Frequently Asked Questions

(FAQs) about the

Symmetry454 and Symmetry010 calendars

Simple perpetual solar calendars that are symmetrical across and between equal quarters yet conserve the traditional 7-day week.

Home Page on the Web: <http://www.sym454.org/>

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Table of Contents

Why reform the Gregorian calendar?

1) **Make the calendar perpetual**: The Gregorian calendar starts on a different weekday every year, so a new calendar has to be produced each year. By contrast, the Symmetry454 and Symmetry010 calendars are always the same every year, perpetually. Every Symmetry454 or Symmetry010 year starts on Monday, in fact every Symmetry454 month starts on Monday. Most perpetual calendar reforms force each year to start on the same weekday by inserting a "null" day that is outside of the traditional 7-day weekly cycle, and by making the leap day a "null" day as well, thereby causing the problem that the perpetual calendar weekday drifts behind the traditional weekly cycle, falling one day further behind whenever a "null" day is inserted. The Symmetry454 and Symmetry010 calendars, however, permanently avoid this problem by employing a **leap week** instead.

2) **Permanently fix the dates and weekdays of holidays and events**: Gregorian calendar holidays that occur on a specified date occur on a different weekday each year, and holidays that occur on a specific weekday within a month occur on a different date each year, whereas the dates and weekdays of holidays and events on the Symmetry454 and Symmetry010 calendars are permanently fixed.

3) **Reduce and delay long-term calendar drift**: The Gregorian calendar mean year, determined by its leap rule, is exactly $365+{}^{97}/_{400} \equiv 365.2425$ days ≡ 365 days 5 hours 49 minutes and 12 seconds, which is almost 12 seconds longer than the average year length between March equinoxes (mean astronomical northward equinoctial year). Therefore, even though the Gregorian Reform intended to correct and prevent the drift of the March equinox, that equinox is still slowly drifting towards earlier dates in March. Furthermore, due to the evolving astronomy of Earth's revolutions and rotations, specifically precession of the equinoxes, the slowly varying tilt of Earth's axis, the advance of perihelion and the steady slowing of the Earth rotation rate due to the tidal transfer of angular momentum from Earth to Moon, the rate of equinox drift will accelerate in the future. I investigated many alternative leap rules for the Symmetry454 calendar and offer a list of superior choices, but rank the simple 293-year cycle with 52 leap weeks as the best choice for the next 4000 years with respect to the March equinox.

4) **Symmetry and uniformity**: The Gregorian calendar has unequal non-symmetrical quarters and a haphazard list of month lengths. The Symmetry454 and Symmetry010 calendars have a simple pattern of month lengths having equal and symmetrical quarters. This symmetry and uniformity enables the development of aesthetically pleasing calendar designs, which is important for a perpetual calendar that will be reused for many years. The symmetrical leap rule of the 293-year leap cycle confers long-term symmetry to the calendar (referring to the smoothly spread symmetrical arrangement of leap years within each 293-year cycle). Furthermore, the moment of the mean March equinox falls at the cycle average in the first year of every cycle, making it simple to carry out astronomical performance evaluations of the calendar, merely by calculating the mean equinox drift for the first year of each cycle and then interpolating between cycles.

Won't it be very costly to switch calendars, especially in computer systems?

A commonly held view is that switching to a new calendar "will make Y2K look like child's play", referring to the substantial resources that went into preparing computers world-wide for the beginning of the year 2000, generally involving upgrade of years in dates from 2 digits to 4 digits and verification of the century leap year rule (century years are Gregorian leap years only if the year number is divisible by 400).

To help with adoption of the Symmetry454 or Symmetry010 calendar I have documented exactly how to implement it in computer systems, and how to interconvert to and from other calendars — see the document entitled *Basic Symmetry454 and Symmetry010 Calendar Arithmetic*, and the *Kalendis* computer program for Windows is freely offered as a demonstration / data verification utility, available from <http://www.sym454.org/kalendis/>.

Isn't it rather radical to have 5 weeks (35 days) in every 3rd month?

This is a common first reaction, but if you look closely then the benefits and logic of this configuration will be compelling. The 4+**5**+4 week structure of the calendar is an inescapable outcome of the design constraints:

Constraint: **The calendar must be perpetual** (starting on the same weekday every year) <u>and</u> **must preserve the traditional 7-day weekly cycle**. No "null" days are allowed outside the 7-day week.

> This is accomplished by having 52 weeks in a regular year, and a leap week at the end of every 6th or 5th year. Placing the leap week at the end of the year simplifies calendar calculations by ensuring that **every ordinal day and week number within the year is permanently fixed**.

Constraint: **The calendar year must be divisible between months into equal quarters**.

> With 52 weeks per regular year, this constrains each quarter to 13 weeks.

Constraint: **Quarters must be symmetrical, with 3 months that are as nearly equal in length as possible**.

> This leads to two possible month length combinations, either 30+**31**+30 **days** or 4+**5**+4 **weeks**.
>
> **Don't underestimate the importance of symmetry in calendar design.** This is not a "gimmick". Makers of calendars will benefit because symmetry enables production of aesthetically pleasing perpetual calendar designs, examples of which are available at the Symmetry454 web site <http://www.sym454.org/>.

Constraint: **Every month must start on Monday**.

> This eliminates the 30+**31**+30 **day** configuration (available in the Symmetry010 calendar), even though it has better-matched month lengths, because only its first month per quarter starts on Monday (the second starts on Wednesday, the third on Saturday).
>
> The 4+**5**+4 **week** configuration has the extra advantage that monthly events or meetings can always be on the same weekday and day of every month. For example, a monthly meeting scheduled on the 3rd Thursday will always be held on the 18th day of every month, which in regular 5-week week months happens to be exactly on the "mid-quarter day".

Not convinced yet? The final constraint ought to settle it: **The leap week must <u>not</u> be a stand-alone week**.

> When the leap week is a separate short 13th month it is conspicuously exceptional and calendar calculations are a bit more complicated. For the 30+**31**+30 **day** configuration, appending the leap week to December makes that quarter 30+31+**37** days — would anybody accept that? (Nevertheless, this configuration is optionally available for demonstration in the Symmetry010 calendar using "Experiment" mode in *Kalendis*.)
>
> The days of the leap week could be distributed to the 30-day months so that every month except December could have 31 days, but the arrangement of weekdays and holidays of leap years would then be very different from non-leap years.
>
> On the other hand, for the 4+**5**+4 **week** arrangement, the 5-week December of the quarter with leap week is no more exceptional than the 5-week November before it, and calendar arithmetic is simplest.

Nevertheless, for those who still believe that the Symmetry454 calendar structure is "too radical", the web site at <http://www.sym454.org/symmetry/> also offers the **Symmetry010 calendar**, which is based on the 30+**31**+30 **days** per quarter design and in leap years has either a 37-day December or a stand-alone leap week at the end of the year. The user of *Kalendis* can experiment with both variants, can choose whether to append the leap week to December or have it stand-alone after December like a 7-day "mini-month", can choose any desired weekday to start the calendar year, and can play with a wide range of experimental leap rules.

Why don't Sym454 / Sym010 quarters start on Sunday instead of Monday?

Although many proposed perpetual calendars start on Sunday, the Symmetry454 and Symmetry010 calendars were originally designed to maintain synchrony with the widely used ISO calendar, which starts on Monday. When Symmetry454 or Symmetry010 is used with the ISO standard leap rule, in every year in which the ISO calendar has a 53rd week there is a corresponding leap week at the end of the Symmetry454 or Symmetry010 calendar. The ISO calendar always starts on the Monday that is closest to Gregorian New Year Day; therefore the Symmetry454 and Symmetry010 calendars must also start on Monday. This fixes Monday to start the first quarter, and the other quarters also start on Monday because all quarters are identical — in fact, in the Symmetry454 calendar every month starts on Monday. Although the ISO leap rule remains available as an "Experiment" option in *Kalendis*, I no longer recommend it, because of its inherently exaggerated mean equinox wobble and because of its excessively long calendar mean year.

The **epoch** of the Symmetry454 and Symmetry010 calendars was **Monday**, January 1 of the year 1 AD, which was also the same date on the proleptic Gregorian calendar.

In the secular world, starting the calendar week on Sunday splits apart the week<u>end</u> days. By starting the calendar week on Monday, which is usually the beginning of the working week anyway, Saturday and Sunday appear in their socially proper place as the week<u>end</u> days.

Some Christians regard Monday as the beginning of the week and Sunday as the Sabbath.

Although North Americans are perhaps more accustomed to calendars having weeks starting on Sunday, elsewhere, especially in Europe, Monday is commonly shown as the first day of the week.

Several statutory holidays are observed at the start of a month. If those land always landed on a weekend day then workers would always get the next regular weekday off anyway, the holiday would permanently become a double-day event. This problem is avoided by making such days land on Monday.

If the Symmetry454 calendar were to start on Sunday, then the 13th of every month would always be Friday! Of course if the 13th of every month were a Friday it would lose its superstitious stigma anyway. Anyhow, by starting the Symmetry454 calendar on Monday the 13th is never a Friday.

Why are you offering multiple leap rules?

There is nothing about any calendar structure that necessarily compels the use of any particular leap rule.

A variety of the alternative *Kalendis* "experiment" mode leap rules are offered for demonstration purposes, to satisfy the curious and to show that a wide range of choices were considered in the design of the calendar.

For the commercial and industrial applications of today, maintaining alignment with the widely used ISO calendar may be more important than solar accuracy, at least until a major proportion of the world is using the Symmetry454 or Symmetry010 calendar.

Astronomical leap rules are advantageous because they are independent of the Gregorian calendar, they permanently avoid long-term drift with respect to the solar cycle, and their leap years are spread as optimally as possible, however their astronomical algorithms are arduous and need to be updated from time-to-time as necessary to incorporate unanticipated changes or improvements in astronomical knowledge. After such algorithm updates, recalculated past dates might become "wrong", and of course future dates can change, so in that sense astronomical leap rules are classified as unpredictable.

Simple fixed arithmetic cycles can closely approximate the astronomy for impressively long durations:

The simple symmetrically spread "52/293" rule is designated as the preferred choice for the present era, and astronomical analysis indicates that it will remain essentially drift-free relative to the March or **northward equinox** for the next **4-5 millennia**.

The simple symmetrically spread "69/389" rule is an attractive alternative, because astronomical analysis indicates that it will remain essentially drift-free relative to the June or **north solstice** for the next **10-11 millennia**.

Why is the 52/293 leap rule preferred?

The 52/293 leap cycle is preferred for the Symmetry454 and Symmetry010 calendars because:

- All other considerations being about equal, a shorter cycle seems preferable.
- There are exactly (7 x 7 x 6 = 294) x 364 days in this 293-year cycle.
  In other words, the total number of days is *exactly* equal to 294 non-leap years.
- The 52 leap weeks in the 293-year cycle *exactly* equal the length of one non-leap year (364 days).
- The average interval between the 52 leap weeks is *exactly* 294 weeks.
- The 293-year leap cycle is ideal for approximating the mean **northward equinox** (March equinox, boreal vernal equinox, northern hemisphere spring equinox) for the past 5 millennia and the next **4-5 millennia**, keeping the average equinox near midnight in Jerusalem at the beginning of the 80th ordinal day of the calendar year (Symmetry454 March 17th or Symmetry010 March 19th).
- Although the 293-year cycle mean year is slightly too short for the present era, that attribute will ensure an extra 5 centuries of excellent northward equinox alignment, until beyond 6000 AD. By then, perihelion will have advanced past that equinox, causing the equinoctial mean year to thereafter get shorter at an accelerating rate — see "The Length of the Seasons (on Earth)" at <http://www.sym454.org/seasons/>.
- The odd number of years per cycle permits the list of leap years to be arranged with perfect symmetry.
- Symmetrically spreading the leap year intervals as smoothly as possible comes very close to minimizing the "mean equinox jitter".
- If the Earth rotation rate doesn't slow down as much as predicted, which seems likely as a consequence of global warming, then that will only prolong the useful range of this leap cycle further into the future.

Won't the insertion of the leap week be disruptive to businesses, *etc.*?

No, in fact many businesses, especially in Europe and in the banking, financial, and manufacturing industries worldwide already use the ISO leap week calendar. Bankers used the leap week "Bankers' calendar" for about a century before ISO published their 8601 standard. Students will enjoy an extra week off between Christmas and New Year Day, during which they will probably go shopping! Many retail outlets that currently offer extended "Boxing Week" sales during this interval will profit greatly from extending the sale for an extra week!

Keep in mind that the leap week is inserted only once in 6 or 5 years, where 6-year intervals are approximately twice the frequency of 5-year intervals, so the impact of inserting a leap week will be rather insignificant.

Over two billion people in oriental countries such as China, Korea, Viet Nam, and Japan use traditional astronomical lunisolar calendars, in which the leap **month** is inserted at **variable positions** in the year, at intervals of 3 or 2 years. An oriental calendar leap month can be inserted between any two months of the year, but due to astronomical calculations there are a few "hot spots" that shift over the ages. If a variably positioned leap month has been proven culturally acceptable to so many people for over a millennium, then why should a mere leap week that is inserted at a fixed position at intervals of only 6 or 5 years be in any way disruptive? **This is just a matter of cultural acceptance.**

What about the impact of the leap week on contract durations?

With the Gregorian calendar, the leap days within the duration of a contract are usually ignored.

With a leap week calendar, however, it could make a 7-day difference to the contract duration, depending on when it starts and ends. There are several possibilities to deal with this, none of which are onerously complex:

1. Both contractual parties agree upon the starting and ending dates, with a prorated leap week adjustment.
2. The contract can span a specified number of weeks, perhaps a multiple of 52.
3. Both contractual parties agree upon the starting and ending dates, without any prorated adjustment for the leap week. If there is an extra week in the time span then that will be slightly disadvantageous to the vendor, but on the other hand it may help "seal the deal".
4. Annual or multi-year contracts can be drawn up as a multiple of 52-week periods, with each included leap week accounted for by a prorated adjustment.

How seasonally accurate are the Symmetry454 and Symmetry010 calendars?

The answer depends on what is considered to be the "gold standard", and on which leap rule is applied.

If alignment with Gregorian New Year Day is taken as the gold standard then the **long-term** average error using the ISO standard leap week rule is **zero**, because the mean ISO Symmetry454 / Symmetry010 year length of $365+{}^{97}/_{400} \equiv 365.2425$ days is exactly equal to the mean Gregorian year length, and the length of the repeating cycle of leap years is also identical (400 years). In the **short term** the <u>maximum</u> deviation between the Symmetry454 or Symmetry010 New Year Day and Gregorian New Year Day is **3** days:

New Year Day Difference: Symmetry454 or Symmetry010 (ISO standard leap week) minus Gregorian

Difference (Days):	**–3**	**–2**	**–1**	**0**	**+1**	**+2**	**+3**	**all ±1**	**all ±2**
Frequency:	14.2%	14.2%	14.5%	14%	14.5%	14%	14.5%	43%	71.2%

For any given 400-year period more than 71% of Symmetry454 or Symmetry010 years start within ±2 days and 43% start within ±1 day of the corresponding Gregorian year. It is no coincidence that each day difference column has almost exactly a ${}^{1}/_{7}$ probability — this is a trivial consequence of the almost equal probability of the Gregorian calendar starting on any weekday. Likewise the ±1 day column is the sum of 3 columns so it has a ${}^{3}/_{7}$ probability, and the ±2 days column is the sum of 5 columns and so it has a ${}^{5}/_{7}$ probability.

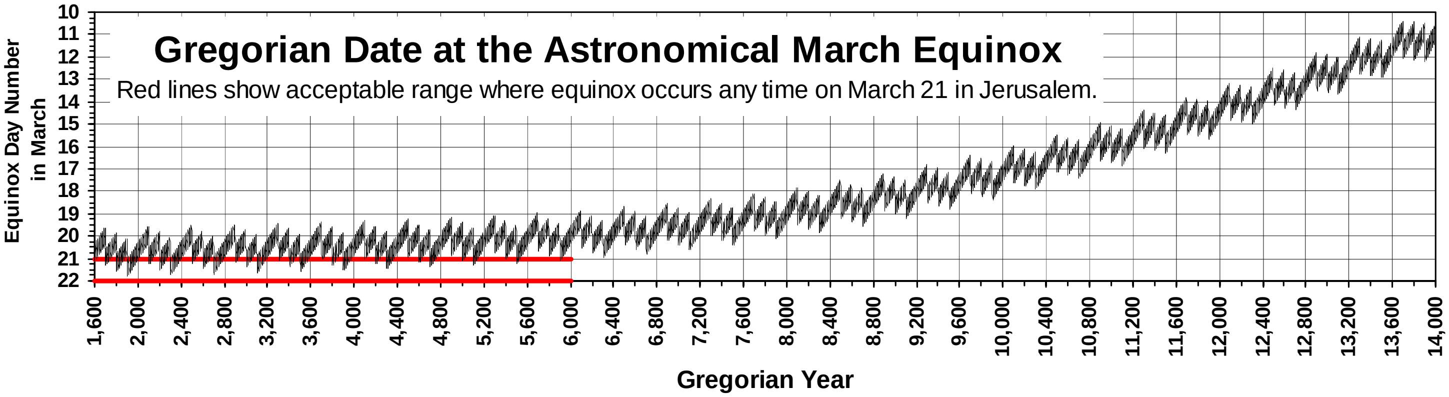
Alternatively, the simple symmetrically and smoothly spread "71/400" leap rule could be employed, also having exactly the same mean year as the Gregorian and ISO calendars. This would have less equinox wobble than the ISO leap rule, but slightly greater New Year Day variance in comparison to the Gregorian table shown above.

If alignment with the **solar cycle** is considered the gold standard, then the Gregorian calendar will be quite far off the mark. The objective of the Gregorian Reform was to keep March 21st close to the March equinox (more precisely the astronomical boreal vernal equinox or northward equinox), so that March 21st, which is the 80th day of a regular year or 81st day of a Gregorian leap year, could be taken as the ecclesiastical equinox date in the Gregorian Easter *computus* (calculation used to reckon the date of Easter).

During most of the present 21st century the northward equinox will land on Gregorian March 20th, but toward the end of the century it will often land on March 19th.

The following chart shows that in the present 400-year Gregorian leap cycle the March equinox will range from almost $1+{}^{1}/_{5}$ day **before** March 21st to about ${}^{3}/_{4}$ day after midnight on March 21st, but in the future the equinox will drift to progressively earlier dates at an accelerating rate. The equinox will first reach the 3-days-early level around year 8076, and after year 9944 it **will always be more than 3 days early**. In about the year 13632 the equinox will reach the 10-days-early level, and after about 14347 it will always be more than 10 days early, just as it was on the Julian calendar at the time of the Gregorian Reform! See my web page entitled "**Why March 21st?**" at <http://www.sym454.org/mar21/>. (The mentioned years are estimates, subject to some astronomical uncertainties that are unavoidable in such long-term projections.)

In the chart below, the zig-zag **2.2 day wobble** of the plotted line around its average path is **due to the non-uniform distribution of leap years in each 400-year Gregorian cycle of years**, see reference 21.



The use of the Symmetry454 or Symmetry010 leap week adds ±3 days of short-term variation to the "equinox wobble" depicted above, but **the date of Easter is <u>unaffected</u> because Easter can only fall on Sunday**.

Any of my proposed **astronomical** Symmetry454 or Symmetry010 leap rules yields **leap years that are as smoothly spread as possible** and **permanently avoid long-term drift** with respect to a specified point in the solar cycle. My fixed arithmetic leap cycles also symmetrically distribute the leap years within each leap cycle.

All long-term drift relative to the March equinox can be eliminated by defining the New Year moment as occurring at a fixed 79 calendar days before the equinox. This is demonstrated by either the **NE79** (actual) or **MNE79** (mean) "gold standard" leap rules in *Kalendis* (where the "NE" prefix is intended as an abbreviation for "northward equinox").

For all astronomical leap rule variants, in the **short term** the <u>maximum</u> deviation between the Symmetry454 or Symmetry010 New Year Day and the astronomical target New Year Day is ±**3** days.

How do the Symmetry454 & Symmetry010 calendars relate to the ISO calendar?

The **I**nternational **O**rganization for **S**tandardization (ISO) publishes the ISO calendar as standard ISO 8601. The ISO calendar is similar to the "Banker's calendar" which by then had been in use for over 100 years as a tool for determining the duration of term deposits, mortgages, loans, *etc.* without complicated calculations.

Today the ISO calendar is widely used in Europe, especially Scandinavia, and worldwide by bankers, financiers and industries. Industries use it to schedule work shifts, production runs, and shipping dates.

The ISO calendar has no months — the date is simply the year, week number within year, and day of the week. The ISO year begins on the Monday that is closest to Gregorian New Year Day (±3 days, Dec 29 to Jan 4).

Most ISO calendar years have exactly 52 weeks, but every 6 or 5 years the ISO calendar needs a 53rd week to get within the required plus or minus 3 days of the next Gregorian New Year Day.

The Symmetry454 or Symmetry010 calendar can optionally be used with the ISO standard leap rule, which is offered as one of the leap rule choices in *Kalendis*. If this is done then New Year Day of the Symmetry454 or Symmetry010 calendar will always be on the same day as the first Monday of the ISO calendar year, and the calendars will remain perfectly aligned at all times, including the Symmetry454 or Symmetry010 leap week matching ISO week 53. The Symmetry454 or Symmetry010 week number, shown beside each week of the Symmetry454 or Symmetry010 calendar, will then be identical to the ISO week number. The Symmetry454 or Symmetry010 calendar ordinal day number will also be identical to the ISO calendar ordinal day number. With the use of the ISO standard leap week the Symmetry454 or Symmetry010 calendar offers a conventional monthly view of the strictly week-oriented ISO calendar.

Isn't the arithmetic for the rule unreasonably complicated?

As explained in the *Basic Symmetry454 and Symmetry010 Calendar Arithmetic* document, the recommended leap rules involve simple arithmetic, which, although perhaps a challenge to do in one's head, are straightforward with the aid of paper and pen or a basic calculator.

In my experience in public dialogues I have found that very few people are able to correctly and fully state the Gregorian calendar leap rule. This is surprising, especially so soon after the so-called "Y2K" crisis, in fact most people are even unable to clearly state what were the concerns about the year 2000. The occasional person is able to vaguely describe concerns about 2-digit years somehow being a problem, and very few individuals are able to explain that the year 2000 was an exceptional century year. Recently a man told me about his unusual property contract, which was originally signed on February 29th, and then he said that "in theory it comes up for renewal only once in *five* years"! I conclude that the general public observes a leap year when they are told to do so, and there is no merit in striving for leap rule that is easy to remember or to reckon by thought alone.

Central authorities compute and publish their calendar decisions for determination of leap years for widely used astronomical calendars such as the traditional oriental lunisolar calendars and the modern Persian calendar (used in Iran). The traditional and modern Hindu calendars widely used in India employ arithmetic that is far too complex for the "man-on-the-street", so the public relies on published lists. The widespread cultural acceptance of published leap year lists shows that there is no merit in striving for a leap rule that is simple enough for anybody to determine without calculation aids, especially if the trade-off will be astronomical wobble or long-term drift.

It is a simple matter to publish the list of leap years, for example imprinting them as a footnote or on the back of the permanent calendar. The same sets of years are leap years within each repeating leap cycle. Lists of Symmetry454 or Symmetry010 leap years (their leap years are the same) that can be generated by *Kalendis* for any selected leap rule, where the year number within the cycle is shown as the left-most column of the report.

What are the official abbreviations for Symmetry454 and Symmetry010?

The official abbreviation of the Symmetry454 calendar name itself is "**Sym454**", as employed in the domain name for the web site at <http://www.sym454.org/>. Likewise, the official abbreviation of the Symmetry010 calendar name is "**Sym010**".

Abbreviations for month names shall always be the first 3 characters of the full month name:
Jan, Feb, Mar, Apr, May, Jun, Jul, Aug, Sep, Oct, Nov, Dec, Irv (applies only if the leap week stands alone).

Abbreviations for weekday names shall always be the first 3 characters of the full weekday name:
Mon, Tue, Wed, Thu, Fri, Sat, Sun.

In numeric short forms the format YYYY•MM•DD is preferred, with the years always having 4 digits and the month and day number always having 2 digits (with a leading zero if necessary). This format yields the same sequence when sorted numerically or as text.

If a numeric short form is desired for weekdays then use Mon=1, Tue=2, Wed=3, Thu=4, Fri=5, Sat=6, Sun=7. This is in compliance with the ISO calendar standard for numbering weekdays.

What is a "solar" calendar?

The orbit of Earth around Sun together with Earth's approximately 23.4° axial tilt determine the annual seasons. On average, a solar calendar ought to remain reasonably synchronized with the seasons over many centuries. By contrast, a lunar calendar stays in step with the lunar phases, a sidereal calendar stays in step with the distant stars (typically the constellations of the zodiac), and a purely arithmetic calendar reckons time either without regard to astronomical cycles or uses fixed or progressive arithmetic expressions that approximate some aspect of the underlying astronomy.

What is a simple perpetual calendar?

A calendar that always starts the year on the same weekday, and in every non-leap year follows a fixed pattern of days in each month, is a simple perpetual calendar, in the sense that a printed copy of the calendar can be reused indefinitely because the calendar will be the same for every year.

A simple perpetual calendar has the advantage that holidays or events that occur on a specified date always fall on the same weekday, and holidays or events that occur on a specific weekday within a specific week of a month always fall on the same calendar date. These attributes would greatly simplify administrative, academic, commercial and industrial scheduling by allowing the same schedule to be applied every year, if desired.

By analogy with flowers, some people like to label perpetual calendars using the term "perennial" to differentiate them from "annual" calendars that must be reprinted each year.

By contrast, the Gregorian calendar starts on a different weekday every year, because its year length of 365 or 366 days is not divisible by 7. Over the long term, each weekday has an almost equal probability of being Gregorian New Year Day. Holidays and events that occur on a specified date fall on a variable weekday. Holidays and events that occur on a specific weekday / week number of a month fall on a variable date. Although it is possible to print a perpetual version of the Gregorian calendar, it would be a complex document, requiring 14 different versions to account for the fact that the Gregorian calendar year can start on any weekday in the 7 types of non-leap years and the 7 types of leap years, and would require an accompanying table listing which variant to employ for each year number.

The Symmetry454 and Symmetry010 calendars are simple perpetual calendars, starting every year, quarter, and week on Monday and ending on Sunday.

The Symmetry454 calendar also starts every month on Monday. Consequently, each day of every Symmetry454 month always falls on the same day of the week, every year, because the number of days in the year is always divisible by 7, either 364 or 371 days (exactly 52 or 53 weeks).

No calendar can truly be "perpetual" in the sense of being usable forever without change, because our world and planetary system are not governed by ideal or constant cycles. The actual astronomical cycles will eventually drift away from any fixed arithmetic calendar. Even calendars based on astronomical algorithms are imperfect because of inescapable limitations of those algorithms and our incomplete knowledge, in particular of the long-term variations in Earth rotation rate, Earth axial tilt, the advance of perihelion, and the gravitational influence of the hundreds of thousands of asteroids in our solar system (see references 23 and 24).

Have other perpetual calendars been used or proposed?

The Enoch calendar or Jubilee calendar, discussed in the biblical era Book of Enoch, had 12 equal 30-day months, a "zero season day" inserted between quarters (intended to fall nominally at each equinox and solstice), and (probably) a leap week. The names of the months and zero season days were chosen to match the seasons of the northern hemisphere. This design followed the commonly held misconception that the equinoxes and solstices are equally spaced with equal-length seasons. The actual astronomical equinoxes and solstices are not equally spaced, however, and their relative positions change slowly over the ages — see my "**Lengths of the Seasons**" web page at <http://www.sym454.org/seasons/>.

Iceland, in the 10th century, used a 52-week calendar and every 7th year was a leap year with 53 weeks, with an average year length of 365 days, which is almost a quarter of a day short per year. This is the equivalent of using the Gregorian calendar with February permanently fixed at 28 days without any leap day, which would cause the seasons to progressively drift later into each calendar year at the rate of almost a day per 4 years!

Although details are sketchy because of little extant documentation, some believe that the "Vikings" used a leap week calendar aligned with respect to the summer (north) solstice, inserting 7 or 8 days every 6 years. Such a calendar would have been semi-perpetual, retaining the same weekdays for one or more cycles of 6 years, but shifting by one weekday after the occasional insertion of an 8th leap day. "The Moose" at wundermoosen.com proposed a "modern Viking calendar" based on two equal halves ("Winter" and "Summer"), with month lengths of 4+4+**5** + **5**+4+4 weeks per half-year, using such a leap cycle (the web page documenting this proposal became inaccessible after the author suffered a stroke). Although during the purported Viking era the north solstitial year length actually changed too rapidly to use with any fixed arithmetic calendar cycle, starting around the year 1000 AD it entered an era of stability which will endure for the next 11 millennia (lasting much longer than the present era of stability of the northward equinoctial year length that will end in only about 4 millennia). In *Kalendis* the user can choose a symmetrically spread leap cycle of 69 leap weeks in 389 years for use with the Symmetry454 or Symmetry010 calendar, which will keep the calendar consistently aligned with respect to the north solstice for the next 10-11 millennia.

The French Revolutionary calendar (see reference 3) was a perpetual calendar that proved to be very unpopular with workers and major religions because of its 10-day week. It had 12 thirty-day months, and 5 or 6 "Revolutionary holidays" were placed at the end of the calendar year, outside of its regular 10-day weekly cycle. Although its Gregorian-like leap rule was originally intended to keep the calendar New Year Day at the southward equinox, it didn't take long for the French to realize that the southward equinoctial year length is substantially shorter than the mean Gregorian year (the present difference is >42 seconds and it is continuing to get progressively shorter). For more information about this changing astronomy, please see my "**Lengths of the Seasons**" web page at <http://www.sym454.org/seasons/>. I have developed a progressive arithmetic leap rule that actually could maintain excellent alignment with the southward equinox in the present era and for thousands of years into the future — please see the "**LASEY**" leap rule (**L**inear **A**pproximation of the **S**outhward **E**quinoctial **Y**ear) on my "**Solar Calendar Leap Rules**" web page at <http://www.sym454.org/leap/>. Although the French did consider a variety of leap rule refinements, they abandoned their calendar before implementing any accurate rule, because workers protested against the longer workweek and the major religions protested against the disruption of the traditional 7-day weekly cycle.

There have been many attempts to base perpetual calendar proposals on adjustments to the 7-day weekly cycle, for example by making the first or last day of the year a special day outside of the traditional 7-day weekly cycle, and likewise for the leap day, wherever it was inserted.

The Thirteen Month calendar was a renowned example (see reference 16), having 13 months of 28 days, each with four 7-day weeks starting on Sunday. December 29th was "Year Day", a day outside of the weekly cycle, and in leap years June 29th was "Leap Day" and was also outside of the weekly cycle. The new 13th month, dubbed "Sol", was inserted between June and July. In addition, the proponents of this calendar proposed fixing the date of Easter, which would also fix all ecclesiastical calendar days that fall at positions counted relative to Easter. Ironically, although this calendar was developed and promoted by businessmen, it became unpopular with businesses and governments because its 13 months were not divisible into equal quarters without cutting into months, and there was strong religious objection to its disruption of the traditional seven-day weekly cycle. Americans also opposed it because American Independence Day would fall on the 17th of Sol instead of the 4th of July. The League of Nations formally rejected the Thirteen-Month calendar in 1937. The Eastman Kodak Company had started to use the Thirteen-Month calendar as an internal corporate calendar in 1928 because George Eastman was a leading advocate for this calendar reform, and the company continued this practice for several decades (until after his death), although in later years it became a leap-week calendar so that it could run parallel to the traditional 7-day weekly cycle and ISO calendar. This was a sensible calendar for Kodak at the time, because their very busy film chemical processing plants ran continuously 24 hours per day, 7 days per week, so a simple perpetual calendar was ideal for scheduling their factory production lines.

In 1931, while the League of Nations was seriously considering a variety of perpetual calendars, Dr. William M. Feldman (see reference 18, page 209-210) briefly described a modification of the 13-Month calendar which replaced the null days (Year Day, Leap Day) with a leap week inserted every 6 or 5 years, thus preserving the traditional weekly cycle. The Feldman 13-month calendar has the advantage of uniform month lengths of 28 days each, but is not divisible into equal quarters. Although he was a mathematician, Dr. Feldman didn't specify the leap cycle for his leap week calendar reform proposal, nor any of the calendar arithmetic.

The World Calendar® (see reference 17) had equal quarters, each of which started on Sunday and ended on Saturday, with quarters comprised of **31**+30+30 days. It had a "Worldsday" at the end of the year that was outside of the weekly cycle, and likewise its "Leapyear Day" was outside of the week, inserted between June and July. The World Calendar® attracted a lot of attention, mainly because of Elisabeth Achelis' lifelong dedication to its cause (possible because of a large inheritance), but in 1955 it was vetoed by the USA at the United Nations, on the grounds of religious objection to its disturbance of the traditional 7-day weekly cycle.

The ISO calendar is perpetual, having either 52 or 53 weeks, but it has no months. The ISO date is expressed as the year, week number within the year, and weekday. The ISO calendar year starts on the Monday closest to Gregorian New Year Day, so its mean year is identical to that of the Gregorian calendar.

The Bonavian calendar (reference 13), proposed by Chris Carrier in 1970, was a leap week calendar with equal quarters having **5**+4+4 weeks (non-symmetrical), with every month starting on Friday. A leap week was to be added at the end of leap years, based on 159 leap weeks in 896 years (a year is leap if it leaves a remainder of 0, 5, 11, 16, or 22 after division by 28, except if the year number is divisible by 896). The calendar mean year was $365+{}^{31}/_{128} \equiv 365.2421875$ days ≡ 365 days 5 hours 48 minutes 45 seconds, which, although it is about 15 seconds too short relative to the present era astronomical mean northward equinoctial year, will nevertheless maintain good (but not excellent) alignment of the March equinox for approximately the next 7000 years. As a demonstration, *Kalendis* can employ a similar but *smoothly spread symmetrical cycle* for the Symmetry454 or Symmetry010 calendar if the user enables the "Experiment" checkbox at the bottom right of the Symmetry window and then chooses the "159/896 = symmetrical Bonavian…" leap rule. Also in this experimental mode, if desired the user can choose to employ the Bonavian **5**+4+4 weeks per quarter structure by clicking on the "Quarters" frame until that option is shown as active, and can choose Friday as the starting weekday. The epoch of this variant will differ from the original Bonavian, however, and likewise the list of leap years will differ due to the symmetrical smoothly spread leap year distribution that *Kalendis* employs.

In the 1970s, Yosef Shteinberg, at that time living in Russia (he later moved to Israel), developed two leap week calendars. His "Calendar With Split Weeks®" employed equal quarters starting on Sunday with 30+30+**31** days per month. In leap years he distributed the 7 days of the leap week throughout the year, so that all months had 31 days except for his 30-day February. Although this strategy has the advantage that the leap week is inconspicuous, the glaring disadvantage is that the dates of weekdays, holidays and events of leap years are different from those of non-leap years. Shteinberg's "Calendar Without Split Weeks®" employed equal quarters with 4+4+**5** weeks per month (non-symmetrical), with each month starting on Sunday, and in leap years July had 5 weeks. Shteinberg selected the same leap rule for both calendar variants, claiming that it was his intention to match the Gregorian mean year length: every year number divisible by 5 is a leap year unless it is divisible by 50. This leap rule yields 72 leap weeks per 400 years, but that is one more than the 71 leap weeks actually required for a leap week calendar mean year to match the Gregorian mean year. One more leap week must be omitted or else his calendar will drift later than the Gregorian by 7 days per 400-year cycle. Shteinberg was apparently unaware of this error, as he wrote to me that my implementation of the ISO leap rule was "missing" one leap week. Furthermore, his non-uniform spread of leap years would have greatly exaggerated the medium-term equinox jitter, as well as its jitter relative to the Gregorian calendar, both spanning more than 2 weeks. Astonishingly, Shteinberg sent me a copy of a letter issued to him by the Vatican, which officially stated that the Catholic Church has no objection to the adoption of his proposed calendar reform!

In 1996-9 Bob McClenon proposed the Reformed Weekly calendar (RWC, see reference 7), with quarter lengths of **31**+30+30 days, each quarter starting on Sunday, and having a leap week. McClenon's leap rule (every year divisible by 5, except if divisible by 40, except if divisible by 400) yields the required 71 leap years per 400-year cycle, but over the course of each 400-year cycle the RWC New Year Day jitters over a 17-day range from 5 days late (year 36 of cycle) to 12 days early (years 365 of cycle) with respect to Gregorian New Year Day.

In contrast to the wild jitter of the Shteinberg and RWC leap rules, the ISO leap rule ensures that the Monday that starts the ISO year is never more than 3 days earlier than or 3 days later than the Gregorian New Year Day.

In 1998 Josef Šuráň proposed a leap week modification of The World Calendar® (referred to in his manuscript as the "Universal Calendar", see reference 22), having quarters of **31**+30+30 days starting on Sunday or of 30+30+**31** days starting on Monday (one wonders why he avoided a symmetrical variant), along with a smoothly spread 62-year leap cycle having 11 leap weeks. Such a leap cycle has a mean year of $365+^{15}/_{62}$ days ≡ 365 days 5 hours 48 minutes $23+^{7}/_{31}$ seconds. Šuráň's leap week, named "Tychon" after the great visual astronomer Tycho Brahe (1546–1601 AD), would be inserted as a stand-alone "mini-month" in the middle of the year after the end of June "so as not to disturb the continuity of months between years", and designated as "00" instead of a month number in numeric dates (this seemingly innocuous feature would actually make calendrical arithmetic extraordinarily complicated). The calendar mean year, intended by Šuráň to approximate the Mean Tropical Year (MTY), was $365+^{15}/_{62}$ days ≡ 365 days 5 hours 48 minutes $23+^{7}/_{31}$ seconds or about 365.24193548387 days. Approximating the MTY is actually not a good target for any calendar, because it doesn't correspond to any observable event, calculation of the MTY length by different methods yields different results, and astronomers express the MTY in terms of fixed-length atomic time days, not the mean solar days that all calendars require. It so happens that such a 62-year cycle can serve as an excellent approximation to the astronomical mean southward equinoctial year from an epoch of 1800 AD until at least 2800 AD.

The Common-Civil-Calendar (CCC, see reference 6), proposed by astronomy Professor Richard Conn Henry has equal quarters starting on Sunday with month lengths of 30+30+**31** days, and it has a leap week called "Newton" inserted every 6 or 5 years as a "mini-month" between June and July. CCC leap years are now the same as the standard ISO calendar.

The Symmetry454 and Symmetry010 calendars have equal quarters starting on Monday, and they conserve the traditional 7-day weekly cycle by inserting a leap week when necessary. They have all of the advantages of other simple perpetual calendars, but in addition they have fully documented public domain calendar arithmetic to facilitate their implementation in computer systems, and the innovation of their structural and long-term symmetry, which brings simple coherency and consistency to these calendars. Positioning the Symmetry454 or Symmetry010 leap week at the end of the calendar year ensures that every date has permanently fixed week-in-year and day-in-year numbers, facilitating administrative, academic, commercial and industrial applications, and simplifying calendar arithmetic. Their superior smoothly spread symmetrical 293-year leap cycle simplifies astronomical performance evaluations (year one of every cycle always falls at the cycle average, so long-term drift can be plotted by simply interpolating from year one of each cycle to the next) and has substantially less short-term equinox wobble than the ISO standard leap week rule. The intentionally slightly short calendar mean year will help the 293-year cycle "hold on" to the northward equinox for about a millennium longer than the ISO cycle.

The Symmetry010 calendar has symmetrically arranged nearly equal month lengths of 30+**31**+30 **days** per month in each quarter. The Symmetry454 calendar is a more radical reform over the traditional calendars and most of the competing calendar reform proposals, with equal quarters each comprising 4+**5**+4 **weeks** per month, but Symmetry454 has compelling additional benefits in that every month starts on the same weekday, every day number of every month always falls on the same weekday, no month ever contains a partial week, and the first four weeks of every month are always identical.

The Home Page for Calendar Reform is a good place to learn more about perpetual calendars (see reference 5), and Karl Palmen's Leap Week Calendars web page (see reference 8) discusses a variety of leap week strategies.

Why are leap weeks every "6 or 5" years rather than every "5 or 6" years?

A leap week calendar must employ some mix of 6- and 5-year intervals between leap years. In any reasonably accurate leap week calendar leap rule the 6-year interval between leap years must occur about twice as frequently as the 5-year interval.

A constant interval of 5 years between leap years is far too frequent. Such a calendar would have mean year of $365+^{2}/_{5}$ ≡ 365.4 days ≡ 365 days 9 hours and 36 minutes per year. That is 3 hours and 47 minutes too long per year, relative to the present era northward equinoctial year of 365 days 5 hours and 49 minutes, corresponding to a drift rate of about another day later than the equinox every $6+{}^{1}/_{3}$ years!

Likewise a constant interval of 6 years between leap years is too infrequent. Such a calendar would have a mean year of $365+{}^{1}/_{6}$ ≡ 365.1666... ≡ 365 days and 4 hours per year, which is 1 hour and 48 minutes too short per year, corresponding to a drift rate of about another day ahead of the equinox every $13+{}^{1}/_{5}$ years!

Therefore to better track the northward equinox, or any desired point in the solar cycle, a leap week calendar must employ some scheme to alternately use 6- and 5-year intervals between leap years, maintaining a certain mix to yield a calendar mean year that closely approximates the mean year of the target point in the solar cycle. Since the drift rate of an every-5-years leap cycle is about twice as fast as that of an every-6-years leap cycle, when expressed as the number of years per day of drift, for the present era all appropriate leap sub-cycle mixes must employ about twice as many 6-year intervals in the mix.

My "Lengths of the Seasons" web page at <http://www.sym454.org/seasons/> is a useful guide for selecting a calendar mean year and thence an appropriate calendar leap cycle.

My "Solar Calendar Leap Rules" web page at <http://www.sym454.org/leap/> details all of the reasonably accurate candidate leap week cycles and shows why the 293-year cycle is the best fixed arithmetic choice for the northward equinox in the present era and near future millennia.

When do you propose starting the Symmetry454 or Symmetry010 calendar?

Many other calendar reforms have proposed leap cycles that start on some inauguration date, so the list of leap years depends on when the reform is adopted. The Symmetry454 calendar leap year list, however, depends only on the leap rule arithmetic, which is always defined relative to the epoch of January 1 of the year 1 AD.

Most other calendar reformers have assumed that international agreement is necessary for adoption of a new calendar, but in truth no organization with universally recognized international calendrical authority exists.

Historically most calendar reforms have been adopted gradually. The Gregorian Reform itself took several centuries to become widespread, and even today there are several countries and many churches still using the old Julian calendar, and some will never use the Gregorian calendar because they relatively recently switched to the Revised Julian calendar, which has shorter calendar mean year of $365+{}^{109}/_{450}$ days ≡ 365 days 5 hours 48 minutes 48 seconds ≡ 365.242222... days. Today the Gregorian mean year is about 12 seconds too long whereas the Revised Julian mean year is about 12 seconds too short, relative to the astronomical mean northward equinoctial year — nevertheless because of future astronomical changes it will be better to have a shorter rather than longer calendar mean year.

The documentation of the Symmetry454 calendar was published in sufficiently complete form by November 2004 so that the first official Symmetry454 New Year Day started in 2005. In that sense the "proposed date" has already passed and anybody can use it today. *Kalendis* or the published Symmetry454 or Symmetry010 calendar arithmetic can be used to interconvert any past, present, or future dates.

When do Gregorian and Symmetry454 or Symmetry010 months match dates?

If the Gregorian calendar year starts on Monday then January always matches the Symmetry454 calendar used with the ISO standard leap week rule. In the same year, if it is a Gregorian leap year then April and July also match, but if it is a non-leap Gregorian year then October is the only additional matching month. Other month matches are possible when the Gregorian calendar starts on other days. The following table summarizes the months that match between the Gregorian and ISO/Symmetry454 calendars, depending on which day of the week is New Year Day ("Gregorian Starts On") and whether the Gregorian year is a non-leap or leap year:

Gregorian Starts On	Matching Months if Gregorian Non-Leap	Matching Months if Gregorian Leap Year
Monday	January, October	January, April, July
Tuesday	April, July	September, December
Wednesday	September, December	June
Thursday	June	March
Friday	February, November	February, August
Saturday	August	May
Sunday	May	October

The following are some observations about Gregorian months matching ISO/Symmetry454 months:

- When they match, the two calendars agree on all weekdays and dates within the month, and they continue to agree for the duration of whichever calendar has the shorter month.
- **Any Gregorian month that starts on Monday matches the corresponding ISO/Symmetry454 month.**
- A Gregorian leap year that starts on Monday has the most matching months (three).
- There is never a year that doesn't have at least one matching month.
- All months except March appear exactly once in the Gregorian non-leap year column.
- All months except November appear exactly once in the Gregorian leap year column.
- Matching for January and February is unaffected by Gregorian leap years because these months start prior to the Gregorian leap day on February 29th. Every ISO/Symmetry454 year begins on Monday, so obviously January always matches when the Gregorian year starts on Monday. Also, since January has only 28 days on the Symmetry454 calendar, February only matches when the Gregorian year starts on the Friday that is 3 days before the Symmetry454 year.

If any alternative reasonably accurate fixed arithmetic Symmetry454 leap rule is employed instead of the ISO standard leap rule then in the present century more than 90% of the ISO matched months will still match.

The freeware Windows application *Kalendis* has a built-in report that lists either the Symmetry454 – Gregorian difference or the Symmetry010 – Gregorian monthly difference for each month of the presently displayed year, and the starting and ending dates of each Symmetry454 or Symmetry010 month along with their corresponding Gregorian and ISO dates (and optionally, if the corresponding calendar window is open: Julian or Revised Julian, the currently selected Oriental calendar, the currently selected Bahá'í calendar variant), to assist with manual date conversions. This command is listed in the "Reports" menu as "Convert 52/293 Symmetry454 List" or "Convert 52/293 Symmetry010 List", depending on the user's choice of Symmetry calendar mode. In "Experiment" mode the user can choose an alternative leap rule instead of 52/293.

Of course the ISO/Symmetry010 calendar more frequently matches the Gregorian calendar, and in general has smaller monthly differences, due to its more conventional 30- or 31-day month lengths. The first, second, or third month of every Symmetry010 quarter matches the Gregorian month when the latter starts on Monday, Thursday, or Saturday, respectively.

Regardless of any month matching patterns, I don't believe that consideration of matching months is relevant to the timing of switching from the Gregorian to the Symmetry454 or Symmetry010 calendar. Rather, it depends on a great many people agreeing to switch, and implementation of global computer software changes to support the switchover. **Legacy computer systems will be the greatest obstacle.**

Therefore, to promote the worldwide adoption of the Symmetry454 or Symmetry010 calendar, I have placed in the public domain detailed information about Symmetry454 and Symmetry010 calendar arithmetic and how to implement it in computer systems, free of any royalty fees or copyright restrictions.

Could the Gregorian calendar be made perpetual, with a leap week?

Yes, for example by fixing February at 28 days and reducing the length of December from 31 to 30 days. The length of the year would then be 364 days, evenly divisible by the length of the 7-day week, with 52 weeks in the year. A leap week would be required every 6 or 5 years, preferably at the end of the year. If the ISO standard leap rule were employed then it would retain the same calendar mean year, allowing continued use of the established Gregorian Easter *computus*.

Nevertheless, the leap week reformed Gregorian calendar would still have unequal quarters (90, 91, 92, 91 days) with an unequal number of regular weekdays and weekend days, no attribute of the calendar would be symmetrical, the non-smoothly spread leap year intervals would amplify the short-term equinox wobble, and the computer programming effort required to implement these changes is no less than what is required to implement the Symmetry454 or Symmetry010 calendar, so why not benefit from the many extra advantages that the Symmetry454 or Symmetry010 calendars offer?

How could one work with Symmetry454 calendar monthly statistics?

Unless an adjustment is made, the $^1/_4$ greater length of Symmetry454 long months (February, May, August, November, and December in leap years) will muddle monthly statistical comparisons with short months. To normalize statistics for comparisons, one can either multiply short month statistics by $^5/_4$ or increase them by 25% to give them the same weight as long months, or multiply long month statistics by $^4/_5$ or decrease them by 20% to give them the same weight as short months. With the Gregorian calendar such weighting would be invalid, because of the varying proportion of weekend days per month, but it is valid for Symmetry454 as the proportion of weekend days per Symmetry454 month is constant.

Will the Symmetry454 calendar lead to two-tiered monthly payments?

Modern banks calculate daily interest on the minimum daily balance, paid monthly. You will continue to receive your fair share of interest each month. In the absence of deposits or withdrawals, the interest paid at the end of a long month (5 weeks) will be about 25% more than interest paid at the end of short months (4 weeks).

Utilities that invoice once monthly and that charge for actual consumption will probably invoice for about 25% more for long months, however that is their fair amount due. Utilities that offer equal billing plans, where a constant amount is billed each month with an annual reconciliation of any discrepancy between the total amounts billed and the actual annual consumption, will most likely continue to offer those plans with an equal amount billed each month. If any do increase the amount by 25% for long months, which would be a fair billing practice, then they would have to revert to an appropriately lower amount for short months.

Financial web sites or personal financial software allow the user to set up regular monthly payments of a fixed amount. If the amount to be paid has to be increased by 25% for long months then either the web site or the software has to be changed to accommodate this feature, or else the user can set up two periodic payment schedules: one to be paid monthly and cover the amount due per short month, and a second schedule to be paid at 3-month intervals and cover only the extra amount due per long month.

Payments for housing rental, long-term loans, and mortgages are typically calculated per annum, with monthly payments calculated as an equal amount paid monthly. This practice could continue with the Symmetry454 calendar. Some landlords and banks may elect for monthly payments to be two-tiered such that the amount is in proportion to the two possible month lengths — that would also be fair practice, and not particularly more complicated than a single-tier payment schedule. The annual amounts paid would be the same.

The current practice of issuing paychecks to employees weekly or bi-weekly ought to continue unchanged. Those employees who receive only monthly salaries ought to be paid 25% more for long months, as that is fair to all parties, and even without formal accounting records this arrangement is simple to calculate.

Alternatively, make such payments according to an on-going 4-week cycle — see the next question.

How will bi-weekly payroll checks be issued?

This will be handled the same way it is with the Gregorian calendar, by keeping track of the week count. To make this easier, the Symmetry Arithmetic documentation explains the simple expression for calculating the week number 1, 2, 3, or 4 of an on-going 4-week cycle, and also our freeware calendrical calculator, *Kalendis*, indicates the week number in this cycle in header row of the table just below the Symmetry date.

Bi-weekly payroll can be issued on a selected weekday of each odd week number (1 or 3) or each even week number (2 or 4), as selected by the organization.

This same cycle can also be used for regular "monthly" payments such as rental, lease payments *etc*., by selecting a specific weekday and week number within the cycle for payments to be due, so that payments will occur every 4 weeks without exception.

Will the Symmetry454 or Symmetry010 calendars have a movie industry impact?

They might, because "Friday the 13^{th}" will never occur!

If every Symmetry454 month starts on Monday then the 13^{th} day of every month is a Saturday.

Likewise, if every Symmetry010 month starts on Monday then the 13^{th} day of the first month of every quarter is a Saturday, the 13^{th} day of the second month of every quarter is a Monday, and the 13^{th} day of the third month of every quarter is a Thursday.