

Screening, Lending Intensity, and the Aggregate Response to a Bank Tax

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Abstract

Understanding how banks allocate their resources across different intermediation activities is crucial for gauging the health of the credit market. In this paper, I show that banks do not fully internalize the effects of their allocation decisions, leading to an inefficiently large amount of low-quality credit. I begin by constructing a model that captures two key features of the financial system. First, competition among lenders means that banks must use resources to attract clients and create credit matches. Second, asymmetric information means that resources also need to be devoted to screening once a client has been attracted. After analyzing how individual banks choose to allocate limited resources across these activities, I establish the existence of a unique steady state and calibrate the model to U.S. data. I find that optimal decisions impart externalities on both the beliefs and the outside options of other lenders, with the direction of these inefficiencies motivating a tax on the matching activity. Steady state results suggest that production exhibits a hump-shaped response to increases in this tax and the model's dynamics indicate that a mild tax can also attenuate business cycle fluctuations.

Keywords: Screening, Lending, Bank Tax, Output

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1 Introduction

Troubled assets have led to huge losses over the past few years, prompting many to explain the Great Recession as a miscalculation of risk by banks. In theory, however, the *raison d'être* of banks is that they are good at intermediation, providing risk sharing in incomplete markets and screening under asymmetric information. How, then, could they have gotten it so wrong? This paper contributes to a growing literature on financial sector inefficiency by investigating an as-yet unexplored margin: inefficiencies arising from the allocation of bank resources across intermediation activities.

I focus on two such activities. First, because of competition among lenders, banks devote some resources to attracting clients. Second, because of asymmetric information, they also devote some resources to screening those clients once attracted. No economic agent has unlimited resources though so a tradeoff between quantity and quality arises. The relevance of this tradeoff provides one interpretation of the recent rise and fall of mortgage-backed securities. In particular, the proliferation of these instruments fostered a credit boom but, *ex post*, it is clear that not enough information underlay the ratings. When viewed in this way, the evolution of mortgage-backed securities may symptomize a more fundamental problem of inefficiently low screening by financial intermediaries.

To understand why and when such inefficiencies can arise, I build a model that formalizes the allocation decision of banks. The economy features a continuum of heterogeneous borrowers differing in production ability. Each borrower needs one unit of capital to produce but this capital can only be intermediated by a mass of *ex ante* identical lenders. As described above, the intermediation process consists of attracting borrowers (i.e., by creating and/or advertising financial products) and screening them. Matches are necessary because credit is needed for production. At the same time though, screening is necessary because low quality borrowers are more likely to destroy capital by running unprofitable projects. Although lenders may want to undertake both matching and screening, the allocation of resources across these activities will be non-trivial if it is either too costly or too time-consuming to

undertake each activity until its marginal return is zero. To ease the exposition, I capture this restriction as a unit resource constraint whereby lenders do not have enough resources to make both activities succeed with probability 1.

After analyzing an individual lender's optimal division of resources between attracting and screening borrowers, I establish the existence of a unique steady state and calibrate the model to U.S. data in order to investigate efficiency in the decentralized equilibrium. I find that the equilibrium is not efficient and identify two key externalities behind the inefficiency. The first operates through the distribution of available borrowers when matches can be preserved over time. Since attracting a borrower today limits the need for matching resources tomorrow, lenders who carry their clients over can devote more of tomorrow's resources towards screening if today's screening efforts are unsuccessful. The eventual rejection of unprofitable borrowers then worsens the pool that currently unmatched lenders will draw from should they try to attract someone later on. The result is an "attract now, screen later" motive that drives the matching activity above the efficient level. This inefficiency is exacerbated by a second externality which arises because unmatched lenders do not take into account that their value function is the outside option of a matched lender. By allocating resources to maximize this value, unmatched lenders increase the opportunity cost of being matched and prompt informed lenders to be more selective in the types they retain. All else constant then, borrowers that a social planner would have deemed good enough to finance are let go in the decentralized market as informed lenders pursue the potential of higher profits. To decrease the prospect of re-matching and thus decrease the endogenous destruction of informed financing, the efficient allocation would prescribe a lower matching intensity.

A corollary of these results is that bank taxes which limit the drive to attract borrowers (for example, regulations on certain aspects of the financial innovation process) can improve social welfare. I investigate a simple version of this policy, namely a proportional tax on the matching activity, and find that steady state production exhibits a hump-shaped response to increases in the tax. I also find that a mild tax can attenuate business cycle fluctuations.

To the extent that it emphasizes financial non-neutrality, my paper is related to the macroeconomic literature on credit channels.¹ It is also related to a more recent branch of this literature which builds on the asset price propagation mechanism of Kiyotaki and Moore (1997) to investigate financial sector inefficiency. In Lorenzoni (2008) and Korinek (2009), for example, fire sales of collateralizable assets impart pecuniary externalities which can culminate in a financial crisis. In my model, however, inefficiency arises even if credit constraints are decoupled from asset prices, thus providing a new justification for regulatory intervention. Since the problem I propose exists at the level of bank decision-making, my paper is also related to previous work on the microfoundations of banking.² Particularly relevant are Direr (2008) and Cao and Shi (2001) who examine screening externalities, Parlour and Rajan (2001) who examine competition externalities with strategic default, and Betsi et al (2009) who examine search frictions in the credit market matching process. None of these studies, however, takes into account the tradeoffs that can arise when lenders engage in both screening and matching so implications at both the bank and aggregate level have yet to be investigated.

The rest of the paper proceeds as follows: Section 2 describes the environment in more detail; Section 3 analyzes individual decisions in the decentralized market; Section 4 establishes the existence of a unique steady state and calibrates the model; Section 5 compares the market equilibrium with the efficient allocation and discusses the externalities; Section 6 proposes a simple corrective tax and presents its effect on both steady state activity and recovery from a crisis; and Section 7 concludes.

2 Environment

All agents are risk neutral and endowed with a unit of effort each period. There is a continuum of firm types, $\omega \in [0, 1]$, with symmetric density function $f(\cdot)$. Types are private

¹See, for example, Gurley and Shaw (1955), Williamson (1987), Bernanke and Gertler (1989), and Kiyotaki and Moore (1997).

²See Freixas and Rochet (1997) for an overview of such models.

information. Each firm has access to a risky production project that requires one unit of external financing (i.e., capital) to operate. A firm that obtains the necessary capital and exerts effort e runs a successful project with probability $(1 + z)e$, where $z \in (-\varepsilon, \varepsilon)$ is an unanticipated mean-zero aggregate productivity shock that is IID over time. z is not contractible and all decisions are made before its realization. An unsuccessful project yields zero while a successful one yields $\theta(\omega)$, where $\theta(\cdot)$ is an increasing and concave function of firm type. Project yields include the original capital input so unsuccessful projects effectively destroy capital. The firm's cost of exerting effort e is $-c \ln(1 - e)$, where $c > 0$ is a constant.

Firms cannot store project output and they do not have direct access to capital so they must borrow from a measure of ex ante identical lenders that also populates the economy. Lenders cannot operate the production project but, in addition to capital, they have access to two technologies that allow them to emerge as intermediaries. First, lenders can create and/or advertise financial products to match firms with capital. The greater the number of matches, the greater the lending intensity. Second, lenders can screen firms to determine whether facilitating such matches is indeed profitable. Although lenders may want to undertake both activities, it is either too costly or too time-consuming to undertake each one until its marginal return is zero. This restriction is captured by a unit resource constraint. In particular, a lender who devotes fraction π of his effort endowment to matching gets a borrower with probability π and discovers that borrower's type with probability $1 - \pi$ immediately thereafter.³ Lenders cannot support more than one match at a time and cannot search "on-the-contract" so the matching technology is only available to unmatched lenders. In contrast, screening can be undertaken by all lenders.

To understand the implications of the resource allocation decision, let us examine how lenders evolve over time. Begin with a lender who is unmatched as of date t . At the beginning of t , the lender chooses π . If he fails to attract a borrower, then he stays unmatched and

³To some extent, the lender's constraint can be thought of as a budget constraint. However, using budgets would complicate the long-run analysis by endogenizing the right-hand side of the constraint without really changing the idea that lenders face a tradeoff between the two activities.

must try again in $t + 1$. If, however, he succeeds in forming a match, then he exerts screening effort $1 - \pi$ right after getting that match. Successful screening means that the lender's information set contains the borrower's true type whereas unsuccessful screening means that it only contains his beliefs about the pool of borrowers from which he drew the match. To keep the analysis tractable, I assume that these beliefs cannot be conditioned on credit history if screening fails.⁴ Given his information set, the newly matched lender must make two more decisions at the beginning of date t . First, he must decide whether to finance the borrower he just attracted or let him go and try again in $t + 1$. Information is clearly important here because only lenders who have successfully screened will be able to gauge how profitable the borrower really is. In contrast, lenders who must rely on their beliefs about the borrower pool can only gauge average profitability across types. In what follows, I denote the retention strategy of a matched and informed lender by $a(\omega)$, where $a(\omega)$ will be an indicator that equals 1 if and only if the lender accepts to finance a type ω borrower.⁵

Conditional on him keeping the borrower, the lender's second decision is what contract terms to offer. I assume no intertemporal commitment so each contract is defined by a one-period loan rate. This rate includes the borrowed unit of capital and must be paid to the lender if the project succeeds. Lenders cannot observe the exact result of a project but can detect the presence of positive output so borrowers repay if and only if their projects are successful. The information on which the lender conditions his loan rate is again important. Since the same rate can induce different ω 's to exert different production effort, the lender's offer affects whether the borrower's project will fail and, thus, whether capital will be destroyed.

Once retention decisions have been made and loan rates set, matched borrowers undertake production. The output of a successful project is then split so that, given loan rate R , the borrower gets $\theta(\omega) - R$ and the lender gets R . Borrowers consume their entire cut. In

⁴That is, discovering credit history requires screening to be at least partly successful.

⁵To simplify the analysis, assume that creating and/or advertising financial products is enforceable in that a lender who has exerted $\pi > 0$ in the current period and attracted a match cannot reject that match unless he can prove that the applicant's ω is too low (i.e., can only discriminate based on ω).

contrast, lenders save $(1 - \delta)R$ as capital for future financing and deplete the rest, δR where $\delta \in (0, 1)$, as operating expenses and/or consumption. In what follows, I assume the existence of an interbank market and denote its market clearing cost of capital by r . Lenders who do not have enough capital in their reserves must borrow at r while lenders who do have enough interpret r as an opportunity cost. Therefore, a lender's gross cost of funds is $1 + r$, where 1 represents the loan made to the borrower. Since each borrower needs only one unit of capital and lenders can finance only one borrower at a time, I can now focus on aggregate rather than individual capital accumulation.

At the end of date t , matches are subject to an exogenous separation probability $\mu \in (0, 1)$.⁶ Separation implies that the lender starts $t + 1$ unmatched. Non-separation implies that he carries his match into $t + 1$ and, therefore, cannot operate the matching technology that period. Since screening is still available to all lenders and an agent's effort endowment is not transferable through time, it then follows that any matched lender who enters $t + 1$ without full information about his borrower's type will undertake complete screening. As a result, uninformedness lasts for at most one period and within-lender credit history is rendered irrelevant. The lender's problem is now the same as that of a matched lender who enters $t + 1$ with full information: at the beginning of the period, he also decides whether to finance the borrower again and, if he accepts to finance, then he also chooses a one-period loan rate. If he rejects, then he enters $t + 2$ unmatched.

3 Optimal Decisions

3.1 Borrowers

Consider a type ω borrower who has obtained financing at loan rate R . With credit history irrelevant and intertemporal incentives precluded, the borrower's problem is a static one. In particular, given R , he chooses how much effort to put into the production project so as to

⁶This ensures that the steady state features both informed and uninformed lending.

maximize his one-period expected utility. Recall that a type ω who exerts production effort e succeeds with probability $(1+z)e$, in which case the project yields output $\theta(\omega)$. Since z is IID with mean 0, expected output is $e\theta(\omega)$ and the borrower's expected utility from consumption is $e[\theta(\omega) - R]$. Taking into account the disutility of effort, the borrower thus solves the following problem:

$$\max_{e \in [0,1]} \{e[\theta(\omega) - R] + c \ln(1 - e)\}$$

Conditional on R , his optimal strategy is:

$$e(\omega, R) = \begin{cases} 0 & \text{if } R > \theta(\omega) - c \\ 1 - \frac{c}{\theta(\omega) - R} & \text{if } R \leq \theta(\omega) - c \end{cases} \quad (1)$$

If the loan rate is higher than the choke rate $\theta(\omega) - c$, then the project will fail with certainty because the borrower has no incentive to exert production effort. On the other hand, if the loan rate is lower than the choke rate, then the borrower's effort is positive but strictly decreasing in R . Note that since $\theta(\omega)$ is an increasing function of the borrower's type, a better borrower is more likely to exert positive effort and his effort in this case will be higher for any given loan rate R .

3.2 Lenders

As described in Section 2, a lender's problem depends on whether he is matched or unmatched and, if matched, it also depends on whether he is informed or uninformed about his borrower's type. Since a lender's choices affect how he evolves over time, I formulate the problem using dynamic programming. In what follows, the aggregate state is summarized by $S \equiv \{K, V(\cdot), V_u(\cdot), \lambda_{-1}(\cdot), \phi_{-1}(\cdot)\}$, where K is the beginning-of-period stock of financing capital, $V(\omega)$ is the value of type ω under informed financing, $V_u(\omega)$ is the value of ω if unmatched, $\lambda_{-1}(\omega)$ is the proportion of ω 's financed by informed lenders last period, and

$\phi_{-1}(\omega)$ is the proportion financed by uninformed lenders.

3.2.1 Informed Lenders

Consider an informed lender matched with a type ω borrower. The lender takes as given S and his individual state $\{\omega, v\}$, where v is the value attained by his borrower. In turn, he must choose whether to keep the borrower (a), what loan rate to charge if he does keep him (R), and what continuation value to offer (v_{+1}). Since the borrower has the option of turning down the contract and hoping for a new lender next period, the continuation value must satisfy the borrower's participation constraint (i.e., the present discounted value of staying cannot be less than $\beta V_{u,+1}(\omega)$). Letting J denote the value function of an informed lender and U the value function of an unmatched lender, the informed problem can be written as:

$$J(\omega, v, S) = \max_{a, R, v_{+1}} \left\{ \begin{array}{l} (1-a) \beta U(S_{+1}, \psi_{+1}) \\ + a \left[\begin{array}{l} \left(1 - \frac{c}{\theta(\omega) - R}\right) R - (1+r(S)) \\ + \beta [(1-\mu) J(\omega, v_{+1}, S_{+1}) + \mu U(S_{+1}, \psi_{+1})] \end{array} \right] \end{array} \right\} \quad (2)$$

subject to

$$a \in [0, 1], \quad R \in [0, \theta(\omega) - c]$$

$$v = \theta(\omega) - R - c + c \ln \left(\frac{c}{\theta(\omega) - R} \right) + \beta [(1-\mu) v_{+1} + \mu V_{u,+1}(\omega)] \geq \beta V_{u,+1}(\omega)$$

$$S_{+1} = \Gamma(S), \quad \psi_{+1} = \mathcal{G}(S_{+1})$$

Let us now work through equation (2). If the lender rejects the borrower, then he gets the discounted value of being unmatched next period (ψ , the individual state of an unmatched lender, will be discussed in the next subsection). If he accepts the borrower, then his current period payoff depends on the borrower's strategy. With $e(\omega, R)$ as in (1), the lender will never want to charge above $\theta(\omega) - c$. Moreover, although higher values of R increase conditional revenue, they also decrease the probability of repayment so the lender would not want to monopolize the borrower irrespective of the participation constraint. Expected revenue is

thus $e(\omega, R)R$ and the lender's gross cost of funds is $1+r(S)$, where the market clearing cost of capital depends on the aggregate state. The lender's future value is then $J(\omega, v_{+1}, S_{+1})$ if the match is not exogenously destroyed and $U(S_{+1}, \psi_{+1})$ otherwise. To complete the problem, the lender's beliefs about the evolution of S and ψ are governed by laws of motion which, as will be discussed in Section 4, must be consistent with aggregate behaviour.

3.2.2 Unmatched Lenders

Consider now an unmatched lender. As discussed earlier, this lender has to choose how to divide resources between getting matches and screening applicants. A lender who devotes π units to attracting a borrower becomes matched and uninformed with probability π^2 , matched and informed with probability $\pi(1-\pi)$, and stays unmatched with probability $1-\pi$. Recall from Section 2 that uninformedness lasts for at most one period so I now define the one-period revenue function of an uninformed lender. Since uninformed lenders cannot discriminate among borrowers, they can only offer a pooled rate $\bar{R}$ which, if below $\theta(\omega) - c$, will induce a type ω to exert effort. Letting $\psi(\omega)$ denote the lender's beliefs about the share of type ω 's in the pool from which he drew, his maximized expected revenue is:

$$\begin{aligned} X(S, \psi) &= \max_{\bar{R}} \int_{\eta(\bar{R})}^1 \left(1 - \frac{c}{\theta(\omega) - \bar{R}}\right) \bar{R} \psi(\omega) d\omega \\ &\quad \text{subject to} \\ &\quad \bar{R} \in [0, \theta(1) - c] \\ &\quad \eta(\bar{R}) = \arg \min_{w \in [0,1]} |\theta(w) - c - \bar{R}| \end{aligned} \tag{3}$$

A newly matched lender who has not discovered his borrower's type gets $X(S, \psi) - (1+r(S))$ in the current period. If the match is exogenously destroyed at the end of the period, then his future value is $U(S_{+1}, \psi_{+1})$. If it is not destroyed, then his future value is $J(\omega, V_{+1}(\omega), S_{+1})$ weighted by $\psi(\omega)$ since ω is not known at the time of the match. We can now write the value function of an unmatched lender:

$$U(S, \psi) = \max_{\pi} \left\{ \begin{array}{l} \pi^2 \left[\begin{array}{l} X(S, \psi) - (1 + r(S)) + \beta \mu U(S_{+1}, \psi_{+1}) \\ + \beta (1 - \mu) \int_0^1 J(\omega, V_{+1}(\omega), S_{+1}) \psi(\omega) d\omega \end{array} \right] \\ + \pi (1 - \pi) \int_0^1 J(\omega, V(\omega), S) \psi(\omega) d\omega \\ + (1 - \pi) \beta U(S_{+1}, \psi_{+1}) \end{array} \right\} \quad (4)$$

subject to

$$\pi \in [0, 1], \quad S_{+1} = \Gamma(S), \quad \psi_{+1} = \mathcal{G}(S_{+1})$$

3.3 Optimal Resource Allocation

The key allocation decision is the unmatched lender's choice of π . The choice clearly depends on the distributions λ , ϕ , and ψ so we must now establish them.

3.3.1 Distributions

Recall that there are two classes of financing - informed and uninformed. The proportion of type ω 's with informed financing is $\lambda(\omega)$ and the proportion with uninformed financing is $\phi(\omega)$. Let Π denote the aggregate lending intensity of unmatched lenders and $A(\omega)$ the aggregate retention strategy of informed lenders. The law of motion for $\lambda(\omega)$ is then:

$$\lambda(\omega) = A(\omega) \left[\begin{array}{l} (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)] \\ + [1 - (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)]] \Pi (1 - \Pi) \end{array} \right] \quad (5)$$

As long as $A(\omega) = 1$, borrowers who were financed by informed lenders last period and who are still around this period again obtain informed financing. The same is true for borrowers who were financed by uninformed lenders last period and who are still around this period. These two statements explain $A(\omega) (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)]$. To see where $A(\omega) [1 - (1 - \mu) [\lambda_{-1}(\omega) + \phi_{-1}(\omega)]] \Pi (1 - \Pi)$ comes from, note that some borrowers obtain informed financing from a new lender. This group is drawn both from borrowers who

were unmatched last period and from borrowers who would have stayed with their last period lender had they not been exogenously separated.

Exogenously separated borrowers are also relevant for $\phi(\omega)$. In particular, some of the borrowers who were exogenously separated last period and who match again this period are not discovered. Moreover, some of the borrowers who were discovered last period (or are sure to be discovered at the start of this period) are not retained by informed lenders yet, if they match again, they may be able to obtain uninformed financing. The same is of course true for previously unmatched borrowers so the law of motion for $\phi(\omega)$ is:

$$\phi(\omega) = [1 - A(\omega)(1 - \mu)[\lambda_{-1}(\omega) + \phi_{-1}(\omega)]] \Pi^2 \quad (6)$$

Indeed, $[1 - A(\omega)(1 - \mu)[\lambda_{-1}(\omega) + \phi_{-1}(\omega)]]$ is the proportion of type ω borrowers looking for new lenders at the beginning of the current period so, in equilibrium, beliefs about the composition of available borrowers must satisfy:

$$\psi(\omega) = \frac{[1 - A(\omega)(1 - \mu)[\lambda_{-1}(\omega) + \phi_{-1}(\omega)]] f(\omega)}{\int_0^1 [1 - A(j)(1 - \mu)[\lambda_{-1}(j) + \phi_{-1}(j)]] dF(j)} \quad (7)$$

3.3.2 Best Response Function

With the distributions in hand, we can examine the optimal choice of π . The following proposition simplifies the analysis by reducing the informed retention strategy $A(\cdot)$ from a function to a scalar:

Proposition 1 *Suppose informed lenders are not bound by the borrower participation constraint. The informed retention strategy can be summarized by a cutoff type ξ , where $A(\omega) = 1$ if and only if $\omega \geq \xi$.*

Proof. See Appendix A. ■

Proposition 2 now establishes the best response of one unmatched lender to the actions of other (symmetric) unmatched lenders, holding the informed side of the market constant. To fix ideas, I focus on steady state.

Proposition 2 *Let $\pi(\Pi)$ denote the steady state best response of π to Π with ξ held fixed. There exists a $\widehat{\xi}$ such that:*

- (i) $\pi(\Pi) = 1$ with $\pi'(\Pi) = 0$ for $\xi < \widehat{\xi}$;
- (ii) $\pi(\Pi) \in (0, 1)$ with $\pi'(\Pi) < 0$ for $\xi \in (\widehat{\xi}, 1)$;
- (iii) $\pi(\Pi) = 0$ with $\pi'(\Pi) = 0$ for $\xi = 1$.

Proof. See Appendix A. ■

A lender has two incentives to learn his borrower's type. First, the information will enable him to reject unprofitable applicants and, second, it will also enable him to extract some rents from the profitable ones. Very high values of ξ mean that only a small group of borrowers are profitable so the desire to identify them drives lending intensity down and, in the extreme case, we observe (iii). On the other hand, very low values of ξ mean that almost all types are profitable so the first incentive is diminished. Moreover, for ξ sufficiently low, the risk of not forming a match this period outweighs the second incentive and π tends to 1 as per (i).

The interesting case is $\xi \in (\widehat{\xi}, 1)$. If $\Pi = 0$, then any unmatched lender who successfully expends $\pi > 0$ will have drawn from the initial distribution of types. As long as this distribution yields profitable expectations (which it must in order for the credit market to get off the ground), the lender will indeed choose $\pi > 0$. However, he will not go as far as $\pi = 1$ since the symmetry of $f(\cdot)$ across good and bad types also makes screening desirable. What happens if Π is slightly positive? Although other lenders only get a few borrowers, they screen them so intensely that at least some good types are pulled off the market while almost all the bad types remain. The average quality of available borrowers thus decreases, increasing any individual lender's incentive to screen and decreasing the choice of π . Consider

now a very high value of Π . A lot of matches are being formed but immediate type discovery is not common among other lenders so both good and bad borrowers are pulled off the market. If uninformed matches were to stay uninformed, beliefs would move back towards the initial distribution, π would increase, and the best response function would be U-shaped. Recall, however, that uninformedness is eventually resolved when lenders can preserve their matches across periods. In turn, high values of Π will translate into worse beliefs about the future and the best response will decrease. As we will see in Section 5, this result underlies one of two important externalities.

4 General Equilibrium

4.1 Market Clearing

Given the strategies and distributions derived above, I now discuss the evolution of the capital base. Suppose the beginning-of-period stock is KS . To get the end-of-period stock KS_{+1} , we must first subtract the amount of capital put into production during the current period. Since each loan transfers one unit of capital to the borrower, the amount of capital used up in financing equals the number of borrowers financed. This is essentially a measure of capital demand and it can be calculated as follows:

$$KD = \int_0^1 [\lambda(\omega) + \phi(\omega)] dF(\omega)$$

If production is unsuccessful, then the borrowed capital cannot be recovered. In contrast, a successful project returns this unit plus some extra capital to the lender and, after depleting a fraction δ , the lender adds back to the base. The law of motion for capital is then:

$$KS_{+1} = KS - KD + (1 - \delta) \left[\int_0^1 e(\omega, R(\omega)) R(\omega) \lambda(\omega) dF(\omega) + \int_0^1 e(\omega, \bar{R}) \bar{R} \phi(\omega) dF(\omega) \right]$$

Note that the cost of funds does not enter this equation directly. If r is interpreted as an opportunity cost that the lender must be compensated for, then it does not enter into aggregate accounting. Alternatively, if r is interpreted as a direct cost - namely the cost of borrowing the required unit from another lender on the interbank market - then it is subtracted from the revenues of the borrowing lender but added to the revenues of the lending lender, effectively washing out. The role of r is thus indirect. In particular, it adjusts to yield choices of A and Π that produce $KD = KS$. With such market clearing, the capital accumulation equation becomes:

$$K_{+1} = (1 - \delta) \left[\int_0^1 e(\omega, R(\omega)) R(\omega) \lambda(\omega) dF(\omega) + \int_0^1 e(\omega, \bar{R}) \bar{R} \phi(\omega) dF(\omega) \right] \quad (8)$$

4.2 Definition and Existence of Equilibrium

To complete the characterization, we need rules for the two remaining state variables: $V(\cdot)$ and $V_u(\cdot)$.⁷ Since an informed lender will never charge above his borrower's choke rate, production effort is always positive under informed financing and the borrower's value satisfies the following functional equation:

$$V(\omega) = \theta(\omega) - R(\omega) - c + c \ln \left(\frac{c}{\theta(\omega) - R(\omega)} \right) + \beta [(1 - \mu) V_{+1}(\omega) + \mu V_{u,+1}(\omega)] \quad (9)$$

In contrast, uninformed lenders can only offer a pooled rate which may or may not induce $e(\omega, R) = 0$ so this borrower's value if unmatched is:

$$\begin{aligned} V_u(\omega) = & \Pi^2 \left[\max \left\{ \theta(\omega) - \bar{R} - c + c \ln \left(\frac{c}{\theta(\omega) - \bar{R}} \right), 0 \right\} + \beta [(1 - \mu) V_{+1}(\omega) + \mu V_{u,+1}(\omega)] \right] \\ & + \Pi (1 - \Pi) V(\omega) + (1 - \Pi) \beta V_{u,+1}(\omega) \end{aligned} \quad (10)$$

An equilibrium in this model is then a set of lender value functions $\{J, U\}$ and sequences

⁷As $V(\omega)$ and $V_u(\omega)$ are really only used to construct the participation constraint that an informed lender must satisfy in order to retain ω , I will just present the rules for types that the lender does indeed want to keep.

of borrower continuation values $\{V, V_u\}$, individual decision rules $\{a, \pi, R, \bar{R}, v_{+1}\}$, aggregate decision rules $\{A, \Pi\}$, distributions $\{\lambda, \phi\}$, financing capital $\{K_{+1}\}$, costs of funds $\{r\}$, and beliefs $\{\psi, \Gamma, \mathcal{G}\}$ satisfying:

1. Lender optimality as per the optimization problems in Section 3.
2. Symmetry (i.e., $A = a$, $\Pi = \pi$, and $V = v$).
3. Capital market clearing.
4. Laws of motion (5), (6), and (8).
5. Functional equations (9) and (10).
6. Consistency of beliefs and, in particular, ψ as given by (7).

The proof of Proposition 1 established the existence of J and U . Moreover, the Theorem of the Maximum implies that the first order condition for π defines a continuous mapping from $\Pi \in [0, 1]$ to $\pi \in [0, 1]$ so there exists at least one symmetric equilibrium in the game between unmatched lenders. Since all unmatched lenders are ex ante identical, restricting attention to symmetry in the lending intensity choice is not unreasonable. Moreover, since all informed lenders matched with a type ω borrower are also ex ante identical, the same can be said for symmetry in their retention strategies and continuation offers. Proposition 3 now addresses the existence and uniqueness of a stationary symmetric equilibrium in the overall economy.⁸

Proposition 3 *If μ is sufficiently high, then there exists a unique non-trivial steady state in the class of symmetric equilibria with non-binding borrower participation constraints.*

Proof. See Appendix A. ■

⁸I restrict attention to non-trivial steady states since even the standard capital accumulation model admits an equilibrium where the economy shuts down.

The remainder of this paper investigates efficiency in the decentralized steady state and the potential for corrective taxes. Since the many interactions between agents make it difficult to obtain closed-form expressions, the next subsection parameterizes the model in order to conduct the analysis.

4.3 Parameterization

I calibrate the model's steady state to match features of the U.S. credit market over the period 1995-2005. Although the Gramm-Leach-Bliley Act did not officially institute broad banking until 1999, the Fed began easing Glass-Steagall in the late 1980s, effectively expanding the range of activities that banks could engage in. As such, I calibrate the model under $\tau = 0$ (i.e., very little to no regulation) then consider policy experiments where $\tau \neq 0$.

Suppose that the initial distribution of firm types is uniform over the unit interval and restrict attention to production functions of the form $\theta(\omega) = y_0 + y_1\omega^\alpha$. I normalize $y_0 = 1$ so that it represents the borrowed unit of capital (i.e., every successful project returns enough to cover its input). I also define the model period to be a quarter and set the discount factor β to match an annualized risk-free interest rate of 4%.

The parameters left to be calibrated are: the exogenous separation probability μ , the lender parameter δ , the borrower disutility parameter c , and the production parameters y_1 and α . I use the following targets:

1. Based on Bharath et al (2009), 71% of business loans come from lenders who recently provided the firm with another loan. The analogous measure in the model is the proportion of loans not in their first period.
2. Define capacity as the production that could be achieved if, all else constant, borrowers exerted effort $e(\omega, 0)$. I use the capacity utilization rate for manufacturing, roughly 0.78 in the FRED database, to target the model's ratio of actual production to capacity.

3. I target K/Y to match the ratio of net business loans to GDP. With net business loans defined as the difference between credit market debt owed by non-farm non-financial businesses and the credit market assets they hold, FRED data suggests a ratio of 0.57.
4. For the period under consideration, the value-added of the financial industry as a fraction of GDP is 0.075 (BEA Economic Accounts). Value-added sums compensation to employees, production taxes, and gross operating surplus so I interpret δK as the model's counterpart. The targeted item is then $\delta K/Y$.
5. From the 1997 Census of Manufactures, Dziczek et al (2008) estimate that the difference between the log labour productivity of the 90th and 10th percentile manufacturing plants is 1.62. I use this figure to target the dispersion of production among successful borrowers.

The resulting parameters are: $\mu = 0.14$, $c = 0.285$, $\delta = 0.13$, $\alpha = 0.5$, and $y_1 = 2.05$.

5 Externalities

Consider now a steady state social planner who holds the entire capital base. He faces the same constraints and intermediation technologies as lenders in the decentralized economy. Subject to aggregate feasibility, the planner chooses lending intensity Π , the informed cutoff ξ , and capital transfers $\mathcal{R} \equiv \{R(\cdot), \bar{R}\}$ to maximize total welfare. Total welfare is taken here to be total present discounted net output. A formal statement of the planner's constrained efficiency problem is presented in Appendix B and the results are summarized in the table below.

Relative to the decentralized market, the planner would devote more resources to screening new matches but be less restrictive in his cutoff once informed. The outcome would be a smaller market for uninformed financing, a lower overall delinquency rate, and higher aggregate welfare.

VARIABLE	PLANNER	MARKET
Lending Intensity (Π)	0.3637	0.4309
Informed Cutoff (ξ)	0.3770	0.4901
Amount of Informed Credit	0.4750	0.4044
Amount of Uninformed Credit	0.0753	0.1169
Average Type Financed	0.6432	0.6578
Average Delinquency Rate	0.3201	0.3426
Aggregate Welfare	80.551	75.100

To understand why unmatched lenders do not achieve the efficient resource allocation, consider first how their choices impact other unmatched lenders. For the relevant values of ξ , recall from Proposition 2 that the best response function $\pi(\Pi)$ is decreasing. As an unmatched lender increases lending intensity, he negatively impacts the beliefs of other unmatched lenders as discussed in Subsection 3.3.2. Unmatched lenders do not take this externality into account and the result is an "attract now, screen later" motive which prompts lending intensity in the decentralized market to be too high.

Consider now how the resource allocation decision affects matched lenders. When an informed lender decides whether or not to keep his borrower, he compares the expected value from that borrower to the value he could get as an unmatched lender. The latter, however, is not exogenous. Instead, unmatched lenders choose lending intensity to maximize that value, thereby increasing the opportunity cost of being matched and prompting informed lenders to be more selective in the types they retain. As a result, borrowers that a social planner would have deemed good enough to fund are let go in the decentralized market. All else constant then, the efficient allocation would prescribe a lower matching intensity to decrease the endogenous destruction of informed financing by decreasing the likelihood of re-matching.⁹

⁹Computing a variant of the decentralized economy where the informed lender's outside option is appropriately exogenized confirms that the outside option externality exacerbates the beliefs externality in this

Overall, unmatched lenders impart externalities on both the beliefs and the outside options of other lenders, leading to an inefficiently large amount of low-quality credit.

6 A Corrective Tax

The direction of the inefficiency identified in Section 5 motivates a tax on lending intensity. In this section, I consider a linear tax which makes activities designed to attract borrowers more costly. The tax rate is denoted by τ and only affects unmatched lenders. In particular, the maximization problem on the right-hand side of equation (4) now includes the term $-\tau\pi$. The tax revenues are then added back to the capital base so that all other equations are unchanged.¹⁰

6.1 Effect on Steady State

Figure 1 illustrates how steady state equilibrium outcomes vary with the lending intensity tax. There are four noteworthy features.

First, higher values of τ lead to lower values of Π and ξ . Since τ makes lending intensity more costly, the decrease in Π is straightforward. The decrease in ξ then follows from the fact that higher taxes and lower re-matching probabilities decrease the outside option of informed lenders, making them less restrictive in their retention of borrowers.

Second, market size (the measure of borrowers financed) exhibits a hump-shaped response to increases in τ . There are two competing forces here. On one hand, the decline in lending intensity decreases match formation but, on the other, the decline in informed selectivity increases match preservation. The second effect dominates at low tax rates but is eventually

direction. Moreover, computing a pseudo problem in which the planner cares about capital rather than net output reveals that any additional distortions stemming from the division of output between lenders and liquidity-constrained borrowers are negligible.

¹⁰Although alternative specifications of τ are certainly possible, I begin with the simple version described here in order to fix ideas, leaving extensions for future work. In what follows, τ can be interpreted as either a direct tax on the number of loans or a regulation which increases the cost of engaging in the matching activity.

overtaken by the first.

Third, higher values of τ increase the average quality of the credit market. Recall that a borrower's default probability depends on both his type and the loan rate he is charged. One of the advantages of informed lenders is that they can give borrowers better incentives to run successful projects. Although the decline in ξ lowers the average type financed, it (along with the increase in $1 - \Pi$) increases the proportion of informed financing and decreases the average delinquency rate.

Finally, at a macroeconomic level, production exhibits a hump-shaped response to increases in the tax. In particular, the shift towards informed financing has a positive effect as long as the frequency of new matches does not become too small. Furthermore, welfare increases as ξ and Π approach the efficient allocation.

6.2 Effect on Dynamics

The response to a temporary negative aggregate productivity shock is shown in Figure 2. I consider an unanticipated one-time shock where z drops to -0.01 at $t = 1$ but reverts to its mean right after. Appendix C describes the algorithm used to compute the response.

A negative shock to the probability of successful production (after Π_1 and ξ_1 have been decided) leads to an immediate drop in aggregate output and the end-of-period capital stock. The fall in capital implies a higher cost of funds in $t = 2$, reducing the incentive to lend and prompting a decline in lending intensity. The increase in r_2 also puts upward pressure on the informed cutoff. At the same time though, higher costs and a lower re-matching probability put downward pressure on this cutoff by deteriorating the informed lender's outside option.

The presence of a small tax reinforces the downward pressure on ξ_2 . Along with providing an additional drag on the value of being unmatched, a small tax prolongs the recovery path of Π_t and, all else constant, deteriorates the outside option further into the future. The net result is a short-term decline in ξ_t so that, by limiting the contraction of informed financing, $\tau = 0.005$ hastens the re-accumulation of capital and fosters a faster recovery in production.

7 Conclusion

This paper has examined the allocation of bank resources across two important intermediation activities: creating credit market matches and screening the borrowers in these matches. I constructed a model to disentangle the implications of this allocation decision in an environment with private information and competing lenders. I then showed that banks do not fully internalize the effects of their resource allocation decisions, leading to an inefficiently large amount of low-quality credit. There are two main externalities behind this result. The first operates through the distribution of available borrowers when matches can be preserved over time while the second arises because unmatched lenders do not take into account that their value function is the outside option of a matched lender. From a policy perspective, these results contribute to the current debate on bank taxes. In particular, the inefficiencies identified by my model suggest that a tax on matching activities (for example, certain regulations on financial innovation) would be more effective than some of the general profit taxes recently tabled. Indeed, steady state results show that production exhibits a hump-shaped response to increases in a matching tax and the model's dynamics indicate that a mild version of this tax can also attenuate business cycle fluctuations.

Figure 1: Steady State Results

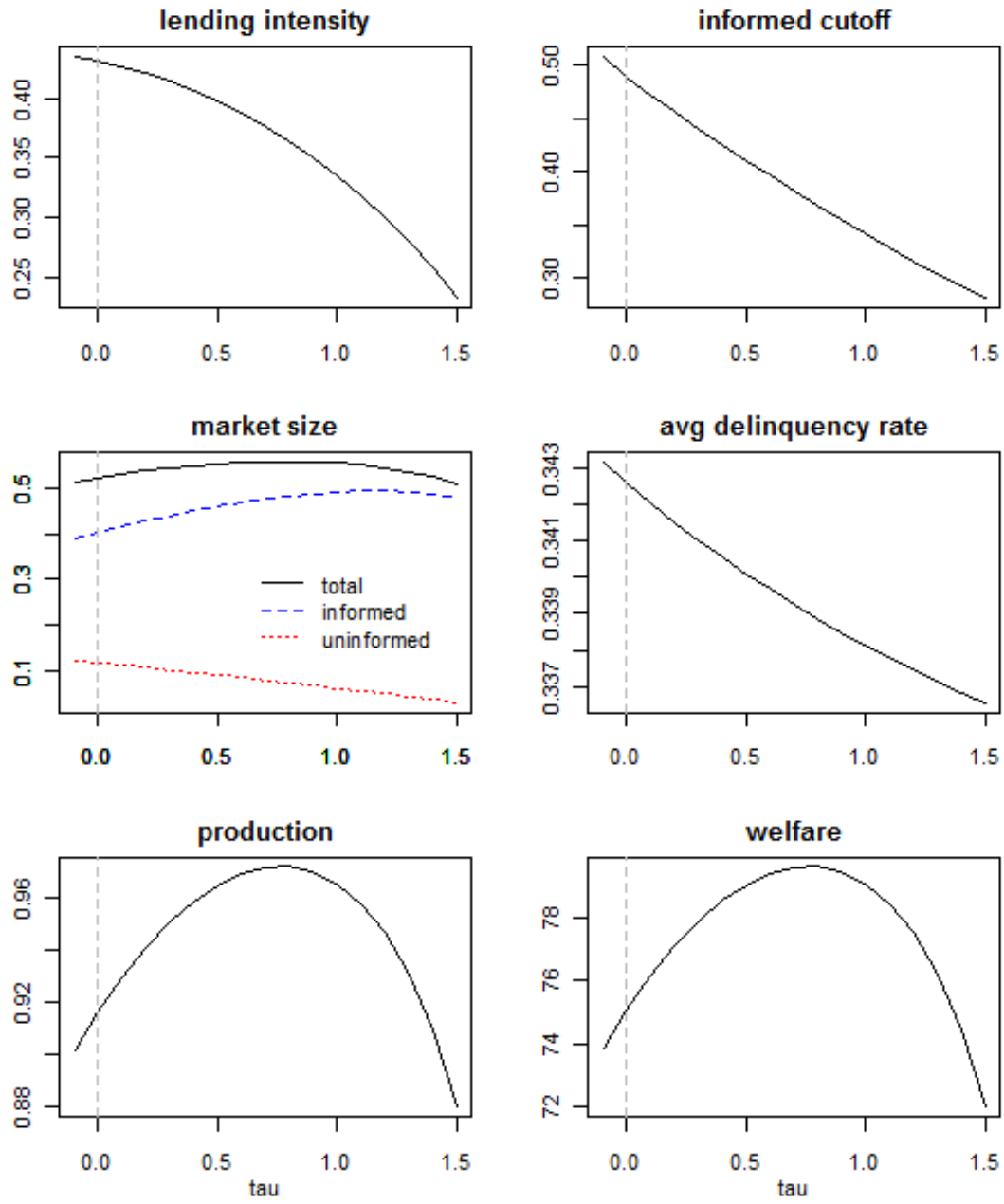
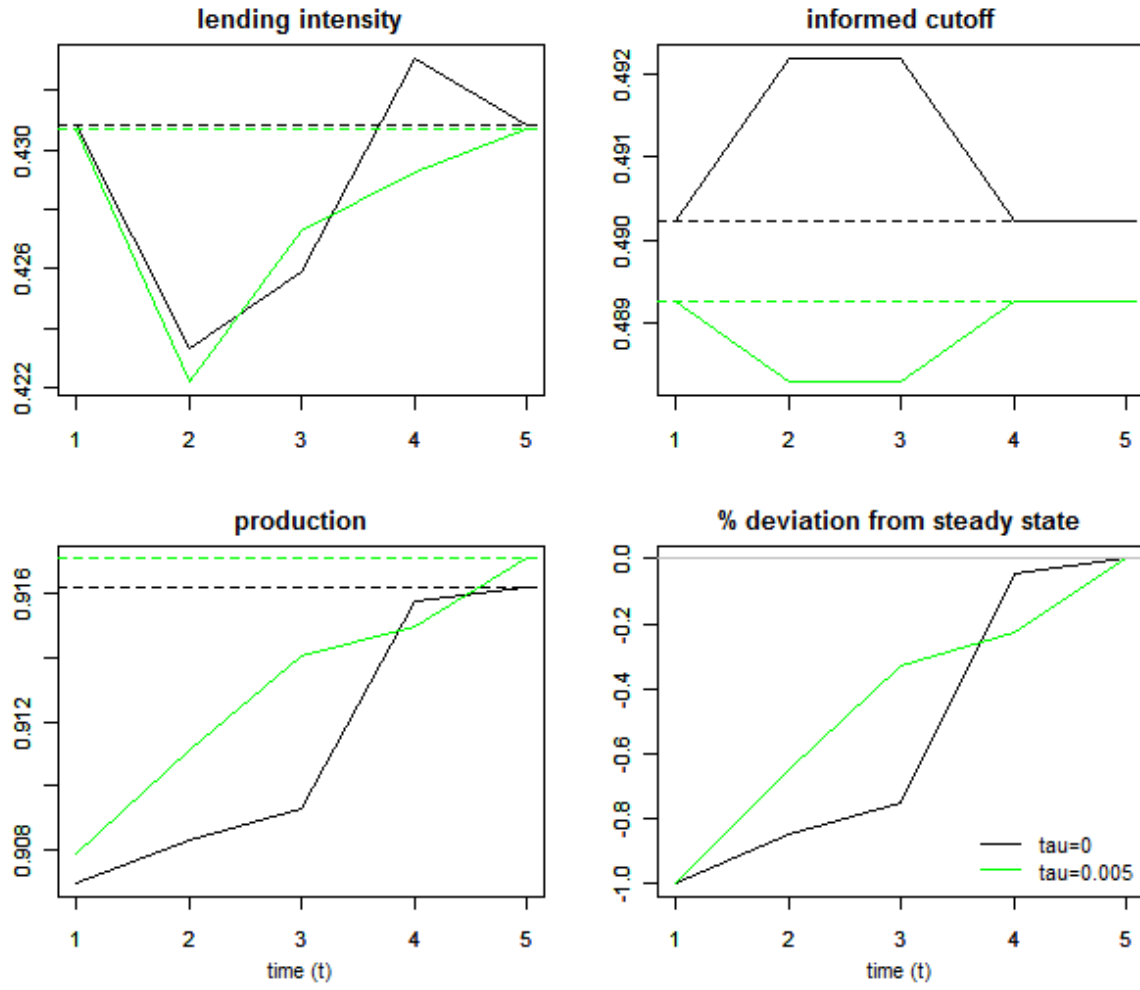


Figure 2: Dynamics After a Temporary Productivity Shock ($z = -0.01$ in $t = 1$)



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Appendix

A. Proofs

Proof of Proposition 1

The proof amounts to showing that J , the informed value function, is increasing in ω . I start by establishing the existence of J . Define indicator i and value function D such that:

$$TD(S, \omega, v, \psi, i) = i * D(S, \omega, v, \psi, 1) + (1 - i) * D(S, 0, 0, \psi, 0)$$

where

$$D(S, \omega, v, \psi, 1) = J(\omega, v, S)$$

$$D(S, 0, 0, \psi, 0) = U(S, \psi)$$

Now suppose D exists in the set of bounded and continuous functions ($\mathcal{C}$). By the Theorem of the Maximum, the right-hand side of equation (2) produces $D(\cdot, 1) \in \mathcal{C}$ while the right-hand side of (4) produces $D(\cdot, 0) \in \mathcal{C}$. Therefore, $TD \in \mathcal{C}$. We can then show that Blackwell's sufficient conditions for a contraction are satisfied so, by the Contraction Mapping Theorem, there does indeed exist a unique $D \in \mathcal{C}$. By implication, J and U exist and are unique, bounded, and continuous. A similar contraction mapping argument can be used to show that J lies in the set of increasing-in- ω functions, completing the proof. ■

Proof of Proposition 2

In what follows, $J(\omega, v, S)$ and $U(S, \psi)$ are shortened to $J(\omega)$ and U while r is used in place of $r(S)$. Let us first examine the value of an informed lender, ignoring the borrower's participation constraint. If he keeps the borrower, then the optimal loan rate is $R^*(\omega) = \theta(\omega) - \sqrt{c\theta(\omega)}$. Defining $g(\omega) \equiv \left(\sqrt{\theta(\omega)} - \sqrt{c}\right)^2$, we can then write $J(\omega)$ as:

$$J(\omega) = \begin{cases} \beta U & \text{if } \omega < \xi \\ \frac{g(\omega) - (1+r) + \beta\mu U}{1 - \beta(1-\mu)} & \text{if } \omega \geq \xi \end{cases}$$

where

$$\xi = \arg \min_{x \in [0,1]} |g(x) - (1+r) - \beta(1-\beta)(1-\mu)U|$$

Turn now to the distribution of borrowers across financing class. In steady state, the expressions in Section 3 simplify to:

$$\lambda(\omega) = \begin{cases} 0 & \text{if } \omega < \xi \\ \frac{\Pi(1-\mu\Pi)}{\mu+(1-\mu)\Pi} & \text{if } \omega \geq \xi \end{cases} \quad \text{and} \quad \phi(\omega) = \begin{cases} \Pi^2 & \text{if } \omega < \xi \\ \frac{\mu\Pi^2}{\mu+(1-\mu)\Pi} & \text{if } \omega \geq \xi \end{cases}$$

Recall that the fraction of type ω 's looking for a new lender is $1 - A(\omega)(1 - \mu)[\lambda(\omega) + \phi(\omega)]$. As a result, an unmatched lender will have the following beliefs about the borrower pool from which he may get a match:

$$\widehat{f}(\omega|\xi, \Pi) = \begin{cases} \frac{\mu+(1-\mu)\Pi}{\mu+(1-\mu)\Pi F(\xi)} & \text{if } \omega < \xi \\ \frac{\mu}{\mu+(1-\mu)\Pi F(\xi)} & \text{if } \omega \geq \xi \end{cases}$$

To ease notation, let $\widehat{f}_L(\xi, \Pi)$ denote the first row of the above equation and let $\widehat{f}_H(\xi, \Pi)$ denote the second. Moreover, let $X(\xi, \Pi)$ denote the maximized one-period revenue of an uninformed lender. The value of an unmatched lender pursuing optimal strategy π^* is then:

$$\begin{aligned} U = & \pi^{*2} \left[X(\xi, \Pi) - (1 + r) + \beta(1 - \mu) \int_0^1 J(\omega) d\widehat{F}(\omega|\xi, \Pi) + \beta\mu U \right] \\ & + \pi^*(1 - \pi^*) \int_0^1 J(\omega) d\widehat{F}(\omega|\xi, \Pi) + (1 - \pi^*)\beta U \end{aligned}$$

Substituting in for $J(\omega)$, we can rearrange the above expression to isolate U . With π^* optimal, $dU/d\pi^* = 0$ so differentiating the isolated expression yields an implicit definition of π^* . The definition can be simplified by combining terms to reconstitute U then using the definition of ξ to sub it out. After some algebra, we get the following expression for optimal lending intensity:

$$\pi^* = \frac{1}{2[1 - \beta(1 - \mu)]} \left(\frac{\int_{\xi}^1 [g(\omega) - g(\xi)] dF(\omega)}{\int_{\xi}^1 [g(\omega) - g(\xi)] dF(\omega) - \left(\frac{X(\xi, \Pi) - g(\xi)}{\widehat{f}_H(\xi, \Pi)} \right)} \right) \equiv s(\xi, \Pi) \quad (\text{A.1})$$

Holding ξ fixed, equation (A.1) defines the best response function $\pi(\Pi)$. If $\xi = 0$, then $s(\xi, \Pi) > 1$ so $\pi(\Pi) = 1$. If $\xi = 1$, then $s(\xi, \Pi) = 0$ so $\pi(\Pi) = 0$. For $\xi < 1$ sufficiently large though, $\pi(\Pi) = s(\xi, \Pi) \in (0, 1)$. To determine the slope of $\pi(\Pi)$ when ξ yields an interior solution, we need to write out the expression for $X(\xi, \Pi)$. Defining $h(\omega, \overline{R}^*) \equiv \left(1 - \frac{c}{\theta(\omega) - \overline{R}^*}\right) \overline{R}^*$, it is:

$$X(\xi, \Pi) = \widehat{f}_L(\xi, \Pi) \int_{\overline{\eta}^*}^{\max\{\overline{\eta}^*, \xi\}} h(\omega, \overline{R}^*) dF(\omega) + \widehat{f}_H(\xi, \Pi) \int_{\max\{\overline{\eta}^*, \xi\}}^1 h(\omega, \overline{R}^*) dF(\omega)$$

Applying the Envelope Theorem yields:

$$\frac{\partial}{\partial \Pi} \left(\frac{X(\xi, \Pi) - g(\xi)}{\hat{f}_H(\xi, \Pi)} \right) = \left(\frac{1 - \mu}{\mu} \right) \left[\int_{\bar{\eta}^*}^{\max\{\bar{\eta}^*, \xi\}} h(\omega, \bar{R}^*) dF(\omega) - g(\xi) F(\xi) \right]$$

Note that $g(\xi)$ must be greater than or equal to $h(\xi, \bar{R}^*)$ since an informed lender can always charge type ξ an amount $\bar{R}^*$. Combined with $h'(\omega, \bar{R}^*) > 0$, this implies that the denominator of equation (A.1) is increasing in Π . Therefore, $\pi'(\Pi) < 0$ when $\xi \in (\hat{\xi}, 1)$. ■

Proof of Proposition 3

For $\Pi > 0$, the steady state market clearing equation can be written as:

$$\Pi = \frac{1}{\mu} \left(\frac{\int_{\xi}^1 [g(\omega) - \frac{1}{1-\delta}] dF(\omega)}{\int_{\xi}^1 [g(\omega) - \frac{1}{1-\delta}] dF(\omega) - \left(\frac{X(\xi, \Pi) - \frac{1}{1-\delta}}{\hat{f}_H(\xi, \Pi)} \right)} \right) \equiv m(\xi, \Pi) \quad (\text{A.2})$$

Proving that there exists a unique symmetric steady state amounts to proving that there is a unique combination of ξ and Π that satisfies $\Pi = s(\xi, \Pi)$ and $\Pi = m(\xi, \Pi)$, where $s(\xi, \Pi)$ and $m(\xi, \Pi)$ are given in equations (A.1) and (A.2) respectively. Let $\Pi_s(\xi)$ denote the solution to $\Pi = s(\xi, \Pi)$ and $\Pi_m(\xi)$ denote the solution to $\Pi = m(\xi, \Pi)$. That there exists one and only one point such that $\Pi_s(\xi) = \Pi_m(\xi)$ is established in a series of steps.

Claim 1 *Suppose the parameters are such that $\int_0^1 g(\omega) dF(\omega) < \frac{1}{1-\delta}$. Any non-trivial steady state must have $\Pi \in (0, 1)$, allowing us to use the interior solution of equation (A.1).*

Proof. Non-triviality rules out $\Pi = 0$. Consider now $\Pi = 1$. At unit lending intensity, the market clearing equation reduces to:

$$(1 - \mu) \int_{\xi}^1 g(\omega) dF(\omega) + [\mu + (1 - \mu) F(\xi)] X(\xi, 1) = \frac{1}{1 - \delta}$$

Since $g(\omega) > 0$, we have $\int_{\xi}^1 g(\omega) dF(\omega) < \int_0^1 g(\omega) dF(\omega)$. Moreover, $h(\omega, \bar{R}^*) \leq g(\omega)$, $\bar{\eta}^* \geq 0$, $\hat{f}_L(\xi, \Pi) \geq \hat{f}_H(\xi, \Pi)$, and $\hat{f}_H(\xi, \Pi) \leq 1$, imply $X(\xi, \Pi) < \int_0^1 g(\omega) dF(\omega)$. Therefore, under the condition assumed in Claim 1, $\Pi = 1$ cannot satisfy market clearing. □

Claim 2 $\Pi_s(\xi)$ and $\Pi_m(\xi)$ intersect at least once.

Proof. Define $\underline{\xi}$ such that $\int_{\underline{\xi}}^1 (g(\omega) - \frac{1}{1-\delta}) dF(\omega) = 0$ and $\bar{\xi}$ such that $g(\bar{\xi}) = \frac{1}{1-\delta}$. With some algebra, we can show $\Pi_s(\underline{\xi}) > \Pi_m(\underline{\xi})$ and $\Pi_s(\xi) < \Pi_m(\xi)$ for all $\xi \geq \bar{\xi}$. □

Claim 3 *All intersections between $\Pi_s(\xi)$ and $\Pi_m(\xi)$ are associated with the same value of Π , labelled Π_0 .*

Proof. Rearrange $\Pi = s(\xi, \Pi)$ and $\Pi = m(\xi, \Pi)$ to get two expressions for $\frac{X(\xi, \Pi)}{f_H(\xi, \Pi)}$. Equating these expressions and rearranging again yields a quadratic in Π , where the roots of this quadratic determine the values Π can achieve at an intersection. Recall from the proof of Claim 2 that intersections require $\xi < \bar{\xi}$ which, given $g'(\cdot) > 0$, is equivalent to $g(\xi) < \frac{1}{1-\delta}$. This fact combined with $\Pi > 0$ can be used to eliminate one of the roots, implying that any intersection must achieve the same value of Π . $\square$

Claim 4 *$\Pi'_s(\xi) < 0$ so there is only one value of ξ such that $\Pi_s(\xi) = \Pi_0$.*

Proof. Totally differentiate equation (A.1) under $\pi = \Pi$. Based on this expression, a sufficient condition for $\Pi'_s(\xi) < 0$ is $X(\xi, \Pi) \leq \frac{1}{1-F(\xi)} \int_{\xi}^1 g(\omega) dF(\omega)$. Since $h(\omega, \bar{R}^*) \leq g(\omega)$, $\bar{\eta}^* \geq 0$, $\hat{f}_L(\xi, \Pi) \geq \hat{f}_H(\xi, \Pi)$, and $\hat{f}_H(\xi, \Pi) \leq 1$, this condition is satisfied. $\square$

Conditional on the participation constraint not binding, we can now conclude that the model has a unique non-trivial steady state. The following claim establishes that μ sufficiently large ensures non-bindingness, completing the proof of Proposition 3.

Claim 5 *If μ is sufficiently large, then $R(\omega) = \theta(\omega) - \sqrt{c\theta(\omega)}$ satisfies the borrower's participation constraint.*

Proof. After some algebra, the steady state participation constraint for a type $\omega \geq \xi$ borrower reduces to:

$$\beta(1-\mu)\Pi^2 \leq \frac{\Theta(\omega, R(\omega))}{\Theta(\omega, \bar{R}) - \Theta(\omega, R(\omega))}$$

where

$$\Theta(\omega, R) = \theta(\omega) - R - c + c \ln\left(\frac{c}{\theta(\omega) - R}\right)$$

With a sufficiently high value of μ , this equation will be satisfied. $\square$

From a computational perspective, Claim 5 can be interpreted as follows: if, for a given μ , we find an equilibrium ignoring the participation constraint then confirm that this equilibrium does indeed induce participation, we have found the unique equilibrium in the class of equilibria where the participation constraint does not bind. $\blacksquare$

B. Planner's Problem

The net output from financing type ω at rate R is:

$$y(\omega, R) = e(\omega, R) \theta(\omega) + c \ln(1 - e(\omega, R))$$

Let $W_I(\omega, \mathcal{R})$ denote the present discounted value of putting ω in an informed match, $W_U(\omega, \mathcal{R})$ the value of putting him in an uninformed match, and $W_N(\omega, \mathcal{R})$ the value of keeping him unmatched. The steady state functional equations are:

$$\begin{aligned} W_I(\omega, \mathcal{R}) &= \begin{cases} \beta W_N(\omega, \mathcal{R}) & \text{if } \omega < \xi \\ y(\omega, R(\omega)) + \beta(1 - \mu)W_I(\omega, \mathcal{R}) + \beta\mu W_N(\omega, \mathcal{R}) & \text{if } \omega \geq \xi \end{cases} \\ W_U(\omega, \mathcal{R}) &= \begin{cases} y(\omega, \bar{R}) + \beta W_N(\omega, \mathcal{R}) & \text{if } \omega < \xi \\ y(\omega, \bar{R}) + \beta(1 - \mu)W_I(\omega, \mathcal{R}) + \beta\mu W_N(\omega, \mathcal{R}) & \text{if } \omega \geq \xi \end{cases} \\ W_N(\omega, \mathcal{R}) &= \begin{cases} \Pi^2 W_U(\omega, \mathcal{R}) + (1 - \Pi^2) \beta W_N(\omega, \mathcal{R}) & \text{if } \omega < \xi \\ \Pi^2 W_U(\omega, \mathcal{R}) + \Pi(1 - \Pi)W_I(\omega, \mathcal{R}) + (1 - \Pi) \beta W_N(\omega, \mathcal{R}) & \text{if } \omega \geq \xi \end{cases} \end{aligned}$$

Solving this system isolates W_I , W_U , and W_N as functions of only ω and the choice variables. Denote the isolated expressions by $W_I^*(\omega, \mathcal{R}, \xi, \Pi)$, $W_U^*(\omega, \mathcal{R}, \xi, \Pi)$, and $W_N^*(\omega, \mathcal{R}, \xi, \Pi)$. In order to construct the aggregate welfare function, we now need the distribution of borrowers across informed financing, uninformed financing, and unmatchedness. For a given Π and ξ , the evolution of borrowers still follows equations (5) and (6), with the steady state versions as in Appendix A. To make the dependence of these distributions on the planner's choices clear, I write $\lambda(\omega|\xi, \Pi)$ and $\phi(\omega|\xi, \Pi)$. Measured at the beginning of the production stage, the objective function is then:

$$\begin{aligned} \mathcal{W}(\mathcal{R}, \xi, \Pi) &= \int_0^1 W_I^*(\omega, \mathcal{R}, \xi, \Pi) \lambda(\omega|\xi, \Pi) dF(\omega) \\ &\quad + \int_0^1 W_U^*(\omega, \mathcal{R}, \xi, \Pi) \phi(\omega|\xi, \Pi) dF(\omega) \\ &\quad + \int_0^1 \beta W_N^*(\omega, \mathcal{R}, \xi, \Pi) [1 - \lambda(\omega|\xi, \Pi) - \phi(\omega|\xi, \Pi)] dF(\omega) \end{aligned}$$

The planner's problem is to choose Π , ξ , $\bar{R}$, and $R(\cdot)$ in order to maximize $\mathcal{W}(\mathcal{R}, \xi, \Pi)$ subject to an aggregate feasibility constraint. The constraint requires that the amount of capital used to finance projects equals the amount of capital available to the planner each period. It is thus equivalent to the market clearing equation.

C. Computational Algorithm for Dynamics

Suppose a one-time aggregate productivity shock hits at $t = 1$. Recall that z is realized after lending and production decisions have been made so all credit market variables are still in steady state at $t = 1$. The capital available for $t = 2$ is then:

$$KS_2 = (1 - \delta)(1 + z) \left[\int_{\xi_{ss}}^1 \left(1 - \sqrt{\frac{c}{\theta(\omega)}}\right) R(\omega) \lambda_{ss}(\omega) dF(\omega) + \int_{\bar{\eta}_{ss}}^1 \left(1 - \frac{c}{\theta(\omega) - \bar{R}_{ss}}\right) \bar{R}_{ss} \phi_{ss}(\omega) dF(\omega) \right] < KS_{ss}$$

Even though z returns to its expected value by $t = 2$, the effects of the $t = 1$ shock are propagated over time due to the change in the stock of financing capital. I start by computing the propagation in absence of the participation constraint. Let $T + 1$ denote the date at which ξ_t returns to ξ_{ss} and let TT denote the date at which the economy returns to steady state. Note that $T + 1 < TT$ since the partition of the type space implied by the evolution of ξ_t must stabilize before the distribution over that space can stabilize. The rest of the transition path is computed in four steps:

1. For $t = 2, \dots, T$:

- Guess Π_t .
- Use Π_t , $\lambda_{t-1}(\cdot)$, and $\phi_{t-1}(\cdot)$ to get $\lambda_t(\cdot)$, $\phi_t(\cdot)$, $\bar{R}_t$, and KS_{t+1} .
- By bisection, find the ξ_t that equates KD_t to the previously determined KS_t .

2. For $t = T + 1, \dots, TT - 1$:

- Use $\lambda_{t-1}(\cdot)$, $\phi_{t-1}(\cdot)$, and $\xi_t = \xi_{ss}$ to get beliefs $\psi_t(\cdot)$.
- Use $J_{t+1}(\xi_{ss}, \cdot) = \beta U_{t+2}(\cdot)$ to get an expression for r_t .
- Recursive substitution of $J_{t+1}(\omega, \cdot)$ into $J_t(\omega, \cdot)$ yields:

$$J_t(\omega, \cdot) = \frac{g(\max\{\omega, \xi_{ss}\}) - g(\xi_{ss})}{1 - \beta(1 - \mu)} + \beta U_{t+1}(\cdot) \text{ for all } t \in [T + 1, TT - 1]$$

- Use $\psi_t(\cdot)$, r_t , and the expression for $J_t(\omega, \cdot)$ to get $U_t(\cdot)$.
- Based on the first order condition for π_t , get the optimal π_t^* .

3. For $t = TT - 1, \dots, T + 1$:

- Recall that the value functions at date TT are the steady state ones. Starting at $t = TT - 1$, use π_t^* as computed in step 2 to get $U_t(\cdot)$ and $J_t(\cdot)$.

- Work back until $t = T + 1$.
4. For $t = T, \dots, 2$
- From step 3, we know the date $T + 1$ value functions. Starting at $t = T$, determine the optimal choice π_t^* then the value functions $U_t(\cdot)$ and $J_t(\cdot)$.
 - Work back until $t = 2$.

Symmetry requires $\Pi_t = \pi_t$ so compare the guess $\{\Pi_t\}_{t=2}^T$ with the result $\{\pi_t^*\}_{t=2}^T$. If the root mean squared error is not sufficiently small, then update the guess in the direction suggested by the result. Repeat until RMSE-convergence then verify that the unconstrained choice of $R(\omega)$ does indeed satisfy the borrower's participation constraint.