The characteristic equation is
\[ r^2 - 2r + 5 = 0. \]

Using the quadratic formula we get
\[ r = \frac{2 \pm \sqrt{4 - 20}}{2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2} = 1 \pm 2i. \]

Therefore one solution is \( e^{(1+2i)t} \) and another is \( e^{(1-2i)t} \). In fact we can just look at the first of these two, because the real and imaginary parts of a solution are each themself a solution.

\[ e^{(1+2i)t} = e^t e^{2it} = e^t (\cos(2t) + i \sin(2t)). \]

The real part of this is \( e^t \cos(2t) \) and the imaginary part of this is \( e^t \sin(2t) \). Then the solutions of the homogeneous equation are
\[ y_1 = e^t \cos(2t) \quad y_2 = e^t \sin(2t). \]

And any solution can be written in the form
\[ y = c_1 e^t \cos(2t) + c_2 e^t \sin(2t) \]
for some \( c_1, c_2 \). We can use the initial condition \( y(\frac{\pi}{2}) = 0 \):
\[ 0 = c_1 e^{\frac{\pi}{2}} \cos(\pi) + c_2 e^{\frac{\pi}{2}} \sin(\pi). \]

Since \( \sin(\pi) = 0 \) and \( \cos(\pi) = -1 \), we have
\[ 0 = -c_1 e^{\frac{\pi}{2}}. \]

It follows that \( c_1 = 0 \). So
\[ y = c_2 e^t \sin(2t), \]
and taking the derivative we get
\[ y' = c_2 e^t \sin(2t) + 2c_2 e^t \cos(2t). \]
And using the initial condition $y'(\frac{\pi}{2}) = 2$, we get

$$2 = c_2 e^{\frac{\pi}{2}} \sin(\pi) + 2c_2 e^{\frac{\pi}{2}} \cos(\pi).$$

Hence,

$$2 = -2c_2 e^{\frac{\pi}{2}},$$

So

$$c_2 = -e^{-\frac{\pi}{2}}.$$ 

(Sometimes $c_2$ might be a little ugly like this, instead of something simple like 1 or $-2$!) Therefore the solution of the initial value problem is:

$$y = -e^{-\frac{\pi}{2}} e^t \sin(2t) = -e^{t-\frac{\pi}{2}} \sin(2t).$$