^2\rho_h^2\sigma_\delta^2}{2}} e^{\frac{3N^2\gamma^2\rho_l^2\sigma_\delta^2}{2}}) \text{ is determined by : } \left(\bar{\delta} - C\gamma + \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l)\right).$$

As a result, the sign of the FOC is determined by:

$$- \left(C\gamma + N\gamma\rho_h\sigma_\delta^2 + N\gamma\rho_l\sigma_\delta^2 + N\gamma\sigma_{\delta j} - \bar{\delta}\right) \times \left(\bar{\delta} - C\gamma + \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l)\right)$$
$$\iff \left(C\gamma + N\gamma\sigma_\delta^2(\rho_h + \rho_l) + N\gamma\sigma_{\delta j} - \bar{\delta}\right)\left(C\gamma + \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l) - \bar{\delta}\right) \text{, the SOC expression.}$$

This allows us to conclude that when $\alpha^* = \frac{1}{2}$ is a minimum, the derivative is increasing over $(\frac{1}{2}, 1)$ and so the maximum occurs at $\alpha^* = 1$, resulting in conditions stated in the proposition.

Proof of Proposition 10

Suppose a firm knows the realization of $\tilde{\rho}_s = \rho_h$.

Choosing to disclose this to investors would result in:

$$RP_i(\tilde{\rho}_s = \rho_h) = \bar{V} - \gamma\sigma_V^2 + \rho_h\bar{\delta} - \gamma N\rho_h^2\sigma_\delta^2.$$

Not disclosing yields:

$$RP_i^{ND} = \bar{V} - \gamma\sigma_V^2 + \bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}\right]$$
$$- \gamma N\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}\right]$$

The firm wants to disclose if and only if $RP_i(\tilde{\rho}_s = \rho_h) > RP_i^{ND}$.

The relevant expression for wanting to disclose ρ_h is:

$$
\begin{aligned}
&\rho_h\bar{\delta} - \gamma N\rho_h^2\sigma_\delta^2 > \bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\
&- \gamma N\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\
\iff &(\rho_h\bar{\delta} - \gamma N\rho_h^2\sigma_\delta^2)(\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&> \bar{\delta}(\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&- \gamma N\sigma_\delta^2(\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
\iff &(\rho_h\bar{\delta})(\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&- \gamma N\rho_h^2\sigma_\delta^2(\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&> \bar{\delta}(\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&- \gamma N\sigma_\delta^2(\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
\iff &(\rho_h\bar{\delta})(\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&- \bar{\delta}(\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&> \gamma N\rho_h^2\sigma_\delta^2(\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&- \gamma N\sigma_\delta^2(\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
\iff &(\bar{\delta})(\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_h e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)} \\
&- \pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} - (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&> \gamma N\sigma_\delta^2(\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_h^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)} \\
&- \pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} - (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
\iff &\bar{\delta}((1-\pi)\rho_h e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)} - (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
&> \gamma N\sigma_\delta^2((1-\pi)\rho_h^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)} - (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}) \\
\iff &\bar{\delta}(\rho_h - \rho_l) > \gamma N\sigma_\delta^2(\rho_h^2 - \rho_l^2) \\
\iff &\bar{\delta} > \gamma N\sigma_\delta^2(\rho_h + \rho_l).
\end{aligned}
$$

As a result, the firm wants to disclose ρ_h if and only if $\bar{\delta} > \gamma N\sigma_\delta^2(\rho_h + \rho_l)$. We can similarly show that the firm wants to disclose ρ_l iff. $\bar{\delta} < \gamma N\sigma_\delta^2(\rho_h+\rho_l)$. Therefore, when $\bar{\delta} > \gamma N\sigma_\delta^2(\rho_h+\rho_l)$, firms observing $\widetilde{\rho} = \rho_h$ will choose to disclose and firms who do not disclose will be inferred as having observed $\widetilde{\rho} = \rho_l$. When when $\bar{\delta} < \gamma N\sigma_\delta^2(\rho_h + \rho_l)$, firms observing $\widetilde{\rho} = \rho_l$ will choose to disclose and firms who do not disclose will be inferred as having observed $\widetilde{\rho} = \rho_h$. We therefore always have full disclosure.

Proof of Proposition 11

Suppose a firm knows $\tilde{\rho_s} = \rho_h$, and $\bar{\delta} > \gamma N \sigma_\delta^2(\rho_h + \rho_l)$, then the following provide sufficient conditions for an interior maximum: $P''(\alpha = \frac{1}{2}) > 0$, and $P(1) \leq P(\frac{1}{2})$.

Note that $P(\alpha)$ is a C^2 function on $[\frac{1}{2}, 1]$. So the Extreme Value Theorem guarantees the existence of a global maximum. $P'(\alpha = \frac{1}{2})$ is 0 and we are assuming $P''(\alpha = \frac{1}{2}) > 0$. By the SOC characterization of minima for C^2 functions, we can conclude $P(\alpha = \frac{1}{2})$ is a strict local minimum. If $\alpha = \frac{1}{2}$ is a strict local min and $P(1) \leq P(\frac{1}{2})$, then we can rule out the maximum being at a boundary point. The max is therefore an interior solution. The proof when $\tilde{\rho_s} = \rho_l$, and $\bar{\delta} < \gamma N \sigma_\delta^2(\rho_h + \rho_l)$ can be similarly proved.

Proof of Corollary 4

Lower bound on K to ensure $\alpha = 1$ is not a maximum.

$$
\begin{aligned}
& Price(1) \leq Price(\frac{1}{2}) \\
\iff & (1-K)R\mathbb{E}[P_j(\alpha = 1; \tilde{m})] \leq R\mathbb{E}[P_j(\alpha = \frac{1}{2}; \tilde{m})] \\
\iff & \mathbb{E}[P_j(\alpha = 1; \tilde{m})] - K\mathbb{E}[P_j(\alpha = 1; \tilde{m})] \leq \mathbb{E}[P_j(\alpha = \frac{1}{2}; \tilde{m})] \\
\iff & 1 - K \leq \frac{\mathbb{E}[P_j(\alpha = \frac{1}{2}; \tilde{m})]}{\mathbb{E}[P_j(\alpha = 1; \tilde{m})]} \\
\iff & 1 - \frac{\mathbb{E}[P_j(\alpha = \frac{1}{2}; \tilde{m})]}{\mathbb{E}[P_j(\alpha = 1; \tilde{m})]} \leq K.
\end{aligned}
$$

Note: for $K \geq 1$, this is always true when we assume the firm wants to disclosure ρ_h.

Upper bound on K to ensure $\alpha = \frac{1}{2}$ is not a maximum.

Sign of SOC determined by: $(1 - K(2\alpha-1)^n)\frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2} - 4nK(2\alpha-1)^{n-1}\frac{\partial \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha} - 4n(n-1)K(2\alpha-1)^{n-2}\mathbb{E}[P_j(\alpha;\tilde{m})]$.

For $n \geq 3$, then $Price''(\alpha = \frac{1}{2}) = \frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2} > 0$ by our assumption that firms want to disclose ρ_h.

For $n = 2$, we can re-write the relevant SOC expression as:

$$\begin{aligned}
& \frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2} - 4n(n-1)K(2\alpha-1)^{n-2}\mathbb{E}[P_j(\alpha;\tilde{m})] > 0 \\
\iff & \frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2} - 8K\mathbb{E}[P_j(\alpha;\tilde{m})] > 0 \\
\iff & \frac{\frac{\partial^2 \mathbb{E}[P_j(\alpha;\tilde{m})]}{\partial \alpha^2}}{8\mathbb{E}[P_j(\alpha;\tilde{m})]} > K.
\end{aligned}$$

Assessing at $\alpha = \frac{1}{2}$ gives us the required condition.

Proof of Proposition 12

We first prove the first part. When firms choose not to voluntarily disclose, it must be because the price upon disclosure is lower. Therefore, if we can show that investors' expected utility increases in prices, then mandatory disclosure will increase investors' expected utility.

Since $\widetilde{W} = W_0 R +_{i=1}^{N} q_i(\widetilde{F}_i - RP_i)$, the derivative of the investor's expected utility with respect to P_i is

$$\begin{aligned}
& \frac{\partial E[-e^{-\gamma \widetilde{W}}]}{\partial P_i} \\
= & -\frac{\partial e^{-\gamma E[\widetilde{W}] + \frac{1}{2}\gamma^2 \sigma^2_{\widetilde{W}}}}{\partial P_i} \\
= & -\frac{\partial e^{-\gamma E[\widetilde{W}]}}{\partial P_i} \\
\propto & \frac{\partial E[\widetilde{W}]}{\partial P_i} < 0.
\end{aligned}$$

We now prove the second part. Relationship destruction harms welfare if and only if $EU_{\frac{1}{2}} - EU_N > 0$, where $EU_{\frac{1}{2}}$ refers to the investor's expected utility when there is no disclosure and relationship formation and EU_N refers to the investor's expected utility when there is no relationship formation, or equivalently:

$$\begin{aligned}
& \pi \exp(F_1(RF_2 + V + F_3) + \delta\rho_h \\
& -(1-\pi)\exp(F_4(RF_5 + V + F_6) + \delta\rho_l) \\
& + e^{NR\gamma\left(V - \gamma\sigma_V^2\right)} \\
> & \ 0,
\end{aligned}$$

where

$$
\begin{aligned}
F_1 &= \frac{N\gamma^2\rho_h^2\sigma_{\delta sq}}{2} - N\gamma, \\
F_2 &= \frac{N\gamma\sigma_{\delta sq}\left(\pi\rho_h^2 e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + \rho_l^2\cdot(1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}\right)}{\pi e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + (1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}}, \\
F_3 &= \frac{\delta\left(\pi\rho_h e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + \rho_l\,(1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}\right)}{\pi e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + (1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}} - \gamma\sigma_V^2, \\
F_4 &= \frac{N\gamma^2\rho_l^2\sigma_{\delta sq}}{2}, \\
F_5 &= \frac{N\gamma\sigma_{\delta sq}\left(\pi\rho_h^2 e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + \rho_l^2\cdot(1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}\right)}{\pi e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + (1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}}, \\
F_6 &= \frac{\delta\left(\pi\rho_h e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + \rho_l\,(1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}\right)}{\pi e^{\frac{N^2\gamma^2\rho_h^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_l} + (1-\pi)\,e^{\frac{N^2\gamma^2\rho_l^2\sigma_{\delta sq}}{2}+N\delta\gamma\rho_h}} - \gamma\sigma_V^2.
\end{aligned}
$$

This difference can be simplified, using Python codes, to a sum that will be positive if the following 3 terms are positive:

$$
\begin{aligned}
&(1) \quad \left(2\bar{\delta}\rho_h - 2\bar{\delta}\rho_l - \gamma\rho_h^2\sigma_\delta^2 + \gamma\rho_l^2\sigma_\delta^2\right) > 0; \\
&(2) \quad (-2N\pi + 2N + \pi - 1) > 0; \\
&(3) \quad \left(2N\gamma\rho_h^2\sigma_\delta^2 - 2\bar{\delta}\rho_h + 2\bar{\delta}\rho_l - \gamma\rho_l^2\sigma_\delta^2\right) > 0.
\end{aligned}
$$

The first term:

$$
\begin{aligned}
&\left(2\bar{\delta}\rho_h - 2\bar{\delta}\rho_l - \gamma\rho_h^2\sigma_\delta^2 + \gamma\rho_l^2\sigma_\delta^2\right) > 0 \\
\iff & 2\bar{\delta}\left(\rho_h - \rho_l\right) > \gamma\sigma_\delta^2\left(\rho_h^2 - \rho_l^2\right) \\
\iff & \bar{\delta} > \frac{\gamma}{2}\sigma_\delta^2(\rho_h + \rho_l).
\end{aligned}
$$

and notice $\frac{\gamma}{2}\sigma_\delta^2(\rho_h + \rho_l) < \frac{N}{2}\gamma\sigma_\delta^2(\rho_h + \rho_l) < \bar{\delta}$ by non-disclosure optimality.

The second term:

$$-2N\pi + 2N + \pi - 1 > 0,$$

is equivalent to

$$(2N - 1)(1 - \pi) > 0.$$

which holds for $\pi < 1$.

The final term:

$$-2N\pi + 2N + \pi - 1 > 0,$$

is equivalent to

$$(2N - 1)(1 - \pi) > 0.$$

$$2N\gamma\rho_h^2\sigma_\delta^2 - 2\overline{\delta}\rho_h + 2\overline{\delta}\rho_l - \gamma\rho_l^2\sigma_\delta^2 > 0,$$

is equivalent to

$$\gamma\sigma_\delta^2(2N\rho_h^2 - \rho_l^2) > 2\overline{\delta}(\rho_h - \rho_l).$$

Note that $\gamma\sigma_\delta^2(2N\rho_h^2 - \rho_l^2) > 2\overline{\delta}(\rho_h - \rho_l)$ as it is exactly equivalent to the condition for non-disclosure to be optimal,

$$\frac{\overline{\delta}}{N\gamma\sigma_\delta^2} < \rho_h + \rho_l.$$

Online Appendix

OA1 Technical details of N-firm extensions

OA1.1 All N firms moving together

We assume that there are N firms, indexed by $i \in \{1, 2, ..., N\}$. Each firm's cash flow is characterized by $\tilde{F}_i = \tilde{V}_i + \tilde{\Delta}$, where $V_i \sim N(\overline{V}, \sigma_V^2)$ is the firm-specific cash flow component, and $\widetilde{\Delta} = \widetilde{\rho}\widetilde{\delta}$ is the common component if firm i chooses to form a relationship. As in the two-firm case, $\tilde{\delta} \sim N(\bar{\delta}, \sigma_\delta^2)$ and $\tilde{\rho} = \rho_h$ with probability π and ρ_l with probability $(1 - \pi)$. We again assume that $\tilde{V}_1, \ldots, \tilde{V}_N, \tilde{\delta}$ are jointly normal and independent of $\tilde{\rho}$.

We then have

$$
\begin{aligned}
EU &= -\pi(e^{-\gamma\mu_W(h)+\frac{\gamma^2}{2}\sigma_W^2(h)}) - (1-\pi)(e^{-\gamma\mu_W(l)+\frac{\gamma^2}{2}\sigma_W^2(l)}),\\
\mu_W(s) &= E[\tilde{W}|\tilde{\rho} = \rho_s] = W_0 R + \rho\bar{\delta}N + \sum_{i=1}^{n} q_i(\bar{V} - RP_i),\\
\sigma_W^2(s) &= V[\tilde{W}|\tilde{\rho} = \rho_s] = \tilde{\rho}^2\sigma_\delta^2 N^2 + \sigma_V^2(\sum_{i=1}^{n} q_i^2).
\end{aligned}
$$

This results in

$$
\begin{aligned}
\frac{\partial\mu_W(s)}{\partial q_i} &= \bar{V} + \rho_s\bar{\delta} - RP_i,\\
\frac{\partial\sigma_W^2(s)}{\partial q_i} &= 2\sigma_V^2 q_i + 2N\rho_s^2\sigma_\delta^2.
\end{aligned}
$$

Divide both sides of the partial by constant: $e^{-\gamma(W_0R+\sum_{i=1}^{n} q_i(\bar{V}-RP_i))+\frac{\gamma^2}{2}\sigma_V^2\sum_{i=1}^{n} q_i^2}$ results in the first order condition being

$$
\begin{aligned}
\frac{\partial EU}{\partial q_i} =& -\pi[e^{-\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)}(-\gamma(\bar{V} + \rho_h\bar{\delta} - RP_i) + \frac{\gamma^2}{2}(2\sigma_V^2 q_i + 2N\rho_h^2\sigma_\delta^2)]\\
&- (1-\pi)[e^{-\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}(-\gamma(\bar{V} + \rho_l\bar{\delta} - RP_i) + \frac{\gamma^2}{2}(2\sigma_V^2 q_i + 2N\rho_l^2\sigma_\delta^2)]\\
=& 0.
\end{aligned}
$$

Plugging in $q_i = 1$ and re-arranging terms results in the price being

$$
\begin{aligned}
RP_i = \bar{V} - \gamma\sigma_V^2 + \bar{\delta}\left[\frac{\pi\rho_h e^{-\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{-\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{-\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{-\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\
- \gamma N\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{-\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{-\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{-\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{-\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\
= \bar{V} - \gamma\sigma_V^2 + \bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right] \\
- \gamma N\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N\rho_l)+\frac{\gamma^2}{2}(\rho_h^2N^2\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N\rho_h)+\frac{\gamma^2}{2}(\rho_l^2N^2\sigma_\delta^2)}}\right].
\end{aligned}
$$

OA1.2 N-1 firms already forming relationship

We now consider the second approach, under which N-1 firms already form relationship with each other and the Nth firm is considering whether to form relationship with the N-1 firms. For the N-1 existing firms, we specify that $\tilde{F}_i = \tilde{V} + \sum_{j\in -I}\tilde{\rho}\tilde{\delta}_{ij}$, where $-I$ denotes $\{1,\ldots,N\}\setminus\{i\}$, $\tilde{V}_i \sim (\bar{V}, \sigma_V^2)$, $i = 1,\ldots,N$, and $\tilde{\delta}_{ij} \sim (\bar{\delta}, \sigma_\delta^2)$, $1 \le i < j \le N$.

We therefore have
$\mathbb{E}[\tilde{F}_i|\tilde{\rho} = \rho_s] = \bar{V} + \rho_s(N-1)\bar{\delta}$,
$\mathbb{V}[\tilde{F}_i|\tilde{\rho} = \rho_s] = \sigma_\delta^2 + \rho_s^2(N-1)\sigma_\delta^2$.

$$
\mathbb{E}[\tilde{W}|\tilde{\rho} = \rho_s] = W_0R + \sum_{i=1}^{N} q_i(\bar{V} - RP_i) + \rho_s(N-1)Q\bar{\delta},
$$

and

$$
\mathbb{V}[\tilde{W}|\tilde{\rho} = \rho_s] = (\sum_{i=1}^{N} q_i^2)\sigma_V^2 + \rho_s^2Q^2\sigma_\delta^2 + \rho_s^2(N-2)(\sum_{i=1}^{N} q_i^2)\sigma_\delta^2,
$$

where

$$
Q = \sum_{i=1}^{N} q_i.
$$

Taking FOC with respect to q_i and set $q_i = 1$ results in the price being

$$
\begin{aligned}
RP_i = \bar{V} - \gamma\sigma_V^2 + (N-1)\bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2N(N-1)\sigma_\delta^2)}}\right] \\
- 2\gamma(N-1)\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2N(N-1)\sigma_\delta^2)}}\right].
\end{aligned}
$$

OA1.2.1 When would the relationship be formed

The conditions regarding when the relationship would be formed differ depending on whether we are looking at the new joiner or the existing firms, as the two types of firms are asymmetric before relationship formation. The conditions when the relationship would be formed would thus be the intersection of the two conditions. We summarize the main conclusions and relegate all the technical details to the appendix.

The new joiner The results regarding when the new joiner would form relationship are summarized in the next Proposition.

Proposition OA1. *There exists a Pareto-dominant relationship equilibrium in which the new joiner will form a relationship with the existing firms with full disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{2\gamma\sigma_{\delta}^2} > \rho_l + \rho_h$ or $\frac{\bar{\delta}}{2\gamma\sigma_{\delta}^2} \in (\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_l+\rho_h}{2})$, with the latter possible if and only if $\pi\rho_h < (1-\pi)\rho_l$. There exists a Pareto-dominant relationship equilibrium in which the new joiner will form a relationship with the existing firms with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if $\frac{\bar{\delta}}{2\gamma\sigma_{\delta}^2} \in (\max(\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}}{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}}, \frac{\rho_l+\rho_h}{2}), \rho_l+\rho_h)$, with $\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}}{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}} < \frac{\rho_l+\rho_h}{2}$ if and only if $\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)} < (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}$.*

Note that when the new joiner discloses the relationship, the conditions for relationship formation are identical to that in Proposition 3, as with relationship disclosure the problem of whether to form a relationship in the first place is essentially a bilateral problem, as in the discussion following Proposition 7. When the joiner chooses not to disclose the relationship, however, the conditions for relationship formation is a generalization of the conditions in Proposition 7 with the latter being the former with $N = 2$. Intuitively, while the problem is essentially a bilateral problem, in the case of no disclosure, the other part consists of N-1 firms instead of one firm in the main setting.

Overall, the conditions for the joiner to disclose relationship and to form relationship in the N-firm setting is qualitatively the same as that in the main setting. In particular, it is straightforward from Proposition OA1 that mandatory relationship disclosure will destroy such relationship formation in the first place for the new joiner when the expected benefit of the relationship relative to the diversification cost is in the intermediate region, i.e., when

$$\max(\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}}{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_{\delta}^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_{\delta}^2)}}, \frac{\rho_h+\rho_l}{2}) < \frac{\bar{\delta}}{2\gamma\sigma_{\delta}^2} < \rho_h + \rho_l.$$

Proof of Proposition OA1

For the new joiner firm, without collaboration:

$$RP_j^N = \bar{V} - \gamma\sigma_V^2.$$

With collaboration and $\alpha^* = 1$:

$$R\mathbb{E}P_j(\alpha^* = 1; \tilde{m}) = \bar{V} - \gamma\sigma_V^2 + (N-1)\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) - 2\gamma(N-1)\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2).$$

With collaboration and $\alpha^* = \frac{1}{2}$:

$$\begin{aligned}
&(N-1)\bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] \\
&> 2\gamma(N-1)\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] \\
\iff &\bar{\delta}\left(\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}\right) \\
&> 2\gamma\sigma_\delta^2\left(\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}\right) \\
&\iff \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\rho_h F_1 + \rho_l F_2}{F_1 + F_2}.,
\end{aligned}$$

When $\alpha^* = 1$, we get $R\mathbb{E}P_j(\alpha^* = 1; \tilde{m}) > RP_j^N$ iff:

$$\begin{aligned}
&(N-1)\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) > 2\gamma(N-1)\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2) \\
\iff &\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) > 2\gamma\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2) \\
&\iff \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}.
\end{aligned}$$

When $\alpha^* = \frac{1}{2}$, we get $R\mathbb{E}P_j(\alpha^* = \frac{1}{2}; \tilde{m}) > RP_j^N$ iff:

$$
\begin{aligned}
&(N-1)\bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] \\
&> 2\gamma(N-1)\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] \\
\iff &\bar{\delta}\left(\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}\right) \\
&> 2\gamma\sigma_\delta^2\left(\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}\right) \\
&\iff \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\rho_h F_1+\rho_l F_2}{F_1+F_2},
\end{aligned}
$$

where

$$F_1 \equiv \pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)},$$

and

$$F_2 \equiv (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}.$$

From Proposition 7 we know that the new joiner will disclose relationship if and only if

$$\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h+\rho_l.$$

Note that

$$\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l} \leq \rho_h+\rho_l,$$

as it is equivalent to

$$\rho_h\rho_l \geq 0.$$

Therefore, the new joiner will establish and disclose relationship with the new joiner if and only if

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_h+\rho_l,$$

or if

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in \left(\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l}, \frac{\rho_h+\rho_l}{2}\right),$$

which requires

$$\frac{\pi\rho_h^2+(1-\pi)\rho_l^2}{\pi\rho_h+(1-\pi)\rho_l} < \frac{\rho_h+\rho_l}{2}.$$

The above inequality holds if and only if

$$\pi\rho_h^2 + (1-\pi)\rho_l^2 - \rho_h\rho_l < 0,$$

which is equivalent to

$$\pi\rho_h < (1-\pi)\rho_l.$$

When $\alpha^* = \frac{1}{2}$, which is equivalent to

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\rho_h F_1 + \rho_l F_2}{F_1 + F_2},$$

From Proposition 7 we know that an existing firm will disclose relationship if and only if

$$\frac{\rho_h + \rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l.$$

Note that

$$\frac{\rho_h F_1 + \rho_l F_2}{F_1 + F_2} \leq \rho_h + \rho_l,$$

as it is equivalent to

$$\rho_h F_2 + \rho_l F_1 \geq 0.$$

Therefore, an existing firm will establish and disclose relationship with the new joiner if and only if

$$\max\left(\frac{F_1 + F_2}{\rho_h F_1 + \rho_l F_2}, \frac{\rho_h + \rho_l}{2}\right) < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l.$$

Note that

$$\frac{\rho_h F_1 + \rho_l F_2}{F_1 + F_2} < \frac{\rho_h + \rho_l}{2}$$

is equivalent to

$$(\rho_h - \rho_l)(F_1 - F_2) < 0,$$

which is equivalent to

$$F_1 < F_2.$$

Existing Firms The results regarding when the existing firms would form relationship with the new joiner are summarized in the next Proposition.

Proposition OA2. *There exists a Pareto-dominant relationship equilibrium in which the existing firms form relationships with the new joiner with full-disclosure ($\alpha^* = 1$) if and only if $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_l + \rho_h$ or $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in \left(\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}, \frac{\rho_l + \rho_h}{2}\right)$, with the latter possible if and only if $\pi\rho_h < (1-\pi)\rho_l$. There exists* *a Pareto-dominant relationship equilibrium in which the existing firms form relationships with the new joiner with non-disclosure ($\alpha^* = \frac{1}{2}$) if and only if* $\frac{\bar{\bar{\delta}}}{2\gamma\sigma_\delta^2} \in$

$$\left(\max\left(\frac{[(N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)]}{[(N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]}, \frac{\rho_l+\rho_h}{2}\right), \rho_l+\rho_h\right).$$

A necessary condition for

$$\frac{[(N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)]}{[(N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)}+(1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]} < \frac{\rho_l+\rho_h}{2}$$

is that $\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} < (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}$.

Note that the conditions for relationship formation when the existing firms choose to disclose is identical to that in Proposition 3. The reason is that with full disclosure, the problem essentially reduces to a bilateral relationship. The conditions for relationship formation when the existing firms choose to not disclose is more complicated and can be viewed as a generalization of the conditions in Proposition 3. The reason is that in a two-firm setting, not disclosing relationship means that each firm has independent cash flows. In a N-firm setting, the existing firms are still intertwined with each other, resulting in the incremental benefit from forming another relationship being different. Nevertheless, the results are qualitatively the same: when the expected benefit from additional relationship relative to the diversification cost is large or small, forming a relationship and disclosing is beneficial; when the expected benefit relative to the cost is in the intermediate region, forming a relationship and not disclosing is beneficial.

The proof of Proposition 8 shows that the conditions stated in the Proposition are the intersection of Propositions OA1 and OA2.

Proof of Proposition OA2

Without new joiner and $\alpha^* = 1$:

$$R\mathbb{E}P_j(\alpha^* = \frac{1}{2}; \tilde{m}) = \bar{V} - \gamma\sigma_V^2 + \bar{\delta}\,(N-2)\,(\pi\rho_h + (1-\pi)\rho_l) - 2\gamma\sigma_\delta^2\,(N-2)\,(\pi\rho_h^2 + (1-\pi)\rho_l^2).$$

Without new joiner and $\alpha^* = \frac{1}{2}$:

$$
\begin{aligned}
R\mathbb{E}P_j(\alpha^* = \frac{1}{2}; \tilde{m}) &= \bar{V} - \gamma\sigma_\delta^2 \\
&+ \bar{\delta}\left(N-2\right) \frac{\left(\pi\rho_h e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \rho_l\left(1-\pi\right)e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}\right)}{\pi e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \left(1-\pi\right)e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}} \\
&- 2\gamma\sigma_\delta^2\left(N-2\right) \frac{\left(\pi\rho_h^2 e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \rho_l^2\cdot\left(1-\pi\right)e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}\right)}{\pi e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \left(1-\pi\right)e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}}.
\end{aligned}
$$

With collaboration and $\alpha^* = 1$:

$$
R\mathbb{E}P_j(\alpha^* = 1; \tilde{m}) = \bar{V} - \gamma\sigma_V^2 + (N-1)\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) - 2\gamma(N-1)\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2).
$$

With collaboration and $\alpha^* = \frac{1}{2}$:

$$
\begin{aligned}
R\mathbb{E}P_j(\alpha^* = \frac{1}{2}; \tilde{m}) = \bar{V} - \gamma\sigma_V^2 \\
&+ (N-1)\bar{\delta}\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right] \\
&- 2\gamma(N-1)\sigma_\delta^2\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]
\end{aligned}
$$

Therefore, when $\alpha^* = 1$, we get $R\mathbb{E}P_j^{collab}(\alpha^* = 1; \tilde{m}) > R\mathbb{E}P_j^{nocollab}(\alpha^* = 1; \tilde{m})$ iff:

$$
\bar{\delta}(\pi\rho_h + (1-\pi)\rho_l) > 2\gamma\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2),
$$

which is equivalent to

$$
\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}.
$$

From Proposition 7 we know that an existing firm will disclose relationship if and only if

$$
\frac{\rho_h + \rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h + \rho_l.
$$

Note that

$$
\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} \leq \rho_h + \rho_l,
$$

as it is equivalent to

$$
\rho_h\rho_l \geq 0.
$$

Therefore, an existing firm will establish and disclose relationship with the new joiner if and only if

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \rho_h + \rho_l,$$

or if

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} \in (\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l}, \frac{\rho_h + \rho_l}{2}),$$

which requires

$$\frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} < \frac{\rho_h + \rho_l}{2}.$$

The above inequality holds if and only if

$$\pi\rho_h^2 + (1-\pi)\rho_l^2 - \rho_h\rho_l < 0,$$

which is equivalent to

$$\pi\rho_h < (1-\pi)\rho_l.$$

When $\alpha^* = \frac{1}{2}$, we get $R\mathbb{E}P_j^{collab}(\alpha^* = \frac{1}{2}; \tilde{m}) > R\mathbb{E}P_j^{nocollab}(\alpha^* = \frac{1}{2}; \tilde{m})$ iff:

$$\bar{\delta}\Bigg((N-1)\left[\frac{\pi\rho_h e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]$$
$$-(N-2)\left[\frac{\left(\pi\rho_h e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \rho_l\,(1-\pi)\,e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}\right)}{\pi e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + (1-\pi)\,e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}}\right]\Bigg)$$
$$> 2\gamma\sigma_\delta^2\Bigg((N-1)\left[\frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}{\pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)} + (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}}\right]$$
$$-(N-2)\left[\frac{\left(\pi\rho_h^2 e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + \rho_l^2\cdot(1-\pi)\,e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}\right)}{\pi e^{\bar{\delta}\gamma\rho_l(N-2)(N-1)+\gamma^2\rho_h^2\sigma_\delta^2(N-2)(N-1)} + (1-\pi)\,e^{\bar{\delta}\gamma\rho_h(N-2)(N-1)+\gamma^2\rho_l^2\sigma_\delta^2(N-2)(N-1)}}\right]\Bigg),$$

which is equivalent to

$$\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} > \frac{(N-1)\left[\frac{\rho_h^2 A + \rho_l^2 B}{A+B}\right] - (N-2)(\pi\rho_h^2 + (1-\pi)\rho_l^2)}{(N-1)\left[\frac{\rho_h A + \rho_l B}{A+B}\right] - (N-2)(\pi\rho_h + (1-\pi)\rho_l)]},$$

where

$$A \equiv \pi e^{\gamma(\bar{\delta}N(N-1)\rho_l)+\gamma^2(\rho_h^2 N(N-1)\sigma_\delta^2)},$$

and

$$B \equiv (1-\pi)e^{\gamma(\bar{\delta}N(N-1)\rho_h)+\gamma^2(\rho_l^2 N(N-1)\sigma_\delta^2)}.$$

From Proposition 7 we know that an existing firm will disclose relationship if and only if

$$\frac{\rho_h+\rho_l}{2} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h+\rho_l.$$

Note that

$$\frac{(N-1)\left[\frac{\rho_h^2 A+\rho_l^2 B}{A+B}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(N-1)\left[\frac{\rho_h A+\rho_l B}{A+B}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]} \leq \rho_h+\rho_l,$$

as it is equivalent to

$$(N-2)\rho_h\rho_l \leq (N-1)\rho_h\rho_l.$$

Therefore, an existing firm will establish and disclose relationship with the new joiner if and only if

$$\begin{aligned} & \max(\frac{(N-1)\left[\frac{\rho_h^2 A+\rho_l^2 B}{A+B}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(N-1)\left[\frac{\rho_h A+\rho_l B}{A+B}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]}, \frac{\rho_h+\rho_l}{2}) \\ < \quad & \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h+\rho_l. \end{aligned}$$

To see the necessary condition for

$$\frac{(N-1)\left[\frac{\rho_h^2 A+\rho_l^2 B}{A+B}\right]-(N-2)(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(N-1)\left[\frac{\rho_h A+\rho_l B}{A+B}\right]-(N-2)(\pi\rho_h+(1-\pi)\rho_l)]} < \frac{\rho_h+\rho_l}{2}, \tag{OA1}$$

first note that when $\frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \rho_h+\rho_l$, $\frac{A}{B} > \frac{\pi}{1-\pi}$ so $\frac{\rho_h A+\rho_l B}{A+B} > \pi\rho_h+(1-\pi)\rho_l$. Therefore the denominator of inequality (OA1) is positive, as

$$\begin{aligned} & (N-1)\frac{\rho_h A+\rho_l B}{A+B} \\ > \quad & (N-1)\pi\rho_h+(1-\pi)\rho_l \\ > \quad & (N-2)(\pi\rho_h+(1-\pi)\rho_l). \end{aligned}$$

Therefore inequality (OA1) is equivalent to

$$(N-2)[\pi\rho_h-(1-\pi)\rho_l] > (N-1)(\rho_h\frac{A}{A+B}-\rho_l\frac{B}{A+B}). \tag{OA2}$$

When $\rho_h A \geq \rho_l B$, then the right hand side of inequality (OA2) is non-negative. Since $\frac{A}{B} > \frac{\pi}{1-\pi}$, this implies $\pi\rho_h - (1-\pi)\rho_l < \rho_h \frac{A}{A+B} - \rho_l \frac{B}{A+B}$. In addition, we have $N-1 > N-2$ so inequality (OA2) cannot be satisfied. Therefore a necessary condition is $\rho_h A < \rho_l B$, which translates to the expression in the proposition.

OA2 Technical details of introducing correlation between firm-specific cash flow and cash flow from relationship formation

Denote Cov($\tilde{V}_i, \tilde{\delta}$) = $\sigma_{\delta i}$, and let C = $\sum_{i=1}^{N} \sigma_{\delta i}$.

We still assume that $\tilde{V}_1, \ldots, \tilde{V}_N, \tilde{\delta}$ are jointly normal, then for $s \in \{h, l\}$,

$$EU = -\pi(e^{-\gamma\mu_W(h)+\frac{\gamma^2}{2}\sigma_W^2(h)}) - (1-\pi)(e^{-\gamma\mu_W(l)+\frac{\gamma^2}{2}\sigma_W^2(l)}),$$

where

$$\mu_W(s) = E[\tilde{W}|\tilde{\rho} = \rho_s] = W_0 R + \rho_s \bar{\delta} N + \sum_{i=1}^{N} q_i(\bar{V} - RP_i),$$

$$\sigma_W^2(s) = V[\tilde{W}|\tilde{\rho} = \rho_s] = \rho_s^2 \sigma_\delta^2 N^2 + \sigma_V^2(\sum_{i=1}^{N} q_i^2) + 2N\rho_s \sum_{i=1}^{N} q_i \sigma_{\delta i}.$$

Take the partial derivative of EU with respect to q_i, then setting $q_i = 1$ results in the price for firm i being

$$P_i = \frac{1}{R}\{\bar{V} - \gamma\sigma_V^2 + \left(\bar{\delta} - C\gamma - N\gamma\sigma_{\delta i}\right) \frac{\left(\pi\rho_h e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l\left(1-\pi\right) e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\pi e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \left(1-\pi\right) e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}}$$
$$- N\gamma\sigma_\delta^2 \frac{\left(\pi\rho_h^2 e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l^2\cdot\left(1-\pi\right) e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\pi e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \left(1-\pi\right) e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}}\}.$$

OA2.1 Relationship disclosure

After sending message $\tilde{m}$, price of firm i with the posterior probability π^m can be similarly expressed as

$$P_i(\alpha;\tilde{m}) = \frac{1}{R}\{\bar{V} - \gamma\sigma_V^2 +$$

$$\left(\bar{\delta} - C\gamma - N\gamma\sigma_{\delta i}\right) \frac{\left(\pi^m \rho_h e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l\left(1-\pi^m\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\pi^m e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \left(1-\pi^m\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}}$$

$$- N\gamma\sigma_\delta^2 \frac{\left(\pi^m \rho_h^2 e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l^2\cdot\left(1-\pi^m\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\pi^m e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \left(1-\pi^m\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}}\}.$$

Each firm j would thus choose disclosure policy α to maximize the expected price

$$\mathbb{E}[P_i(\alpha;\tilde{m})] = P_i(\alpha;\tilde{m}=h)Pr(\tilde{m}=h) + P_i(\alpha;\tilde{m}=l)Pr(\tilde{m}=l).$$

OA2.2 Relationship formation

Comparing the prices, we have that when $\alpha^* = 1$, $\mathbb{E}P_i(\alpha^*=1;\tilde{m}) > P_i^N$ if and only if:

$$\left(\bar{\delta} - C\gamma - N\gamma\sigma_{\delta i}\right)\left(\pi\rho_h + (1-\pi)\rho_l\right) > N\gamma\sigma_\delta^2(\pi\rho_h^2 + (1-\pi)\rho_l^2),$$

or, equivalently,

$$\frac{\bar{\delta}}{N\gamma\sigma_\delta^2} > \frac{\pi\rho_h^2 + (1-\pi)\rho_l^2}{\pi\rho_h + (1-\pi)\rho_l} + C\gamma + N\gamma\sigma_{\delta i}.$$

When $\alpha^* = \frac{1}{2}$, $\mathbb{E}P_i(\alpha^* = \frac{1}{2};\tilde{m}) > P_i^N$ if and only if:

$$\left(\bar{\delta} - C\gamma - N\gamma\sigma_{\delta i}\right)\left(\pi\rho_h e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l\left(1-\pi\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)$$

$$> N\gamma\sigma_\delta^2\left(\pi\rho_h^2 e^{N\bar{\delta}\gamma\rho_l + \frac{\gamma^2\cdot\left(2CN\rho_h + N^2\rho_h^2\sigma_\delta^2\right)}{2}} + \rho_l^2\cdot\left(1-\pi\right)e^{N\bar{\delta}\gamma\rho_h + \frac{\gamma^2\cdot\left(2CN\rho_l + N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right),$$

or, equivalently,

$$\frac{(\pi\rho_h^2 + (1-\pi)\rho_l^2)}{(\pi\rho_h + (1-\pi)\rho_l)} > \frac{\bar{\delta}}{\gamma N\sigma_\delta^2} > \frac{\pi\rho_h^2 e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)\rho_l^2 e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}{\pi\rho_h e^{\gamma(\bar{\delta}N\rho_l) + \frac{\gamma^2}{2}(\rho_h^2 N^2\sigma_\delta^2)} + (1-\pi)\rho_l e^{\gamma(\bar{\delta}N\rho_h) + \frac{\gamma^2}{2}(\rho_l^2 N^2\sigma_\delta^2)}}.$$

Therefore mandatory disclosure destroys relationship destruction if and only if

$$\begin{aligned}\frac{(\pi\rho_h^2+(1-\pi)\rho_l^2)}{(\pi\rho_h+(1-\pi)\rho_l)} &> \frac{\left(\bar{\delta}-C\gamma-N\gamma\sigma_{\delta j}\right)}{N\gamma\sigma_\delta^2} \\ &> \frac{\left(\pi\rho_h^2 e^{N\bar{\delta}\gamma\rho_l+\frac{\gamma^2\cdot\left(2CN\rho_h+N^2\rho_h^2\sigma_\delta^2\right)}{2}}+\rho_l^2\cdot(1-\pi)\,e^{N\bar{\delta}\gamma\rho_h+\frac{\gamma^2\cdot\left(2CN\rho_l+N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}{\left(\pi\rho_h e^{N\bar{\delta}\gamma\rho_l+\frac{\gamma^2\cdot\left(2CN\rho_h+N^2\rho_h^2\sigma_\delta^2\right)}{2}}+\rho_l\,(1-\pi)\,e^{N\bar{\delta}\gamma\rho_h+\frac{\gamma^2\cdot\left(2CN\rho_l+N^2\rho_l^2\sigma_\delta^2\right)}{2}}\right)}.\end{aligned}$$

OA3 Other variations

OA3.1 Optimal collaboration intensity

In this subsection, we study the optimal collaboration intensity $\bar{\delta}$ from a firm's point of view. We continue to assume that each firm's objective function is to maximize the expected asset price. Since the two firms are symmetric, we focus on one firm without loss of generality. To illustrate the result most transparently, we assume that there is no cost of the firms to change $\bar{\delta}$, which is a measure for intensive margin of firm relationships.

For the case without matching uncertainty, by pricing function (4), firms would choose the largest $\bar{\delta}$ possible (i.e., with unbounded support, firms would choose $\bar{\delta}$ to be infinity).

We now consider the case with matching uncertainty and in this case, we show that the optimal intensity $\bar{\delta}$ is interior. For simplicity, we assume that $\rho_l = 0$. As mentioned before, in this special setting, uncertainty about matching intensity $\tilde{\rho}$ can also be interpreted as uncertainty about the existence of firm relationship. The asset prices for $j = \{A, B\}$ are just a special case of the prices in the conceptual framework given by equation (6):

$$\begin{aligned}P_j &= \frac{\bar{V}-\gamma\sigma_V^2}{R} \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{2\gamma^2\rho_h^2\sigma_\delta^2}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2}+(1-\pi)e^{2\gamma\rho_h\bar{\delta}}}\right] \\ &- \frac{2\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{2\gamma^2\rho_h^2\sigma_\delta^2}}{\pi e^{2\gamma^2\rho_h^2\sigma_\delta^2}+(1-\pi)e^{2\gamma\rho_h\bar{\delta}}}\right]. \qquad \text{(OA3)}\end{aligned}$$

Firms choose their collaboration intensity $\bar{\delta} \in [0, \infty)$ to maximize their asset prices (OA3).

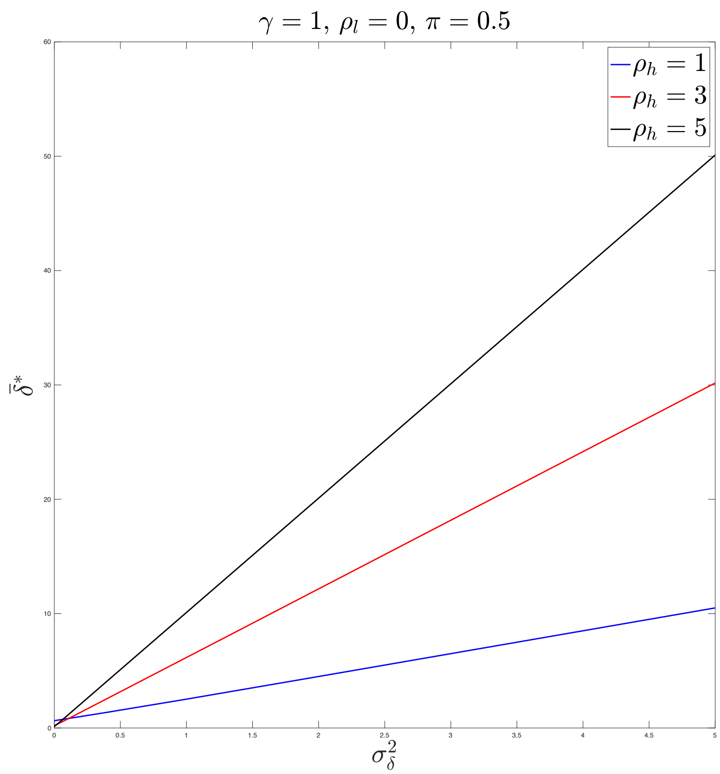
An increase in $\bar{\delta}$ has two effects on the price (OA3). There is a direct effect that increases the price due to an increase in expected cash flows. There is also an indirect effect that decreases the price due to the risk associated with an increase in the asset demand of both assets caused by the direct effect. A high asset demand adds risk to the investor's portfolio as she believes with probability $1 - \pi$ that firms A and B have no relationship ($\tilde{\rho} = \rho_l = 0$). In other words, an increase in $\bar{\delta}$ increases the downside risk and thus the kurtosis term as captured in the third term of equation (OA3). When the investor is uncertain about the matching intensity between two firms, it is optimal for firms to have a limited collaboration intensity, in contrast to the case in which the investor is certain about the matching intensity.

Proposition OA3 (Optimal Collaboration Intensity)**.** *The optimal collaboration intensity* $\bar{\delta} = \bar{\delta}^*$ *is uniquely determined by the solution to the following equation:*

$$\pi(1-\pi)\rho_h + \rho_h \pi^2 e^{2\gamma\rho_h(\gamma\rho_h\sigma_\delta^2 - \bar{\delta})} + 4\gamma^2\pi(1-\pi)\rho_h^3\sigma_\delta^2 - 2\gamma\pi(1-\pi)\rho_h^2\bar{\delta} = 0.$$

In Figure OA1, we report the optimal collaboration intensity $\bar{\delta}^*$ for several parameter specifications. Specifically, we assume $\gamma = 1$ and $\pi = 0.5$, set several values for ρ_h, and depict $\bar{\delta}^*$ against σ_δ^2. We can see that the optimal collaboration intensity $\bar{\delta}^*$ is interior and increasing in σ_δ^2.

Figure OA1: Optimal Collaboration Intensity



Proof of Proposition OA3

Taking the FOC of the asset price under uncertainty (6) with respect to $\bar{\delta}$ and re-arranging terms, we obtain

$$\frac{\pi(1-\pi)\rho_h + \rho_h\pi^2 e^{2\gamma\rho_h(\gamma\rho_h\sigma_\delta^2-\bar{\delta})} + 4\gamma^2\pi(1-\pi)\rho_h^3\sigma_\delta^2 - 2\gamma\pi(1-\pi)\rho_h^2\bar{\delta}}{Re^{2\gamma^2\rho_h^2\sigma_\delta^2+2\gamma^2\rho_l^2\sigma_\delta^2+2\bar{\delta}\gamma\rho_h+2\bar{\delta}\gamma\rho_l}\left((1-\pi)e^{2\gamma(\gamma\rho_l^2\sigma_\delta^2+\bar{\delta}\rho_h)} + \pi e^{2\gamma(\gamma\rho_h^2\sigma_\delta^2+\bar{\delta}\rho_l)}\right)^2} = 0.$$

OA3.2 Disclosure of exposure to factors by one firm

This subsection aims at further comparing our paper to Heinle et al. (2018) by connecting the disclosure of an exposure to a risk factor as in their study to a disclosure of a relationship as in our paper. Assume there is only one firm with payoffs $\tilde{F}_A = \tilde{V}_A + \tilde{\Delta}$, where $\tilde{V}_A \sim N(\bar{V}, \sigma_V^2)$ and $\tilde{\Delta}$ is given by

$$\tilde{\Delta} = \tilde{\rho}\tilde{\delta}, \text{ with } \tilde{\delta} \sim N\left(\bar{\delta}, \sigma_\delta^2\right) \text{ and } \tilde{\rho} = \begin{cases} \rho_h & \text{with probability } \pi, \\ \rho_l & \text{with probability } 1-\pi, \end{cases} \tag{OA4}$$

where $\rho_h > \rho_l \geq 0$ and $\bar{\delta} \geq 0$. Investors are uncertain about the exposure of the firm to the factor $\tilde{\delta}$. The price of the asset is given by

$$\begin{aligned} P_A &= \frac{\bar{V} - \gamma\sigma_V^2}{R} \\ &+ \frac{\bar{\delta}}{R}\left[\frac{\pi\rho_h e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2+\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2+\gamma\rho_h\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2+\gamma\rho_l\bar{\delta}} + (1-\pi)e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2+\gamma\rho_h\bar{\delta}}}\right] \\ &- \frac{\gamma\sigma_\delta^2}{R}\left[\frac{\pi\rho_h^2 e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2+\gamma\rho_l\bar{\delta}} + (1-\pi)\rho_l^2 e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2+\gamma\rho_h\bar{\delta}}}{\pi e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2+\gamma\rho_l\bar{\delta}} + (1-\pi)e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2+\gamma\rho_h\bar{\delta}}}\right]. \end{aligned} \tag{OA5}$$

There are three changes with respect to the price under matching uncertainty with two firms given by equation (6). First, the third term is no longer multiplied by 2, which accounts for the diversification cost, as there is no diversification cost with a single firm. Second, there is a change in the relative importance of the variance of the common factor in the exponential weights relative to the mean of this common factor. The variance of the common factor now is relatively more important than the mean relative to the two firms case. Third, there is a change in the absolute importance of weights. The exponential weights are smaller than in the two firms case due to the absence of correlation in a single firm case.

As in Section 4, we now specify that the firm can commit to a disclosure policy such that the firm can choose the following probability $\alpha \in [\frac{1}{2}, 1]$ at zero cost:

$$\begin{aligned} Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_l) = \alpha, \\ Pr(\tilde{m} = l \mid \tilde{\rho} = \rho_h) &= Pr(\tilde{m} = h \mid \tilde{\rho} = \rho_l) = 1 - \alpha. \end{aligned} \tag{OA6}$$

As before, when $\alpha = 1$, the firm provides perfect disclosure of the realization of $\tilde{\rho}$. Instead, when $\alpha = 1/2$, the firm provides no information about the realization of $\tilde{\rho}$. Any α in between will generate only partial disclosure about the realization of $\tilde{\rho}$. The following proposition shows under which conditions the firm will choose to opt for a full disclosure policy or a non-disclosure policy, which is a counterpart for Proposition 2. The intuition is similar to the model in our main setting but the results are qualitatively different. Specifically, the region where no disclosure is optimal shrinks. This is because no disclosure is more costly due to the absence of diversification cost. In addition, since there is only one firm, we are not able to answer questions related to the real effects of mandatory relationship disclosure on relationship formation.

Proposition OA4. *The optimal disclosure policy $\alpha = \alpha^*$ is a corner solution and is given by*

$$\alpha^* = \begin{cases} \frac{1}{2}, & \text{if } \frac{\rho_l + \rho_h}{4} < \frac{\bar{\delta}}{2\gamma\sigma_\delta^2} < \frac{\rho_l + \rho_h}{2}, \\ 1, & \text{otherwise} . \end{cases} \tag{OA7}$$

Proof of Proposition OA4

Following the same steps as in the proof of Proposition 2, the FOC of the expected asset price (OA5) with respect to α equals zero at $\alpha = \frac{1}{2}$ is:

$$\left.\frac{\partial E[P_A(\alpha; \tilde{m})]}{\partial \alpha}\right|_{\alpha = \frac{1}{2}} = 0.$$

The disclosure policy $\alpha = \frac{1}{2}$ is a maximium if and only if these two conditions are satisfied: $\bar{\delta} < \gamma(\rho_l + \rho_h)\sigma_\delta^2$ and $e^{\frac{1}{2}\gamma^2\rho_h^2\sigma_\delta^2 + \gamma\rho_l\bar{\delta}} < e^{\frac{1}{2}\gamma^2\rho_l^2\sigma_\delta^2 + \gamma\rho_h\bar{\delta}}$.