

Relationship Lending and the Transmission of Monetary Policy*

Kinda Hachem[†]
University of Toronto

First Version: September 2008

This Version: January 2010

Abstract

This paper demonstrates that the banking notion of relationship lending matters for the transmission of monetary policy. I first construct an asymmetric information model with a continuum of heterogeneous borrowers and the possibility of lender learning through repeated interactions. I then derive the optimal credit contracts in this environment and analyze their implications for aggregate output. A variety of contracts are observed in equilibrium, with sufficiently good borrowers entering into multi-period lending relationships and economies that can sustain these relationships exhibiting a smoother steady state output profile and a more gradual response to certain monetary shocks. The results are consistent with empirical evidence so the model provides a basis for investigating the proportion of cross-country differences in monetary transmission that can be explained by cross-country differences in relationship lending.

JEL Classifications: D82, D83, E37, E44

Keywords: Asymmetric Information, Credit Channels, Learning, Relationship Lending, Monetary Transmission, Aggregate Output

*I am indebted to my supervisor, Professor Shouyong Shi, for his guidance and support during this research. I have also received useful comments from seminar participants at the University of Toronto and the Federal Reserve Bank of Chicago. Financial support from the Social Sciences and Humanities Research Council of Canada as well as Shouyong Shi's Bank of Canada Fellowship and Canada Research Chair are gratefully acknowledged. Any remaining errors are my own.

[†]Department of Economics, University of Toronto, 150 St. George Street, Toronto, Ontario, Canada, M5S 3G7. E-mail: kinda.hachem@utoronto.ca.

1 Introduction

Over the past few decades, the macroeconomic literature has emphasized the importance of financial intermediation for real activity. Bernanke (1983) and Diamond and Dybvig (1983), for example, have argued that financial disruption intensified the Great Depression by curtailing the credit needed for productive investment and, since then, formal credit channel models of the business cycle have been developed. In this paper, I study the implications of relationship lending, a concept not widely examined outside of the banking literature, for the propagation of monetary shocks. The theoretical results suggest that cross-country differences in relationship lending are a good candidate for explaining cross-country differences in monetary transmission.

One of the first papers to formalize the interaction between intermediation and business cycles is Williamson (1987). In his model, the key friction is costly state verification. In particular, the outcome of an investment project is only freely observable by the borrower who undertook it. The lender, on the other hand, must incur a monitoring cost to ensure that the borrower does not evade repayment by falsely claiming failure and, once embedded into a general equilibrium context, such costs generate a credit rationing effect that propagates shocks. Bernanke and Gertler (1989) consider a similar environment but focus on how the effects of costly state verification depend on borrower balance sheets. Borrowers with higher net worth can finance more investment internally, reducing the risk faced by their lenders and inducing lower agency costs. Shocks that affect net worth thus affect access to credit and this then compounds the response of spending. A similar prediction arises in Kiyotaki and Moore (1995) but the friction there is limited commitment, not private information. Lenders simply cannot force borrowers to repay unsecured debt so they demand collateral and, since net worth determines how much collateral a borrower can pledge, it also determines how much credit he will be extended. These models, however, abstract from multi-period credit relationships so some work has also been done to examine the implications of long-term credit contracts. In Gertler (1992), for example, on-going credit relationships permit debt

rescheduling so that credit constraints are determined by both current and expected future profits. Alternatively, in Khan and Ravikumar (2001) and Smith and Wang (2006), long-term contracts are used to give borrowers intertemporal incentives to report truthfully. While the last two papers suggest that such incentives improve economic outcomes, the volatile nature of expected profits leads the relationships in Gertler (1992) to amplify the output response to certain shocks.

In this paper, I consider another feature of multi-period lending relationships that has yet to be fully addressed in a macroeconomic context: learning. As defined in the banking literature, relationship lending is the provision of credit by financial intermediaries that acquire proprietary information about their borrowers over multiple interactions.¹ Lenders grant credit with an eye to whether or not project outcomes will permit them to be repaid and, in reality, project outcomes depend on private borrower characteristics. Therefore, the banking concept of relationship lending can change the nature of the game when multi-period credit contracts are an option.² This is not to say that relationship lending undoes the credit channel but, rather, that assuming away the propensity of credit markets to turn out such relationships also assumes away a potentially important source of cross-country transmission differences. Recent empirical evidence supports this argument. Across the major European economies, Ehrmann et al (2001) establish that relationship lending is very prevalent in Germany and Italy but not in Spain and France. Incidentally, they also find that the quantity of bank loans responds less severely to a monetary contraction in the first two countries than in the last two. A similar pattern obtains on the pricing side. Borio and Fritz (1995), for example, find that policy rate increases translate more slowly into loan rate increases in Germany and Italy than in Spain. Additional support for the impact of relationship lending in Germany is provided by Weth (2002) and Iacoviello and Minetti (2008)

¹Boot (2000).

²Multiple periods are important both because they permit learning and because learning has long-term implications. Although Bose and Cothren (1997) present a growth model where lenders invest in learning about borrower types, the information acquired cannot be used in future contracts since agents are two-period-lived overlapping generations and thus only enter into credit contracts in their first period.

while additional evidence for Italy is provided by Gambacorta (2004). Moreover, based on US survey data, Berger and Udell (1995) conclude that American borrowers with larger banking relationships pay lower interest rates and are less likely to pledge collateral. Taken together, these studies support the contention that relationship banking changes the way credit responds to monetary shocks. That this change ultimately figures in the transmission process is evidenced by the Mojon and Peersman (2003) finding that the peak decline in investment following a monetary contraction is smaller in Germany and Italy than in Spain and France.

While the empirical evidence supports a correlation between the extent of relationship lending and the effects of monetary policy, more theoretical work is needed for a causal understanding. Building on the environments presented in Schmeits (2005) and Van Tassel (2002), I first construct an asymmetric information model with heterogeneous borrowers and the possibility of lender learning through multiple interactions. In particular, all borrowers have access to investment projects but, depending on their type, succeed with different probabilities. Types are private information unless successive periods of financing are obtained from the same lender, in which case that lender becomes more informed about his borrower. Other lenders are not privy to this information but they can update their beliefs based on what they do observe. Unlike the banking literature, however, I analyze how changes in the policy rate affect aggregate output through the optimal contracts. I also consider a continuum of firm types and allow for different firm exit rates since the informational properties of relationship lending can lead to improved credit terms for certain borrowers, altering their ability to overcome adverse, idiosyncratic shocks. A variety of contracts arise in equilibrium, with sufficiently good borrowers entering into multi-period lending relationships and economies that can sustain these relationships exhibiting a smoother steady state output profile and a more gradual response to certain monetary shocks. This prediction is consistent with the empirical correlations discussed above so the model provides a basis for investigating the proportion of cross-country differences in monetary transmission that

can be explained by cross-country differences in relationship lending. The rest of the paper proceeds as follows. Section 2 describes the environment in more detail. The baseline model is analyzed in Section 3 and extended in Section 4. Section 5 concludes.

2 Environment

Consider an economy with a continuum of firm types, denoted by $\omega \in [0, 1]$ and distributed according to a non-degenerate probability density function $f(\cdot)$. All types have access to the same production technology but differ in their probabilities of successful operation. In particular, a type ω firm is able to produce θ_1 units of output with probability $p(\omega)$ and zero units with probability $1 - p(\omega)$ where $p : [0, 1] \rightarrow [0, 1]$ is a continuously differentiable, strictly increasing function. As an outside option, firms also have access to a riskier, type-independent project which yields θ_2 with probability q and zero with probability $1 - q$. Assume that $\theta_1 < \theta_2$ and $p(0)\theta_1 = q\theta_2$.³ In what follows, I refer to the production technology as *P1* and the outside option as *P2*. In order to undertake either project, firms need one unit of capital. I assume that output is not storable so this capital must be borrowed and repayment depends only on the current period's project. In particular, the borrowed unit is always recovered by the lender since capital is not destroyed in the production process but interest payments can only be made by borrowers with successful projects. I assume that lenders cannot observe the exact output of a project but they can detect the presence of consumption so borrowers pay interest at the end of the period if and only if their projects are successful. Each period then, lenders have to decide how much to charge while borrowers have to decide which contracts to accept and which projects to then undertake. For now then, there is no quantity rationing.

When borrowers first enter the credit market, their type is private information and they

³ $p(0)\theta_1 > q\theta_2$ requires more algebra but yields similar conclusions. Either way, the outside option is riskier in the sense that it is second order stochastic dominated by the production technology. The presence of a riskier option introduces moral hazard and creates a trade-off in the lender's problem. Moreover, the type-independent nature of this project allows for a pure strategy equilibrium in the simultaneous game between second period lenders.

choose among perfectly competitive lenders. After providing credit to a borrower for one period though, it is likely that the chosen lender – the insider – knows more about that particular borrower than do all the other lenders. I abstract from the process through which lenders acquire information, summarizing it instead by a positive probability of type discovery. In other words, insiders learn their borrower’s type with probability $\phi \in (0, 1]$ before deciding on a second period loan rate. In order to simplify the exposition, I take $\phi = 1$. Other lenders – that is, the outsiders – are not privy to this information and do not observe the insider’s offer when they make their own offers. They do, however, find out if the borrower defaulted on his first period loan and can revise their beliefs about the borrower’s type conditional on this information.⁴ Since default occurs when the borrower’s project yields no output, the probability of default is just the probability of project failure. Given that the insider also knows his borrower’s default history, he can form expectations about the outsider offer. After receiving both the insider and outsider offers, the borrower decides whether to continue with the insider in the second period or move to an outsider.

At the end of the borrower’s second period in the credit market, I assume that his type is revealed to everyone. This is done to avoid carrying credit history throughout the model and, therefore, to keep the state space finite. Also starting at the end of the second period though is a positive probability that the borrower is exogenously separated from the credit market. This separation eliminates all information about him and requires that he draw a new type and re-enter the market as a first-time borrower next period. Therefore, even though information is revealed after two periods, there are always first-time, second-time, and advanced borrowers at any date t . In what follows, I use *periods* $k = 1, 2, \dots$ to denote time in the credit market and *dates* $t = 1, 2, \dots$ to denote time in the general economy. The probability of exogenous separation is $\mu - \varepsilon I \in (0, 1)$ where I equals 1 if the borrower stays with his insider and 0 otherwise. In Section 3, $\varepsilon = 0$ so that the insider’s only advantage

⁴This is the only cost of default in the model. If, in addition, the borrower is forced to wait a few periods before his next contract, the marginal type that chooses the risky project may fall but the qualitative conclusions of the model are unlikely to change.

over an outsider is informational. The results there support the argument that, on average, the informational properties of relationship lending lead to improved credit terms. To the extent that firms with better terms are better able to overcome adverse, idiosyncratic shocks, relationship lending may also be consistent with $\varepsilon > 0$. This case is considered in Section 4.

3 Baseline Model ($\varepsilon = 0$)

3.1 Period $k \geq 3$

Since types are revealed to everyone in the third period, the problem is one of perfect information for $k \geq 3$. Project choice does not affect future outcomes as borrowers either start anew with exogenous probability μ or continue to period $k + 1$ with exogenous probability $1 - \mu$. Consequently, each borrower will choose the project that yields him a higher expected return in the current period. A trade-off arises, however, since $P1$ generates more expected revenue but also increases the likelihood of interest payments. At high loan rates then, the borrower may have an incentive to choose the riskier project. More precisely, the returns to $P1$ and $P2$ for a type ω borrower are $p(\omega)[\theta_1 - R]$ and $q[\theta_2 - R]$ respectively and the loan rate that makes him indifferent between the two is:

$$\bar{R}(\omega) = \frac{p(\omega)\theta_1 - q\theta_2}{p(\omega) - q} \quad (1)$$

where $\bar{R}(0) = 0$ and $\bar{R}'(\omega) > 0$. Type ω borrowers thus choose $P1$ if charged $R \leq \bar{R}(\omega)$ and $P2$ otherwise. I summarize this strategy as follows:

$$\bar{\gamma}(R|\omega) = \begin{cases} p(\omega) & \text{if } R \leq \bar{R}(\omega) \\ q & \text{if } R > \bar{R}(\omega) \end{cases} \quad (2)$$

Now, given the borrower's optimal project choice, lenders choose the loan rate.⁵ The

⁵Since an advanced borrower operates in a perfect information environment, he attracts the same offer from every lender when $\varepsilon = 0$ and is thus indifferent among them. Without loss of generality, I complete the borrower's strategy by assuming that he stays with his second period lender for all $k \geq 3$.

policy rate r enters the model as the lender's cost of funds so the value of a lender with information set Ω about a k^{th} period borrower is denoted by $J_k(r|\Omega)$. In each period k , the lender offers the loan rate $R_k^*(r|\Omega)$ that maximizes expected profits and generates this value. With probability μ , lenders in $k \geq 3$ are separated from their borrowers and must start the next period with a first-timer. All lenders have the same information set in period 1 so competition forces expected profits there to zero. Symmetric information in periods $k \geq 3$ also means zero profits, making the value function of a lender with an advanced borrower:

$$J_{k \geq 3}(r|\omega) = \max_R \{ \bar{\gamma}(R|\omega) R - r \} \longrightarrow 0 \quad (3)$$

The solution to this problem is given by Proposition 1:

Proposition 1 *For each $k \geq 3$, the loan rate offered to a type ω borrower is:*

$$R_{k \geq 3}^*(r|\omega) = \begin{cases} r/p(\omega) & \text{if } r \leq p(\omega) \bar{R}(\omega) \\ r/q & \text{if } r > p(\omega) \bar{R}(\omega) \end{cases} \quad (4)$$

Consistent with their more costly nature then, the average loan rate offered to a low type exceeds that offered to a high one.

Proof. *I begin by analyzing $\bar{\gamma}(R|\omega) R$. First, note that the highest loan rate sustainable as an equilibrium is θ_2 : anything above θ_2 and the borrower will not want to undertake either project. Second, not all loan rates below θ_2 are optimal. Consider the expected revenue associated with charging $\bar{R}(\omega)$ versus $\bar{R}(\omega) + \delta$ for $\delta > 0$. At $\bar{R}(\omega)$, the borrower chooses P1 and the lender receives $p(\omega) \bar{R}(\omega)$ while, at $\bar{R}(\omega) + \delta$, he chooses P2 and the lender receives $q(\bar{R}(\omega) + \delta)$. The second expression is less than the first for $\delta < (p(\omega) - q) \bar{R}(\omega) / q \equiv \tilde{\delta}_1$. Since $p(\omega) > q$, the set $(0, \tilde{\delta}_1)$ is non-empty and, therefore, all loan rates between $\bar{R}(\omega)$ and $\bar{R}(\omega) + \tilde{\delta}_1$ are dominated by $\bar{R}(\omega)$. Substituting in the value for $\tilde{\delta}_1$, the set of dominated loan rates simplifies to $(\bar{R}(\omega), \hat{R}(\omega))$ where $\hat{R}(\omega) \equiv p(\omega) \bar{R}(\omega) / q$. The optimized revenue function of a perfectly informed lender is illustrated by the bold line in Figure 1.*

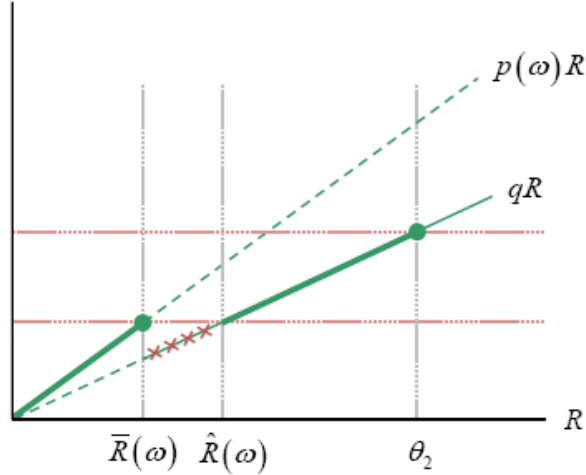


Figure 1: Optimized revenue function of an informed lender

In equilibrium though, competition forces $\bar{\gamma}(R|\omega)R = r$ so we can re-interpret the vertical axis in Figure 1 as the policy rate. Reflecting the graph over the 45° line so that r is now on the horizontal yields $R_{k \geq 3}^*(r|\omega)$ as given in equation (4). The last part of the proposition follows from the fact that $p'(\omega) > 0$. ■

Proposition 1 implies that $R_{k \geq 3}^*(r|\omega) \leq \bar{R}(\omega)$ if $r \leq p(\omega)\bar{R}(\omega)$ and $R_{k \geq 3}^*(r|\omega) > \bar{R}(\omega)$ if $r > p(\omega)\bar{R}(\omega)$. To determine the output produced by advanced borrowers under these loan rates, I normalize the population in each period to one and define type $\tilde{\omega}(r)$ such that $p(\tilde{\omega}(r))\bar{R}(\tilde{\omega}(r)) = r$. Both $p(\cdot)$ and $\bar{R}(\cdot)$ are strictly increasing functions so, for all $\omega < \tilde{\omega}(r)$, it follows that $p(\omega)\bar{R}(\omega) < r$ and, therefore, $R_{k \geq 3}^*(r|\omega) > \bar{R}(\omega)$. Recalling the optimal strategy summarized by equation (2), this implies that all types below $\tilde{\omega}(r)$ choose *P2*. In a similar manner, it can be shown that all types above $\tilde{\omega}(r)$ choose *P1*, making the total output of advanced borrowers:

$$Y_{k \geq 3}(r) = \int_0^{\tilde{\omega}(r)} q\theta_2 dF(x) + \int_{\tilde{\omega}(r)}^1 p(x)\theta_1 dF(x) \quad (5)$$

With $\tilde{\omega}'(r) > 0$ and $p(x)\theta_1 > q\theta_2$ for $x \in (0, 1]$, equation (5) defines an output function that is monotonically decreasing in the policy rate.

3.2 Period $k = 2$

In the second period, first period credit histories are made public. Denote default by $d = D$ and non-default by $d = N$. While this is the only information observed by outside lenders, insiders also learn their borrower's type before making an offer. Since, however, this type is also ultimately revealed to outsiders in the third period, the borrower's future outcomes are independent of current project choice and the optimal strategy of a type ω borrower is still given by $\bar{\gamma}(R|\omega)$.

Consider first an inside lender who has discovered that his borrower is type ω . With competition driving future profits to zero, the insider's expected revenue is just $\bar{\gamma}(R|\omega)R$ as illustrated in Figure 1. Now, however, the insider has an informational advantage over all other lenders so competition will not necessarily eliminate current profits. Noting that the insider can charge above the outsider rate and lose the borrower, his value is:

$$J_2(r|\omega, d) = \max \left\{ 0, \max_R \bar{\gamma}(R|\omega)R - r \text{ s.t. } R \leq R_2^*(r|d) \right\} \quad (6)$$

Without loss of generality, I assume that insiders only keep borrowers which net them positive expected profit.⁶ The following proposition establishes the structure of the $k = 2$ equilibrium:

Proposition 2 *The second period credit market separates neatly between insiders and outsiders. In particular, for a given credit history d , outsiders attract $\omega \in [0, c_d(r)]$ and insiders retain $\omega \in (c_d(r), 1]$ where $c_d(r)$ satisfies $J_2(r|c_d(r), d) = 0$.*

Proof. *Suppose types ω_0 and $\omega_0 + \delta$ have the same credit history d . Since credit history is the only information that the outsider can condition on, he offers ω_0 and $\omega_0 + \delta$ the same loan rate $R_2^*(r|d)$. Consider an insider who finds it optimal to keep ω_0 . Retaining ω_0 when $\varepsilon = 0$ implies that the insider must be charging $R_2^*(r|\omega_0, d) \leq R_2^*(r|d)$. Moreover, the fact that*

⁶This assumption simplifies the exposition aimed at here but is innocuous. Assuming instead that the insider keeps types for which he is indifferent yields the same loan rates and output functions derived below but the split between borrowers may occur within the insider rather than across insiders and outsiders.

the insider finds it optimal to keep ω_0 implies that he must be making positive profit on this borrower. With $\delta > 0$ and $\bar{R}'(\omega) > 0$, equation (2) establishes that $\bar{\gamma}(R|\omega_0 + \delta) \geq \bar{\gamma}(R|\omega_0)$ for any R . Therefore, the insider could offer $\omega_0 + \delta$ loan rate $R_2^*(r|\omega_0, d)$, keep him, and make at least as much as he is making on ω_0 . Since ω_0 and δ were chosen arbitrarily, Proposition 2 follows by induction. ■

Consider now an outsider who has attracted a $k = 2$ borrower and represent his beliefs about the borrower's type by a cumulative distribution function $\hat{F}_d(\cdot)$ defined over the interval $[0, c_d(r)]$ according to Bayes' rule. Whatever the borrower's type actually is, it will be known to everyone in the next period so future profits will be competed away. The expected profit of an outsider who charges his class d borrower R_d is thus:

$$\int_0^{c_d(r)} \bar{\gamma}(R_d|x) R_d d\hat{F}_d(x) - r \longrightarrow 0 \quad (7)$$

In equilibrium, the outsider's beliefs will depend on R_d . Moreover, all outsiders must have the same beliefs so competition also drives $J_2(r|d)$ down to zero.

Proposition 3 *When $k = 2$, outside lenders offer $R_2^*(r|d) = r/q$ and get $\omega \in [0, \tilde{\omega}(r)]$. Inside lenders keep $\omega \in (\tilde{\omega}(r), 1]$ and offer:*

$$R_2^*(r|\omega, d) = \begin{cases} \bar{R}(\omega) & \text{if } \omega \in (\tilde{\omega}(r), \hat{\omega}(r)) \\ r/q & \text{if } \omega \in [\hat{\omega}(r), 1] \end{cases} \quad (8)$$

where $\tilde{\omega}(r)$ is as defined in Section 3.1 and $\hat{\omega}(r)$ is such that $q\bar{R}(\hat{\omega}(r)) = r$.

Proof. Let $R_d(r)$ denote the as yet undetermined solution to the outsider's problem and define type $\omega_d(r)$ such that $\bar{R}(\omega_d(r)) = R_d(r)$. By definition, all types above $\omega_d(r)$ choose P1 and all types below it choose P2 so equation (7) can be written as:

$$R_d(r) = \begin{cases} r / \left[\int_0^{\omega_d(r)} q d\hat{F}_d(x) + \int_{\omega_d(r)}^{c_d(r)} p(x) d\hat{F}_d(x) \right] & \text{if } \omega_d(r) < c_d(r) \\ r/q & \text{if } \omega_d(r) \geq c_d(r) \end{cases} \quad (9)$$

Lemma 1: $\widehat{F}_d(\cdot)$ is the CDF of a non-degenerate distribution.

Proof: Using Bayes' Rule, $\widehat{F}_d(x)$ is given by:

$$\widehat{F}_d(x) \equiv \Pr(\omega \leq x|d) = \frac{\Pr(d|\omega \leq x) \Pr(\omega \leq x)}{\Pr(d)}$$

All first-time borrowers advance to the second period so the unconditional type distribution is still $F(\cdot)$. Note, however, that $\Pr(\omega \leq x) = F(x)/F(c_d(r))$ since the outsider only gets $\omega \in [0, c_d(r)]$ in equilibrium. If $c_d(r) \leq \xi(r)$, where $\xi(r)$ is the lowest type that chose P1 in the first period, then the outsider's beliefs are given by:

$$\widehat{F}_d(x) = \frac{F(x)}{F(c_d(r))} \text{ for } x \in [0, c_d(r)]$$

If, however, $c_d(r) > \xi(r)$, Bayesian updating yields:

$$\widehat{F}_N(x) = \begin{cases} \frac{qF(x)}{qF(\xi(r)) + \int_{\xi(r)}^{c_N(r)} p(z) dF(z)} & \text{if } x \in [0, \xi(r)] \\ \frac{qF(\xi(r)) + \int_{\xi(r)}^x p(z) dF(z)}{qF(\xi(r)) + \int_{\xi(r)}^{c_N(r)} p(z) dF(z)} & \text{if } x \in [\xi(r), c_N(r)] \end{cases}$$

$$\widehat{F}_D(x) = \begin{cases} \frac{(1-q)F(x)}{(1-q)F(\xi(r)) + \int_{\xi(r)}^{c_D(r)} (1-p(z)) dF(z)} & \text{if } x \in [0, \xi(r)] \\ \frac{(1-q)F(\xi(r)) + \int_{\xi(r)}^x (1-p(z)) dF(z)}{(1-q)F(\xi(r)) + \int_{\xi(r)}^{c_D(r)} (1-p(z)) dF(z)} & \text{if } x \in [\xi(r), c_D(r)] \end{cases}$$

With $F(\cdot)$ well-behaved, $\widehat{F}_d(\cdot)$ is the CDF of a non-degenerate distribution. \square

Lemma 2: $\omega_d(r) \geq c_d(r)$.

Proof: Suppose $\omega_d(r) < c_d(r)$. In this case, a type $c_d(r)$ borrower chooses P1 if offered loan rate $R_d(r)$, permitting his insider an expected profit of $\Pi = p(c_d(r))R_d(r) - r$. Given equation (9) and Lemma 1, $\omega_d(r) < c_d(r)$ also implies $R_d(r) > r/p(c_d(r))$ and, therefore, $\Pi > 0$. From Proposition 2, however, $J_2(r|c_d(r), d) = 0$ so it should be the case that $\Pi \leq 0$. By contradiction then, $\omega_d(r) \geq c_d(r)$. \square

Taken together, Lemma 2 and equation (9) yield $\bar{R}(c_d(r)) \leq R_2^*(r|d) = r/q$. To determine the value of $c_d(r)$, note that an insider with a type $c_d(r)$ borrower can make an expected profit of $p(c_d(r))\bar{R}(c_d(r)) - r \leq J_2(r|c_d(r), d) = 0$ by charging him $\bar{R}(c_d(r))$. If this expression is strictly negative, then there exists a $\delta > 0$ such that $\bar{R}(c_d(r) + \delta) < r/q$ and $p(c_d(r) + \delta)\bar{R}(c_d(r) + \delta) - r < 0$. Therefore, the best an insider can do on a type $c_d(r) + \delta$ borrower is charge $R_2^*(r|d) = r/q$ and break even but, given that insiders only keep borrowers which yield them positive profit, this contradicts the fact that they keep all $\omega > c_d(r)$. In equilibrium then, $c_D(r)$ and $c_N(r)$ are implicitly defined by $p(\cdot)\bar{R}(\cdot) = r$. This is the same equation that defines $\tilde{\omega}(r)$, implying $c_D(r) = c_N(r) = \tilde{\omega}(r)$. Since we now know that the insider only keeps $\omega > \tilde{\omega}(r)$, we can restrict attention to $r < p(\omega)\bar{R}(\omega)$ in order to prove equation (8). Consider first $r \in (q\bar{R}(\omega), p(\omega)\bar{R}(\omega))$. In this case, the outsider offer of $R_2^*(r|d) = r/q$ falls between $\bar{R}(\omega)$ and $\hat{R}(\omega)$ where $\hat{R}(\omega) \equiv p(\omega)\bar{R}(\omega)/q$. Recall from the proof of Proposition 1 that an informed lender would rather charge $\bar{R}(\omega)$ than any $R \in (\bar{R}(\omega), \hat{R}(\omega))$. Therefore, since the insider cannot charge above $R_2^*(r|d)$ and keep the borrower, he charges $R_2^*(r|\omega, d) = \bar{R}(\omega)$ and gets $J_2(r|\omega, d) = p(\omega)\bar{R}(\omega) - r > 0$. Consider now $r \leq q\bar{R}(\omega)$. In this case, the outsider's offer falls below the borrower's reservation loan rate and the best the insider can do is match it, yielding $R_2^*(r|\omega, d) = r/q$ and $J_2(r|\omega, d) = (p(\omega) - q)r/q$. Recalling that $r \in (q\bar{R}(\omega), p(\omega)\bar{R}(\omega))$ corresponds to $\omega \in (\tilde{\omega}(r), \hat{\omega}(r))$ and $r \leq q\bar{R}(\omega)$ corresponds to $\omega \geq \hat{\omega}(r)$ completes the proof. ■

The proof of Proposition 3 establishes that all types below $\tilde{\omega}(r)$ move to an outsider and pay $R_2^*(r|d) > \bar{R}(\omega)$ while all types above $\tilde{\omega}(r)$ stay with their insiders and pay $R_2^*(r|\omega, d) \leq \bar{R}(\omega)$. As a result, $Y_2(r)$ equals $Y_{k \geq 3}(r)$ as given in equation (5). In other words, when the insider discovers his borrower's type with certainty, the perfect information level of output is achieved. This, however, would not be the case with $\phi < 1$ since $\phi < 1$ implies a positive probability that no one is informed about the borrower's type in $k = 2$, making credit history the only piece of information available to the market. Proposition 3 also establishes that all types above $\tilde{\omega}(r)$ benefit from maintaining lending relationships with better ones benefiting

over wider ranges of the policy rate. The first part of this statement follows from the fact that types $\omega \in (\tilde{\omega}(r), 1]$ stay with their insiders and are consequently charged a flat rate of $\bar{R}(\omega)$ over the interval $r \in (q\bar{R}(\omega), p(\omega)\bar{R}(\omega))$ instead of the monotonically increasing function of r that arises in a pooled equilibrium. The second part follows from the fact that the length of this interval is increasing in ω . Insiders also benefit from entering into lending relationships with better firms since $p(\tilde{\omega}(r))\bar{R}(\tilde{\omega}(r)) - r = 0$ implies that they make positive profits on types above $\tilde{\omega}(r)$. Lower types, in contrast, do not establish banking relationships when their insiders are informed, consistent with the empirical finding of Memmel et al (2007) that high quality firms are more likely to choose relationship lenders.

It is interesting to note the role of competition here. In the presence of outside lenders, the second-period insider is forced to solve a constrained optimization problem. On the one hand, he cannot extract monopoly rents but, on the other, competition does not fully erode his informational advantage and, in equilibrium, the insider finds it optimal to use his finer information set and tailor contracts around the borrower's reservation rate in the manner discussed above. As in Schemits (2005) then, competition mitigates the hold-up problem. The result that competition can help sustain a mutually beneficial second-period credit contract contrasts somewhat with the Petersen and Rajan (1995) argument that concentration increases the value of lending relationships so the treatment of information appears critical in analyzing interactions between credit market structure and credit market outcomes. This view is supported by Cao and Shi (2001).

3.3 Period $k = 1$

Recall from the previous section that second period loan rates do not depend on credit history when $\phi = 1$. Therefore, the reservation rate of a first-time borrower is still $\bar{R}(\omega)$ and his optimal strategy can once again be summarized by $\bar{\gamma}(R|\omega)$. Moreover, we can use $J_2(r|\omega)$ rather than $J_2(r|\omega, d)$ to denote the lender's valuation of a second period credit relationship with a type ω borrower. Assuming that the current policy rate is the best predictor of the

future policy rate, first period lenders obtain the following expected profit from charging their borrowers R_1 :

$$\int_0^1 \bar{\gamma}(R_1|x) R_1 dF(x) + \beta \int_{\tilde{\omega}(r)}^1 J_2(r|x) dF(x) - r \longrightarrow 0 \quad (10)$$

With a continuum of types, the discontinuity of $\bar{\gamma}(R_1|x)$ makes it impossible to get a closed-form expression for the first period loan rate so I employ the following computational procedure. Let $R_1(r)$ denote the as yet undetermined solution to equation (10) and define a type $\xi(r)$ such that $\bar{R}(\xi(r)) = R_1(r)$. By this definition, all types above $\xi(r)$ choose $P1$ in the $k = 1$ equilibrium and all types below it choose $P2$. Substituting in the $J_2(r|x)$'s implied by Proposition 3, equation (10) can be re-written as:

$$\bar{R}(\xi(r)) = \frac{\left[\frac{1}{\beta} + [1 - F(\tilde{\omega}(r))] - \int_{\tilde{\omega}(r)}^1 \frac{p(x)}{q} dF(x) \right] r - \int_{\tilde{\omega}(r)}^{\tilde{\omega}(r)} p(x) \bar{R}(x) dF(x)}{\frac{1}{\beta} \left[\int_0^{\xi(r)} q dF(x) + \int_{\xi(r)}^1 p(x) dF(x) \right]} \quad (11)$$

Given r , I run a grid search over the unit interval to find the $\xi(r)$ that satisfies this equation. The first period loan rate is then $\bar{R}(\xi(r))$ and total first period output is:

$$Y_1(r) = \int_0^{\xi(r)} q \theta_2 dF(x) + \int_{\xi(r)}^1 p(x) \theta_1 dF(x) \quad (12)$$

3.4 Aggregate Output

At any date t , existing borrowers can be divided into three groups: those that entered the credit market at date t , those that entered at date $t - 1$, and those that entered before date $t - 1$. The first group produces according to $Y_1(r)$, the second according to $Y_2(r)$, and the third according to $Y_{k \geq 3}(r)$. In order to determine aggregate output, it is necessary to determine how many borrowers are in each group. Re-normalizing the entire population to one, this is equivalent to determining the probability that a given borrower falls into one of these groups. Let $\Psi_{1,t}$, $\Psi_{2,t}$, and $\Psi_{k \geq 3,t}$ denote the date t weights of these groups. Aggregate

output can then be written as:

$$Y(r, t) = \Psi_{1,t}Y_1(r) + \Psi_{2,t}Y_2(r) + \Psi_{k \geq 3,t}Y_{k \geq 3}(r)$$

With the possibility of exogenous separation beginning at the end of the second period, the group weights evolve according to:

$$\begin{aligned}\Psi_{1,t+1} &= \mu(\Psi_{2,t} + \Psi_{k \geq 3,t}) \\ \Psi_{2,t+1} &= \Psi_{1,t} \\ \Psi_{k \geq 3,t+1} &= 1 - \Psi_{1,t+1} - \Psi_{2,t+1}\end{aligned}$$

Substituting $\Psi_{k \geq 3,t}$ into the expression for $\Psi_{1,t+1}$, the evolution of Ψ_1 is determined by a one-dimensional difference equation and, with $\mu \in (0, 1)$, the entire system is asymptotically stable. Starting from any initial distribution then, the group weights converge to $\Psi_1 = \Psi_2 = \frac{\mu}{1+\mu}$ and $\Psi_{k \geq 3} = \frac{1-\mu}{1+\mu}$ and steady state aggregate output equals:

$$Y(r) = \frac{\mu}{1+\mu}Y_1(r) + \frac{1}{1+\mu}Y_{k \geq 3}(r) \quad (13)$$

where $Y_1(r)$ and $Y_2(r) = Y_{k \geq 3}(r)$ are given by equations (12) and (5) respectively.

In addition to comparative statics, it will be instructive to compare $Y(r)$ with the output function generated by a standard credit channel model where exogenous separation occurs with certainty every period and private information is never revealed. In this context, a representative lender would expect the following profit from charging R_S :

$$\int_0^1 \bar{\gamma}(R_S|x) R_S dF(x) - r \longrightarrow 0$$

Representing the solution to this problem by $R_S(r) = \bar{R}(\xi_S(r))$, we can characterize $\xi_S(r)$ by $\left[\int_0^{\xi_S(r)} q dF(x) + \int_{\xi_S(r)}^1 p(x) dF(x) \right] \bar{R}(\xi_S(r)) = r$ and use it to get the following output function for the standard model:

$$Y_S(r) = \int_0^{\xi_S(r)} q \theta_2 dF(x) + \int_{\xi_S(r)}^1 p(x) \theta_1 dF(x)$$

3.5 Numerical Results

Figures 2 and 3 compare the steady state output functions for two parameterizations. In Figure 2, $\beta = 0.96$, $\theta_1 = 5$, $\theta_2 = 6$, $q = 0.525$, and $p(\omega) = 0.63 + 0.27\omega$. In Figure 3, I keep β , θ_1 , and θ_2 unchanged and narrow the dispersion of probabilities so that $q = 0.65$ and $p(\omega) = 0.78 + 0.12\omega$. This ensures that values for $\xi(r)$ and $\xi_S(r)$ can always be found on the unit interval without loosening their defining equations. The distribution of types under both parameterizations is assumed to be uniform.

The grey lines are the output functions generated by the standard credit model. The blue and red lines are the functions for periods 1 and $k \geq 3$ generated by the model with relationship lending. Aggregate output in the relationship lending model is then illustrated by the black lines. The solid black line is based on $\mu = 0.1$ while the dashed one is based on $\mu = 0.5$. The extent of relationship lending is given by $(1 - \tilde{\omega}(r)) / (1 + \mu)$ and is decreasing in the policy rate and the rate of exogenous separation.

Even though the first period of the relationship lending model is characterized by the same information frictions as the standard model, $Y_1(r)$ lies above $Y_S(r)$. In anticipation of the second period insider profits afforded by a lending relationship, first period lenders compete more fiercely for borrowers, pushing the pooled rate down further and making it profitable for a greater number of types to choose *PI*. This effect is most pronounced over moderate policy rates. Since only types above $\tilde{\omega}(r)$ stay with their insiders and $\tilde{\omega}'(r) > 0$, high values of r are associated with fewer lending relationships, lowering the expectation of future profits and weakening the drive to attract borrowers. The divergence between $Y_1(r)$ and $Y_S(r)$ increases as r decreases but, for very low values of r , the scope for further reductions in the pooled rate is limited and the difference between the two lines retrenches but does not entirely disappear. Relationship lending thus leads to a marked improvement in first period output.

As established in Section 3.2, relationship lending also pushes second period output towards the more temperate perfect information profile. The difference between $Y_{k \geq 3}(r)$

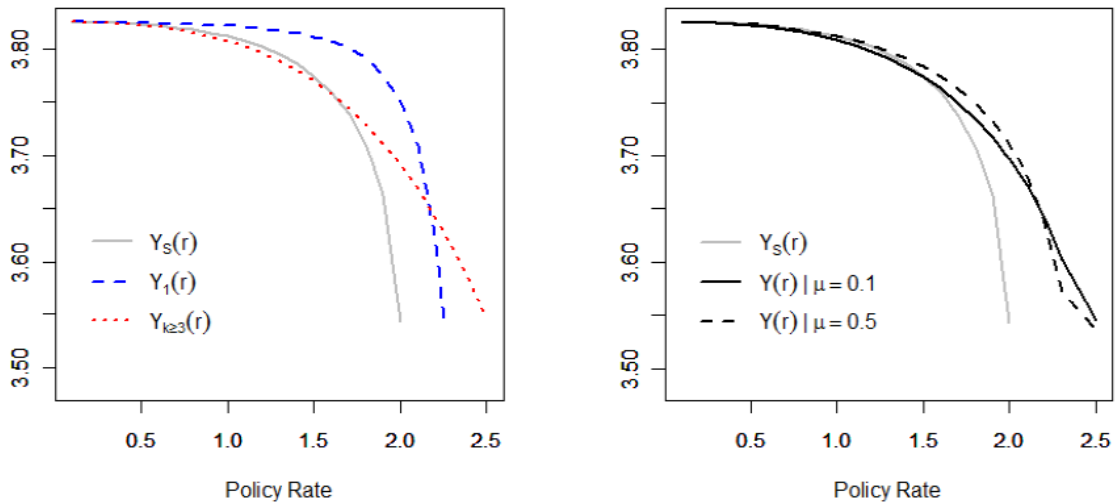


Figure 2: Steady state output under $\varepsilon = 0$ and $q = 0.525$

and $Y_S(r)$ is intuitive. At low policy rates, informed lenders can grant favourable credit terms (i.e., terms that induce $P1$) to a larger proportion of borrowers without suffering a loss. The same is true for uninformed lenders in the standard model but, since they calculate profits and losses in expected terms, they overcompensate. In particular, the fact that they can cover their cost of funds with a relatively low loan rate that incidentally compels most borrowers to choose the safe project leads them to expect a higher probability of repayment. Competition then induces a lower pooled rate and we observe $Y_S(r) \geq Y_{k \geq 3}(r)$. In contrast, when the cost of funds is sufficiently high, this mechanism has the opposite effect so we observe $\xi_S(r) > \tilde{\omega}(r)$ and $Y_S(r) < Y_{k \geq 3}(r)$.

Aggregating $Y_1(r)$ and $Y_{k \geq 3}(r)$ and comparing to a standard credit model then, we can conclude that relationship lending leads to a smoother aggregate output function, with a greater measure of these relationships enabling a greater degree of smoothing. To demonstrate that the preceding results are not just a by-product of the parameters chosen, the Appendix solves the model analytically for the case of two borrower types. The conclusion of the continuum model holds: economies poised to foster longer-term credit relationships are also poised for a smoother output profile.

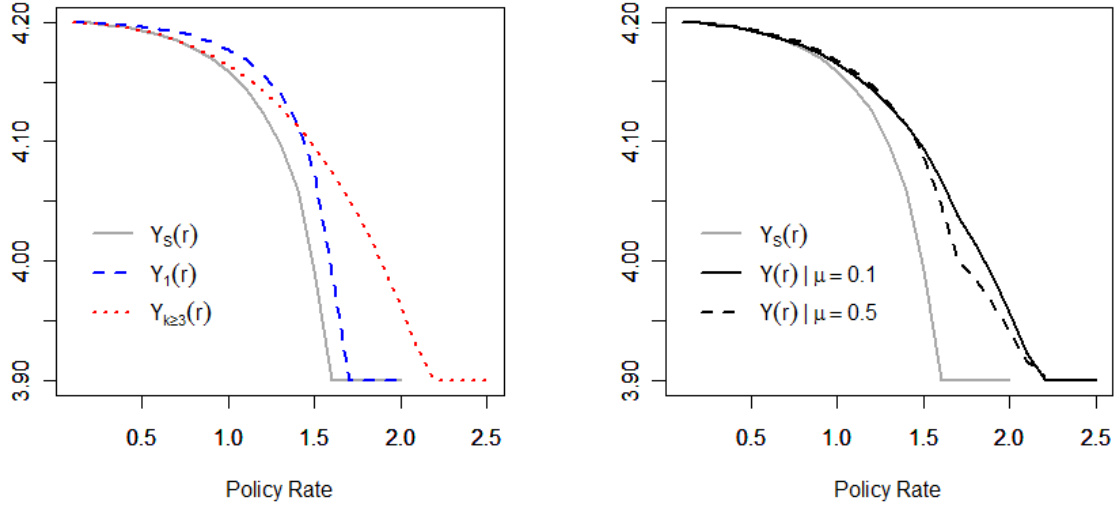


Figure 3: Steady state output under $\varepsilon = 0$ and $q = 0.65$

A corollary of this result is that the institutional parameters affecting an economy’s ability to foster lending relationships also affect its real response to policy shocks. Higher rates of exogenous separation, for example, lead to a greater proportion of first-time borrowers, cutting into the smoothing benefits of relationship lending at high policy rates.⁷ Here, μ can be interpreted as the firm death rate and, as suggested by Adachi and Aidis (2007), its magnitude is influenced by the regulatory environment (i.e., enforcement of property rights, anti-trust laws, hiring and firing restrictions, predatory tax practices, inspection agencies, etc.). In a more general version of the model, both μ and ϕ would qualify as institutional parameters with ϕ representing average lender quality. In addition, μ could be made at least partially endogenous to allow for exit rates that vary with the business cycle and/or interact with the labour market.

⁷To see that this conclusion is not just a product of the timing of exogenous separation in the relationship lending model or the fact that information is eventually revealed to all lenders, suppose that exogenous separation occurs with probability μ at the end of the first period and probability 1 at the end of the second. The $k = 2$ problem is unchanged so $Y_2(r)$ still equals the perfect information level of output. The $k = 1$ problem, on the other hand, is slightly different since the second term in equation (10) must now be multiplied by $(1 - \mu)$. Once again, higher values of μ imply that the economy is able to sustain fewer lending relationships so weight shifts from $Y_2(r) = Y_{k \geq 3}(r)$ to $Y_1(r)$ and the aggregate output function steepens. Moreover, with exogenous separation beginning at the end of period 1, fewer lending relationships now also mean that first period lenders assign a lower probability to future profits, pushing $\xi(r)$ closer to $\xi_S(r)$, $Y_1(r)$ closer to $Y_S(r)$, and actually hastening the fall in output.

4 Extended Model ($\varepsilon > 0$)

4.1 Key Differences

The results of Section 3 suggest that, on average, the informational properties of relationship lending lead to improved credit terms. To the extent that firms with better terms are better able to overcome adverse, idiosyncratic shocks, relationship lending may also be consistent with different firm exit rates. To implement this possibility, I now allow $\varepsilon > 0$. In particular, a borrower experiences exogenous separation with probability $\mu - \varepsilon$ as long as he stays with his insider. If or once he switches to an outside lender, separation occurs at rate μ . Types are still redrawn from a uniform distribution upon exit and, since the first period loan rate is a pooling one, lower types prefer higher separation rates that allow them to re-enter the market as first-timers. The opposite is true at the top of the distribution so the second period credit market should still separate between inside and outside lenders. For higher types, however, the added benefit of staying with an insider means that insiders can now charge slightly above the outsider rate without losing these borrowers. The same is true for $k \geq 3$ so insiders with advanced borrowers may be able to make positive profits and may not always charge $R_{k \geq 3}^*(r|\omega)$ as given by equation (4). As a result, the proofs in Section 3.2 no longer apply and market separation may not occur according to $\tilde{\omega}(r)$. Moreover, with $\varepsilon > 0$, the fact that r affects which second-time borrowers stay with their insiders means that it now also affects the distribution of borrowers across periods, allowing us to consider transition dynamics.

Outsiders still compete against each other and make zero expected profits so their $k = 2$ and $k \geq 3$ value functions are of the same form as equations (7) and (3). In contrast, the $k = 2$ value function of an insider is now:

$$J_{2,I}(r|\omega, d) = \max \left\{ 0, \left\{ \begin{array}{l} \max_R \quad \bar{\gamma}(R|\omega) R - r + \beta(1 - \mu + \varepsilon) J_{k \geq 3, I}(r|\omega) \\ s.t. \quad V_{2,I}(\omega|R) \geq V_{2,O}(\omega|R_{2,O,d}) \end{array} \right\} \right\} \quad (14)$$

where $V_{2,I}(\omega|R)$ is the value of a second-time type ω borrower who stays with his insider and pays loan rate R while $V_{2,O}(\omega|R_{2,O,d})$ is the value of this borrower should he move to an outsider charging $R_{2,O,d}$. The lender's value function for any $k \geq 3$ is also given by the right hand side of equation (14) but with $V_{k \geq 3,I}(\omega|R) \geq V_{k \geq 3,O}(\omega|R_{k \geq 3,O})$ as the borrower participation constraint. Under $\varepsilon = 0$, it was proven that second-time borrowers are not separated according to default history but, in the absence of an analytical solution for $\varepsilon > 0$, there is no presumption that this is still the case. Therefore, second period loan rates are not restricted to be history-independent and the first period borrower strategy is now denoted by $\bar{\gamma}_1(R|\omega)$ in order to distinguish it from $\bar{\gamma}(R|\omega)$. The value function of a first period lender is then similar to equation (10) except that $\bar{\gamma}_1$ is used instead of $\bar{\gamma}$ and expected future profits are determined based on equation (14) and the market separation that results.

For a first-time borrower facing loan rate R , the expected payoffs associated with choosing $P1$ and $P2$ are given by equations (15) and (16) respectively:

$$p(\omega) [\theta_1 - R + \beta V_2(\omega|R_{2,I,N}^*, R_{2,O,N}^*)] + (1 - p(\omega)) \beta V_2(\omega|R_{2,I,D}^*, R_{2,O,D}^*) \quad (15)$$

$$q [\theta_2 - R + \beta V_2(\omega|R_{2,I,N}^*, R_{2,O,N}^*)] + (1 - q) \beta V_2(\omega|R_{2,I,D}^*, R_{2,O,D}^*) \quad (16)$$

The borrower's value function, $V_1(\omega|R)$, is given by the maximum of these two equations and his strategy is $\bar{\gamma}_1(R|\omega) = p(\omega)$ if and only if (15) is greater than (16). The borrower's second period value is $V_2(\omega|R_{2,I,d}, R_{2,O,d}) = \max\{V_{2,I}(\omega|R_{2,I,d}), V_{2,O}(\omega|R_{2,O,d})\}$ where:

$$\begin{aligned} V_{2,I}(\omega|R) &= \max\{p(\omega) [\theta_1 - R], q [\theta_2 - R]\} + \beta(\mu - \varepsilon) \int_0^1 V_1(x|R_1) dF(x) \\ &\quad + \beta(1 - \mu + \varepsilon) \max\{V_{k \geq 3,I}(\omega|R_{k \geq 3,I}), V_{k \geq 3,O}(\omega|R_{k \geq 3,O})\} \end{aligned} \quad (17)$$

$$\begin{aligned} V_{2,O}(\omega|R) &= \max\{p(\omega) [\theta_1 - R], q [\theta_2 - R]\} \\ &\quad + \beta \left[\mu \int_0^1 V_1(x|R_1) dF(x) + (1 - \mu) V_{k \geq 3,O}(\omega|R_{k \geq 3,O}) \right] \end{aligned} \quad (18)$$

and $V_{k \geq 3,I}(\omega|R)$ and $V_{k \geq 3,O}(\omega|R)$ are also given by the right hand sides of (17) and (18).

Suppose an unanticipated, permanent increase in r occurs at date t . Lenders with advanced borrowers can adjust immediately to the new steady state but this may or may not be true for $k = 2$. Recall that the only piece of information available to a second period outsider is whether or not the borrower defaulted on his first period loan and, at date t , this outcome depends on the loan rate charged at $t - 1$ (i.e., the pre-shock policy rate). In the insider's problem, however, expected future profits depend on the loan rate at date $t + 1$ (i.e., the post-shock policy rate). An equilibrium for $k = 2$ is reached when each lender's offer is a best response to the other's and, since the post-shock steady state is conditioned entirely on the post-shock policy rate, the second period loan rates that prevail at date t may not be the same as those that prevail in the new steady state. Now, if first period lenders at date t expect a full adjustment by date $t + 1$, then they will adjust immediately to the new steady state. As a result, outsider information and insider profits at date $t + 1$ will both be conditioned on the new policy rate and the $k = 2$ equilibrium reaches the new steady state, consistent with the time t expectations of $k = 1$ lenders. In what follows, I will focus on this case. That is, all contracts adjust to the new steady state by date $t + 1$.⁸ Note, however, that even with a quick contract response, the effects of the policy rate shock continue to be propagated through the distribution. To see why, define an indicator function $I_{2,d,t}(\omega)$ that equals 1 if a second-time borrower with type ω and default history d stays with his insider at date t . Similarly, define $I_{k \geq 3,t}(\omega)$ so that $I_{k \geq 3,t}(\omega) = 1$ if $V_{k \geq 3,I}(\omega | R_{k \geq 3,I,t}) \geq V_{k \geq 3,O}(\omega | R_{k \geq 3,I,t})$. The mass of first period borrowers at date $t + 1$ is now:

$$\Psi_{1,t+1} = (\mu - \varepsilon) \int_0^1 [\psi_{2,I,t}(x) + \psi_{k \geq 3,I,t}(x)] dx + \mu \int_0^1 [\psi_{2,O,t}(x) + \psi_{k \geq 3,O,t}(x)] dx$$

where $\psi_{2,I,t}(\cdot)$ is the distribution of borrower types across $k = 2$ insiders and $\psi_{2,O,t}(\cdot)$ is the distribution of borrower types across $k = 2$ outsiders. The corresponding distributions for $k \geq 3$ are denoted by $\psi_{k \geq 3,I,t}(\cdot)$ and $\psi_{k \geq 3,O,t}(\cdot)$ and the laws of motion are as follows:

⁸Other assumptions about the time it takes for contracts to adjust would be ad hoc at this point. Issues of contract "stickiness" are thus left for future work.

$$\begin{aligned}\psi_{2,I,t+1}(\omega) &= \Psi_{1,t} \left\{ \begin{aligned} &\bar{\gamma}_1(R_{1,t}|\omega) I_{2,N,t+1}(\omega) \\ &+ [1 - \bar{\gamma}_1(R_{1,t}|\omega)] I_{2,D,t+1}(\omega) \end{aligned} \right\} \\ \psi_{2,O,t+1}(\omega) &= \Psi_{1,t} \left\{ \begin{aligned} &\bar{\gamma}_1(R_{1,t}|\omega) [1 - I_{2,N,t+1}(\omega)] \\ &+ [1 - \bar{\gamma}_1(R_{1,t}|\omega)] [1 - I_{2,D,t+1}(\omega)] \end{aligned} \right\}\end{aligned}$$

$$\begin{aligned}\psi_{k \geq 3,I,t+1}(\omega) &= (1 - \mu + \varepsilon) [\psi_{2,I,t}(\omega) + \psi_{k \geq 3,I,t}(\omega)] I_{k \geq 3,t+1}(\omega) \\ \psi_{k \geq 3,O,t+1}(\omega) &= \left\{ \begin{aligned} &(1 - \mu) [\psi_{2,O,t}(\omega) + \psi_{k \geq 3,O,t}(\omega)] \\ &+ (1 - \mu + \varepsilon) [\psi_{2,I,t}(\omega) + \psi_{k \geq 3,I,t}(\omega)] [1 - I_{k \geq 3,t+1}(\omega)] \end{aligned} \right\}\end{aligned}$$

Shocks to the policy rate affect the terms offered by various lenders and changes in these terms then affect which borrowers choose to stay with their insiders (i.e., $I_{2,N}$, $I_{2,D}$, and $I_{k \geq 3}$ respond). When ε is positive, types that stay with their insiders become more persistent so changes in r alter the distribution of borrower types in and across periods. As these distributions evolve to their new steady states, aggregate dynamics are observed well beyond time t .

In order to compute the equilibrium quantities, I discretize the type space and the set of possible loan rates and initialize the loan rate functions and the value functions. Given the loan rates, I determine the borrowers' strategies by iterating on their value functions then, based on these strategies, I iterate on the loan rates to find the optimal lender responses. The equilibrium is determined by iterating on the outer loop until the starting and ending loan rate functions converge. To execute the iterations, I use the parameterization associated with $q = 0.65$ and, unless otherwise specified, set $\mu = 0.3$ and $\varepsilon = 0.065$.

4.2 Numerical Results

Figure 4 shows the steady state profiles of aggregate output and the extent of relationship lending under different values of ε . Figure 5 illustrates the second period loan rates and

Figure 6 decomposes the extent of relationship lending across steady states. Figures 7 and 8 illustrate transition dynamics.

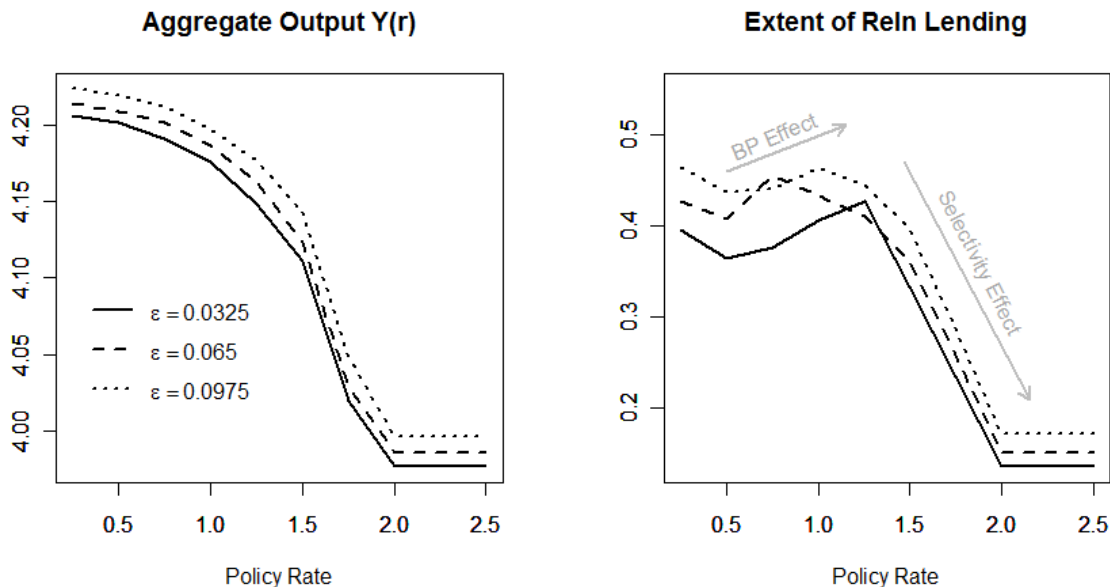


Figure 4: Steady state output under $\varepsilon > 0$, $q = 0.65$, and $\mu = 0.3$

Higher values of ε make better types more persistent, skewing the distribution of borrowers rightward and shifting the steady state output profile up. From Figure 4, we can also see that the steady state measure of relationship lending now exhibits a hump-shaped response to increases in r instead of a monotonic decline. A higher policy rate increases the cost of lending and, all else constant, the lowest type on which the insider breaks even so, as before, insiders become more selective in their retention of borrowers and fewer lending relationships are formed. Now, however, the additional “bargaining power” that ε gives the insider over better types means that more of the necessary break even can be accommodated by increases in the loan rate, stemming the restriction of insider credit. The bargaining power effect plays out initially but is eventually dominated by the selectivity effect.

The bargaining and selectivity effects are also useful for understanding why credit history can matter with $\varepsilon > 0$ (Figure 5). At higher policy rates, the increase in insider selectivity means that more types have to resort to outsider credit. This increases outsider uncertainty and makes credit history a natural screening mechanism. The informativeness of credit

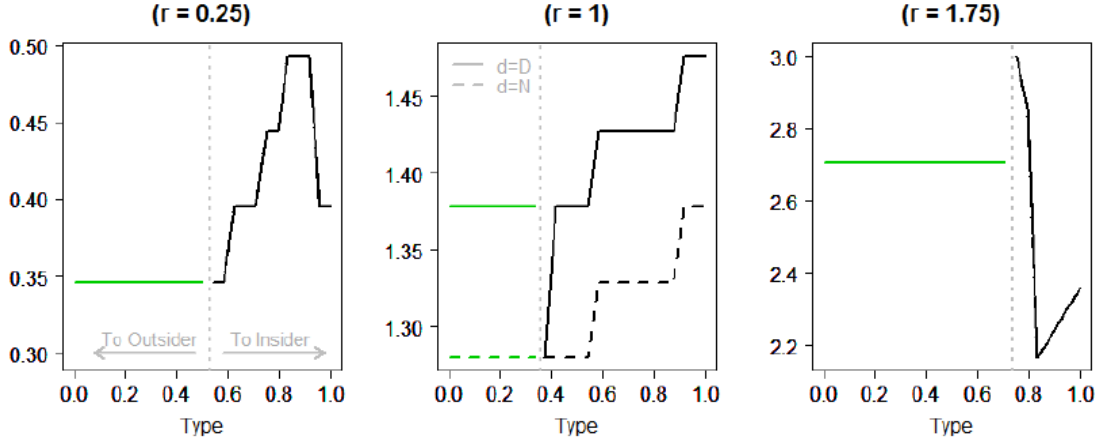


Figure 5: Realized second period loan rates under $\varepsilon = 0.065$, $q = 0.65$, and $\mu = 0.3$

history, however, depends on the first period loan rate. In particular, a very high R_1 induces most types to choose $P2$ in the first period and implies high default probabilities across the board. The opposite is true when R_1 is very low so, by getting good firms to choose $P1$ and bad firms to choose $P2$, moderate loan rates generate the most informative credit histories. For credit history to matter then, we need a relatively high value of r but a relatively moderate value of R_1 . This configuration can be achieved under $\varepsilon > 0$ since the bargaining power afforded to insiders over high types increases the expectation of future profits and, for any r , competitive first period lenders settle on a lower value of R_1 . In contrast, under $\varepsilon = 0$, the policy rates that increased outsider uncertainty and created the need for a sorting mechanism were high enough that they also diminished the informativeness of credit history, resulting in a second period equilibrium that was independent of d .

Turning now to dynamics, Figures 7 and 8 demonstrate that the extent of relationship lending tends to overshoot along the transition path. To see why, define the “extensive” margin in period k as the total number of borrowers in that period and the “intensive” margin as the proportion of these borrowers that enter into multi-period lending relationships. The extent of relationship lending in any given period is approximately equal to the product of its intensive and extensive margins and the extent of relationship lending at date t is the sum of

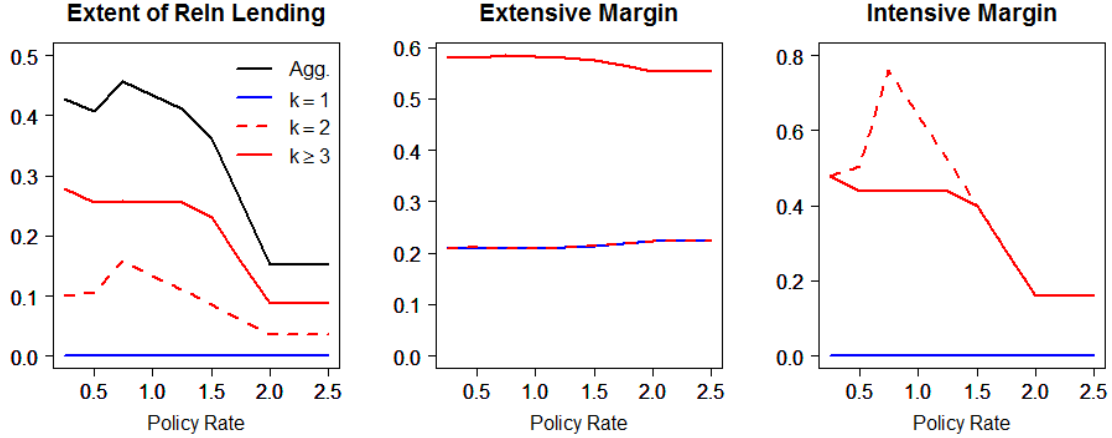


Figure 6: Steady state margins under $\varepsilon = 0.065$, $q = 0.65$, and $\mu = 0.3$

the extents for periods 2 and above.⁹ The right panel of Figure 6 reveals that $k = 2$ is critical for the analysis. When r increases from 0.5 to 0.75, the bargaining power effect drives up the second period intensive margin and, with the extensive margin still at the original steady state, the net result is an immediate increase in the extent of relationship lending as shown in Figure 7. Recall, however, that borrowers in lending relationships experience a lower rate of exogenous separation so the distribution of borrowers ultimately shifts towards $k \geq 3$, leading to a fall in all other extensive margins. With the intensive margin for advanced borrowers unchanged, the extent of relationship lending declines along the transition path. Overall then, the increase in relationship lending overshoots its new steady state and the decrease in output undershoots. As set up at the end of Section 3.4, the standard credit model adjusts to its new steady state immediately so, *relative* to a model without relationship lending, the model presented in this paper generates a smoother response to certain monetary shocks.

In contrast, an increase in the policy rate from 0.75 to 1 causes the decline in output to overshoot (Figure 8). For this range of r , the insider’s selectivity effect dominates, pushing the second period intensive margin back down. The immediate decrease in relationship lending eventually increases the number of young borrowers (i.e., the pool of potential relationship borrowers) and, over time, this offsets part of the initial decline in relationship

⁹The result is an approximation for the second period since it aggregates across default histories.

lending and, in turn, output. Even at its trough, however, aggregate output in Figure 8 exceeds the standard credit model's steady state of $Y_S(1) \approx 4.16$.

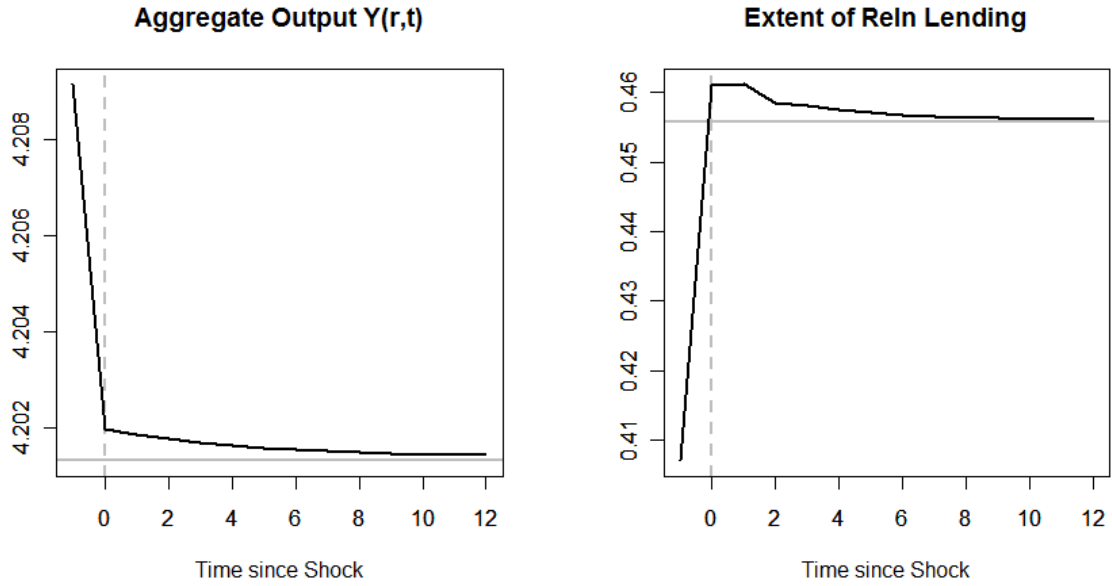


Figure 7: Transition between steady states, $r = 0.5$ to $r = 0.75$

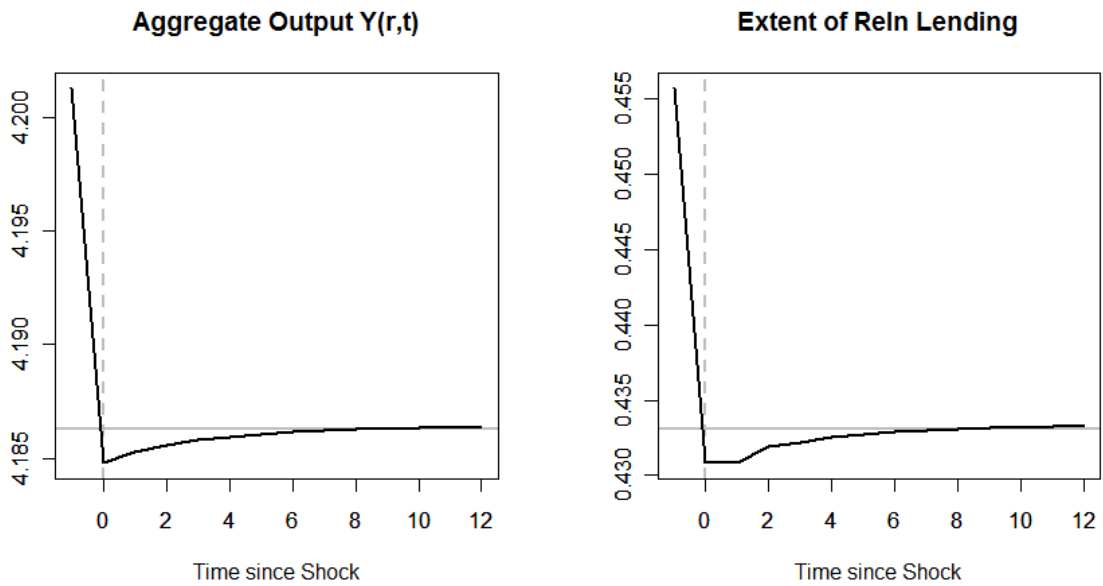


Figure 8: Transition between steady states, $r = 0.75$ to $r = 1$

5 Concluding Remarks

This paper has argued that the banking notion of relationship lending matters for the transmission of monetary policy. I began by setting up an asymmetric information model with a continuum of heterogeneous borrowers and the possibility of lender learning through repeated interactions. I then derived the optimal credit contracts in this environment and analyzed how they transmit changes in the policy rate to aggregate output. A variety of contracts are observed in equilibrium, with sufficiently good borrowers entering into multi-period lending relationships and economies that can sustain these relationships exhibiting a smoother steady state output profile and a more gradual response to certain monetary shocks. This prediction is consistent with empirical evidence so the model provides a basis for investigating the proportion of cross-country differences in monetary transmission that can be explained by cross-country differences in relationship lending. Future work will be directed at calibrations to quantify this effect and, to this end, a fundamental source of shock persistence will be necessary to generate longer-lived dynamics through the contracts and investigate the interaction of these contracts with other market frictions.

6 Appendix: Baseline Model with Two Borrower Types

Consider the model of Section 3 with only two borrower types, $i \in \{L, H\}$. Both types have access to the same projects as before except that $P1$ now succeeds with probability p_i if operated by a type i borrower. Let λ_0 denote the fraction of type H borrowers in the economy and assume that $p_H > p_L > q$, $\theta_1 < \theta_2$, and $p_L\theta_1 > q\theta$.

6.1 Period $k \geq 3$

Reservation loan rates and borrower strategies are as in equations (1) and (2) with i subscripts replacing ω so $R_{k \geq 3}^*(r|i)$ equals r/p_i if $r \leq p_i\bar{R}_i$ and r/q otherwise. When the policy rate is less than or equal to $p_i\bar{R}_i$, type i borrowers are charged at most \bar{R}_i and, therefore, choose $P1$. The opposite is true for $r > p_i\bar{R}_i$ and, since both p_i and \bar{R}_i are greater for $i = H$, output for each $k \geq 3$ is given by:

$$Y_{k \geq 3}(r) = \begin{cases} [\lambda_0 p_H + (1 - \lambda_0) p_L] \theta_1 & \text{if } r \leq p_L \bar{R}_L \\ \lambda_0 p_H \theta_1 + (1 - \lambda_0) q \theta_2 & \text{if } r \in (p_L \bar{R}_L, p_H \bar{R}_H] \\ q \theta_2 & \text{if } r > p_H \bar{R}_H \end{cases} \quad (19)$$

Equation (19) is illustrated by the red line in Figure 9. Although discontinuities emerge with a discrete number of types, output still contracts with increases in the policy rate.

6.2 Period $k = 2$

Consider an outsider facing a borrower he believes is type H with probability $\pi \in [0, 1]$. Conditional on π , expected outsider and insider profits are given respectively by

$$\max_R [\pi \bar{\gamma}_H(R) + (1 - \pi) \bar{\gamma}_L(R)] R - r \longrightarrow 0 \quad (20)$$

$$J_2(r|i, \pi) = \max \left\{ 0, \max_R \bar{\gamma}_i(R) R - r \text{ s.t. } R \leq R_2^*(r|\pi) \right\} \quad (21)$$

Analyzing the outsider's expected revenue along the lines of the Proposition 1 proof, it can be shown that his equilibrium offer is:

$$R_2^*(r|\pi) = \begin{cases} r/[\pi p_H + (1-\pi)p_L] & \text{if } r \leq \bar{r}_L(\pi) \\ r/[\pi p_H + (1-\pi)q] & \text{if } r \in (\bar{r}_L(\pi), \bar{r}_H(\pi)] \\ r/q & \text{if } r > \bar{r}_H(\pi) \end{cases}$$

where $\bar{r}_L(\pi) \equiv [\pi p_H + (1-\pi)p_L]\bar{R}_L$ and $\bar{r}_H(\pi) \equiv [\pi p_H + (1-\pi)q]\bar{R}_H$. The insider's strategy is then summarized by the following proposition:

Proposition 4 *Consider an insider who has discovered his second-time borrower's type. If $i = L$, it is optimal for the insider to offer above $R_2^*(r|\pi)$ and lose the borrower. If, on the other hand, $i = H$, it is optimal for him to keep the borrower by offering:*

$$R_2^*(r|H, \pi) = \begin{cases} r/[\pi p_H + (1-\pi)p_L] & \text{if } r \leq \bar{r}_L(\pi) \\ r/[\pi p_H + (1-\pi)q] & \text{if } r \in (\bar{r}_L(\pi), \bar{r}_H(\pi)] \\ \bar{R}_H & \text{if } r \in (\bar{r}_H(\pi), p_H \bar{R}_H] \\ r/q & \text{if } r > p_H \bar{R}_H \end{cases}$$

Proof. Define $f(x) \equiv \pi p_H + (1-\pi)x$ and consider $i = L$ and $i = H$ in turn.

Proof for $i = L$: (i) If $r < \bar{r}_L(\pi)$, then $R_2^*(r|\pi)$ is less than \bar{R}_L . Charging \bar{R}_L thus costs the insider the borrower and yields him zero profit in the current period. Charging $R_2^*(r|\pi)$, on the other hand, yields him $p_L R_2^*(r|\pi) - r$. This expression simplifies to $-\pi(p_H - p_L)r/f(p_L)$ which is zero for $\pi = 0$ and negative for $\pi \in (0, 1]$. Therefore, the insider is at least as well off if he charges a loan rate above $R_2^*(r|\pi)$. (ii) If $r = \bar{r}_L(\pi)$, then $R_2^*(r|\pi)$ equals \bar{R}_L . Charging \bar{R}_L yields the insider $p_L \bar{R}_L - \bar{r}_L(\pi)$ which simplifies to $-\pi(p_H - p_L)\bar{R}_L \leq 0$ so he is still at least as well off if he charges a loan rate above $R_2^*(r|\pi)$. (iii) If $r \in (\bar{r}_L(\pi), \bar{r}_H(\pi)]$, then $R_2^*(r|\pi)$ is greater than \bar{R}_L . Charging \bar{R}_L yields the insider $p_L \bar{R}_L - r < p_L \bar{R}_L - \bar{r}_L(\pi) \leq 0$ while charging $R_2^*(r|\pi)$ yields him $qR_2^*(r|\pi) - r$. The latter simplifies to

$-\pi(p_H - q)r/f(q) \leq 0$ so the insider is at least as well off if he charges a loan rate above $R_2^*(r|\pi)$. (iv) If $r > \bar{r}_H(\pi)$, then $R_2^*(r|\pi)$ is again greater than \bar{R}_L . Charging \bar{R}_L yields negative profits as in the previous case but charging $R_2^*(r|\pi)$ now yields $qR_2^*(r|\pi) - r = 0$. Therefore, the insider is no worse off if he charges above $R_2^*(r|\pi)$ and loses the borrower.

Proof for $i = H$: (i) If $r \leq \bar{r}_L(\pi)$, then $R_2^*(r|\pi)$ is less than \bar{R}_H . Charging \bar{R}_H thus costs the insider the borrower and yields him zero profit. Charging $R_2^*(r|\pi)$, on the other hand, yields him $p_H R_2^*(r|\pi) - r$. This expression simplifies to $(1 - \pi)(p_H - p_L)r/f(p_L)$ which is positive for $\pi \in [0, 1)$ and zero for $\pi = 1$. Therefore, the insider is at least as well off charging $R_2^*(r|\pi)$ and keeping the borrower as he is charging above $R_2^*(r|\pi)$ and losing him. (ii) If $r \in (\bar{r}_L(\pi), \bar{r}_H(\pi))$, then $R_2^*(r|\pi)$ is again less than \bar{R}_H . Charging \bar{R}_H thus yields no profit while charging $R_2^*(r|\pi)$ now yields $p_H R_2^*(r|\pi) - r = (1 - \pi)(p_H - q)r/f(q) \geq 0$. Therefore, the insider is still at least as well off charging $R_2^*(r|\pi)$ and keeping the borrower. (iii) If $r = \bar{r}_H(\pi)$, then $R_2^*(r|\pi)$ equals \bar{R}_H . Charging \bar{R}_H yields the insider $p_H \bar{R}_H - \bar{r}_H(\pi)$ which simplifies to $(1 - \pi)(p_H - q)\bar{R}_H \geq 0$ so he is at least as well off charging $R_2^*(r|\pi) = \bar{R}_H$ and keeping the borrower as he is charging above $R_2^*(r|\pi)$ and losing him. (iv) If $r \in (\bar{r}_H(\pi), p_H \bar{R}_H]$, then $R_2^*(r|\pi)$ is greater than \bar{R}_H . Charging \bar{R}_H yields the insider a profit of $p_H \bar{R}_H - r \geq 0$ while charging $R_2^*(r|\pi)$ yields him $qR_2^*(r|\pi) - r = 0$. Therefore, the insider is at least as well off charging \bar{R}_H and keeping the borrower as he is charging a loan rate greater than or equal to $R_2^*(r|\pi)$. (v) If $r > p_H \bar{R}_H$, then $R_2^*(r|\pi)$ is again greater than \bar{R}_H . Charging $R_2^*(r|\pi)$ yields the same expected profit as in the previous case but charging \bar{R}_H now yields $p_H \bar{R}_H - r < 0$. Therefore, the insider is better off charging $R_2^*(r|\pi)$ than he is charging \bar{R}_H and no worse off charging $R_2^*(r|\pi)$ than he is charging above $R_2^*(r|\pi)$ and losing the borrower. ■

With the optimal insider responses as argued above, the only second-time borrowers that an outsider can attract are those that have been discovered as type L . In equilibrium then, $\pi = 0$ and the loan rates paid by type L and H simplify to $R_2^*(r|0)$ and $R_2^*(r|H, 0)$

respectively. As before, better borrowers benefit from multi-period relationships with their insiders over moderate policy rates and the output function that results is $Y_2(r) = Y_{k \geq 3}(r)$.

6.3 Period $k = 1$

With a finite number of types, the first period equilibrium can be solved for analytically. In particular, two borrower types implies three options for the lender: (i) charge below \bar{R}_L , (ii) charge between \bar{R}_L and \bar{R}_H , or (iii) charge above \bar{R}_H . To simplify notation, I set $\beta = 1$. Along the lines of Section 3.3, the loan rate associated with option $j \in \{1, 2, 3\}$ is:

$$R_{1j}(r) = \frac{r - \lambda_0 J_2(r|H, 0)}{\lambda_0 \bar{\gamma}_H(R) + (1 - \lambda_0) \bar{\gamma}_L(R)} \quad (22)$$

where $\{\bar{\gamma}_L(R), \bar{\gamma}_H(R)\}$ equals $\{p_L, p_H\}$ for $j = 1$, $\{q, p_H\}$ for $j = 2$, and $\{q, q\}$ for $j = 3$. Although $R_{11}(r)$, $R_{12}(r)$, and $R_{13}(r)$ are all candidates for $R_1^*(r|\lambda_0)$, there is a consistency issue that needs to be addressed. In particular, these loan rates are based on the assumption that $R_{11}(r) \leq \bar{R}_L$, $R_{12}(r) \in (\bar{R}_L, \bar{R}_H]$, and $R_{13}(r) > \bar{R}_H$. Any $R_{1j}(r)$ that does not satisfy these inequalities is inconsistent and cannot be an equilibrium. The equilibrium loan rate for $k = 1$ is characterized in the following proposition:

Proposition 5 *Suppose $p_H < q(1 + \lambda_0)/\lambda_0$ and define the following critical λ :*

$$\bar{\lambda} \equiv \frac{q\bar{R}_H - p_L\bar{R}_L}{(p_H - q)\bar{R}_H + (p_H - p_L)\bar{R}_L}$$

Also define cutoff policy rates, r_s and $r_1 < r_2 < r_3$, such that:

$$r_s \equiv \begin{cases} r_1 \equiv q\bar{R}_L \left[\frac{\lambda_0 p_H + (1 - \lambda_0) p_L}{q - \lambda_0(p_H - q)} \right] & \text{if } \lambda_0 < \bar{\lambda} \\ r_2 \equiv \frac{[\lambda_0 p_H + (1 - \lambda_0) p_L] \bar{R}_L + \lambda_0 p_H \bar{R}_H}{1 + \lambda_0} & \text{if } \lambda_0 \geq \bar{\lambda} \end{cases}$$

$$r_3 \equiv \frac{[2\lambda_0 p_H + (1 - \lambda_0) q] \bar{R}_H}{1 + \lambda_0}$$

The first period equilibrium loan rate, $R_1^*(r|\lambda_0)$, is less than or equal to \bar{R}_L for $r \leq r_s$, between \bar{R}_L and \bar{R}_H for $r \in (r_s, r_3]$, and greater than \bar{R}_H for $r > r_3$.

Proof. I start by proving the following Lemma:

Lemma 3: Given two consistent candidates for the first period loan rate, R_A and R_B such that $R_A < \bar{R}_i < R_B$, R_B cannot be an equilibrium.

Proof: Suppose R_B is an equilibrium and consider the alternative offer $R_A + \delta < \bar{R}_i$ where $\delta > 0$. Since $R_A + \delta < R_B$, the lender still gets the borrower. Moreover, $R_A + \delta$ and R_A have the same ranking relative to \bar{R}_L and \bar{R}_H so $\bar{\gamma}_i(R_A + \delta) = \bar{\gamma}_i(R_A)$ for $i \in \{L, H\}$. The expected profit from charging $R_A + \delta$ is thus:

$$\begin{aligned} E\Pi(R_A + \delta) &= E\Pi(R_A) + [\lambda_0 \bar{\gamma}_H(R_A) + (1 - \lambda_0) \bar{\gamma}_L(R_A)] \delta \\ &= [\lambda_0 \bar{\gamma}_H(R_A) + (1 - \lambda_0) \bar{\gamma}_L(R_A)] \delta > 0 \end{aligned}$$

The expected profit from charging R_B , however, is $E\Pi(R_B) = 0$ so R_B cannot be optimal, contradicting the assumption that it is an equilibrium. \square

Given several consistent candidates for the first period loan rate, Lemma 3 establishes that $R_1^*(r|\lambda_0)$ equals the lowest one. Since $R_{1j}(r)$ is increasing in j , I will begin by determining $R_{11}(r)$ for each range of the policy rate, proceeding to $R_{12}(r)$ or $R_{13}(r)$ only in the event of an inconsistency:

(i) If $r \leq p_L \bar{R}_L$, then insiders charge r/p_L and obtain $J_2(r|H, 0) = (p_H - p_L) r/p_L$. Substituting this expression into equation (22) for $j = 1$ yields:

$$R_{11}(r) = \left[\frac{p_L - \lambda_0 (p_H - p_L)}{\lambda_0 p_H + (1 - \lambda_0) p_L} \right] \frac{r}{p_L} \quad (23)$$

The condition $p_H < q(1 + \lambda_0)/\lambda_0$ ensures a positive numerator and, therefore, consistency of $R_{11}(r)$ just requires:

$$r \leq p_L \bar{R}_L \left[\frac{\lambda_0 p_H + (1 - \lambda_0) p_L}{p_L - \lambda_0 (p_H - p_L)} \right] \quad (24)$$

Since $p_H > p_L$, the term in square brackets is greater than one so equation (24) is satisfied for the policy rates under consideration. By Lemma 3 then, $R_1^*(r|\lambda_0)$ equals $R_{11}(r) \leq \bar{R}_L$ for $r \leq p_L \bar{R}_L$ where $R_{11}(r)$ is as in equation (23).

(ii) If $r \in (p_L \bar{R}_L, p_H \bar{R}_H]$, then insiders charge r/q and obtain $J_2(r|H, 0) = (p_H - q)r/q$. Substituting into equation (22) for $j = 1$ yields:

$$R_{11}(r) = \left[\frac{q - \lambda_0 (p_H - q)}{\lambda_0 p_H + (1 - \lambda_0) p_L} \right] \frac{r}{q} \quad (25)$$

where the condition $p_H < q(1 + \lambda_0)/\lambda_0$ once again ensures a positive numerator. Consistency of $R_{11}(r)$ in this case amounts to $r \leq r_1$. With some algebra, it can be shown that r_1 falls between $p_L \bar{R}_L$ and $q \bar{R}_H$ if $\lambda_0 < \bar{\lambda}$ and above $q \bar{R}_H$ otherwise. In other words, if $\lambda_0 \geq \bar{\lambda}$, $R_1^*(r|\lambda_0)$ equals $R_{11}(r) \leq \bar{R}_L$ for $r \in (p_L \bar{R}_L, p_H \bar{R}_H]$ where $R_{11}(r)$ is as in equation (25). If, on the other hand, λ_0 is less than $\bar{\lambda}$, this characterization is only true for $r \in (p_L \bar{R}_L, r_1]$; to see what happens over the interval $(r_1, q \bar{R}_H]$, it is necessary to consider $R_{12}(r)$. Substituting $J_2(r|H, 0)$ into equation (22) for $j = 2$ yields:

$$R_{12}(r) = \left[\frac{q - \lambda_0 (p_H - q)}{\lambda_0 p_H + (1 - \lambda_0) q} \right] \frac{r}{q} \quad (26)$$

That $R_{12}(r)$ is greater than \bar{R}_L here is easily established from $R_{11}(r) > \bar{R}_L$ so consistency of $R_{12}(r)$ just requires:

$$r \leq q \bar{R}_H \left[\frac{\lambda_0 p_H + (1 - \lambda_0) p_L}{q - \lambda_0 (p_H - q)} \right] \quad (27)$$

Since $p_H > q$, the term in square brackets is greater than one so equation (27) is satisfied for the policy rates here. Therefore, when $\lambda_0 < \bar{\lambda}$, $R_1^*(r|\lambda_0)$ equals $R_{12}(r) \in (\bar{R}_L, \bar{R}_H]$ for $r \in (r_1, q \bar{R}_H]$ where $R_{12}(r)$ is as in equation (26).

(iii) If $r \in (q\bar{R}_H, p_H\bar{R}_H]$, then insiders charge \bar{R}_H and obtain $J_2(r|H, 0) = p_H\bar{R}_H - r$. Substituting into equation (22) for $j = 1$ yields:

$$R_{11}(r) = \frac{(1 + \lambda_0)r - \lambda_0 p_H \bar{R}_H}{\lambda_0 p_H + (1 - \lambda_0) p_L} \quad (28)$$

Consistency of $R_{11}(r)$ now amounts to $r \leq r_2$. With some algebra, it can be shown that $r_2 \leq q\bar{R}_H$ if $\lambda_0 < \bar{\lambda}$ and $r_2 \in (q\bar{R}_H, p_H\bar{R}_H)$ otherwise. Therefore, if $\lambda_0 \geq \bar{\lambda}$, then $R_1^*(r|\lambda_0)$ equals $R_{11}(r) \leq \bar{R}_L$ for $r \in (p_L\bar{R}_L, r_2]$ where $R_{11}(r)$ is as in equation (28). However, if $\lambda_0 < \bar{\lambda}$, then $R_{11}(r)$ cannot be an equilibrium for any $r \in (q\bar{R}_H, p_H\bar{R}_H]$. Consider now $R_{12}(r)$. Substituting $J_2(r|H, 0)$ into equation (22) for $j = 2$ yields:

$$R_{12}(r) = \frac{(1 + \lambda_0)r - \lambda_0 p_H \bar{R}_H}{\lambda_0 p_H + (1 - \lambda_0) q} \quad (29)$$

Since $R_{1j}(r)$ is increasing in j and it has been established that $R_{11}(r)$ exceeds \bar{R}_L when $R_{12}(r)$ is relevant, $R_{12}(r)$ must also exceed \bar{R}_L for these policy rates. To be consistent, however, $R_{12}(r)$ must not exceed \bar{R}_H . This condition is equivalent to $r \leq r_3$ and it can be shown that r_3 lies between r_2 and $p_H\bar{R}_H$ for $\lambda_0 \in [0, 1]$. Therefore, $R_1^*(r|\lambda_0)$ equals $R_{12}(r) \in (\bar{R}_L, \bar{R}_H]$ for $r \in (q\bar{R}_H, r_3]$ if $\lambda_0 < \bar{\lambda}$ and $r \in (r_2, r_3]$ if $\lambda_0 \geq \bar{\lambda}$. The relevant expression for $R_{12}(r)$ is given in equation (29). Since neither $R_{11}(r)$ nor $R_{12}(r)$ is consistent for $r \in (r_3, p_H\bar{R}_H]$, we must now proceed to $R_{13}(r)$. Substituting $J_2(r|H, 0)$ into equation (22) for $j = 3$ yields:

$$R_{13}(r) = [(1 + \lambda_0)r - \lambda_0 p_H \bar{R}_H] / q \quad (30)$$

That $R_{13}(r)$ is greater than \bar{R}_H is easily established from $R_{12}(r) > \bar{R}_H$ so consistency of $R_{13}(r)$ just requires $r \leq (q\theta_2 + \lambda_0 p_H \bar{R}_H) / (1 + \lambda_0)$ which is satisfied by $r \leq p_H \bar{R}_H$ since $q\theta_2 > p_H \bar{R}_H$. Therefore, $R_1^*(r|\lambda_0)$ equals $R_{13}(r) > \bar{R}_H$ for $r \in (r_3, p_H \bar{R}_H]$ where $R_{13}(r)$ is as in equation (30).

(iv) If $r > p_H \bar{R}_H$, then insiders charge r/q and obtain $J_2(r|H, 0) = 0$. Following the

same procedure as above, it can be shown that the only consistent candidate is $R_{13}(r)$ and, therefore, $R_1^*(r|\lambda_0)$ equals $R_{13}(r) > \bar{R}_H$ for $r \in (p_H \bar{R}_H, q\theta_2]$ where $R_{13}(r) = r/q$. ■

Given that type i borrowers choose $P1$ as long as they are not charged above \bar{R}_i , the first period output function implied by Proposition 5 is:

$$Y_1(r) = \begin{cases} [\lambda_0 p_H + (1 - \lambda_0) p_L] \theta_1 & \text{if } r \leq r_s \\ \lambda_0 p_H \theta_1 + (1 - \lambda_0) q \theta_2 & \text{if } r \in (r_s, r_3] \\ q \theta_2 & \text{if } r > r_3 \end{cases} \quad (31)$$

Equation (31) is illustrated by the blue line in Figure 9. As in the continuum model, the first period output function is steeper than $Y_{k \geq 3}(r)$. Moreover, based on the definition of r_s and the fact that $r_2 > r_1$, this pattern is more pronounced for higher values of λ_0 .

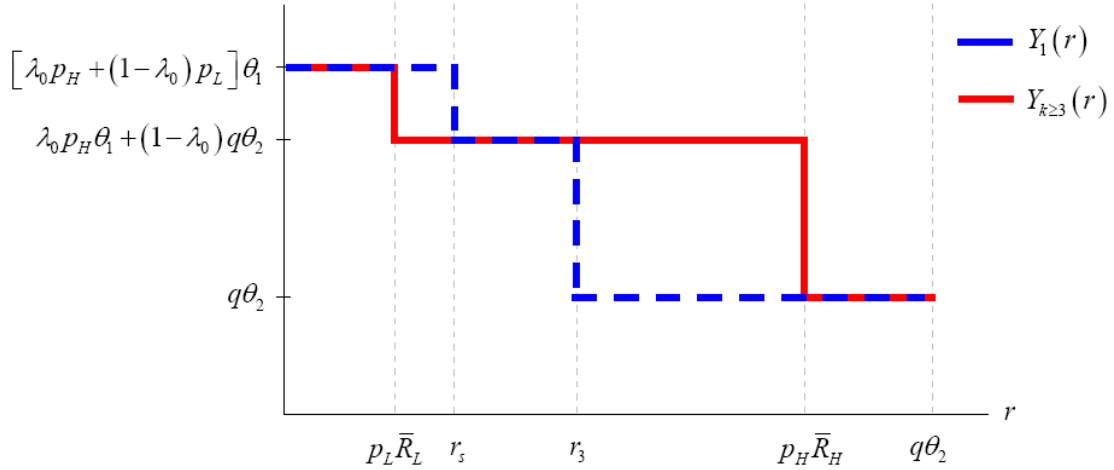


Figure 9: Output functions in the two-type model

6.4 Aggregation

Equation (13) is unchanged by a reduction in the number of types so aggregation proceeds as before. Substituting equations (31) and (19) for $Y_1(r)$ and $Y_{k \geq 3}(r)$ yields the following

steady state aggregate output function in the two-type case:

$$Y(r) = \begin{cases} Y_{\max} & \text{if } r \leq p_L \bar{R}_L \\ Y_{\max} - \left(\frac{1-\lambda_0}{1+\mu} \right) (p_L \theta_1 - q \theta_2) & \text{if } r \in (p_L \bar{R}_L, r_s] \\ Y_{\text{mid}} & \text{if } r \in (r_s, r_3] \\ Y_{\min} + \left(\frac{\lambda_0}{1+\mu} \right) (p_H \theta_1 - q \theta_2) & \text{if } r \in (r_s, p_H \bar{R}_H] \\ Y_{\min} & \text{if } r > p_H \bar{R}_H \end{cases} \quad (32)$$

where $Y_{\max} \equiv [\lambda_0 p_H + (1 - \lambda_0) p_L] \theta_1$, $Y_{\text{mid}} \equiv \lambda_0 p_H \theta_1 + (1 - \lambda_0) q \theta_2$, and $Y_{\min} \equiv q \theta_2$. This result is illustrated by the black line in Figure 10. Lower values of μ decrease the second line in equation (32) but increase the fourth one, thereby flattening the output profile. Turning to the standard credit model, the representative lender's problem is now given by equation (20) with λ_0 instead of π so $R_2^*(r|\lambda_0)$ always prevails, generating $Y_S(r)$ as shown by the grey line below. The conclusion of the continuum model holds: economies poised to foster longer-term credit relationships are also poised for a smoother output profile.

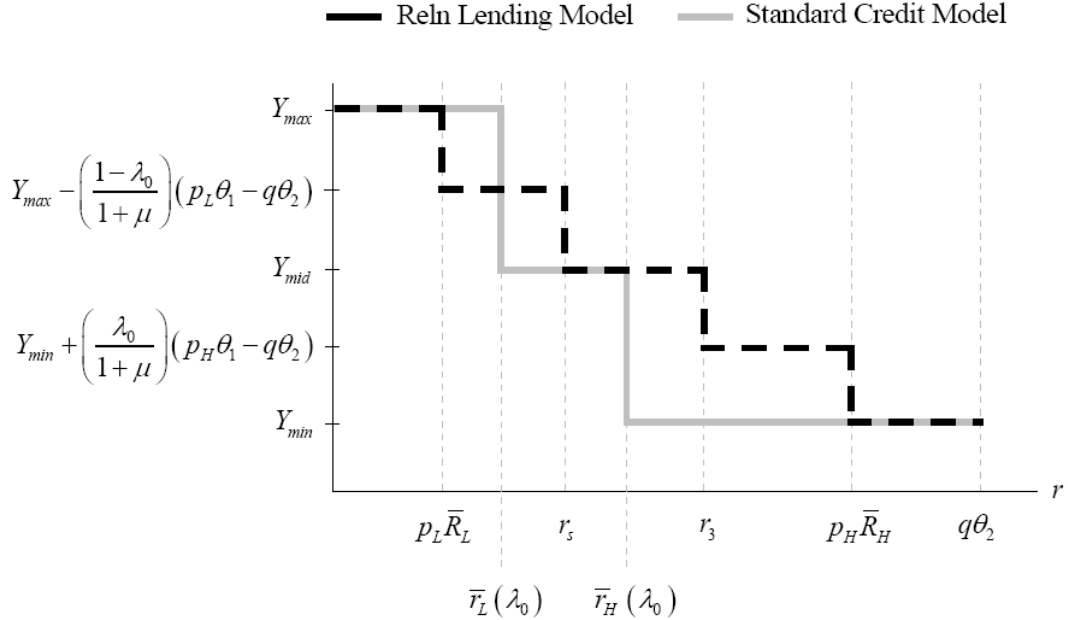


Figure 10: Aggregate output comparison for the two-type model

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