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Quantum metaphysical indeterminacy

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Abstract On a wide variety of presently live interpretations, quantum mechanics violates the classical supposition of 'value definiteness', according to which the properties ('observables') of a given particle or system have precise values at all times. Here we consider whether two recent approaches to metaphysical indeterminacy—a metaphysical supervaluationist account, on the one hand, and a determinable-based account, on the other—can provide an intelligible basis for quantum metaphysical indeterminacy (QMI), understood as involving quantum value indefiniteness. After identifying three sources of such QMI, we show that previous arguments (Darby in Australas J Philos 88:227–245, 2010; Skow in Philos Q 60:851–858, 2010) according to which supervaluationism cannot accommodate QMI are unsuccessful; we then provide more comprehensive arguments for this conclusion, which moreover establish that the problems for supervaluationism extend far beyond the orthodox interpretation. We go on to argue that a determinable-based approach can accommodate the full range of sources of QMI.

Keywords Quantum mechanics · Metaphysical indeterminacy · Quantum indeterminacy · Metaphysical supervaluationism · Determinable-based metaphysical indeterminacy

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1 Can we make metaphysical sense of quantum indeterminacy?

As Feynman (1982) observed, “we always have had a great deal of difficulty in understanding the world view that quantum mechanics represents” (471). Among the perplexing aspects of quantum mechanics is its seeming, on a wide variety of currently live interpretations (including but not limited to the so-called ‘orthodox’ interpretation), to violate the classical supposition of ‘value definiteness’, according to which the properties—a.k.a. ‘observables’—of a given particle or system have precise values at all times.[1] Indeed, value indefiniteness lies at the heart of what is supposed to be distinctive about quantum phenomena, as per the following classic cases:

- Prior to detection, the location of a particle in a double-slit experiment is indeterminate
- Prior to opening the box, Schrödinger’s cat is neither determinately alive nor determinately dead
- A particle cannot have precise values of both position and momentum at the same time
- A particle measured as spin-x-up at t has indeterminate spin-y and spin-z values at t
- The components of a spin-entangled state do not have determinate values of spin

On the interpretations in question, the indeterminacy in such cases is taken to be metaphysical, not merely epistemological, much less semantic. As Wolff (2015) notes, on an orthodox reading, “quantum mechanics suggests *metaphysical* indeterminacy, not (merely) epistemic indeterminacy” (380), and Darby (2010) notes, more generally, that “there are obstacles in the way of [...] interpreting such indefiniteness as merely epistemic or representational [...]. Quantum mechanics, then, looks a likely source of examples of genuine metaphysical indeterminacy” (227).

The question before us, then, is this: Can we make sense of quantum indeterminacy as being genuinely metaphysical, and if so, how?

In Feynman’s time, there were no developed accounts of metaphysical indeterminacy (henceforth: MI). Recently, however, two new approaches to MI have been proposed, each of which aims to provide an intelligible basis for this phenomenon.[2]

- On the metaphysical supervaluationist approach developed by Akiba (2004), Barnes (2006, 2010), Williams (2008), Barnes and Williams (2011), and others, MI involves the world’s being primitively unsettled about which of some range of completely determinate options obtains. Though on this approach MI is taken

[1] It has also been suggested that quantum objects may be metaphysically indeterminate in failing to have determinate identity; see, e.g., Lowe (1994) and French and Krause (2003). Here we focus only on property value indeterminacy.

[2] Other accounts aiming to characterize MI include those in Smith and Rosen (2004) and Torza (2017); beyond some notes, we leave discussion of these alternatives for another day.

to be primitive, proponents model this primitive along lines familiar from supervaluationist treatments of semantic indeterminacy, with metaphysical indeterminacy reflecting unsettledness not between linguistic precisifications, but between precisificationally possible worlds or states of affairs.

- On the determinable-based approach developed by Wilson (2013, 2016), MI involves the obtaining of an indeterminate state of affairs, in which (in the simplest case) an entity (object, system, etc.) has a determinable property, but no *unique* determinate of that determinable. As we'll discuss, there are two ways that a determinable can fail to be uniquely determined: first, if there are too many candidate determinates (corresponding to 'glutty' MI); second, if there are none at all (corresponding to "gappy" MI).

Three further points of difference are worth noting:

1. On a metaphysical supervaluationist approach, MI involves its being *indeterminate* which *determinate* (precise) state of affairs obtains; on a determinable-based approach, MI involves its being *determinate* (or just plain true) that an *indeterminate* (imprecise) state of affairs obtains.
   Reflecting this structural difference, we follow Wilson (2013) in sometimes heuristically characterizing supervaluationist accounts as 'meta-level' accounts, and determinable-based accounts as 'object-level' accounts.
2. On a metaphysical supervaluationist approach, MI generates *propositional* indeterminacy in, e.g., certain propositions expressing that a given determinate/precise state of affairs or precisificationally possible world obtains, or that a given object (system, etc.) has a given property; this propositional indeterminacy is then treated by means of a new indeterminacy operator. On a determinable-based approach, MI does not generate any propositional indeterminacy, and so no indeterminacy operator is required. Rather, MI involves a certain pattern of instantiation of determinable and determinate properties; consequently, propositions expressing the obtaining of any given state of affairs (whether precise or imprecise) or the having of any given property (whether determinate or determinable) will, if meaningful, be determinately (i.e., straightforwardly) true or determinately false, as per classical semantic usual.[3]
3. On a metaphysical supervaluationist approach, and as is familiar from discussions of semantic supervaluationism, it is possible (if truth is 'super-truth'—truth on every precisification) to preserve certain theorems of classical logic, though certain classical laws of inference (including contraposition) must be rejected; whether the classical semantic principle of bivalence is preserved depends on whether there is a privileged precisification (as per 'non-standard' supervaluationism, and as is endorsed by Barnes and Williams 2011). On a determinable-based approach, and again reflecting that this approach does not generate propositional indeterminacy, no revisions to classical logic or semantics are required.

[3] See Wilson (2016) for reasons to reject taking MI to generate propositional indeterminacy.

In this paper, we consider whether either a metaphysical supervaluationist approach or a determinable-based approach can accommodate quantum MI (QMI), understood as involving quantum value indefiniteness. We start by discussing the usual theoretical indication of QMI, and distinguishing three seemingly different sources of QMI (Sect. 2). We then show that previous arguments for the conclusion that metaphysical supervaluationism cannot accommodate QMI, due to Darby (2010) and Skow (2010), are unsuccessful, in leaving open several supervaluationist responses. We go on to provide more comprehensive argumentation for the negative conclusion. Here, among other results, we establish that the problems for supervaluationism extend far beyond the concern that is the focus of Darby's and Skow's discussions (according to which a supervaluationist approach is incompatible with the orthodox interpretation, in light of the Kochen–Specker theorem) to also attach to common understandings of other interpretations on which there is supposed to be QMI (Sect. 3).[4] We then argue that a determinable-based account can successfully accommodate all three varieties of QMI, considering in each case whether any prima facie advantages accrue to a gappy or rather a glutty determinable-based approach (Sect. 4). We close by observing the positive mutual bearing of our results on the coherence and intelligibility of both quantum mechanics and metaphysical indeterminacy (Sect. 5).

2 Preliminaries: EEL, and three sources of quantum MI

In this section we discuss the linking principle that has standardly been taken to underlie attributions of QMI, and highlight three seemingly distinct sources of QMI.

2.1 The eigenstate–eigenvalue link

Suggestions that there is QMI typically advert to a linking principle taken to underlie attributions of determinate properties, and conversely, judgments of value indeterminacy. For example, in discussing QMI in orthodox quantum mechanics, Wolff (2015) says:

> Orthodox quantum mechanics assumes that a system is in an eigenstate for some observable *O* iff that observable takes one of its eigenvalues. While some observables are compatible in the sense that a system can at the same time be in eigenstates with respect to these observables [...], many observables are incompatible. This is true in particular of position and momentum, and of the different components of spin. When a system is in a state of superposition

[4] There are, of course, many understandings (readings, versions) of each 'interpretation', appealing, e.g., to different underlying ontologies. Though we can't consider the prospects for supervaluationism *vis-á-vis* every variation on a given interpretational theme, for dialectical purposes it will suffice to show that the problems for supervaluationism do in fact attach to the orthodox interpretation and moreover extend to common understandings of several live non-orthodox interpretations.

with respect to some observable O, the system has no eigenvalues with respect to that observable. (380)

Here Wolff appeals to the *Eigenstate–Eigenvalue Link* (EEL), linking eigenstates with value determinateness:

(EEL): A quantum system has a definite value v for an observable O *iff* it is in an eigenstate of O having eigenvalue v.

More generally, as Frigg (2009) notes, the question of how to move from the quantum formalism to the properties of a given system is "commonly answered by appeal to the so-called Eigenstate–Eigenvalue Rule" (266), and as Lewis (2016) observes, EEL represents a "fairly standard way of understanding quantum states" (76). Correspondingly, in what follows we present our taxonomy of sources of QMI, and our later comparative assessment of whether and how the aforementioned accounts of MI can treat these forms of QMI, in terms primarily referring to EEL, though as we will see down the line we will have occasion to mention certain alternative linking principles.[5]

We will also usually focus on observables with discrete spectra, thus admitting eigenvectors and hence application of EEL. However, we sometimes follow other commentators in discussing incompatible observables with continuous spectra (notably, position and momentum), on the assumption that (as per, e.g., Vernaz-Gris et al. 2014) there are strategies for discretizing such spectra.

2.2 Three sources of quantum MI

There are three seemingly distinct sources of QMI, operative in (i) *superposition of states*, (ii) *incompatible observables* and (iii) *entanglement*. These sources are clearly related—most saliently, as involving properties which are interdependent in ways resulting in one or other failure of value definiteness. But in certain respects the sources are also interestingly different, in ways that enter into both critical and constructive portions of our future discussion.

1. *Superposition*. Consider a system S having a non-degenerate observable O, with distinct eigenstates $|\psi\rangle$, $|\varphi\rangle$ having distinct associated eigenvalues. Any linear combination $|\omega\rangle = c_1|\psi\rangle + c_2|\varphi\rangle$ is a permissible 'superposition' state of S. If the eigenvalues of $|\psi\rangle$, $|\varphi\rangle$ are different, then $|\omega\rangle$ is not an eigenstate of O. It thus follows from EEL that if S is in $|\omega\rangle$, S does not have a definite value of O.

[5] Wallace (ms.) claims that one might resist the supposition that even the orthodox interpretation is committed to EEL, on grounds that the practice of physicists doesn't rely on it. Wallace clarifies that the considerations he raises aren't aimed at showing that the best interpretation of quantum mechanics is one which abandons EEL, but in any case attention to the considerations he raises must await another occasion; meanwhile we continue on under the assumption that, as above, indications of QMI commonly proceed by way of EEL.

Case in point: Schrödinger's cat, prior to opening the box.

2. *Incompatible Observables*. Consider a system S and two of its observables, O_1 and O_2.[6] As a first approximation, observables are represented by self-adjoint/Hermitian operators—for any two distinct operators, we can define their commutator as follows: $[O_1, O_2] = O_1O_2 - O_2O_1$. Two observables O_1, O_2 are *incompatible* iff $[O_1, O_2] \neq 0$. Now, if O_1, O_2 are incompatible, some eigenstates of O_1 are not eigenstates of O_2, and *vice versa*. It follows from EEL that if S is in such an eigenstate of, say, O_1, then S does not have a definite value of O_2.

   Case in point: any two or more observables subject to a generalized uncertainty principle, including position and momentum, and distinct components of spin.

3. *Entanglement*. Consider two quantum systems S_1, S_2 that compose system S_{12}. The Hilbert space H_{12} of the composite system is the tensor product space of the Hilbert spaces H_1 and H_2, associated with S_1 and S_2, respectively. There are vectors $|\omega\rangle \in H_{12}$ that cannot be written as $|\psi\rangle \otimes |\varphi\rangle$, with $|\psi\rangle \in H_1$ and $|\varphi\rangle \in H_2$. In this case the state of S_{12} is *entangled*. Given this understanding, consider a pure entangled state $|\omega\rangle \neq |\psi\rangle \otimes |\varphi\rangle$. Since it is a pure state, it is an eigenstate of some operator. This operator is a sum of operators representing observables of S_1 and S_2—e.g., $O = O_1 - O_2$. It follows from EEL that S_{12} has a definite value of O. Yet $|\omega\rangle$ is an eigenstate of neither O_1 nor O_2. It follows from EEL that the component parts S_1, S_2 do not have definite values for O_1 and O_2, respectively.

   Case in point: the singlet EPR-state (i.e., $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2)$, in which the quantum component systems each lack a determinate spin value.

By lights of EEL, these three kinds of cases all involve QMI, associated with one or more systems' failing to be in one or more eigenstates of (have one or more determinate values of) of one or more observables. The three cases are interestingly different, however, both metaphysically and mathematically:

- Metaphysically, cases of superposition MI involve the failure of a system to be in any eigenstate of (have any determinate value of) a single observable; cases of incompatible observable MI involve the failure of a system to be in any eigenstate of (have any determinate value of) one of its observables, given that it is in an eigenstate of (has a determinate value of) some other (incompatible) observable; cases of entanglement MI involve the failure of components of a composite system to be in eigenstates of (have determinate values of) certain observables, given that the composite system is in an eigenstate of (has a determinate value of) a related observable.[7]

[6] In what follows, we use the same notation for both observables and operators; strictly speaking, operators are mathematical objects representing observables.

[7] Entanglement might also be present in a simple system (see Hasegawa 2012), in which case the characterization of entanglement MI would need tweaking.

- Mathematically, the indeterminacy in each of the three cases is underpinned by different mathematical features: linearity in the case of superposition, non-commutative operators in the case of incompatible operators, and tensor product laws in the case of entanglement.

As we will see, there are cases to be made that not all these forms of QMI can be treated alike, whether supervaluationist or determinable-based MI is at issue.

3 Can a supervaluationist account accommodate quantum MI?

3.1 Metaphysical supervaluationism

A metaphysical supervaluationist account takes a 'meta-level' approach to MI, according to which MI involves its being indeterminate which state of affairs, of some range of determinate/precise states of affairs, obtains. As Barnes (2010) expresses the general idea:

> It's perfectly determinate that everything is precise, but [...] it's indeterminate which precise way things are. (622)

Somewhat more specifically, Barnes and Williams (2011) say:

> When *p* is metaphysically indeterminate, there are two possible (exhaustive, exclusive) states of affairs—the state of affairs that *p* and the state of affairs that not-*p*—and it is simply unsettled which in fact obtains.

Note that the sense of a 'possible' state of affairs (more generally, world) here is one that is restricted to possibilities that are compatible with what is actually the case, since otherwise it would be settled that such an (incompatible) state of affairs (possibility) does *not* obtain.

Again, metaphysical supervaluationists take MI to be primitive, but aim to explicate the phenomenon by exploiting a structural similarity to semantic supervaluationist accounts of vagueness. On semantic accounts, indeterminacy is taken to reflect our not yet having settled on a fixed interpretation of certain expressions in our language. Such indeterminacy is modeled by appeal to a range of admissible precisifications of our language, each compatible with existing (determinate) usage of our terms, and in each of which all semantic indeterminacy has been resolved; indeterminacy is reflected in there being admissible precisifications which differ as regards the extension of a given expression (e.g., 'bald'). On metaphysical supervaluationist accounts (and here drawing on Barnes and Williams 2011), the appeal is not to a range of admissible precisifications of our language, but to a space of admissible "precisificationally possible worlds", each compatible with existing (determinate) facts, and in each of which all metaphysical indeterminacy has been resolved. Instead of our language being unsettled as between different precisifications, our world is unsettled as regards "which world is actualized". Rather than indeterminacy reflecting linguistic precisifications' differing as regards, e.g., the extensions of certain expressions, indeterminacy reflects precisificationally possible worlds' differing as regards whether a given state of affairs (e.g., p) obtains. And like semantic supervaluationists, metaphysical supervaluationists formalize the indeterminacy at issue via what Darby (2010) calls a "modal operator approach":

> [Here] indeterminacy may be captured by a modal operator. By this I mean that a 'definitely' operator D may be prefixed to formulas; that the indeterminacy operator $\triangledown$ is to D as contingency is to necessity ($\triangledown\varphi$ iff $\neg D\varphi \wedge \neg D\neg\varphi$) [...]. (228)

Given these structural and formal similarities, it is no surprise that metaphysical supervaluationist accounts are, like semantic supervaluationist accounts, able (again, if truth is 'supertruth') to preserve the tautologies of classical logic—an advantage Barnes and Williams (2011) take to hinge on precisificational worlds' being "maximal and classical":

> Importantly, given our picture of indeterminacy, all the worlds in the space of precisifications are themselves maximal and classical. For any p, each precisification will opt for one of p or $\neg p$, and thus, every precisification will represent as true the law of excluded middle, $p \vee \neg p$—and similarly for every classical tautology.

3.2 Take one: the failure of a supervaluationist treatment of QMI

As Darby (2010) discusses, one might initially see paradigm cases of seeming MI as inviting characterization in terms of a metaphysical supervaluationist (henceforth, just 'supervaluationist') approach, as in the case of Schrödinger's cat:

> [There is] a suggestive parallel between the terms in the superposition and the idea [...] of precisifications. One of the terms in the superposition [...] is a term where the cat is alive, the other is not; that is reminiscent of multiple ways of drawing the extension of 'alive', on some of which 'the cat is alive' comes out true, on some, false. (235)

The supervaluationist might more generally suggest that, as per EEL, when a system is in an eigenstate for some observable, it has a determinate value for that observable, and when it isn't in an eigenstate for an observable, it is indeterminate for each determinate value of the observable whether the system has it, notwithstanding that the observable determinately has exactly one of those determinate values.[8]

Nonetheless, as Darby (2010) and Skow (2010) independently argue, in characterizing MI in terms of unsettledness between fully determinate worlds, supervaluationism cannot accommodate the 'deep'—i.e., insuperable—QMI characteristic of the orthodox interpretation, on which "it is inconsistent to suppose that every observable has a definite value" (Darby, 237). As Skow puts it:

[8] Again, we advert to EEL as reflecting a convenient and "fairly standard" way of thinking about when quantum observables have determinate values; but when relevant will discuss the bearing of other linking principles.

> In the Barnes and Williams model each actual [precisificationally possible] world attributes to each quantum system a value for each determinable property, and all actual worlds agree on the values assigned to properties which have determinate values (854).
> But it is possible for there to be metaphysical indeterminacy even when it is impossible to precisify reality completely. The orthodox interpretation of quantum mechanics illustrates this possibility. So this theory of MI is not adequate. (851)

Why think QMI is insuperable? Darby and Skow each cite the Kochen–Specker theorem, according to which, on the orthodox interpretation, the assumption of complete value determinacy leads to contradiction.[9] More specifically: in a Hilbert space of dimension $d \geq 3$ it is impossible to assign a definite value of 1 or 0 to every projection operator P_i such that, if a set of commuting P_i satisfies $\sum P_i = 1$ then the values $v(P_i)$ associated with such projectors satisfy $\sum v(P_i) = 1$.[10] Bokulich (2014) succinctly describes the result for the case of spin:

> [O]ne can measure the spin of a quantum particle, such as a photon (which is a spin-1 particle and so a boson), in three orthogonal directions: S_x, S_y, and S_z. When we measure the square of the spin component in each of these three directions, quantum mechanics requires that one of these directions gets the value 0, while the other two directions get the value 1 (because we know $S_x^2 + S_y^2 + S_z^2 = 2$). The Kochen–Specker theorem then shows that there is no consistent way to assign zeros and ones to all the possible spin directions, such that this constraint is satisfied; we run into the contradiction that one and the same spin direction needs to be assigned two incompatible values. (466)

Hence it is that on the orthodox interpretation, QMI is insuperable, hence incompatible with a 'shallow' conception of MI as indeterminacy between fully determinate worlds, as per a supervaluationist treatment.

3.3 Remaining supervaluationist responses

How might a supervaluationist respond? Skow considers and rejects two responses. In what follows, we first observe that Skow's replies to these responses are not compelling; we also draw attention to a third response, which neither he nor Darby address.

3.3.1 The rejectionist response

The first supervaluationist response is to reject the orthodox interpretation whose commitments are incompatible with allowing all observables of a system to be given determinate values, and to adopt an alternative interpretation instead.

[9] See Darby (2010, pp. 237–238), whose presentation follows Hughes (1989), and Skow (2010, pp. 854–856). For a detailed introduction, see Held (2018).

[10] See, e.g., Peres (2002).

Skow replies that what matters is not that the orthodox interpretation is correct, but that it is possibly correct. Skow's reply is less than compelling, however. To start, there are cases to be made that the orthodox interpretation is not, or at least is not clearly, metaphysically possible. Hence Albert (1992) maintains that the notion of measurement at issue in this theory is so ill-posed that it is unclear what the theory asserts, much less that it represents an empirically adequate metaphysical possibility, and Barrett (2010) says, in discussing the 'standard collapse formulation of quantum mechanics",

> The quantum measurement problem, however, arises as a result of the conflict between [the formulation's] two dynamical laws. If we suppose that measuring devices are physical systems like any other, then the standard collapse theory is inconsistent because the incompatible laws might be applied to the same evolution; on the other hand, if measuring devices are somehow special, the standard theory is incomplete since it does not tell us what interactions should count as measurements. (226)

Even granting the metaphysical possibility of a quantum scenario in line with the orthodox interpretation, it remains that insofar as this interpretation implausibly locates collapse in acts of measurement by observers, many suppose that it is not, by present scientific lights, a live theoretical possibility.[11] These concerns with the orthodox interpretation—which in now being widely rejected is 'orthodox' in name only—provide the supervaluationist with a principled independent basis for rejecting the interpretation, along with Skow's claim that incompatibility with this interpretation suffices to show that a supervaluationist treatment of QMI is inadequate.

3.3.2 Partial precisifications

A second supervaluationist response involves revising their approach so that its application does not require *perfectly* precise worlds. Here, the thought goes, so long as the precisifications are *more* precise than whatever unsettled world or state of affairs is at issue, one can implement the supervaluationist strategy: indeterminacy would be unsettledness between *more* rather than *maximally* precise options.[12] Skow considers and rejects this strategy, as follows:

> [S]uppose we [...] replace perfectly precise possible worlds with imprecise possible worlds (sets of sentences from a language which suffers from semantic indeterminacy). Even when there is no metaphysical indeterminacy,

[11] Thanks to Nina Emery for this point. Note also that this sort of restriction on which quantum theories or associated possibilities are relevant to assessing the adequacy of a given account of QMI isn't in tension with the goal of offering a metaphysical account of such indeterminacy, contrasting with semantic or epistemic accounts of such indeterminacy.

[12] Torza (2017) offers an account of MI which explicitly abandons complete precisifications; consideration of this account and of whether it is a version of the supervaluationist strategy under consideration is a substantive question, which as noted we leave for another day.

> we can expect it to happen that several imprecise possible worlds do not determinately misrepresent reality. (858)

Skow's reply does not block the 'partial precisification' response, however, since the supervaluationist has ways of ensuring that imprecision in worlds tracks metaphysical rather than merely semantic indeterminacy. One strategy might be to endorse a non-semantic conception of possible worlds, maintaining that whenever worlds so understood are imprecise then multiple actuality does entail MI. Another—more in line with the supposition of recent proponents of metaphysical supervaluationism according to which the worlds between which the actual world is unsettled are 'ersatz'—would be to allow that possible worlds are sets of sentences, but to maintain that MI is defined in term of ersatz possible worlds for which all semantic indeterminacy has been resolved. Indeed, metaphysical supervaluationists typically schematically characterize MI in just these terms, as when, e.g., Barnes and Cameron (2016) say "By worldly indeterminacy we mean indeterminacy that remains even once we've specified exactly what proposition it is we're asking about" (121). Moreover, the supervaluationist might maintain that quantum mechanical constraints on maximal assignments of properties provide a principled way of determining which partial precisifications are relevant to a supervaluationist treatment of QMI.[13]

3.3.3 *Non-actual laws*

A third supervaluationist response remains—namely, to deny that the worlds used to model QMI would have to be ones where the quantum laws are operative.[14] Such an approach might be motivated on grounds that what is most important is to model 'local' cases of QMI (involving sub-world systems), and that for such local cases, the right account of the QMI at issue is one appealing to classical rather than quantum worlds—not least, because we experience the world as classical.[15] Neither Darby nor Skow consider this response, however.

3.4 Take two: the failure of a supervaluationist treatment of QMI

We now provide more comprehensive argumentation for the conclusion that a supervaluationist account of MI cannot accommodate QMI.

3.4.1 *The rejectionist strategy*

As above, the supervaluationist has principled reason to resist taking incompatibility with the orthodox interpretation to decisively establish a problem for their approach

[13] Thanks to Michael Miller here. To be sure, there are other difficulties with the partial precisification strategy, which we will highlight down the line. Our present point is simply that there are available supervaluationist responses to Skow's specific concern with this strategy.

[14] This response was suggested by Ross Cameron (p.c.).

[15] Thanks to an anonymous referee for suggesting this motivation.

to QMI. After all, there remain numerous other live interpretations on which there is QMI, and for all Darby and Skow establish, the supervaluationist approach might properly accommodate QMI on these interpretations. As we'll now argue, however, a supervaluationist treatment of QMI is also at odds with common understandings of the Ghirardi–Rimini–Weber (GRW) and Everettian interpretations, such that the 'rejectionist' strategy is clearly unworkable. To prefigure: what is core to the difficulties for supervaluationism is the presence of dependencies among certain observables preventing these from all being given determinate values. To be sure, as Skow (2010) notes, "The Kochen–Specker theorem shows that there are not complete precisifications of reality which respect the dependencies among properties in orthodox quantum mechanics" (858). But common understandings of non-orthodox interpretations are also committed to property interdependence preventing complete precisifications, even where the Kochen–Specker theorem does not apply.

First, consider the GRW interpretation, a collapse theory on which said dependencies remain in place. Roughly speaking, GRW replaces the deterministic dynamics of the Schrödinger equation with a new stochastic dynamics; according to the new dynamics, every system has a small probability of undergoing a "hit" resulting in its state collapsing to a state with a more determinate value of position. Let us focus on the position of a particle. In undergoing a hit, the particle's wavefunction is multiplied by a narrow Gaussian function with tails going to infinity.[16] Hence the particle's state does not collapse into a precise position eigenstate; indeed, given these tails, the state is not an eigenstate of being confined to any finite range. Thus GRW, together with EEL, will not ascribe any determinate position to the particle; and similarly for other quantum observables.[17] This is enough to spell trouble for the supervaluationist; for every *maximally precise* world they appeal to in characterizing QMI will fail to be compatible with GRW.

One might wonder if this incompatibility reflects its being inappropriate to apply EEL to GRW, in light of the fact that the form of collapse on GRW leaves a lingering 'tail' of indeterminacy. Albert and Loewer (1992) suggest, for example, that given this residual indeterminacy, EEL should here be replaced by the *Fuzzy Link* (FL):

> (FL): A quantum system has a definite value v for a particular observable O *iff* the square projection of its state into an eigenstate of O is greater than $1 - P$, for some P.

For similar reasons, Lewis (2016) suggests that proponents of GRW or other interpretations involving residual indeterminacy might usefully avail themselves of the 'Vague Link' (VL):

[16] For discussion of why the tails are necessary, see Albert and Loewer (1992).

[17] Thanks to Nina Emery and Heather Demarest here. As Emery observed in her AOC comments: "[the] fuzzy link will give rise to MI in the paradigm case in just the same way as [...] the eigenstate-eigenvalue link".

(VL): A system has a determinate value for a given determinable to the extent that the square projection of its state onto an eigenstate of the corresponding operator is close to 1.

While full discussion of these alternative linking principles is beyond the scope of this paper, here we make two observations which suggest that such discussion would not undercut our present point. First, whether EEL should be replaced by an alternative principle in GRW is controversial (see, e.g., Frigg 2009); hence even if one is inclined towards an understanding of GRW incorporating such a replacement, it will remain that a supervaluationist treatment of QMI is incompatible with a common understanding of GRW. Second and more importantly, it is natural to see these alternative principles as offering merely pragmatic means of glossing over the fact that on the interpretations in question, the operative means of rendering states more determinate nonetheless leaves residual indeterminacy, in which case appeal to an alternative linking principle will not in fact render GRW compatible with a supervaluationist account. Such a pragmatic reading is suggested by Frigg's observation that the primary motivation for GRW—namely, that it avoids appeal to measurement as a means of gaining determinate values—is achieved at the price of its form of collapse not *really* gaining determinate values, and his characterization of the 'common wisdom' response to this residual indeterminacy which proceeds by "pointing out that GRW post-hit states are close to eigenstates and positing that being close to an eigenstate is as good as being an eigenstate" (268).

A seemingly more metaphysical response to residual QMI on the GRW interpretation proceeds not by endorsing an alternative linking principle but rather by endorsing an alternative fundamental ontology and associated properties and operators—e.g., an ontology on which the fundamental properties of quantum systems are mass-density distributions (as per, e.g., Ghirardi et al. 1995; for discussion see Egg and Esfeld 2015) or 'flashes' (as per Bell 1987; for discussion see Esfeld and Gisin 2014)—whose values are completely determinate. A full consideration of alternative understandings of GRW appealing to non-standard fundamental ontologies is again beyond the scope of this paper. Here we register, first, that in any case such understandings of the GRW interpretation are controversial; second, that it is unlikely that Bell's flash interpretation will be of use to the supervaluationist, since this interpretation (like Bohm's, about which more anon) appears to remove rather than accommodate QMI;[18] third, that while the mass-density interpretation of Ghirardi et al. provides a basis for collapse resulting in completely precise distributions of mass-density, there remains indeterminacy in,

[18] Hence in describing GRW with flashes, Esfeld (2014) says, "The flashes are all there is in space-time. That is to say, apart from when it spontaneously localizes, the temporal development of the wave-function in configuration space does not represent the distribution of matter in physical space. It represents the objective probabilities for the occurrence of further flashes, given an initial configuration of flashes. As in [Bohmian Mechanics], there hence are no superpositions of anything existing in physical space" (100).

e.g., the positions of macroscopic objects,[19] which at least renders unclear the compatibility of this interpretation with a supervaluationist approach.

Next, consider an Everettian interpretation. To appreciate what is at stake in this interpretation, consider a simple spin-x measurement of a system S. Let S be in the superposition state $\frac{1}{\sqrt{2}}(|\downarrow\rangle_S + |\uparrow\rangle_S)$, and let the measuring apparatus M be in its ready state $|Ready\rangle$. Quantum mechanics predicts that after a measurement interaction, the composite system will be in state $\frac{1}{\sqrt{2}}(|\downarrow\rangle_S|\downarrow\rangle_M + |\uparrow\rangle_S|\uparrow\rangle_M)$. As Wallace (2013) describes the Everettian approach:

> Macroscopically indefinite states like $[\frac{1}{\sqrt{2}}(|\downarrow\rangle_S|\downarrow\rangle_M + |\uparrow\rangle_S|\uparrow\rangle_M)]$ are physically reasonable after all, and should be understood as describing a multiplicity: a situation in which there are two pointers (or sets of pointers), one pointing left [tracking spin-x-up] and one pointing right [tracking spin-x-down], and with each dynamically separated from the other.[20] (210)

Accordingly, the Everettian will maintain that the measurement interaction results in the existence of two dynamically robust and causally separated world-branches: one in which exists a spin-x-down particle and a measurement device registering that outcome, and one in which exists a spin-x-up particle and a measurement device registering that outcome.

It might seem that the supervaluationist can accommodate QMI on an Everettian interpretation, by taking world-branches to correspond to precisificationally possible worlds, such that, e.g., the aforementioned entangled system might be taken to involve multiple worlds in which spin-x properties are perfectly determined. But the impression of accommodation is incorrect, for three reasons.

First, assimilating branches to precisificationally possible worlds is in tension with a supervaluationist treatment of MI. Recall that the supervaluationist takes MI to consist in its being indeterminate which maximally precise world obtains. But on the Everettian interpretation, it is never indeterminate which branch obtains, however ‘obtaining’ is understood: either it is determinate that they all obtain (taking a kind of ‘meta-world’ perspective), or else it is determinate which one of them obtains (taking a local perspective, from within a given branch). Thus, despite positing a multiplicity of worlds, an Everettian interpretation is unsuited for supervaluationist treatment—effectively, because this interpretation makes no room for the primitive meta-level indeterminacy posited by the supervaluationist.

Second, in any case the phenomenon of branching pertains to macroscopic, not microscopic systems, as indicated by Wallace’s remarks, above. Wallace more specifically says:

> Multiplicity, in the Everett interpretation, is an emergent, high-level notion. The theory is a “many-worlds” theory in the same sense that modern astrophysics is a “many-stars” theory: in both cases, the objects being

[19] As Esfeld (2014) notes, on this approach, macroscopic objects are “well localized” (100) but not determinately so.

[20] The text in brackets has been altered for compatibility with our discussion.

> multiplied are not represented in the fundamental structure of the theory. [...] Contemporary defences of the Everett interpretation, almost exclusively, restrict multiplicity to the emergent level. (2013, p. 217)

As such, even if the phenomenon of branching could be understood in supervaluationist terms, this treatment would be restricted to MI associated with macroscopic systems.

Third, even if branching were to apply to microscopic as well as macroscopic systems, there would remain cases of incompatible observable MI within a branch that could not be given a supervaluationist treatment. Consider a branch containing a single particle, that can be in only two position states x_1 and x_2 and two momentum states p_1 and p_2, and which has been measured to be in p_1. The incompatible observable MI in this case cannot be treated by appeal to branches where the indeterminate values are rendered more determinate; for here there are no 'more' determinate worlds to branch into.[21]

A similar concern attaches to any incompatible observables, such as position/momentum or different components of spin. These will obey the generalized uncertainty principle; e.g., for position/momentum it will be the case that $\sigma_{O1}^2\sigma_{O2}^2 = \left(\frac{1}{2i}\langle[O_1, O_2]\rangle\right)^2$, where σ_O^2 is the variance of observable O. Hence any maximally precise determination of one observable in an incompatible set will entail *an infinite variance* for the other observables in the set, such that once one observable in the set receives a maximally determinate value, the generalized uncertainty principle rules out even the slightest precisification of any incompatible observable in the set. In any such case, there will be no more determinate worlds for the supervaluationist to appeal to—at least, none compatible with this principle. One might wonder whether the supervaluationist could accommodate such irresolvable imprecision at a world by appeal to the partial precisification strategy, discussed above. We will shortly provide reasons for thinking that this strategy fails; in the meantime, it remains that modulo this strategy, a supervaluationist approach cannot generally accommodate incompatible observable MI on an Everettian interpretation—for reasons, we observe, not depending on the Kochen–Specker theorem.

Two morals can be drawn from attention to the GRW and Everettian interpretations. First, one need not invoke the Kochen–Specker theorem to identify difficulties for a supervaluationist treatment of QMI. Again, the difficulties generated by the theorem turn on there being dependencies between properties rendering it impossible for them to all to be given determinate values; but such dependences remain in place on common readings of these non-orthodox interpretations. Second, in the Everettian interpretation, we have our first case-in-point of the usefulness of distinguishing between different sources of QMI, for as we have just seen, on an Everettian interpretation these sources are not, in general, given a uniform treatment.

Finally, it is worth noting that a Bohmian interpretation, on which "the configuration of a system of particles evolves via a deterministic motion

[21] Bokulich (2014, p. 465) uses a similar case to illustrate why position-momentum MI cannot be given an epistemic interpretation.

choreographed by the wave function" (Goldstein 2017, p. 1), is also unsuited for supervaluationist purposes. To be sure, a supervaluationist can endorse a Bohmian interpretation without contradiction; but that won't be to the point of showing that supervaluationism can accommodate QMI, since Bohm's theory is "completely determinate" (see, e.g., Goldstein 1996, p. 148), such that any seeming indeterminacy is at best epistemic.

More precisely, this is true for position, the privileged observable on Bohm's account. One might wonder whether properties beside position can be subject to value indeterminacy, since as Lewis (2016) observes, "although the Bohmian strategy arguably makes all the properties we directly observe determinate, it does not thereby make *all* properties determinate" (101–102). Lewis mentions spin, by way of example. But given that on a Bohmian interpretation position is standardly seen as the only physically fundamental property,[22] and this property is standardly seen as determinate, it is unclear how Bohmian mechanics can vindicate the existence of genuine QMI, as supervaluationists aim to do.

To sum up: metaphysical supervaluationism is incompatible with common understandings of the orthodox, GRW, and Everettian interpretations (and its compatibility with a Bohmian interpretation is not to the point of accommodating QMI).[23] These interpretations and their common understandings comprise the bulk of the standard slate of options for which QMI might be at issue. Their joint rejection would thus be fatally ad hoc, and so the supervaluationist strategy of rejecting any interpretations conflicting with their approach is clearly unworkable.

3.4.2 Partial precisifications

As above, the supervaluationist might aim to respond to the Darby–Skow objection by endorsing a non-standard version of supervaluationism, on which QMI involves the world's being unsettled between more precise rather than fully precise worlds; and they moreover have resources for responding to Skow's concern that an appeal to partial precisifications would be unable to distinguish cases of semantic supervaluationism from cases of genuinely metaphysical indeterminacy. Nonetheless, as we'll now argue, the partial precisification response fails, for two reasons.

First, in allowing that some imprecise possible worlds cannot be further precisified, the supervaluationist means of ensuring compatibility with the tautologies of classical logic is undermined—recall Barnes and Williams's remarks concerning the importance of the precisifications' being "maximal and classical"

[22] Hence: "In the Bohmian mechanical version of nonrelativistic quantum theory, quantum mechanics is fundamentally about the behavior of particles; the particles are described by their positions, and Bohmian mechanics prescribes how these change with time" (Goldstein 2017, p. 13).

[23] Considerations of space prevent our discussing modal interpretations, on which only a proper subset of quantum properties are taken to have definite values, but in our view supervaluationism will also run afoul of such interpretations, due to a modal variant of the Kochen-Specker-theorem (see Domenech et al. 1981). We plan to discuss modal interpretations in more detail in future work.

for this purpose—and so in turn is a primary stated motivation for a supervaluationist approach.[24]

Second, if supervaluationism is revised to appeal to worlds which are metaphysically (and not just semantically) imprecise, the supervaluationist will need to treat this indeterminacy in such worlds by some or other account of MI. But there is no prospect of generally accounting for such MI just by appeal to supervaluationism. One concern with such a strategy (raised by an anonymous referee) is that the need to prevent complete precisification (in order to accommodate 'deep' QMI, in particular) threatens to generate a potentially infinite regress of supervaluationist analyses. But even if such regresses are taken to be unproblematic, in general the application of quantum constraints will entail that some worlds cannot be further precisified, for reasons that echo our previous remarks about an Everettian interpretation. Hence in a world containing a single particle with a maximally determinate position (hence for which it is determinately true that the particle has such-and-such location), there are no more precise worlds available to accommodate the particle's indeterminacy in momentum; similarly, and more generally, for any worlds in which one observable in an incompatible set is maximally determined.[25]

Consequently, on the partial precisificationist strategy, at least some cases of MI in only partially precise worlds will need to be treated in some non-supervaluationist (presumably, object-level) fashion, so that a supervaluationist treatment of QMI ultimately will require supplementation by a second account of MI. Such a two-pronged treatment of QMI is both ontologically costly and unsystematic. Given that (as we will argue) a determinable-based object-level account can itself accommodate the full range of QMI, the partial precisification strategy ultimately invites rejecting a supervaluationist meta-level account.

[24] Torza (2017) moreover argues that implementing the partial precisification strategy (a version of which he endorses) requires rejecting classical compositional semantics. Note that there is no associated difficulty for a determinable-based object-level account: on this account MI does not generate any propositional indeterminacy, so compatibility with classical logic and semantics does not need to be regained, so to speak, by appeal to what is true in all 'maximal and classical' precisificationally possible worlds.

[25] It may be worth noting that these considerations bear negatively on Robbie Williams' suggestion (p.c.) that the supervaluationist could avoid running afoul of the Kochen–Specker theorem by singling out a proper subset of observables as physically relevant, and appealing to partial precisifications where only these observables were determinate. Implementing the suggestion would require that, for every set of incompatible observables, the supervaluationist identify only one as physically relevant; but in general—notably, for components of spin—there will be no plausible non-arbitrary way of doing this. To be sure, there remains the radical strategy of taking only *one* quantum observable to be physically relevant, which would indeed sidestep our arguments, since (as prefigured) these rely on there being certain dependencies between *different* (physically relevant) quantum observables. But in removing dependencies between real quantum observables the supervaluationist throws the baby out with the bathwater, for any QMI there may be stems from these dependencies. Hence it is that the salient interpretation on which there is a sole privileged observable—namely, Bohmian mechanics—is one on which the observable is always determinately-valued, and any seeming indeterminacy is merely epistemic.

3.4.3 Non-actual laws

As above, the supervaluationist might aim to respond to the Darby–Skow objection by taking QMI to be modeled by appeal to precisificationally possible worlds not subject to value indeterminacy—i.e., in which classical (non-quantum) laws are operative—on grounds that such classical precisifications would serve to model QMI of a 'local' variety (both spatiotemporally and with respect to local choices of observables), especially in light of the seemingly classical nature of our experience. Neither Darby nor Skow consider this response; however, as we'll now argue, the response is problematic, for two reasons.

First, notwithstanding that much of our experience is classical (and in particular, that we typically experience determinate values upon measurement), we can and do experience quantum phenomena. For example, not long ago there was experimental confirmation of quantum behaviour of comparatively large systems, such as carbon-60 molecules (see Arndt et al. 1999).

Second, taking precisifications to be ones in which classical laws are operative violates supervaluationist constraints on admissible precisifications—namely, that precisifications cannot be determinately incompatible with (cannot determinately misrepresent) the actual world. In particular, the true claim that 'the position and momentum of a system cannot be jointly fully precise' is determinately true if the actual world is, as we are assuming, a quantum world; but classical worlds in which every system has determinate position and momentum will be worlds in which this claim is false, not true; hence any such world would fail to be an admissible precisification. Relatedly, it is unclear how to make sense, on the present suggestion, of the supervaluationist claim that QMI involves the actual world's being primitively unsettled between certain more determinate options, given that these (classical, non-quantum) options are determinately not the case.

4 Can a determinable-based account accommodate quantum MI?

4.1 A determinable-based account of MI

An object-level approach to MI places indeterminacy in states of affairs themselves, with the basic idea being that MI involves the having of an indeterminate property. Wilson (2013) suggests, more specifically, that MI involves, in the first instance, a state of affairs whose constitutive entity (object, system, etc.) has a determinable property, but no *unique* determinate of that determinable:

> *Determinable-based MI*: What it is for a state of affairs to be MI in a given respect R at a time t is for the state of affairs to constitutively involve an object (more generally, entity) O such that (i) O has a determinable property P at t, and (ii) for some level L of determination of P, O does not have a unique level-L determinate of P at t.

Why look to determinables for insight into MI? The motivation reflects that determinables are distinctively *unspecific* properties which admit of specification by determinate properties. Other kinds of properties admit of specification: disjunctions are less specific than disjuncts, conjuncts are less specific than conjunctions; a genus is less specific than a species. But nothing prevents these specifiable properties from being themselves precise, or ontologically reducible to precise properties. By way of contrast, as Wilson (2012) argues, determinables are *irreducibly* imprecise, and in particular are not ontologically reducible to any complex combinations of determinates; so they represent a promising basis for characterizing worldly indeterminacy.

Now, traditionally it has been assumed that when something has a determinable property at a time, it also has a unique determinate at that time, for every level of determination. But as argued in Wilson (2013) (see also the discussion in Wilson 2017), the assumption of unique determination is too strong, and should be rejected as a general feature of determinables and determinates. The failure of the traditional assumption reflects that there can be cases where a determinable instance is not uniquely determined due either to there being *too many* candidate determinate instances, or due to there being *no* candidate determinate instances. Cases involving multiple instantiated determinates correspond to a 'glutty' implementation of a determinable-based account; cases involving no instantiated determinates correspond to a 'gappy' implementation of the account. In the remainder of this paper we aim to argue that both gappy and glutty implementations can be seen as live options for treating the varieties of QMI, and to identify certain potential costs and benefits of each approach, as a basis for further investigation.

As set-up for this discussion, it is worth saying a bit more about glutty and gappy implementations of a determinable-based account. To start, there are two ways in which the conditions in *Determinable-based MI* might be satisfied in glutty fashion. (Actually, there are three—the third is new, and will be discussed after setting out the usual glutty and gappy approaches.)

First, and as discussed in Wilson (2013), glutty satisfaction of the conditions might proceed by way of 'multiple relativized determination', as illustrated by (a reasonable interpretation of) the case of an iridescent feather—red from one perspective, blue from another, and where the differences in determinate colours reflect interference phenomena as opposed to the feather's having parts which are different specific shades of colour. In such a case, for any given time *t*, the feather has the determinable property *colour* at *t*; but it would be metaphysically arbitrary to take one of the determinates of this determinable—either *red* or *blue*—to be "the" shade had by the feather at *t*. Rather, the determinate shades are had by the feather in relativized fashion. Depending on the account of colour at issue, the phenomenon of relativization at issue might advert to actual or possible mental observers, or rather to 'perspectives' understood objectively (e.g., as rays from spatial locations in the vicinity of the feather to the feather); in the former case the relativized determinates might be, and in the latter case the relativized determinates are, concurrently instantiated at a time *t*. Either way, the dependence of the determinable's determination on multiple available circumstances that are in some salient sense on a par undercuts the uniqueness assumption.

A second way in which the conditions in *Determinable-based MI* might be satisfied in glutty fashion, also discussed in Wilson (2013), is illustrated by the case of indeterminate macro-object boundaries. Here the macro-object—a cloud, say—has a determinable boundary property, but no unique determinate of that boundary property, not (or at least, not clearly) due to relativization phenomena, but rather due to the determinable property's being concurrently multiply realized by the precise boundary properties of multiple micro-configurations in the vicinity of the cloud, and where (as per Unger's 1980 'problem of the many') no one of these micro-configurations (or associated comparatively precise boundary properties) is appropriately taken to be identical with or 'the' realizer of the cloud (or its boundary property). As we see it, this sort of case is interestingly different from the feather case, in that, even if one is inclined (which one might not be) to suppose that the cloud has the determinate boundary properties in relativized fashion, in any case the multiple determinates of the determinable property are had by objects (here, micro-configurations) different from the one (here, the cloud) entering into the indeterminate states of affairs. And as in the case of an objective take on the perspectives at issue in the feather case, the determinates at issue in a given case of macro-object boundary MI at a time *t* are concurrently instantiated at *t*.

Turning to gappy implementations: the failure of unique determination at the heart of *Determinable-based MI* is also present in cases in which an object has a determinable property at a time *t*, but *no* determinates of the determinable are instantiated at *t*, even as a relativized matter of fact, or as possessed by some other object(s). Wilson (2013, 2016) argues that cases of the genuinely open future are properly seen as involving gappy determinable-based MI, and as we will see, some find it natural to see certain cases of QMI as involving (completely) undetermined determinables.

Finally, we here observe a new way in which the conditions of *Determinable-based MI* might be satisfied—namely, if instantiation can come in degrees, and it suffices for a determinable to not be uniquely determined that none of its determinates are instantiated to degree 1. The degree-theoretic approach here is superficially similar to but importantly different from that in Smith and Rosen (2004); in particular, we reject three claims that Smith and Rosen accept, including that all fundamental properties are maximally precise, that MI involves an object's being an 'intermediate instance' of a precise property, and that 'fuzzy logic' is the correct logic of MI. Though we cannot fully enter into a comparative discussion here, the general idea is that in cases where instantiation can come in degrees, claims of the form 'object *O* has property *P*' are not truth-evaluable, since incomplete; rather, what is truth-evaluable are claims of the form 'object *O* has property *P* to degree *d*', and such claims are either true or false, as per usual.[26] On this understanding, no special 'fuzzy' logic is required. While one might aim to apply a degree-theoretic approach to either glutty or gappy determinable-based MI,

[26] Compare an endurantist (or 'three-dimensionalist') position as regards the persistence of objects, according to which bare claims of the form 'Object *O* has property *P*' are not truth evaluable, since incomplete; rather, what is truth-evaluable are (on one salient strategy; see Haslanger (1989) for discussion) claims of the form 'Object *O* has property *P* at time *t*'.

in what follows our discussion of this approach as applied to QMI will focus on glutty implementations, for reasons to be made clear down the line.[27]

4.2 Can a determinable-based approach accommodate quantum MI?

It is natural to see quantum observables as having a determinable/determinate structure:

> [T]he determinable/determinate model is one of the most commonly used ways of understanding classical physical quantities like mass [...] so a natural move would be to turn to this model for the case of quantum properties. (Wolff 2015, 379)

It is also natural to see this structure as operative in QMI; as Skow (2010) says, in such cases "the wavefunction does not determine a value for all of the system's determinable properties" (857). Indeed, Lewis (2016) presents EEL in terms of determinables and determinates, in the 'Strict Link' (SL):

> (SL) A system has a determinate value for a given determinable property if and only if its state is an eigenstate of the operator corresponding to the property, and the determinate value is the eigenvalue for that eigenstate.

Again, Lewis sees this as a 'fairly standard' way of understanding quantum states, operative in both philosophy and physics; and he too sees this understanding as naturally suggesting "that quantum mechanics postulates indeterminacy in the world" (76).

Now, as noted, there are linking principles alternative to EEL (and SL, etc.); but to the extent that these are consonant with there being QMI, they will likely be compatible with taking quantum observables to have determinable/determinate structure—after all, their differences turn on how to understand what it is for a property to have a determinate value, not on rejecting such structure. Hence it seems reasonable to proceed on the assumption that there is no in-principle barrier to applying a determinable-based account to the quantum cases at hand.

4.3 Superposition MI

Recall two paradigmatic cases of superposition—Schrödinger's cat in the state $\frac{1}{\sqrt{2}}(|alive\rangle + |dead\rangle)$, and a single electron in the two-slit experiment, in the state $\frac{1}{\sqrt{2}}(|S_1\rangle + |S_2\rangle)$, where S_1 and S_2 represent having gone through the left or right slits, respectively. What are the determinable properties in these cases? These are given, as in Lewis's discussion above, by whatever observable is at issue—in the case of Schrödinger's cat, something along lines of the property *having a certain life status*, with determinates *being alive* and *being dead*, and in the case of the electron, something along lines of the property *having traveled from the emitter to the*

[27] See especially note 30.

detector, with determinates *having traveled through the left slit* and *having traveled through the right slit*.

Now, in these cases, as per EEL, the system has the determinable property—the cat in the box has a life-status, the electron traveled between emitter and detector—but no unique determinate of that property, as is registered by the system's being in a state of superposition of the relevant determinates. As such, these cases satisfy the conditions in *Determinable-based MI*.

So far, so good. Next, is a gappy or rather a glutty implementation of a determinable-based approach best suited to such cases of superposition MI? We note three *prima facie* reasons to prefer a glutty implementation.

First, taking the formalism at face value in these cases, superpositions involve additive combinations of determinate states. Descriptions often reflect this, as when Einstein (1939) describes Schrödinger's case in glutty terms: "At a fixed time parts of the Ψ-function correspond to the cat being alive and other parts to the cat being pulverized".

Second, interference of a particle with itself, as seems to occur in the double-slit case, seems naturally understood as involving interacting determinate states; hence Dirac (1930) says, "So long as the photon is partly in one beam and partly in the other, interference can occur when the two beams are superposed" (8-9). It's unclear how to make sense of such interference in gappy terms, given that on a gappy approach no determinates of the determinable are instantiated, either in relativized or unrelativized fashion. Even if there is some broadly theoretical way of predicting the interference patterns associated with the experiment, one might naturally seek to understand these patterns as grounded—as in the classical case—in some actually occurring physical phenomena. A glutty implementation, involving as it does the occurrence of multiple determinates, provides a substantive physical basis for these patterns.

Third, one might worry that a gappy implementation will wash away important quantum information stored in the coefficients of the superposition state. To see the concern, consider the following quantum states: $|\omega_1\rangle = \frac{1}{\sqrt{2}}|\psi\rangle + \frac{1}{\sqrt{2}}|\varphi\rangle$ and $|\omega_2\rangle = \sqrt{0.1}|\psi\rangle + \sqrt{0.9}|\varphi\rangle$; and suppose $|\psi\rangle$ and $|\varphi\rangle$ are eigenstates of observable O having eigenvalues 1 and -1, respectively. If a gappy implementation leaves one unable to say more than that system S has no determinate value of O, the concern is that this conflates, so to speak, states $|\omega_1\rangle$ and $|\omega_2\rangle$; for this much is true of both states, yet they carry importantly different information, encoded in the coefficients of the superposition terms.

It remains to consider whether and how a glutty implementation might be applied to cases of superposition. As above, there are at least three different routes to glutty satisfaction of the conditions of *Determinable-based MI*, reflecting whether the determinable at issue admits of multiple relativized determination in which the determinates, when instantiated, are possessed by the same object (as with the iridescent feather), whether the determinable at issue admits of non-relativized multiple determination in which the determinates are concurrently possessed by different objects (as with the determinate boundaries of micro-aggregates in the vicinity of a determinable macro-object boundary), and one according to which the failure of unique determination reflects that multiple determinates of the determinable are instantiated to degree less than 1 (but greater than 0). We address the possibility of each approach in turn, focusing on what would be needed for such an implementation to appropriately accommodate interference interference effects characteristic of the double-slit experiment.

To start, might superposition MI be understood as involving multiple relativized determination? We think so, if the application satisfies three associated conditions. First, in order to provide an occurrent ground for the observed interference patterns, we must require that the multiple determinates be concurrently and not just potentially instantiated. Second, we must be careful to avoid thinking of the relativization at issue as tracking what would be observed if a measurement were performed.[28] Given that the determinates are relative to some or other circumstances, these must rather be understood as pertaining to whatever physical circumstances are associated with the electron's occupying different positions. An advantage of understanding superposition MI as involving multiple relativized (concurrently instantiated) determination is that relativizing the positions of the electron blocks a kind of concern to the effect that no one object can have incompatible properties—a concern that is more salient in certain cases of superposition (such as Schrödinger's cat). Of course, more needs to be said about the nature of the relativizations at issue, but ultimately this is a matter for physicists rather than metaphysicians. In any case, on the present approach, the electron concurrently travels through both slits, albeit in relativized fashion, and that this is so is responsible for the interference effects manifested on the detecting screen.[29]

It is less clear to us that superposition MI can be understood as involving non-relativized multiple determination. Such an application would require that there be multiple bearers of the multiple determinates, each different from the electron. Perhaps this can be seen to make sense, but we will not pursue that strategy here.

The third glutty approach, on which instantiation can come in degrees, strikes us as promising. Here the general suggestion is that one can extract the degree of instantiation of a particular determinate—i.e., eigenvalue—from the square moduli of the coefficient of the corresponding eigenvector in a given quantum state, as per the following degree-theoretic variation on EEL:

> DEEL: A quantum system S has a definite value v for an observable O to a degree y iff $\sqrt{y}$ is the absolute value of the coefficient of the associated eigenvector having eigenvalue v in the quantum state of S.

By way of schematic illustration, consider a system S and observable O with eigenvectors $|\psi\rangle$ and $|\varphi\rangle$, with eigenvalues 1 and -1, respectively. Given that the

[28] Here we agree with Wolff (2015) (and also with an anonymous referee) that an appeal to perspectives or measurement contexts invites an (incorrect) reading of the multiple determination at issue according to which "different determinates of the same determinable [...] could be instantiated at the same time, but would need to be 'looked at' from different perspectives" (384).

[29] It may be worth observing that the present suggestion can be understood as a variation on a many-worlds approach to macro-superpositions, where the relative states are (a) instantiated within a single world, and (b) apply to micro-superpositions as well as macro-superpositions.

state of S is $|\omega\rangle$, first write the state of S using the eigenvectors of O as a basis, along the following lines: $|\omega\rangle = c_1|\psi\rangle + c_2|\varphi\rangle$. Then extract the degree of instantiation of the different eigenvalues from the coefficients of the respective eigenvectors. Here, S instantiates $O = 1$ to degree $\left|c_1^2\right|$, and S instantiates $O = -1$ to degree $\left|c_2^2\right|$.[30]

More specifically, we can understand superposition MI in the case of the double-slit experiment as follows. Let $|1\rangle$ be the vector representing the state according to which the particle passes through slit 1, and let $|2\rangle$ be the vector representing the state according to which the particle passes through slit 2. In the double slit case the quantum state would then be, say: $|\psi\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle)$. According to the degree theoretic treatment of glutty MI, the particle instantiates both determinates (passing through slit 1 and passing through slit 2) to degree 1/2.[31] That each determinate is instantiated to a degree larger than zero provides, in turn, an occurrent physical basis for the interference effect.

Two further points are worth noting. First, as discussed, cases of QMI are typically associated with properties whose interdependence prevents them from concurrently taking on determinate values. At least in the paradigmatic cases of superposition MI, this interdependence and associated fact of mutual exclusion can be seen as the familiar variety associated with determinates of a single determinable: just as nothing can be both red and blue (simpliciter) all over, neither can Schrödinger's cat be both alive and dead (simpliciter), and nor can the particle go through both slits (simpliciter). At best the determinate values must be had in relativized or degree-theoretic fashion.

Second, by way of comparison with a supervaluationist treatment, and independent of whether cases of superposition MI are treated using a gappy or glutty implementation of a determinable-based account, there is no danger of running afoul of the Kochen–Specker theorem, or, more generally, of violating constraints stemming from interdependent properties. This is clear for a gappy implementation, since here the more determinate properties are not at all instantiated, but it is also true for a glutty implementation, for neither multiply relativized, non-relativized multiply occurrent, nor degree-theoretic versions of such an implementation rely on taking it to be possible, much less actual, that all QMI is resolved. Similar remarks apply to a determinable-based treatment of other sources of QMI.

[30] On the suggestion at hand, it is unclear that it makes sense to apply a degree-theoretic approach to gappy QMI, at least for cases where no determinates of the determinate are instantiated, on pain of violating certain quantum constraints; hence we do not pursue this strategy here.

[31] To be clear: this description makes sense on the supposition that the superposition is not collapsed via measurement or otherwise resolved. It is important to register that on a degree-theoretic understanding of (e.g.,) superposition-based QMI of the sort operative in the double-slit experiment, the claim is not that, insofar as the particle instantiates both determinates to a certain specific degree (<1) when in a superposition state, one will thereby be in position to measure the particle as passing through a given slit to the associated degree. Such a result would violate well-known empirical results, but in any case on the degree-theoretic approach under consideration here, the operative assumption is that, as per usual, measurements resolve the superposition in such a way that one only ever measures the particle as having gone through one of the slits (that is, as instantiating the relevant determinate to degree 1). Thanks to Michael Miller for discussion here.

4.4 Incompatible observable MI

Paradigmatic cases of incompatible observable MI involve position and momentum, and different components of spin. Bokulich (2014) endorses a gappy implementation of *Determinable-based MI* as accommodating position-momentum MI:

> On the determinable-based account of MI, the position of the particle [...] is a vague property: while the particle possesses the determinable property of position, it does not possess a determinate value for that determinable.[32] (467)

Bokulich also sees a gappy implementation as promising for spin MI. So does Wolff (2015):

> Assigning a determinate spin value to a particle in a particular direction [say, 'up' to 'spin-*z*'] necessarily leaves the spin values of that particle in other directions indeterminate. [...] Permitting the instantiation of determinables without determinates helps to describe this phenomenon, because we can say that *x*-spin and *y*-spin are determinables with two determinates each, and neither of these determinates is instantiated even though the determinables 'spin-*x*' and 'spin-*y*' are. (385)

As Bokulich's and Wolff's discussions indicate, both cases satisfy the conditions of *Determinable-based MI*: in Bokulich's case, the system has the determinable position, but (thanks to the system's also having a somewhat determinate value of momentum) no unique determinate of that determinable; in Wolff's case, the system has the determinables spin-*x* and spin-*y*, but (thanks to the system's also having a determinate value of spin-*z*) no unique determinates of either determinable.

As we'll see, Wolff has some reservations about whether a determinable-based approach suitably illuminates the case of spin MI. These reservations aside, we agree with Bokulich and Wolff correct that a gappy approach to these cases is in any case promising.[33]

What are the prospects for a glutty implementation of incompatible observable MI? Here again we are inclined to think that either a glutty implementation on which the failure of unique determination is associated with multiple relativized determinates which are concurrently instantiated in suitably objective, non-perspectival, fashion, or a glutty implementation involving degrees of instantiation, will serve to accommodate such QMI.

Common to both glutty approaches, as directed at the case of spin MI, is that what it comes to for the spin value of a particle in a given direction to be MI (given that the spin value in an orthogonal direction has received a determinate value) is for

[32] It is also true that on glutty implementations, the object having the determinable property "fails to have a definite value" of the determinable—at least, *simpliciter*, or to degree 1. In context, Bokulich is clear that she has in mind a gappy implementation.

[33] Torza (2017) objects that a gappy implementation violates the supposition that, e.g., claims that a system has a determinable position should be formalized in existential terms as the having of some determinate position; but we reject this supposition as building in the reducibility of determinables to determinates.

the system to have a determinable spin value in that direction (e.g., the determinable spin-x), but for the system not to have a unique determinate of that determinable, due to the concurrent instantiation of the determinates of that determinable (e.g., spin-x-up and spin-x-down). The concurrent instantiation of these multiple determinates would, on the first approach, be relativized (*nota bene*: to some physical states of affairs, having nothing in particular to do with perspectival or measurement-involving contexts). So a case of spin MI generated, e.g., by a measurement in the spin-z direction would involve four concurrently instantiated relativized determinates: spin-x-up and spin-x-down (relative to the x direction) and spin-y-up and spin-y-down (relative to the y direction).

Alternatively, a glutty account of spin MI might involve associated degrees of instantiation rather than relativization. Such an application is straightforward, for any eigenstate of an observable that is a member of an incompatible set will be a superposition of eigenstates of its incompatible observable(s). The implementation thus requires only that the quantum state be expressed using the other observable's eigenstates as a basis, at which point the degrees of instantiation can be extracted from the coefficients, as per the previous section.

The latter observation serves to provide some additional support for a glutty treatment of incompatible observable MI, of one or other of the aforementioned varieties; for given that such an expression will be a superposition, the aforementioned advantages of a glutty treatment of superposition will also be inherited by a glutty treatment of incompatible observable MI.

4.5 Entanglement MI

Finally, consider the case of entanglement MI paradigmatically associated with the so-called singlet EPR-state (i.e., $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2-|\downarrow\rangle_1|\uparrow\rangle_2)$, in which the quantum component systems S_1 and S_2 lack a determinate spin value. Note that there seems to be nothing special about spin in this case, so that we might also consider, e.g., a two-particle system entangled in the position degree of freedom, as per state $\frac{1}{\sqrt{2}}(|x_1\rangle_1|x_2\rangle_2-|x_2\rangle_1|x_1\rangle_2)$. In this case we would say that particles S_1 and S_2 do not have any determinate position.

Once again, a gappy implementation seems natural. Bokulich (2014) endorses a gappy implementation for the case of spin entanglement:

> Consider again two particles A and B. Suppose that these two particles have become entangled in the spin degree of freedom. This means that in this situation spin is a vague property. What makes entanglement different from the failure of value definiteness discussed in Section 4.3 [involving incompatible observables], is that these vague properties now also exhibit (nonlocal) correlations. (Bokulich 2014, pp. 468–469).

We have no reservations about a gappy implementation that are pertinent specifically to this or related cases.

Can entanglement be treated in glutty terms? It seems so, in either its relativization or degree-theoretic variants. A natural way of implementing a glutty relativization approach would draw on considerations advanced in non-standard Everettian interpretations denying the multiplicity of world-branches (e.g., that defended in Conroy 2012). In the case of spin, for example, we could say that relative to the first particle's having spin-up, the second particle has spin-down, and that relative to the first particle's having spin-down, the second particle has spin-up (and *vice versa*).[34]

As for the degree of instantiation variant: here we would first need to calculate the states of the component parts. These will be mixed states; we can then make use of the fact that every mixed state can be written as a weighted sum of pure states. These weights will represent the degrees of instantiation of the relevant determinates, as per the coefficient-based recipe of the previous sections.

5 Concluding remarks

Quantum mechanics and metaphysical indeterminacy are deeply connected. Indeed, some take QMI to represent the best case for thinking that there is or could be properly metaphysical indeterminacy, and some take the intelligibility of quantum mechanics to rest in part on whether we can make sense of seeming QMI. But as Darby (2010) correctly notes, "this connection is going to work only if quantum-mechanical indefiniteness really is a species of indeterminacy as explored by metaphysicians, and it is not immediately obvious that this is so. On the contrary, the connection is sensitive to the precise way in which indeterminacy is understood" (228).

Darby's and Skow's reasons for rejecting a metaphysical supervaluationist understanding of QMI do not succeed, in leaving open several supervaluationist responses. Still, we have argued, they are right that QMI is not properly understood in supervaluationist terms; for the property dependencies characteristic of quantum phenomena, which rule out taking QMI to involve indeterminacy between determinate options, are present not just on the orthodox interpretation but also on (common understandings of) all the main non-orthodox interpretations conceiving of quantum indeterminacy in metaphysical terms.

Luckily, as Bokulich and Wolff suggested, and as we've here confirmed and developed, QMI can be properly understood in determinable-based terms, and more specifically can be seen as involving either gappy or glutty implementations of a determinable-based approach. As we've discussed, there might be advantages to treating some sources of QMI in glutty rather than gappy terms. A full assessment of which treatments are best suited to which sources of QMI must await another day, however. Our main constructive goal here has been to show that one or other implementation of a determinable-based account is available as providing a basis for accommodating the full spectrum of sources of QMI, as indicated on most (common understandings of) live interpretations of quantum mechanics. In the process, we

[34] This proposal suggests a reading of the quantum state in thoroughly relational terms, bearing some similarity to the relational interpretation advanced in Rovelli (1996) and discussed in Laudisa and Carlo (2013).

hope to have contributed to the project of rendering these interpretations metaphysically intelligible: at least so far as commitment to MI is concerned, quantum mechanics is not, after all, as mysterious as it has sometimes been thought to be.

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