A trigonometric polynomial of degree $n$ is an expression of the form

$$\sum_{k=-n}^{n} c_k e^{ikt}, \quad c_k \in \mathbb{C}.$$

Using the identity $e^{it} = \cos t + i \sin t$, we can write a trigonometric polynomial of degree $n$ in the form

$$a_0 + \sum_{k=1}^{n} a_k \cos kt + \sum_{k=1}^{n} b_k \sin kt, \quad a_k, b_k \in \mathbb{C}.$$

For $1 \leq p < \infty$ and for a $2\pi$-periodic function $f$, we define the $L^p$ norm of $f$ by

$$\|f\|_p = \left( \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p dt \right)^{1/p}.$$

For a continuous $2\pi$-periodic function $f$, we define the $L^\infty$ norm of $f$ by

$$\|f\|_\infty = \max_{0 \leq t \leq 2\pi} |f(t)|.$$

If $f$ is a continuous $2\pi$-periodic function, then there is a sequence of trigonometric polynomials $f_n$ such that $\|f - f_n\|_\infty \to 0$ as $n \to \infty$ [31, p. 54, Corollary 5.4].

If $1 \leq p < \infty$ and $f$ is a continuous $2\pi$-periodic function, then

$$\|f\|_p = \left( \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p dt \right)^{1/p} \leq \left( \frac{1}{2\pi} \int_0^{2\pi} \|f\|_\infty^p dt \right)^{1/p} = \|f\|_\infty.$$

Jensen’s inequality [16, p. 44, Theorem 2.2] (cf. [30, p. 113, Problem 7.5]) tells us that if $\phi : [0, \infty) \to \mathbb{R}$ is convex, then for any function $h : [0, 2\pi] \to [0, \infty)$ we have

$$\phi \left( \frac{1}{2\pi} \int_0^{2\pi} h(t) dt \right) \leq \frac{1}{2\pi} \int_0^{2\pi} \phi(h(t)) dt.$$
If $1 \leq p < q < \infty$, then $\phi : [0, \infty) \to \mathbb{R}$ defined by $\phi(x) = x^{q/p}$ is convex. Hence, if $1 \leq p < q < \infty$ then for any $2\pi$-periodic function $f$,

$$
\|f\|_p = (\phi(\|f\|_p))^{1/q}
$$

$$
= \left( \phi \left( \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^p dt \right) \right)^{1/q}
$$

$$
\leq \left( \frac{1}{2\pi} \int_0^{2\pi} \phi(|f(t)|^p) dt \right)^{1/q}
$$

$$
= \left( \frac{1}{2\pi} \int_0^{2\pi} |f(t)|^q dt \right)^{1/q}
$$

$$
= \|f\|_q.
$$

The *Dirichlet kernel* $D_n$ is defined by

$$
D_n(t) = \sum_{k=-n}^{n} e^{ikt} = 1 + 2 \sum_{k=1}^{n} \cos kt.
$$

One can show [14, p. 71, Exercise 1.1] that

$$
\|D_n\|_1 = \frac{4}{\pi} \cdot \log n + O(1).
$$

(On the other hand, it can quickly be seen that $\|D_n\|_\infty = 2n + 1$, and it follows from Parseval’s identity that $\|D_n\|_2 = \sqrt{2n + 1}$.)

Pólya and Szegő [27, Part VI] present various problems about trigonometric polynomials together with solutions to them. A result on $L^\infty$ norms of trigonometric polynomials that Pólya and Szegő present is for the sum $A_n(t) = \sum_{k=1}^{n} \frac{\sin kt}{k}$. The local maxima and local minima of $A_n$ can be explicitly determined [27, p. 74, no. 23], and it can be shown that [27, p. 74, no. 25]

$$
\|A_n\|_\infty \sim \int_0^{\pi} \frac{\sin t}{t} dt.
$$

1 \hspace{1cm} L^p \text{ norms}

If $1 \leq p < q < \infty$, then [14, p. 123, Exercise 1.8] (cf. [7, p. 102, Theorem 2.6]) there is some $C(p,q)$ such that for any trigonometric polynomial $f$ of degree $n$, we have

$$
\|f\|_q \leq C(p,q)n^{\frac{1}{p} - \frac{1}{q}} \|f\|_p.
$$

This inequality is sharp [33, p. 230]: for $1 \leq p < q < \infty$ there is some $C'(p,q)$ such that if $F_n(t) = \frac{1}{n} \sum_{k=0}^{n-1} D_k(t)$ ($F_n$ is called the *Fejér kernel*) then

$$
\|F_n\|_q > C'(p,q)n^{\frac{1}{p} - \frac{1}{q}} \|F_n\|_p.
$$
Let $X_n = \{a_0 + \sum_{k=1}^{n} a_k \cos kt + b_k \sin kt : a_k, b_k \in \mathbb{R}\}$, the real vector space of real valued trigonometric polynomials of degree $n$, have norm

$$
\|f\|_{X_n} = \max\{|a_0|, |a_1|, \ldots, |a_n|, |b_1|, \ldots, |b_n|\}.
$$

Let $Y_{n,p}$ be the same vector space with the $L^p$ norm. Ash and Ganzburg [1] give upper and lower bounds on the operator norm of the map $i : X_n \to Y_{n,p}$ defined by $i(f) = f$.

Bernstein’s inequality [14, p. 50, Exercise 7.16] states that for $1 \leq p \leq \infty$, if $f$ is a trigonometric polynomial of degree $n$, then

$$
\|f\|_p \leq n \|f\|_p.
$$

In the other direction, if $f \in C^1$ then

$$
\begin{align*}
\frac{1}{2\pi} \int_0^{2\pi} f(s)ds &= \frac{1}{2\pi} \int_0^{2\pi} s f'(s)ds + \frac{1}{2\pi} \int_t^{2\pi} (s-2\pi)f'(s)ds \\
&= \frac{1}{2\pi} \int_0^{2\pi} f(s)ds + \frac{1}{2\pi} \int_0^{2\pi} s f'(s)ds - \int_t^{2\pi} f'(s)ds \\
&= \frac{1}{2\pi} \int_0^{2\pi} f(s)ds + \frac{1}{2\pi} s f(s)\bigg|_0^t - \frac{1}{2\pi} \int_0^{2\pi} f(s)ds - f(s)\bigg|_t^{2\pi} \\
&= f(t).
\end{align*}
$$

Hence

$$
|f(t)| \leq \frac{1}{2\pi} \int_0^{2\pi} |f(s)|ds + \frac{1}{2\pi} \int_0^{2\pi} s|f'(s)|ds + \frac{1}{2\pi} \int_t^{2\pi} (2\pi - s)|f'(s)|ds \\
\leq \frac{1}{2\pi} \int_0^{2\pi} |f(s)|ds + \int_0^{t} |f'(s)|ds + \int_t^{2\pi} |f'(s)|ds \\
= \|f\|_1 + 2\pi \|f'\|_1,
$$

so

$$
\|f\|_1 \leq \|f\|_1 + 2\pi \|f'\|_1.
$$

This is an instance of the Sobolev inequality [26].

It turns out that for a trigonometric polynomial the mass cannot be too concentrated. More precisely, the number of nonzero terms of a trigonometric polynomial restricts how concentrated its mass can be. Let $d\mu = \frac{dt}{2\pi}$. Thus $\mu([0, 2\pi]) = 1$. A result of Turán [20, p. 89, Lemma 1] states that if $\lambda_1, \ldots, \lambda_N \in \mathbb{Z}$ and $T(t) = \sum_{n=1}^{N} b_n e^{i\lambda_n t}$, $b_n \in \mathbb{C}$, then for any closed arc $I \subset [0, 2\pi]$,

$$
\|T\|_\infty \leq \left( \frac{2e}{\mu(I)} \right)^{N-1} \max_{t \in I} |T(t)|.
$$

Nazarov [11, p. 452] shows that there is some constant $A$ such that if $E$ is a closed subset of $[0, 2\pi]$ (not necessarily an arc), then

$$
\|\hat{T}\|_1 \leq \left( \frac{A}{\mu(E)} \right)^{N} \max_{t \in E} |f(T)|.
$$
Nazarov [23] proves that there exists some constant $C$ such that if $0 \leq q \leq 2$ and $\mu(E) \geq \frac{1}{3}$, then

$$\|T\|_q \leq e^{C(N-1)(1-\frac{\mu(E)}{2\pi})} \left( \frac{1}{2\pi} \int_E |T(t)|^q dt \right)^{1/q}.$$ 

These results of Turan and Nazarov are examples of the uncertainty principle [9], which is the general principle that a constrain on the support of the Fourier transform of a function constrains the support of the function itself.

In [10], Hardy and Littlewood present inequalities for norms of $2\pi$-periodic functions in terms of certain series formed from their Fourier coefficients. Let $c_k \in \mathbb{C}$, $k \in \mathbb{Z}$, be such that $c_k \to 0$ as $k \to \pm \infty$, and define $c_k^*, c_1^*, c_2^*, c_3^*, \ldots$ to be the absolute values of the $c_k$ ordered in decreasing magnitude. For real $r > 1$, define

$$S^*_r(c) = \left( \sum_{k=-\infty}^{\infty} c_k^r (|k| + 1)^{r-2} \right)^{1/r}.$$ 

For instance, if $c_k = 1$ for $-N \leq k \leq N$ and $c_k = 0$ for $|k| > N$, then $S^*_r(c) = \left( 1 + 2 \sum_{k=2}^{N+1} k^{r-2} \right)^{1/r}$. Hardy and Littlewood state the result [10, p. 164, Theorem 2] that if $1 < p \leq 2$ then there is some constant $A(p)$ such that for any sequence $c$, with $c_k \to 0$ as $k \to \pm \infty$, if $f(t) = \sum_{k=-\infty}^{\infty} c_k e^{ikt}$ and $\|f\|_p < \infty$ then

$$S^*_p(c) \leq A(p)\|f\|_p.$$ 

A proof of this is given in Zygmund [35, vol. II, p. 128, chap. XII, Theorem 6.3].

Asking if this inequality holds for $p = 1$ suggests the following question that Hardy and Littlewood pose at the end of their paper [10, p. 168]: Is there a constant $A$ such that for all distinct positive integers $m_k, k = 1, \ldots, N$, we have

$$\| \sum_{k=1}^{N} \cos m_k t \|_1 > A \log N?$$ 

McGehee, Pigno and Smith [18] prove that there is some $K$ such that for all $N$, if $n_1, \ldots, n_N$ are distinct integers and $c_1, \ldots, c_N \in \mathbb{C}$ satisfy $|c_k| \geq 1$, then

$$\| \sum_{k=1}^{N} c_k e^{int} \|_1 > K \log N.$$ 

Thus

$$\| \sum_{k=1}^{N} \cos m_k t \|_1 = \frac{1}{2} \| \sum_{k=1}^{N} e^{im_k t} + e^{-im_k t} \|_1 \geq \frac{1}{2} \cdot K \log(2N).$$ 

For $k \geq 2$, define $T_N(t) = \sum_{n=1}^{N} e^{int}$. Since $\|T_N\|_\infty = N$, for each $p \geq 1$ we have $\|T_N\|_p \leq N$. Hua’s lemma [22, p. 116, Theorem 4.6] states that if $\epsilon > 0$, then

$$\|T_N\|_{2^k} = O \left( N^{1-\frac{k}{2^k} + \epsilon} \right).$$
Hua’s lemma is used in additive number theory. The number of sets of integer solutions of the equation
\[ f(x_1, \ldots, x_n) = N, \quad a_r \leq x_r \leq b_r \]
is equal to (cf. [12, p. 151])
\[ \sum_{a_1 \leq x_1 \leq b_1} \cdots \sum_{a_n \leq x_n \leq b_n} \int_0^1 e^{2\pi i(f(x_1,\ldots,x_n) - N)t} dt. \]

Borwein and Lockhart [4]: what is the expected \( L^p \) norm of a trigonometric polynomial of order \( n \)? Kahane [13, Chapter 6] also presents material on random trigonometric polynomials.

Nursultanov and Tikhonov [25]: the sup on a subset of \( T \) of a trigonometric polynomial \( f \) of degree \( n \) being lower bounded in terms of \( \|f\|_\infty, n \), and the measure of the subset.

2 \( \ell^p \) norms

For a \( 2\pi \)-periodic function \( f \), we define \( \hat{f} : \mathbb{Z} \rightarrow \mathbb{C} \) by
\[ \hat{f}(k) = \frac{1}{2\pi} \int_0^{2\pi} e^{-ikt} f(t) dt. \]

For \( 1 \leq p < \infty \), we define the \( \ell^p \) norm of \( \hat{f} \) by
\[ \| \hat{f} \|_p = \left( \sum_{k=\infty}^{-\infty} |\hat{f}(k)|^p \right)^{1/p}, \]
and we define the \( \ell^{\infty} \) norm of \( \hat{f} \) by
\[ \| \hat{f} \|_\infty = \max_{k \in \mathbb{Z}} |\hat{f}(k)|. \]

Parseval’s identity [31, p. 80, Theorem 1.3] states that \( \|f\|_2 = \|\hat{f}\|_2 \).
If \( 1 \leq p < \infty \), then
\[ \| \hat{f} \|_\infty \leq \left( \cdots + \|\hat{f}\|_\infty^p + \cdots \right)^{1/p} = \|\hat{f}\|_p. \]

If \( 1 \leq p < q < \infty \), then, since for each \( k \), \[ \frac{|\hat{f}(k)|}{\|\hat{f}\|_q} \leq 1, \]
\[ 1 = \left( \sum_{k=\infty}^{-\infty} \left( \frac{|\hat{f}(k)|}{\|\hat{f}\|_q} \right)^q \right)^{1/q} \leq \left( \sum_{k=\infty}^{-\infty} \left( \frac{|\hat{f}(k)|}{\|\hat{f}\|_q} \right)^p \right)^{1/q} = \|\hat{f}\|_p/\|\hat{f}\|_q. \]

Hence for \( 1 \leq p < q \leq \infty \),
\[ \|\hat{f}\|_q \leq \|\hat{f}\|_p. \]
For $1 \leq p < \infty$, if $f$ is a trigonometric polynomial of degree $n$ then

$$\|\hat{f}\|_p = \left( \sum_{k=-n}^{n} |\hat{f}(k)|^p \right)^{1/p} \leq \left( \sum_{k=-n}^{n} \|\hat{f}\|_\infty^p \right)^{1/p} = (2n+1)^{1/p}\|\hat{f}\|_\infty.$$  

For $1 \leq p < q < \infty$, we have [30, p. 123, Problem 8.3] (this is Jensen’s inequality for sums)

$$\left( \sum_{k=-n}^{n} \frac{1}{2n+1} |\hat{f}(k)|^p \right)^{1/p} \leq \left( \sum_{k=-n}^{n} \frac{1}{2n+1} |\hat{f}(k)|^q \right)^{1/q},$$  

i.e.

$$(2n+1)^{-1/p}\|\hat{f}\|_p \leq (2n+1)^{-1/q}\|\hat{f}\|_q.$$  

Hence for $1 < p < q < \infty$,

$$\|\hat{f}\|_p \leq (2n+1)^{\frac{q-p}{pq}}\|\hat{f}\|_q.$$  

For any $t$,

$$|f(t)| = \left| \sum_{k=-\infty}^{\infty} \hat{f}(k)e^{ikt} \right| \leq \sum_{k=-\infty}^{\infty} |\hat{f}(k)e^{ikt}| = \sum_{k=-\infty}^{\infty} |\hat{f}(k)| = \|\hat{f}\|_1.$$  

Hence

$$\|f\|_\infty \leq \|\hat{f}\|_1.$$  

For any $k \in \mathbb{Z}$,

$$|\hat{f}(k)| = \left| \frac{1}{2\pi} \int_0^{2\pi} e^{-ikt} f(t) dt \right| \leq \frac{1}{2\pi} \int_0^{2\pi} |f(t)| dt = \|f\|_1.$$  

Hence

$$\|\hat{f}\|_\infty \leq \|f\|_1.$$  

The Hausdorff-Young inequality [32, p. 57, Corollary 2.4] states that for $1 \leq p \leq 2$ and $\frac{1}{p} + \frac{1}{q} = 1$, if $f \in L^p$ then

$$\|\hat{f}\|_q \leq \|f\|_p.$$  

The dual Hausdorff-Young inequality [32, p. 58, Corollary 2.5] states that for $1 \leq p \leq 2$ and $\frac{1}{p} + \frac{1}{q} = 1$, if $f \in L^q$ then

$$\|f\|_q \leq \|\hat{f}\|_q.$$  

A survey on the Hausdorff-Young inequality is given in [6])

For $M+1 \leq k \leq M+N$, let $a_k \in \mathbb{C}$ and let $S(t) = \sum_{k=M+1}^{N+1} a_k e^{ikt}$. Let $t_1, \ldots, t_R \in \mathbb{R}$, and let $\delta$ be such that if $r \neq s$ then

$$\|t_r - t_s\| \geq \delta.$$
where \( \|t\| = \min_k |t - k| \) is the distance from \( t \) to a nearest integer. *The large sieve* [19] is an inequality of the form

\[
\sum_{r=1}^R |S(2\pi t_r)|^2 \leq \Delta(N, \delta) \sum_{k=M+1}^{M+N} |a_k|^2.
\]

A result of Selberg [19, p. 559, Theorem 3] shows that the large sieve is valid for \( \Delta = N - 1 + \delta^{-1} \).

Kristiansen [15]
Boas [2]

For \( F : \mathbb{Z}/n \to \mathbb{C} \), its Fourier transform \( \hat{F} : \mathbb{Z}/n \to \mathbb{C} \) (called the discrete Fourier transform) is defined by

\[
\hat{F}(k) = \frac{1}{n} \sum_{j=0}^{n-1} F(j) e^{-2\pi i jk/n}, \quad 0 \leq k \leq n - 1,
\]

and one can prove [31, p. 223, Theorem 1.2] that

\[
F(j) = \sum_{k=0}^{n-1} \hat{F}(k) e^{2\pi i kj/n}, \quad 0 \leq j \leq n - 1.
\]

One can also prove Parseval’s identity for the Fourier transform on \( \mathbb{Z}/n \) [31, p. 223, Theorem 1.2]. It states

\[
\sum_{k=0}^{n-1} |\hat{F}(k)|^2 = \frac{1}{n} \sum_{j=0}^{n-1} |F(j)|^2.
\]

Let \( P(t) = \sum_{k=0}^{n-1} a_k e^{ikt} \). Define \( F : \mathbb{Z}/n \to \mathbb{C} \) by

\[
F(j) = \sum_{k=0}^{n-1} a_k e^{2\pi i kj/n}, \quad 0 \leq j \leq n - 1.
\]

(That is, \( \hat{F}(k) = a_k \).) We then have

\[
\sum_{k=0}^{n-1} |a_k|^2 = \frac{1}{n} \sum_{j=0}^{n-1} |F(j)|^2 = \frac{1}{n} \sum_{j=0}^{n-1} |P(\frac{2\pi j}{n})|^2.
\]

Thus

\[
\|P\|_2 = \left( \frac{1}{n} \sum_{j=0}^{n-1} |P(\frac{2\pi j}{n})|^2 \right)^{1/2}.
\]

The Marcinkiewicz-Zygmund inequalities [35, vol. II, p. 28, chap. X, Theorem 7.5] state that there is a constant \( A \) such that for \( 1 \leq p \leq \infty \), if \( f \) is a trigonometric polynomial of degree \( n \) then

\[
\left( \frac{1}{2n+1} \sum_{k=0}^{2n} |f(\frac{2\pi k}{2n+1})|^p \right)^{1/p} \leq A(2\pi)^{1/p} \|f\|_p,
\]
and for each $1 < p < \infty$ there exists some $A_p$ such that if $f$ is a trigonometric polynomial of degree $n$ then

$$
\|f\|_p \leq A_p \left( \frac{1}{2n+1} \sum_{k=0}^{2n} |f\left( \frac{2\pi k}{2n+1} \right)|^p \right)^{1/p}.
$$

Máté and Nevai [17, p. 148, Theorem 6] prove that for $p > 0$, if $S_n$ is a trigonometric polynomial of degree $n$ then

$$
\|S_n\|_\infty \leq A_n \left( \frac{(1 + np)e^{2}}{2} \right)^{1/p} \|S_n\|_p.
$$

Máté and Nevai [17] prove a version of Bernstein’s inequality for $0 < p < 1$, and their result can be sharpened to the following [34]: For $0 < p < 1$, if $T_n$ is a trigonometric polynomial of order $n$ then

$$
\|T'_n\|_p \leq n \|T_n\|_p.
$$

Let $\text{supp} \hat{f} = \{ k \in \mathbb{Z} : \hat{f}(k) \neq 0 \}$. A subset $\Lambda$ of $\mathbb{Z}$ is called a Sidon set [28, p. 121, §5.7.2] if there exists a constant $B$ such that for every trigonometric polynomial $f$ with $\text{supp} \hat{f} \subseteq \Lambda$ we have

$$
\|\hat{f}\|_1 \leq B \|f\|_\infty.
$$

Let $B(\Lambda)$ be the least such $B$. A sequence of positive integers $\lambda_k$ is said to be lacunary if there is a constant $\rho$ such that $\lambda_{k+1} \geq \rho \lambda_k$ for all $k$. If $\lambda_k$ is a lacunary sequence, then $\{ \lambda_k \}$ is a Sidon set [21, p. 154, Corollary 6.17]. If $\Lambda \subset \mathbb{Z}$ is a Sidon set, then [28, p. 128, Theorem 5.7.7] (cf. [21, p. 157, Corollary 6.19]) for any $2 < p < \infty$, for every trigonometric polynomial $f$ with $\text{supp} \hat{f} \subseteq \Lambda$ we have

$$
\|f\|_p \leq B(\Lambda) \sqrt{p} \|f\|_2,
$$

and

$$
\|f\|_2 \leq 2B(\Lambda) \|f\|_1.
$$

Let $0 < p < \infty$. A subset $E$ of $\mathbb{Z}$ is called a $\Lambda(p)$-set if for every $0 < r < p$ there is some $A(E, p)$ such that for all trigonometric polynomials $f$ with $\text{supp} \hat{f} \subset E$ we have

$$
\|f\|_p \leq A(E, p) \|f\|_2.
$$

$\Lambda(p)$ sets were introduced by Rudin, and he discusses them in his autobiography [29, Chapter 28]. A modern survey of $\Lambda(p)$-sets is given by Bourgain [5].

Bochkarev [3] proves various lower bounds on the $L^1$ norms of certain trigonometric polynomials. Let $c_k \in \mathbb{C}$, $k \geq 1$. If there are constants $A$ and $B$ such that

$$
A \frac{(\log k)^s}{\sqrt{k}} \leq |c_k| \leq B \frac{(\log k)^s}{\sqrt{k}}, \quad k \geq 1,
$$

then

$$
\|f\|_p \leq A(E, p) \|f\|_2.
$$

Bochkarev [3] proves various lower bounds on the $L^1$ norms of certain trigonometric polynomials. Let $c_k \in \mathbb{C}$, $k \geq 1$. If there are constants $A$ and $B$ such that

$$
A \frac{(\log k)^s}{\sqrt{k}} \leq |c_k| \leq B \frac{(\log k)^s}{\sqrt{k}}, \quad k \geq 1,
$$

then

$$
\|f\|_p \leq A(E, p) \|f\|_2.
$$
then [3, p. 58, Theorem 19]
\[ \| \sum_{k=1}^{n} c_k e^{ikt} \|_1 \gg \begin{cases} (\log n)^{s - \frac{1}{2}}, & s > \frac{1}{2}, \\ \log \log n, & s = \frac{1}{2}. \end{cases} \]

If \( P(t) = \sum_{k=0}^{n} a_k e^{ikt} \) with \( a_k \in \{-1, 1\} \), then by the Cauchy-Schwarz inequality and Parseval’s identity we have
\[ \| P \|_1 = \frac{1}{2\pi} \int_{0}^{2\pi} |P(t)| dt \leq \| 1 \|_2 \cdot \| P \|_2 = 1 \cdot \| \hat{P} \|_2 = \sqrt{n+1}. \]

Newman [24] shows that in fact we can do better than what we get using the Cauchy-Schwarz inequality and Parseval’s identity:
\[ \| P \|_1 < \sqrt{n + 0.97}. \]

A Fekete polynomial is a polynomial of the form \( \sum_{k=1}^{l-1} \left( \frac{k}{l} \right) z^k \), \( l \) prime, where \( \left( \frac{k}{l} \right) \) is the Legendre symbol. Let \( P_l(t) = \sum_{k=1}^{l-1} \left( \frac{k}{l} \right) e^{ikt} \). Erdélyi [8] proves upper and lower bounds on \( \left( \frac{1}{\pi} \int_{I} |P_l(t)|^q dt \right)^{1/q} \), \( q > 0 \), where \( I \) is an arc in \([0, 2\pi]\).

References


