Mnemonics for the Gradient, Divergence, and Curl Operator

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The gradient, divergence, and curl are operators commonly encountered when we study electromagnetic theory. This communication introduces three mnemonics to remember their expressions in the cartesian, cylindrical, and spherical coordinate systems. The mnemonics are expressed in the matrix forms to emphasize the linearity of the operators. In particular, the gradient of a scalar field f defined in terms of the limit is [1, Ch. 1]

$$\nabla f = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \left[\oint f d\vec{s} \right].$$

The matrix form of the gradient operator is

$$\nabla f = \begin{bmatrix} \frac{1}{h_1} & 0 & 0 \\ 0 & \frac{1}{h2} & 0 \\ 0 & 0 & \frac{1}{h3} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial u_1} & 0 & 0 \\ 0 & \frac{\partial}{\partial u_2} & 0 \\ 0 & 0 & \frac{\partial}{\partial u_3} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} f. \tag{1}$$

The divergence of a vector field $\vec{f}$ is defined as

$$\nabla \cdot \vec{f} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \left[\oint_s \vec{f} \cdot d\vec{S} \right].$$

The matrix form of the divergence operator is

$$\nabla \cdot \vec{f} = \frac{1}{h_1 h_2 h_3} \begin{bmatrix} \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \end{bmatrix} \begin{bmatrix} h_2 h_3 & 0 & 0 \\ 0 & h_3 h_1 & 0 \\ 0 & 0 & h_1 h_2 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}. \tag{2}$$

The curl of a vector field $\vec{f}$ is defined as

$$\nabla \times \vec{f} = \lim_{\Delta v \to 0} \frac{1}{\Delta v} \left[\oint_s d\vec{s} \times \vec{f} \right].$$

The matrix form of the curl operator is[1]

$$\nabla \times \vec{f} = \begin{bmatrix} \frac{1}{h_2 h_3} & 0 & 0 \\ 0 & \frac{1}{h_3 h_1} & 0 \\ 0 & 0 & \frac{1}{h_1 h_2} \end{bmatrix} \begin{bmatrix} 0 & -\frac{\partial}{\partial u_3} & \frac{\partial}{\partial u_2} \\ \frac{\partial}{\partial u_3} & 0 & -\frac{\partial}{\partial u_1} \\ -\frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_1} & 0 \end{bmatrix} \begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}. \tag{3}$$

Eq. (1), (2), (3) are the mnemonics. The coordinate specific parameters are listed below.

	h_1	h_2	h_3	u_1	u_2	u_3	$\vec{n}_1$	$\vec{n}_2$	$\vec{n}_3$
Cart.	1	1	1	x	y	z	$\vec{a}_x$	$\vec{a}_y$	$\vec{a}_z$
Cyl.	1	ρ	1	ρ	ϕ	z	$\vec{a}_\rho$	$\vec{a}_\phi$	$\vec{a}_z$
Sph.	1	r	$r\sin\theta$	r	θ	ϕ	$\vec{a}_r$	$\vec{a}_\theta$	$\vec{a}_\phi$

References

[1] J. Jin, *The Finite Element Method in Electromagnetics*, 2nd ed. New York, NY: John Wiley & Sons, Inc., 2002.

[1] The curl operator can also be remembered in terms of the determinent of the following matrix

$$\nabla \times \vec{f} = \frac{1}{h_1 h_2 h_3} \begin{vmatrix} h_1 \vec{a}_1 & h_2 \vec{a_2} & h_3 \vec{a_3} \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1 f_1 & h_2 f_2 & h_3 f_3 \end{vmatrix}.$$