Quantum Metaphysical Indeterminacy*

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Can we make metaphysical sense of quantum indeterminacy?

As Feynman (1982) observed, "we always have had a great deal of difficulty in understanding the world view that quantum mechanics represents" (471). Among the perplexing aspects of quantum mechanics is its seeming, on a wide variety of presently live realist interpretations (including but not limited to the so-called 'orthodox' interpretation), to violate the classical supposition of 'value definiteness', according to which the properties—a.k.a. 'observables'—of a given particle or system have precise values at all times.[1] Indeed, value indefiniteness lies at the heart of what is supposed to be distinctive about quantum phenomena, as per the following classic cases:

- Prior to detection, the location of a particle in a double-slit experiment is indeterminate
- Prior to opening the box, Schrödinger's cat is neither determinately alive nor determinately dead
- A particle cannot have precise values of both position and momentum at the same time
- A particle measured as spin-x-up at t has indeterminate spin-y and spin-z values at t
- The components of a spin-entangled state do not have determinate values of spin

On the interpretations in question, the indeterminacy in such cases is taken to be metaphysical, not merely epistemological, much less semantic. As Wolff (2015) notes, on an orthodox reading,

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[1] It has also been suggested that quantum objects may be metaphysically indeterminate in failing to have determinate identity; see, e.g., Lowe 1994 and French and Krause 2003. Here we focus only on property value indeterminacy.

"quantum mechanics suggests *metaphysical* indeterminacy, not (merely) epistemic indeterminacy" (380), and Darby (2010) notes, more generally, that "there are obstacles in the way of [...] interpreting such indefiniteness as merely epistemic or representational [...] Quantum mechanics, then, looks a likely source of examples of genuine metaphysical indeterminacy" (227).

The question before us, then, is this: Can we make sense of quantum indeterminacy as being genuinely metaphysical, and if so, how?

In Feynman's time, there were no developed accounts of metaphysical indeterminacy (henceforth: MI). Recently, however, two new approaches to MI have been proposed, each of which aims to provide an intelligible basis for this phenomenon.[2]

- On the metaphysical supervaluationist approach developed by Akiba (2004), Barnes (2006, 2010), Williams (2008), Barnes and Williams (2011), and others, MI involves the world's being primitively unsettled about which of some range of completely determinate options obtains. Though on this approach MI is taken to be primitive, proponents model this primitive along lines familiar from supervaluationist treatments of semantic indeterminacy, with metaphysical indeterminacy reflecting unsettledness not between linguistic precisifications, but between precisificationally possible worlds or states of affairs.

- On the determinable-based approach developed by Wilson (2013, 2017), MI involves the obtaining of an indeterminate state of affairs, in which (in the simplest case) an entity (object, system, etc.) has a determinable property, but no *unique* determinate of that determinable. As we'll discuss, there are two ways that a determinable can fail to be uniquely determined: first, if there are too many candidate determinates (corresponding to 'glutty' MI); second, if there are none at all (corresponding to "gappy" MI).

Three further points of difference are worth noting:

1. On a metaphysical supervaluationist approach, MI involves its being *indeterminate* which *determinate* (precise) state of affairs obtains; on a determinable-based approach, MI involves its being *determinate* (or just plain true) that an *indeterminate* (imprecise) state of affairs obtains.

   Reflecting this structural difference, we follow Wilson (2013) in sometimes heuristically characterizing supervaluationist accounts as 'meta-level' accounts, and determinable-based accounts as 'object-level' accounts.

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[2] Other accounts aiming to characterize properly metaphysical indeterminacy include those in Smith and Rosen 2004 and Torza in progress; beyond notes 15, 17, and 19, we leave discussions of these alternatives for another day.

2. On a metaphysical supervaluationist approach, MI generates *propositional* indeterminacy in, e.g., certain propositions expressing that a given determinate/precise state of affairs or precisificationally possible world obtains, or that a given object (system, etc.) has a given property; this propositional indeterminacy is then treated by means of a new indeterminacy operator. On a determinable-based approach, MI does not generate any propositional indeterminacy, and so no indeterminacy operator is required. Rather, MI involves a certain pattern of instantiation of determinable and determinate properties; consequently, propositions expressing the obtaining of any given state of affairs (whether precise or imprecise) or the having of any given property (whether determinate or determinable) will, if meaningful, be determinately (i.e., straightforwardly) true or determinately false, as per classical semantic usual.[3]

3. On a metaphysical supervaluationist approach, and as is familiar from discussions of semantic supervaluationism, it is possible (if truth is 'super-truth'—truth on every precisification) to preserve certain theorems of classical logic, though certain classical laws of inference (including contraposition) must be rejected; whether the classical semantic principle of bivalence is preserved depends on whether there is a privileged precisification (as per 'non-standard' supervaluationism, and as is endorsed by Barnes and Williams 2011). On a determinable-based approach, and again reflecting that this approach does not generate propositional indeterminacy, no revisions to classical logic or semantics are required.[4]

In this paper, we consider whether either a metaphysical supervaluationist approach or a determinable-based approach can accommodate quantum MI (QMI), henceforth restrictedly understood as involving quantum value indefiniteness. We start by discussing certain broadly theoretical indications of QMI, and distinguishing three seemingly different sources of QMI (§1). We then show that previous arguments for the conclusion that metaphysical supervaluationism cannot handle QMI, due to Darby (2010) and Skow (2010), are unsuccessful, in leaving open several supervaluationist responses (§2). We go on to provide more comprehensive argumentation for the negative conclusion; here, among other salient results, we establish that the problems for supervaluationism extend far beyond the orthodox interpretation that is the focus of Darby's and Skow's discussions (§3). We then argue that a determinable-based account can successfully accommodate all three varieties of QMI, with certain sources being most naturally treated via a glutty implementation, and others being most naturally treated via a gappy implementation; here we follow Bokulich (2014) in certain respects and depart from her in others (§4). We close by responding to

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[3] See Wilson (2017) for reasons to reject taking MI to generate propositional indeterminacy.

[4] See Calosi and Wilson in progress for further discussion.

certain concerns with a determinable-based approach to QMI, either in its glutty implementation (Bokulich 2014) or in general (Wolff 2015) (§5). We close by observing the positive mutual bearing of our results on the coherence and intelligibility of both quantum mechanics and metaphysical indeterminacy (§6).

1 Preliminaries: linking principles and sources of QMI

In this section we discuss the linking principles that have been taken to underlie attributions of QMI, and highlight three seemingly distinct sources of QMI.

1.1 The eigenstate-eigenvalue link and its variants

Suggestions that there is QMI typically advert to certain linking principles that have been taken to underlie attributions of determinate properties, and conversely, judgments of value indeterminacy, as in Wolff's (2015) remarks:

> Orthodox quantum mechanics assumes that a system is in an eigenstate for some observable O iff that observable takes one of its eigenvalues. While some observables are compatible in the sense that a system can at the same time be in eigenstates with respect to these observables [...], many observables are incompatible. This is true in particular of position and momentum, and of the different components of spin. When a system is in a state of superposition with respect to some observable O, the system has no eigenvalues with respect to that observable. (380)

Here Wolff appeals to the *Eigenstate-Eigenvalue Link* (EEL), linking eigenstates with value determinateness:

> (EEL): A quantum system has a definite value v for a particular observable O *iff* it is in an eigenstate of O belonging to eigenvalue v.

As Lewis (2016) observes, EEL represents a "fairly standard way of understanding quantum states" (76); correspondingly, in what follows we present our taxonomy of sources of QMI in terms referring to EEL.

There are, however, alternative linking principles that one might prefer, depending on one's commitments and which interpretation is at issue. Here we note certain of these and briefly register how their endorsement might bear on attributions of quantum MI. One alternative is *Von Neumann's Principle* (VNP):

> (VNP): A quantum system has a definite value v for a particular observable O *iff* a measurement of O is certain to result in value v.

We see VNP as an operational variant on EEL, naturally seen as presupposing something like the orthodox interpretation, on which measurement induces collapse.

Two other linking principles are associated with interpretations that either do not involve collapse or on which collapse does not result in a fully determinate value (as per the GRW interpretation, on which 'collapse' leaves a lingering 'tail'). One of these is the *Fuzzy Link* (FL), discussed by Albert and Loewer (1992):

> (FL): A quantum system has a definite value v for a particular observable O *iff* the square projection of its state into an eigenstate of O is greater than $1 - P$, for some P.

Another is the *Vague Link* (VL), discussed by Lewis (2016):

> (VL): A system has a determinate value for a given determinable to the extent that the square projection of its state onto an eigenstate of the corresponding operator is close to 1.

A concern about FL and VL for purposes of characterizing quantum MI is that there might be thought to be a conventional element in the choice of parameter P (in FL) or in what it is for a projection to be 'close' to 1 (in VL), in tension with the aim of characterizing properly metaphysical quantum MI. Properly understood, however, there is no deep difficulty here. Indeed, Lewis is clear that VL is aimed at accommodating properly metaphysical indeterminacy: "The vague link, as its name implies, explicitly posits vagueness in the world" (89).

We will later return to VL and certain variants of EEL (§3). Meanwhile, we operate under the assumption that while it is convenient (and again, "fairly standard") to appeal to EEL in characterizing quantum MI, alternative linking principles would also suffice for this, so long as their attributions of determinate values (or the absence thereof) can be seen as reflecting properly metaphysical facts about determinacy and indeterminacy.[5]

1.2 Three sources of quantum MI

There are three seemingly distinct sources of QMI, operative in (i) *superposition of states*, (ii) *incompatible observables* and (iii) *entanglement*. These sources are clearly related—most saliently, as involving properties which are interdependent in ways resulting in one or other failure of value definiteness. But in certain respects the sources are also interestingly different, in ways that enter into both critical and constructive portions of our future discussion.

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[5] For similar reasons, we restrict our attention in §1.2 to observables with discrete spectra, thus admitting eigenvectors.

1. *Superposition.* Consider a system S, and one of its observables O, understood as having two different eigenstates $|\psi\rangle$, $|\varphi\rangle$ belonging to different eigenvalues. Any linear combination $|\omega\rangle = c_1 |\psi\rangle + c_2 |\varphi\rangle$ is a permissible 'superposition' state of S. If the eigenvalues of $|\psi\rangle$, $|\varphi\rangle$ are different, then $|\omega\rangle$ is not an eigenstate of O. It thus follows from EEL that if S is in $|\omega\rangle$, S does not have a definite value of O.

   Case in point: Schrödinger's cat, prior to opening the box.

2. *Incompatible Observables.* Consider a system S and two of its observables, O_1 and O_2.[6] As a first approximation, observables are represented by self-adjoint/Hermitian operators—For any two distinct operators, we can define their commutator as follows: $[O_1, O_2] = O_1O_2 - O_2O_1$. Two observables O_1, O_2 are *incompatible* iff $[O_1, O_2] \neq 0$. Now, if O_1, O_2 are incompatible, some eigenstates of O_1 are not eigenstates of O_2, and *viceversa.* It follows from EEL that if S is in such an eigenstate of, say, O_1, then S does not have a definite value of O_2.

   Case in point: any two or more observables subject to a generalized uncertainty principle, including position and momentum, and distinct components of spin.

3. *Entanglement.* Consider two quantum systems S_1, S_2 that compose system S_{12}. The Hilbert space H_{12} of the composite system is the 'tensor product space' of the Hilbert spaces H_1 and H_2, associated with S_1 and S_2, respectively. Famously there are vectors such as e.g. $|\omega\rangle \in H_{12}$ that cannot be written as $|\psi\rangle \otimes |\varphi\rangle$, with $|\psi\rangle \in H_1$ and $|\varphi\rangle \in H_2$. In this case the state of S_{12} is *entangled.* Given this orthodox understanding, consider a pure entangled state $|\omega\rangle \neq |\psi\rangle \otimes |\varphi\rangle$. Since it is a pure state, it is an eigenstate of some operator. This operator is a sum of operators representing observables of S_1 and S_2—e.g., $O = O_1 - O_2$. It follows from EEL that S_{12} *has* a definite value of O. Yet $|\omega\rangle$ is an eigenstate of neither O_1 nor O_2. It follows from EEL that the component parts S_1, S_2 do not have definite values for O_1 and O_2, respectively.

   Case in point: the singlet EPR-state (i.e., $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$, in which the quantum component systems each lack a determinate spin value.

By lights of the usual linking principle(s), these three kinds of cases all involve QMI, associated with one or more systems' failing to be in one or more eigenstates of (have one or more determinate values of) of one or more observables. The three cases are interestingly different, however, both metaphysically and mathematically:

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[6] In what follows, we use the same notation for both observables and operators; strictly speaking, operators are mathematical objects representing observables.

- Metaphysically, cases of superposition MI involve the failure of a (typically, simple) system to be in any eigenstate of (have any determinate value of) a single observable; cases of incompatible observable MI involve the failure of a (typically, simple) system to be in any eigenstate of (have any determinate value of) one of its observables, given that it is in an eigenstate (has a determinate value of) some other (incompatible) observable; cases of entanglement MI involve the failure of components of a composite system to be in eigenstates of (have determinate values of) certain observables, given that the composite system is in an eigenstate of (has a determinate value of) a related observable.[7]
- Mathematically, the indeterminacy in each of the three cases is underpinned by different mathematical facts: linearity in the case of superposition, non-commutative operators in the case of incompatible operators, and tensor product laws in the case of entanglement.

One might wonder whether, in spite of these differences, there is a single source underlying these sources of seeming QMI, or whether one of these is more fundamental than the others. Perhaps so, but for present purposes we needn't enter into these further metaphysical details. Even if there is a deeper commonality, what is important for our purposes is that there are also respects in which these cases are different, as there clearly seem to be; for as we will see, there are cases to be made that not all cases of quantum MI can be treated alike, whether supervaluationist or determinable-based MI is at issue.

2 Can a supervaluationist account accommodate quantum MI?

2.1 Metaphysical supervaluationism

As above, a metaphysical supervaluationist account takes a 'meta-level' approach, according to which MI involves its being indeterminate which state of affairs, of some range of determinate/precise states of affairs, obtains. As Barnes (2010) expresses the general idea:

> It's perfectly determinate that everything is precise, but [. . .] it's indeterminate which precise way things are. (622)

Somewhat more specifically, Barnes and Williams (2011) say:

> When *p* is metaphysically indeterminate, there are two possible (exhaustive, exclusive) states of affairs—the state of affairs that *p* and the state of affairs that not-*p*—and it is simply unsettled which in fact obtains.

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[7] Entanglement might also be present in a simple system (see Hasegawa 2012, in which case the characterization of entanglement MI would need tweaking.

Note that the sense of a 'possible' state of affairs (more generally, world) here is one that is restricted to possibilities that are compatible with what is actually the case, since otherwise it would be settled that such an (incompatible) state of affairs (possibility) does *not* obtain.

Again, metaphysical supervaluationists take MI to be primitive, but aim to explicate the phenomenon by exploiting a structural similarity to semantic supervaluationist accounts. On semantic accounts, indeterminacy is taken to reflect our not yet having settled on a fixed interpretation of certain expressions in our language. Such indeterminacy is modeled by appeal to a range of admissible precisifications of our language, each compatible with existing (determinate) usage of our terms, and in each of which all semantic indeterminacy has been resolved; indeterminacy is reflected in there being admissible precisifications which differ as regards the extension of a given expression (e.g., 'bald'). On metaphysical supervaluationist accounts (and here drawing on Barnes and Williams 2011), the appeal is not to a range of admissible precisifications of our language, but to a space of admissible "precisificationally possible worlds", each compatible with existing (determinate) facts, and in each of which all metaphysical indeterminacy has been resolved. Instead of our language being unsettled as between different precisifications, our world is unsettled as regards "which world is actualized". Rather than indeterminacy reflecting linguistic precisifications' differing as regards, e.g., the extensions of certain expressions, indeterminacy reflects precisificationally possible worlds' differing as regards whether a given state of affairs (e.g., p) obtains. And like semantic supervaluationists, metaphysical supervaluationists formalize the indeterminacy at issue via what Darby (2010) calls a "modal operator approach":

> [Here] indeterminacy may be captured by a modal operator. By this I mean that a 'definitely' operator D may be prefixed to formulas; that the indeterminacy operator ∇ is to D as contingency is to necessity ($\nabla\varphi$ iff $\neg D\varphi \wedge \neg D\neg\varphi$) [. . .]. (228)

Given these structural and formal similarities, it is no surprise that metaphysical supervaluationist accounts are, like semantic supervaluationist acconts, able (again, if truth is 'supertruth') to preserve the theorems of classical logic—an advantage Barnes and Williams (2011) take to hinge on precisificational worlds' being "maximal and classical':

> Importantly, given our picture of indeterminacy, all the worlds in the space of precisifications are themselves maximal and classical. For any p, each precisification will opt for one of p or $\neg p$, and thus, every precisification will represent as true the law of excluded middle, $p \vee \neg p$—and similarly for every classical tautology.

2.2 Can metaphysical supervaluationism accommodate quantum MI?

As Darby (2010) discusses, one might initially see paradigm cases of seeming MI as inviting characterization in terms of a metaphysical supervaluationist (henceforth, just 'supervaluationist') approach, as in the case of Schrödinger's cat:

> [There is] a suggestive parallel between the terms in the superposition and the idea [...] of precisifications. One of the terms in the superposition [...] is a term where the cat is alive, the other is not; that is reminiscent of multiple ways of drawing the extension of 'alive', on some of which 'the cat is alive' comes out true, on some, false. (235)

The supervaluationist might more generally suggest that, as per EEL, when a system is in an eigenstate for some observable, it definitely has a determinate value for that observable, and when it isn't in an eigenstate for an observable, it is indeterminate for each determinate value of the observable whether the system has it, notwithstanding that the observable determinately has exactly one of those determinate values.[8]

2.2.1 The Darby-Skow objection

Nonetheless, as Darby (2010) and Skow (2010) independently note, in characterizing MI in terms of unsettledness between fully determinate worlds, supervaluationism fails to accommodate the 'deep'—i.e., insuperable—quantum MI characteristic of the orthodox interpretation, on which "it is inconsistent to suppose that every observable has a definite value" (Darby, 237). As Skow puts it:

> In the Barnes and Williams model each actual [precisificationally possible] world attributes to each quantum system a value for each determinable property, and all actual worlds agree on the values assigned to properties which have determinate values (854).
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> But it is possible for there to be metaphysical indeterminacy even when it is impossible to precisify reality completely. The orthodox interpretation of quantum mechanics illustrates this possibility. So this theory of MI is not adequate. (851)

Why think quantum MI is insuperable? Darby and Skow cite the Kochen-Specker theorem, according to which, on the orthodox interpretation, the assumption of complete value determinacy leads to contradiction.[9] More specifically: in a Hilbert space of dimension $d \geq 3$ it is impossible to assign a definite value of 1 or 0 to every projection operator P_i such that, if a set of commuting P_i

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[8] Again, we advert to EEL as reflecting a convenient and "fairly standard" way of thinking about when quantum observables have determinate values; but when relevant will discuss the bearing of other linking principles.

[9] See Darby 2010, 237–238, whose presentation follows Hughes 1989, and Skow 2010, 854-56. For a detailed introduction, see Held 2016.

satisfies $\sum P_i = 1$ then the values $v(P_i)$ associated with such projectors satisfy $\sum v(P_i) = 1$.[10] As Bokulich (2014) nicely describes the result for the case of spin:

> [O]ne can measure the spin of a quantum particle, such as a photon (which is a spin-1 particle and so a boson), in three orthogonal directions: S_x, S_y, and S_z. When we measure the square of the spin component in each of these three directions, quantum mechanics requires that one of these directions gets the value 0, while the other two directions get the value 1 (because we know $S_x^2 + S_y^2 + S_z^2 = 2$). The Kochen-Specker theorem then shows that there is no consistent way to assign zeros and ones to all the possible spin directions, such that this constraint is satisfied; we run into the contradiction that one and the same spin direction needs to be assigned two incompatible values. (466)

Hence it is that on the orthodox interpretation, quantum MI is insuperable, hence incompatible with a 'shallow' conception of MI as indeterminacy between fully determinate worlds, as per supervaluationism.

How might a supervaluationist respond? Skow considers and rejects two responses. In what follows, we first observe that Skow's replies to these responses are not compelling; we also draw attention to a third response, which neither he nor Darby address.

The rejectionist response. The first supervaluationist response is to reject the orthodox interpretation whose commitments are incompatible with allowing all observables of a system to be given determinate values.

Skow replies that what matters is not that the orthodox interpretation is correct, but that it is possibly correct. Skow's reply is uncompelling, however, for metaphysicians regularly respond to charges that they cannot accommodate a given possibility by denying the possibility, as a kind of modus tollens. Moreover, given that the orthodox interpretation is commonly seen as implausible in taking collapse to be induced by measurement, the supervaluationist can maintain that its possibility is sufficiently distant that it is safely ignored.[11]

Partial precisifications. A second supervaluationist response involves revising their approach so that its application does not require *perfectly* precise worlds. Here, the thought goes, so long as the precisifications are *more* precise than whatever unsettled world or state of affairs is at issue, one can implement the supervaluationist strategy: indeterminacy would be unsettledness between *more* rather than *maximally* precise options.[12] Skow considers and rejects this strategy, as follows:

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[10] See, e.g., Peres 2002.

[11] Thanks to Nina Emery for this point.

[12] Torza (in progress) offers an account of MI which explicitly abandons complete precisifications; consideration of this account and of whether it is a version of the supervaluationist strategy under consideration is a substantive question, which as noted we leave for another day.

> [S]uppose we [. . .] replace perfectly precise possible worlds with imprecise possible worlds (sets of sentences from a language which suffers from semantic indeterminacy). Even when there is no metaphysical indeterminacy, we can expect it to happen that several imprecise possible worlds do not determinately misrepresent reality. (858)

Skow's reply does not block the 'partial precisification' response, however, since there are ways to ensure that imprecision in worlds tracks metaphysical rather than merely semantic indeterminacy—e.g., by endorsing a non-semantic conception of possible worlds, and maintaining that whenever worlds so understood are imprecise then multiple actuality does entail MI, or by allowing that possible worlds are non-maximal sets of sentences, but maintaining that when language cannot be made fully precise, this is always due to MI.

Non-actual laws. A third supervaluationist response remains—namely, to model MI by appeal to worlds with non-quantum laws.[13] Neither Darby nor Skow consider this response.

2.2.2 The failure of supervaluationist responses

We now provide more comprehensive argumentation for the conclusion that a supervaluationist account of MI cannot accommodate QMI.

The rejectionist strategy. As above, the supervaluationist has principled reason to resist taking incompatibility with the orthodox interpretation to decisively establish a problem for their approach to QMI. After all, there remain numerous other live interpretations on which there is QMI; and for all Darby and Skow establish, the supervaluationist approach might properly accommodate QMI on these interpretations. As we'll now argue, however, a supervaluationist treatment of quantum MI is also at odds with the GRW and Everettian interpretations, such that the 'rejectionist' strategy is clearly unworkable. To prefigure: what is core to the difficulties for supervaluationism is the presence of dependencies among certain observables preventing these from all being given determinate values. To be sure, as Skow (2010) notes, "The Kochen-Specker theorem shows that there are not complete precisifications of reality which respect the dependencies among properties in orthodox quantum mechanics" (858). But other interpretations are also committed to property interdependence preventing complete precisifications, even where EEL is relaxed, or where the Kochen-Specker theorem does not apply.

First, consider GRW, a collapse theory on which said dependencies remain in place. Roughly speaking, GRW introduces a new stochastical physical law, running parallel to Schrödinger's equation; according to this law, every system has a small probability of undergoing a 'hit', resulting

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[13] This response was suggested by Ross Cameron, p.c.

in its state collapsing to a more determinate state. Let us focus on the position of a particle. In undergoing a hit, the particle's wavefunction is multiplied by a narrow Gaussian function; since the function is narrow, but still three-dimensional, it has 'tails' going to infinity. Hence the particle's does not collapse into a precise position eigenstate; indeed, given these tails, the state is not an eigenstate of being confined to any finite range. Thus GRW, together with EEL, will not ascribe any determinate position to the particle; and similarly for other quantum observables. This is enough to spell trouble for the supervaluationist; for every *maximally precise* world they appeal to in characterizing quantum MI will fail to be compatible with GRW.

Nor can this difficulty be overcome by replacing EEL with something like FL or VL; for even if one aims, by endorsing such an alternative linking principle, to be pragmatically flexible or generous about what counts as a determinate property attribution, from the point of view of the metaphysical facts pertaining to the so-called 'tail problem', GRW is committed to quantum MI's being insuperable. Hence the supervaluationist aiming to reject interpretations at odds with their approach will be obliged to reject GRW. And since collapse on this interpretation occurs spontaneously rather than via measurement, this rejection canot be motivated on grounds that GRW need not be taken seriously.

Next, consider an Everettian interpretation. To appreciate what is at stake in this interpretation, consider a simple spin$_x$ measurement of a system S. Let S be in the superposition state $\frac{1}{\sqrt{2}}(|{\downarrow}\rangle_S + |{\uparrow}\rangle_S)$, and let the measuring apparatus M be in its ready state $|Ready\rangle$. Quantum mechanics predicts that after a measurement interaction, the composite system will be in state $\frac{1}{\sqrt{2}}(|{\downarrow}\rangle_S |{\downarrow}\rangle_M + |{\uparrow}\rangle_S |{\uparrow}\rangle_M)$. According to an Everettian interpretation,

> Macroscopically indefinite states like $\frac{1}{\sqrt{2}}({\downarrow}\rangle_S |{\downarrow}\rangle_M + |{\uparrow}\rangle_S |{\uparrow}\rangle_M)$ are physically reasonable after all, and should be understood as describing a multiplicity: a situation in which there are two pointers (or sets of pointers), one pointing left [tracking spin$_x$-up] and one pointing right [tracking spin$_x$-down], and with each dynamically separated from the other. (Wallace 2013, 210; notation altered)

Accordingly, the Everettian will maintain that the measurement interaction results in the existence of two dynamically robust and causally separated world-branches: one in which exists a spin$_x$-up particle and a measurement device registering that outcome, and one in which exists a spin$_x$-down particle and a measurement device registering that outcome.

It might seem that the supervaluationist can accommodate superposition and entanglement MI on an Everettian interpretation, with world-branches corresponding to precisificationally possible worlds—e.g., the aforementioned entangled system might be taken to involve multiple worlds in which spin$_x$ properties are perfectly determined. But this impression is misleading. For crucially, the supervaluationist takes MI to consist in its being *indeterminate* which maximally precise world obtains. But on the Everettian interpretation, it is never indeterminate which branch obtains, however 'obtaining' is understood: either it is determinate that they all obtain (taking a kind of 'meta-world' perspective), or else it is determinate which one of them obtains (taking a local perspective, from within a given branch). Thus, despite being similar in each positing a multiplicity of worlds, an Everettian interpretation is not suited for supervaluationist treatment, even where entanglement or superposition MI is concerned—effectively, because this interpretation makes no room for the primitive meta-level indeterminacy posited by the supervaluationist. Indeed, one might see an Everettian interpretation as denying that there is any genuine quantum MI of either superposition or entanglement varieties.

That said, there remains quantum MI on an Everettian interpretation; for if we want to retain EEL (or something like it) relative to each world-branch, as Everett himself did, cases of incompatible observables will give rise to MI *within* branches. Once again, however, the supervaluationist will not be able to generally accommodate it. For a simple example, consider the state: $\frac{1}{\sqrt{2}}(|x_1\rangle_1 |x_2\rangle_2 - |x_2\rangle_1 |x_1\rangle_2)$. Within each branch, the particle's momentum is maximally indeterminate, and so due to general uncertainty principles of the sort operative on an Everettian interpretation, there will be no more determinate worlds for the supervaluationist to appeal to—at least, none compatible with these principles. The same applies to all incompatible observables (e.g., different spin components). Hence the supervaluationist aiming to reject interpretations at odds with their approach will be obliged to reject the Everettian interpretation—again, for reasons that cannot be pinned on its involving an implausible account of collapse.

Two morals can be drawn from attention to the Everettian interpretation. First, one need not invoke the Kochen-Specker theorem to identify difficulties for a supervaluationist treatment of quantum MI. Again, the difficulties generated by the theorem turn on there being dependencies between properties rendering it impossible for them to all to be given determinate values; but incompatible observables within a single branch are mutually dependent in just this way, in conformity to general uncertainty principles. Second, here we have our first case-in-point of the usefulness of distinguishing between different sources of quantum MI, for as we have just seen, on an Everettian interpretation these sources are not given a uniform treatment.

Finally, it is worth noting that a Bohmian interpretation, on which "the configuration of a system of particles evolves via a deterministic motion choreographed by the wave function" (Goldstein 2012, 1), is also unsuited for supervaluationist purposes. To be sure, a supervaluationist can endorse a Bohmian interpretation without contradiction; but that won't be to the point of showing that supervaluationism can accommodate quantum MI, since Bohm's theory is "completely determinate" (see, e.g., Goldstein 1996, 148), such that any seeming indeterminacy is at best epistemic.

More precisely, this is true for position, the privileged observable on Bohm's account. One might wonder whether properties beside position can be subject to value indeterminacy, since as Lewis (2016) observes, "although the Bohmian strategy arguably makes all the properties we directly observe determinate, it does not thereby make *all* properties determinate" (101–102). Lewis mentions spin, by way of example. But given that on a Bohmian interpretation position is standardly seen as the only physically relevant property,[14] and this property is standardly seen as determinate, it is unclear how Bohmian mechanics can vindicate the existence of genuine quantum MI, as supervaluationists aim to do.

To sum up: metaphysical supervaluationism is incompatible with the orthodox, GRW, and Everettian interpretations (and its compatibility with a Bohmian interpretation is not to the point of accommodating quantum MI).[15] These interpretations comprise the bulk of the standard slate of options for which quantum MI might be at issue; moreover, for the GRW and Everettian interpretations there is no clear independent motivation for their rejection. The joint rejection of all these interpretations would thus be fatally ad hoc, and so the supervaluationist strategy of rejecting any interpretations conflicting with their approach is clearly unworkable.

Partial precisifications. As above, the supervaluationist can respond to the Darby-Skow objection by endorsing a non-standard version of supervaluationism, appealing to partial rather than full precise worlds; and they moreover have resources for responding to Skow's concern that an appeal to partial precisifications would be unable to distinguish cases of semantic supervaluationism from cases of genuinely metaphysical indeterminacy. Nonetheless, as we'll now argue, the partial precisification response fails, for three reasons.

First, if supervaluationism is revised to appeal to imprecise worlds, the point of a meta-level account dissipates. Why understand quantum MI as indeterminacy between less-but-still-indeterminate options? Relatedly, to admit imprecise worlds is effectively to admit object-level MI in addition to primitive meta-level MI. That's an ontological cost; and given that (as we will argue) a determinable-based object-level account can accommodate the full range of quantum MI, would invite rejecting a supervaluationist meta-level account.

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[14] Hence: "In the Bohmian mechanical version of nonrelativistic quantum theory, quantum mechanics is fundamentally about the behavior of particles; the particles are described by their positions, and Bohmian mechanics prescribes how these change with time" (Goldstein 2012, 13).

[15] Considerations of space prevent our discussing modal interpretations, on which only a proper subset of quantum properties are taken to have definite values, but in our view supervaluationism will also run afoul of such interpretations, due to a modal variant of the KS-theorem (see Domenech *et al.* 1981).

Second, in allowing that possible worlds may be imprecise, the supervaluationist means of ensuring compatibility with the theorems of classical logic is undermined—recall Barnes and Williams's remarks concerning the importance of the precisifications' being "maximal and classical" for this purpose—and so in turn is a primary stated motivation for a supervaluationist approach.

Third, for reasons similar to those generating incompatibility with the Everett interpretation, partial precisifications won't generally accommodate quantum MI. Consider a world containing a single particle, that can be in only two position states x_1 and x_2 and two momentum states p_1 and p_2, and which has been measured to be in p_1. Bokulich (2014, 465) uses this case to illustrate why position-momentum MI cannot be given an epistemic interpretation. Here it illustrates the failure of a partial precisificationist version of supervaluationism; for here there are no 'more' determinate worlds to appeal to.

More generally, a similar concern attaches to any incompatible observables, such as position/momentum or different components of spin. These will obey the generalized uncertainty principle; e.g., for position/momentum we will have that $\sigma_{O1}^2\sigma_{O2}^2 = (\frac{1}{2i}\langle[O_1, O_2]\rangle)^2$, where σ_O^2 is the variance of observable O. Hence any maximally precise determination of one observable in an incompatible set will entail *an infinite variance* for the other observables in the set, such that once we have a maximally determined value for one observable, even the slightest precisification of any incompatible observables in the set will violate the generalized uncertainty principle. The revised supervaluationist strategy will be unable to accommodate any such case—for reasons, we observe, not depending on the Kochen-Specker theorem.

This argument bears on the suggestion that the supervaluationist could avoid running afoul of the Kochen-Specker theorem by singling out a proper subset of observables as physically relevant, and appealing to partial precisifications where only these observables were determinate. What the previous argument shows is that implementing the suggestion would require that, for every set of incompatible observables, the supervaluationist identify only one as physically relevant; but in general—notably, for components of spin—there will be no plausible non-arbitrary way of doing this. To be sure, there remains the radical strategy of taking only *one* quantum observable to be physically relevant, which would indeed sidestep our arguments, since (as prefigured) these rely on there being certain dependencies between *different* (physically relevant) quantum observables. But in removing dependencies between real quantum observables the supervaluationist throws the baby out with the bathwater, for any quantum MI there may be stems from these dependencies. Hence it is that the salient interpretation on which there is a sole privileged observable—namely, Bohmian mechanics—is one on which the observable is always determinately-valued, and any seeming indeterminacy is taken to be merely epistemic.

Non-actual laws. As above, the supervaluationist can respond to the Darby-Skow objection that QMI can be modeled by appeal to worlds not subject to value indeterminacy; and insofar as neither Darby nor Skow address this objection, it remains on the table. Nonetheless, as we'll now argue, a supervaluationist response appealing to non-actual (in particular, non-quantum) laws is problematic, for two reasons.

First, on a supervaluationist treatment—whether semantic or metaphysical—the precisifications between which language or the world are supposed to be unsettled are supposed to be 'admissible', in the sense of being candidates for being actual. In particular, on a metaphysical supervaluationist account, the primitive indeterminacy at issue is supposed to reflect our world's being unsettled as regards "which world is actualized". But worlds with non-quantum laws will determinately fail to be candidates for being "actualized", and so be besides the point of a supervaluationist treatment.

Second, even were this restriction to be relaxed, a model of MI appealing to worlds with non-quantum laws would fail to illuminate distinctively *quantum* MI. Here it is worth contrasting the situation faced by a metaphysical supervaluationist aiming to implement the 'non-actual laws' strategy with a similar but less problematic situation that might be faced by a semantic supervaluationist. Suppose that, for contingent and ultimately uninteresting reasons, our language could never be made fully precise. In that case, we might nonetheless find it useful to model semantic indeterminacy as indeterminacy between perfectly precise languages. In the case of quantum MI, however, the failure of quantum worlds to admit of full precisification is not a contingent, uninteresting fact, having nothing deep to do with the phenomenon being modeled; rather, this fact lies at the very heart of quantum phenomena. If the best a metaphysical supervaluationist can do by way of modeling QMI is to appeal to precisificationally possible worlds that are not in fact candidates for being actual, there would remain little or no reason for seeeing supervaluationism as providing genuine metaphysical insight into this phenomenon.

3 Can a determinable-based account accommodate quantum MI?

3.1 A determinable-based account of MI

An object-level approach to MI places indeterminacy in states of affairs themselves, with the basic idea being that MI involves the having of an indeterminate property. Wilson (2013) suggests, more specifically, that MI involves, in the first instance, a state of affairs whose constitutive entity (object, system, etc.) has a determinable property, but no unique determinate of that determinable:

> *Determinable-based MI*: What it is for a state of affairs to be MI in a given respect R at a time t is for the state of affairs to constitutively involve an object (more generally, entity) O such that (i) O has a determinable property P at t, and (ii) for some level L of determination of P, O does not have a unique level-L determinate of P at t.

Why look to determinables for insight into MI? The motivation reflects that determinables are distinctively *unspecific* properties which admit of specification by determinate properties. Other kinds of properties admit of specification: disjunctions are less specific than disjuncts, conjuncts are less specific than a conjunction; a genus is less specific than a species. But nothing prevents these specifiable properties from being themselves precise, or ontologically reducible to precise properties. By way of contrast, as Wilson (2012) argues, determinables are *irreducibly* imprecise, and in particular are not ontologically reducible to any complex combinations of determinates; so they represent a promising basis for characterizing worldly indeterminacy.

Now, traditionally it has been assumed that when something has a determinable property at a time, it also has a unique determinate at that time, for every level of determination. But as argued in Wilson 2013, the assumption of unique determination is too strong even for the paradigmatic case of colors, and should be rejected as a general feature of determinables and determinates.

For example, the case of an iridescent feature—red from one perspective, blue from another—suggests that determination may be a *relativized* phenomenon, in ways undermining the assumption of unique determination: the feather has the determinable property *color* at t; but it would be inappropriately arbitrary to pick one of the determinates of this determinable—either *red* or *blue*—as "the" shade had by the feather at t; and this would be inappropriate whether or not the determinates were relativized to perspectives. Nor, as per usual, can both determinate shades be attributed to the feather in unrelativized fashion, on pain of contradiction. Of course, there are other interpretations of the feather case, but for purposes of showing that the traditional assumption of unique determination is too strong it is enough that one could reasonably interpret it as involving an determinable that is not uniquely determined.

The feather case illustrates one way for the conditions in *Determinable-based MI* to be satisfied, where the determinable is not uniquely determined due to there being too many candidate determinates; such satisfaction corresponds to 'glutty' MI. A second way for a determinable to fail to be uniquely determined is if there are no candidate determinate(s); such satisfaction corresponds to 'gappy' MI. Wilson (2013) and (2017) argues that cases of the genuinely open future involve gappy MI, and as we will see, some find it natural to see certain cases of quantum MI as involving (completely) undetermined determinables.

3.2 Can a determinable-based approach accommodate quantum MI?

It is natural to see quantum observables as having a determinable/determinate structure:

> [T]he determinable/determinate model is one of the most commonly used ways of understanding classical physical quantities like mass [. . .] so a natural move would be to turn to this model for the case of quantum properties. (Wolff 2015, 379)

It is also natural to see this structure as operative in quantum MI; as Skow says, in such cases "the wavefunction does not determine a value for all of the system's determinable properties" (857). Indeed, Lewis (2016) presents EEL in terms of determinables and determinates, in the 'Strict Link' (SL):

> (SL) A system has a determinate value for a given determinable property if and only if its state is an eigenstate of the operator corresponding to the property, and the determinate value is the eigenvalue for that eigenstate.

Again, Lewis sees this as a 'fairly standard' way of understanding quantum states, operative in both philosophy and physics; and he too sees this understanding as naturally suggesting "that quantum mechanics postulates indeterminacy in the world" (page).

Now, as noted, there are linking principles alternative to EEL (and SL, etc.); but to the extent that these are consonant with there being quantum MI, they will likely be compatible with taking quantum observables to have determinable/determinate structure—after all, their differences turn on how to understand what it is for a property to have a determinate value, not on rejecting such structure. Hence it seems reasonable to proceed on the assumption that there is no in-principle barrier to applying a determinable-based account to the quantum cases at hand.

3.3 Superposition MI

Recall two paradigmatic cases of superposition—Schrödinger's cat in the state $\frac{1}{\sqrt{2}}(|alive\rangle + |dead\rangle)$, and a single electron in the two-slit experiment, in the state $\frac{1}{\sqrt{2}}(|S_1\rangle + |S_2\rangle)$, where S_1 and S_2 represent having gone through the left or right slits, respectively. What are the determinable properties in these cases? These are given, as in Lewis's discussion above, by whatever observable at issue—in the case of Schrödinger's cat, something along lines of the property of having a certain life status, having as determinates *being alive* and *being dead*, and in the case of the electron, something along lines of the property of *having traveled from the emitter to the detector*, having as determinates *having traveled through the left slit* and *having traveled through the right slit.*

Now, in these cases, as per EEL or its variants, the system has the determinable property—the cat in the box has a life-status, the electron traveled between emitter and detector—but no unique determinate of that property, as is registered by the system's being in a state of superposition of these determinates. As such, these cases satisfy the conditions in *Determinable-based MI.*

So far, so good. Next, is a gappy or rather a glutty implementation of a determinable-based approach best suited to such cases of superposition MI? We note three *prima facie* reasons to prefer a glutty implementation.

First, taking the formalism at face value in these cases, superpositions involve additive combinations of determinate states. Descriptions often reflect this; Einstein (1939) describes Schrödinger's case in glutty terms: "At a fixed time parts of the Ψ-function correspond to the cat being alive and other parts to the cat being pulverized".

Second, interference of a particle with itself, as seems to occur in the double-slit case, seems naturally understood as involving interacting determinate states; hence Dirac (1930) says, "So long as the photon is partly in one beam and partly in the other, interference can occur when the two beams are superposed" (8-9). Indeed, it's unclear how to make sense of such interference in gappy terms, given that on this approach no determinates of the determinable are instantiated, even in relativized fashion.

Third, one might worry that a gappy implementation will wash away important quantum information stored in the coefficients of the superposition state. To see the concern, consider the following quantum states: $|\omega_1\rangle = \frac{1}{\sqrt{2}}|\psi\rangle + \frac{1}{\sqrt{2}}|\varphi\rangle$ and $|\omega_2\rangle = \sqrt{0.1}\,|\psi\rangle + \sqrt{0.9}\,|\varphi\rangle$; and suppose $|\psi\rangle$ and $|\varphi\rangle$ are eigenstates of observable O belonging to eigenvalues 1 and -1. If a gappy implementation leaves one unable to say more than that system S has no determinate value of O, the concern is that this conflates, so to speak, states $|\omega_1\rangle$ and $|\omega_2\rangle$; for this much is true of both states, yet they carry importantly different information, encoded in the coefficients of the superposition terms.

Turning to a glutty implementation, it remains to spell out how such an implementation might accommodate interference effects characteristic of the double-slit experiment. We see two strategies.

The first draws on the sort of multiple relativized determination operative in the iridescent feather case. A feature of this case—interesting, if not essential, for reasons we will later discuss—is that multiple perspectives and associated relativized determinations can concurrently occur: two persons can synchronically look at the feather from different perspectives, one seeing red, one seeing blue. While such multiple relativized determination prevents attributing a unique determinate shade to the feather, there remains a sense in which the feather can be, albeit in relativized fashion, both red and blue at a time; and indeed, such mutual relativized determination is apparently responsible for certain visual 'interference' effects involving shimmering and the like. Similarly, one might suggest, for the case of the electron in the double-slit experiment: the electron concurrently travels through both slits, albeit in relativized fashion, and that this is so is responsible for the interference effects manifested on the detecting screen.

The second strategy draws on a view on which instantiation can come in degrees. Such a view is compatible with satisfaction of the conditions in *Determinable-based MI*, supposing that it suffices for a determinable to not be uniquely determined that none of its determinates are instantiated to degree 1.[16] The suggestion is then to extract the degree of instantiation of a particular determinate—i.e., eigenvalue—from the square moduli of the coefficient of the corresponding eigenvector in a given quantum state, as per the following degree-theoretic variation on EEL, (DEEL):

> DEEL: A quantum system instantiates $O = x$ to a degree y iff $\sqrt{y}$ is the absolute value of the coefficient of the x's eigenvector in the quantum state of S.

(Note that DEEL might also be seen as filling in VL.) By way of schematic illustration, consider a system S and observable O with eigenvectors $|\psi\rangle$ and $|\varphi\rangle$, belonging to eigenvalues 1 and -1, respectively. Given that the state of S is $|\omega\rangle$, first write the state of S using the eigenvectors of O as a basis, along the following lines: $|\omega\rangle = c_1 |\psi\rangle + c_2 |\varphi\rangle$. Then extract the degree of instantiation of the different eigenvalues from the coefficients of the respective eigenvectors. Here, S instantiates $O = 1$ to degree $\left|c_1^2\right|$, and S instantiates $O = -1$ to degree $\left|c_2^2\right|$.

Two further points are worth noting. First, as discussed, cases of quantum MI are typically associated with properties whose interdependence prevents them from being mutually instantiated. At least in the paradigmatic cases of superposition MI, this interdependence and associated fact of mutual exclusion can be seen as the familiar variety associated with determinates of a single determinable: just as nothing can be both red and blue (simpliciter) all over, neither can Schrödinger's cat be both alive and dead (simpliciter), and nor can the particle go through both slits (simpliciter). At best the determinate values must be had in relativized or degree-theoretic fashion.

Second, by way of comparison with a supervalutionist treatment, and whether cases of superposition MI are treated using a gappy or glutty implementation of a determinable-based account, there is no danger of running afoul of the Kochen-Specker theorem, or, more generally, of violating constraints stemming from interdependent properties. This is clear for a gappy implementation, since here the more determinate properties are not at all instantiated, but it is also true for the glutty implementations, for neither relativized or degree-theoretic versions of such an implementation rely on taking it to be possible, much less actual, that all quantum MI is resolved. Similar

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[16] The degree-theoretic approach here is superficially similar to but importantly different from that in Smith and Rosen 2004; in particular, we reject three claims that Smith and Rosen accept, including that all fundamental properties are maximally precise, that MI involves an object's being an 'intermediate instance' of a precise property, and that 'fuzzy logic' is the correct logic of MI. See [. . .] .

remarks apply to a determinable-based treatment of other sources of quantum MI.

3.4 Incompatible observable MI

Paradigmatic cases of incompatible observable MI involve position and momentum, and different components of spin. Bokulich (2014) endorses a gappy implementation of *Determinable-based MI* as accommodating position-momentum MI:

> On the determinable-based account of MI, the position of the particle [. . .] is a vague property: while the particle possesses the determinable property of position, it does not possess a determinate value for that determinable.[17] (467)

Bokulich also sees a gappy implementation as promising for spin MI. So does Wolff (2015):

> Assigning a determinate spin value to a particle in a particular direction [say, 'up' to 'spin-z'] necessarily leaves the spin values of that particle in other directions indeterminate. [. . .] Permitting the instantiation of determinables without determinates helps to describe this phenomenon, because we can say that x-spin and y-spin are determinables with two determinates each, and neither of these determinates is instantiated even though the determinables 'spin-x' and 'spin-y' are. (385)

As Bokulich's and Wolff's discussions indicate, both cases satisfy the conditions of *Determinable-based MI*: in Bokulich's case, the system has the determinable *position*, but (thanks to the system's also having a somewhat determinate value of momentum) no unique determinate of that determinable; in Wolff's case, the system has the determinables *spin-x* and *spin-y*, but (thanks to the system's also having a determinate value of *spin-z*) no unique determinates of either determinable.

As we'll see, Wolff has some reservations about whether a determinable-based approach suitably illuminates the case of spin MI. These reservations aside, are Bokulich and Wolff correct that a gappy approach to these cases is most promising? Indeed, it is natural to see the MI in these cases in gappy terms.[18] We register just one caveat. Consider the claims that (i) system S does not have a determinate *spin-x*, even as a relativized matter, and (ii) the square of the *spin-x* component of S should be assigned 0. Friends of a gappy implementation should make sure that (i) does not entail (ii), for it will follow from that entailment that a gappy implementation would not satisfy the quantum mechanical constraint $S_x^2 + S_y^2 + S_z^2 = 2$. We are optimistic that the entailment can be blocked, but in any event, something needs to be said.

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[17] It is also true that on glutty implementations, the object having the determinable property "fails to have a definite value" of the determinable. In context, Bokulich is clear that she has in mind a gappy implementation.

[18] Torza (in progress) objects that a gappy implementation violates the supposition that, e.g., claims that a system has a determinable position should be formalized in existential terms as the having of some determinate position; but we reject this supposition as building in reducibility of determinables to determinates. See [. . .].

A glutty implementation does not invite this difficulty, and so it is worth considering the prospects of such an implementation. Here again, a glutty approach might involve either relativized determinates or degrees of instantiation.

The relativization approach would, as previously, require specification of what exactly the determinations are relative to—perspectives, circumstances, measurements, or what-have you. Wolff claims that this sort of approach to spin MI would not be properly metaphysical:

> The perspectival reading of complementarity suggests that the different eigenstates of an observable, understood as different determinates of the same determinable, could be instantiated at the same time, but would need to be 'looked at' from different perspectives. This looks too much like an epistemic reading of quantum mechanical uncertainty. (384)

Wolff's complaint seems to be motivated by the fact that, as mentioned above, different perspectives on and associated determinate colors of an iridescent feather could be concurrently instantiated. But this feature of the feather case is inessential to its usefulness in motivating glutty satisfaction of the conditions in *Determinable-based MI*. What is crucial is that it would be arbitrary to attribute a unique color to the feather; for this it suffices that the feather *could* be differently determined in different circumstances. The same goes for a glutty treatment of spin MI: what is important for purposes of establishing glutty satisfaction of the conditions is that different perspectives on or associated measurements of different spin components *could* be taken, which point is compatible with a non-epistemic reading.

Moving on to a degree of instantiation approach: this application is straightforward, for any eigenstate of an observable that is a member of an incompatible set will be a superposition of eigenstates of its incompatible observable(s). The implementation thus requires only that the quantum state be expressed using the other observable's eigenstates as a basis, at which point the degrees of instantiation can be extracted from the coefficients, as per the previous section.

3.5 Entanglement MI

Finally, consider the case of entanglement MI paradigmatically associated with the so-called singlet EPR-state (i.e., $\frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 - |\downarrow\rangle_1 |\uparrow\rangle_2)$, in which the quantum component systems S_1 and S_2 lack a determinate spin value. Note that there seems to be nothing particular to spin in this case, so that we could consider e.g. a two-particles system entangled in the position degree of freedom, as per state $\frac{1}{\sqrt{2}}(|x_1\rangle_1 |x_2\rangle_2 - |x_2\rangle_1 |x_1\rangle_2)$. In this case we would say that particles S_1 and S_2 do not have any determinate position.

Once again, a gappy implementation seems natural. Bokulich (2015) suggests a gappy implementation both in the case of entanglement in the spin-degree:

> Consider again two particles A and B. Suppose that these two particles have become entangled in the spin degree of freedom. This means that in this situation spin is a vague property. What makes entanglement different from the failure of value definiteness discussed in Section 4.3, is that these vague properties now also exhibit (nonlocal) correlations. (Bokulich 2014, 468–469).

We have no reservations about a gappy implementation that are pertinent specifically to this case.

Can entanglement be treated in glutty terms? It seems so, in either its relativization and degree-theoretic variants. A natural way of implementing a glutty relativization approach would draw on considerations advanced in Everettian interpretations denying the multiplicity of world-branches (e.g., that defended in Conroy 2012). In the case of spin, for example, we could say that relative to the first particle's having spin up, the second particle has spin down, and that relative to the first particle's having spin down, the second particle has spin up (and *viceversa*).

As for the degree of instantiation variant, we first would need to calculate the states of the component parts. They will be mixed states. We then need to make use of the fact that every mixed state can be written as a weighted sum of pure states. These weights will then represent the degree of instantiation, as per the previous recipe.

4 Concluding remarks

Quantum mechanics and metaphysical indeterminacy are deeply connected. Indeed, some take quantum MI to represent the best case for thinking that there is or could be properly metaphysical indeterminacy, and some take the intelligibility of quantum mechanics to rest in part on whether we can make sense of seeming quantum MI. But as Darby (2010) correctly notes, "this connection is going to work only if quantum-mechanical indefiniteness really is a species of indeterminacy as explored by metaphysicians, and it is not immediately obvious that this is so. On the contrary, the connection is sensitive to the precise way in which indeterminacy is understood" (228).

Darby's and Skow's reasons for rejecting a metaphysical supervaluationist understanding of QMI do not succeed, in leaving open several supervaluationist responses. Still, as we have argued, these responses are unsuccessful, and so Darby's and Skow's conclusion stands: QMI is not properly understood in supervaluationist terms, as involving indeterminacy between determinate (or more determinate) options. Luckily, as Bokulich and Wolff suggested, and as we've also here confirmed and developed, quantum MI can be properly understood in object-level terms, and more specifically can be seen as involving either gappy or glutty determinable-based MI. As we've discussed, some sources of quantum MI may be better treated in gappy terms, others in glutty terms. Our main goal, however, has been to show that one or other implemention of a determinable-based account is available as providing a basis for accommodating the full spectrum of sources of QMI, one or more of which are indicated on most live interpretations of quantum mechanics. In the process, we hope to have contributed to the project of rendering these interpretations metaphysically intelligible: at least so far as commitment to MI is concerned, quantum mechanics is not, after all, as mysterious as it has sometimes been thought to be.

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