

A Hybrid Control Approach to Multi-Robot Coordinated Path Following

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Abstract—The paper studies the coordinated path following problem, namely, steering a group of unicycles to a given path while achieving an inter-vehicle formation pattern. A novel hybrid control approach is proposed in the paper to solve the problem despite that the neighbors of each vehicle may change over time due to limited sensing capabilities. The state space of each vehicle is partitioned into several regions relative to the path. Then a control law is synthesized to steer all the vehicles into a region where another control law is devised to achieve both path following and motion coordination. Thus, with the state-dependent switching control law, a group of vehicles are evenly spaced on the path and move as a whole eventually.

I. INTRODUCTION

A group of agents can be used to carry out tasks that are too difficult or simply inefficient for a single agent to perform alone. Moreover, fault tolerance and robustness are additional advantages of multi-agent systems. All these factors motivate researchers to work on some specific problems of multi-agent systems including cooperative formation control, rendezvous, consensus, cooperative target tracking, coordinated path following, and topology control of distributed sensor networks. For a complete review, see [3]–[5], [13], [14], [16].

The problem of steering a group of wheeled vehicles along given spatial paths while achieving a desired inter-vehicle formation pattern is referred to as the coordinated path following problem. It has been a topic of interest over the last few years. Motivated in military applications, the previous work on the coordinated path following problem has been restricted to marine robots and spacecrafts [2], [12], [17]. The common strategy for coordinated path following can be divided into two parts: path following and multi-vehicle coordination. Although many schemes are available for the path following problem recently, there is a lack of complete solutions when coordination among vehicles is required. In [18], a group of vehicles modeled as Newtonian particles are steered to generate patterns on close smooth curves, which has been applied for ocean sampling. In [8], [9], [17], a parameterized path is generated for each vehicle with respect to a desired spacial path. Each vehicle then executes a pure path following along its assigned path with dynamical path variable synchronization. Ghabcheloo et al. [6], [7] propose a control methodology for coordinated path following over a class of pre-specified paths by decoupling path following (in space) and inter-vehicle coordination (in time).

All the above methods consider both vehicle coordination and path following no matter how far the vehicles are. However, due to limited sensing capabilities and communication budgets, each vehicle may not be possible to have the information of the path and its neighbors all the time.

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Instead, for vehicles far away from the path, only brief information of some neighbor's position is available to it without knowing any information of the path, and vehicles already close to the path may obtain more information about the path and their neighbors so that they can follow the given path and coordinate their motion with other vehicles also close to the path to generate a desired formation pattern. As a result, for vehicles far away from the path, the primary objective is to get closer to the path rather than coordinate their motion with others. For vehicles close to the path, they are able to sense the path and more neighbor vehicles around the path, so the primary objective is to follow the path and achieve a desired inter-vehicle formation. This problem is addressed in the paper and referred to as the coordinated path following problem. A novel approach based on reachability specifications and hybrid control is proposed to solve the problem. The state space of each vehicle is partitioned into several regions relative to the path. Then control laws are synthesized to steer all the vehicles into a region where another control law is devised to achieve both path following and motion coordination. Thus, with the state-dependent switching control law, a group of vehicles are evenly spaced on the path and move as a whole eventually despite that the neighbors of each vehicle may change over time due to limited sensing range. In the process of coordinated path following, though each vehicle may sense several other vehicles, only the local measurement of one neighbor called pre-neighbor is used in our control strategy for the coordinated control purpose, which makes the information measurement at a minimum. As a first contribution, we propose a new methodology based hybrid control approach to synthesize distributed control laws for coordinated path following problems. Second, we take into account limited sensing capabilities and we show that the coordinated path following problem can be solved by our control strategy using only local information and minimum links though the neighboring relationship may be dynamically changed. Finally, global results are obtained with rigorous analysis using reachability and set invariance theory together with Lyapunov theory and ultimate boundedness results.

Several proofs are omitted in the paper due to space limitations and can be found in [11].

II. PRELIMINARIES AND PROBLEM STATEMENT

A. Vehicle Kinematic Model

Consider a group of unicycles labeled 1 through n in the plane. The posture of each unicycle is described by $q_i = (x_i, y_i, \theta_i)^T \in \mathbb{R}^2 \times [-\pi, \pi)$, called the state of unicycle i , where (x_i, y_i) denotes the position of its representing point defined in an inertia coordinate frame \mathcal{W} , θ_i is its orientation with respect to the x -axis of \mathcal{W} . The kinematic model for

unicycle i with pure rolling and non-slipping is given as:

$$\dot{q}_i = \begin{pmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{pmatrix} = \begin{pmatrix} v_i \cos \theta_i \\ v_i \sin \theta_i \\ \omega_i \end{pmatrix}. \quad (1)$$

The control inputs v_i and ω_i stand for its linear speed and angular speed, respectively. In the paper, we denote $\mathcal{Z} := \{q | q \in \mathbb{R}^2 \times [-\pi, \pi)\}$ the state space of each vehicle.

B. Local Sensing Information

Let Γ be a spatial smooth path on the plane, which is assumed to have a direction (see Fig. 1). The curvature at a point $p \in \Gamma$ is denoted by $\chi(p)$. We assume that the smooth path Γ satisfies $|\chi(p)| < \chi_0$ for all $p \in \Gamma$, where χ_0 is a constant. Define $R_0 = \frac{1}{\chi_0}$.

When the distance from vehicle i to the path Γ is less than R_0 , the nearest point on Γ to vehicle i is unique and is denoted by p_i . Then we define $\phi_i = (\rho_i, \psi_i)$ the path following error to Γ , where $\rho_i \in \mathbb{R}$ called the *location difference* is the distance from vehicle i to the nearest point p_i on Γ but has a sign (that is, $\rho_i > 0$ when vehicle i is on the left side of Γ in the direction of the path and $\rho_i < 0$ when it is on the other side), $\psi_i \in [-\pi, \pi)$ is the *orientation difference* defined as the difference between the heading of vehicle i and the tangent direction of Γ at p_i . For two vehicles i and j , we let l_{ij} be the arc length between p_i and p_j and we call it *arc distance* between vehicle i and vehicle j . An illustration is given in Fig. 1.

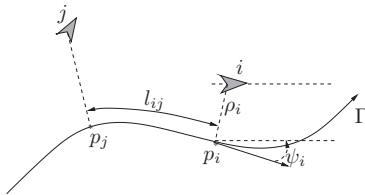


Fig. 1. A path Γ with a direction, path following errors (ρ_i, ψ_i) , and arc distance l_{ij} .

Define $\mathcal{Z}_1 = \{q = (x, y, \theta) | \|(x, y)\|_{\Gamma} \leq R_0, \theta \in [-\pi, \pi)\}$, where $\|\cdot\|_{\Gamma}$ is the distance from a point to the set Γ . We may say, a vehicle i is in \mathcal{Z}_1 or outside of \mathcal{Z}_1 (by slightly misusing the notion), meaning that its state is in \mathcal{Z}_1 or not in \mathcal{Z}_1 , or equivalently that the distance from vehicle i to the path Γ is less than R_0 or greater than R_0 . Suppose that the sensing radius of each vehicle is R ($R > R_0$). Then, for vehicle i in \mathcal{Z}_1 , it can sense the path in the sense that current location difference ρ_i and orientation difference ψ_i are available to vehicle i as well as the curvature $\chi(p_i)$. Moreover, if there is another vehicle j in its sensing region, then the arc distance l_{ij} is available to vehicle i , too.

For vehicles outside of \mathcal{Z}_1 , we assume that they can obtain the position information (distance and bearing angle) of at least one vehicle that is in \mathcal{Z}_1 or a landmark on the path Γ through other manners. A schematic representation is given in Fig. 2, where i represents a vehicle outside of \mathcal{Z}_1 , and j represents a vehicle in \mathcal{Z}_1 or a landmark on the path Γ . In the example, the distance d_{ij} and the bearing angle $\alpha_{ij} \in [-\pi, \pi)$ are available to the vehicle i .

C. Problem Statement

The *coordinated path following problem* consists of finding distributed control laws for a group of vehicles using only

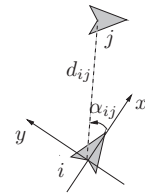


Fig. 2. Distance d_{ij} and bearing angle α_{ij} .

local available information such that the group of vehicles eventually converge to follow a given path and in addition, the vehicles that are connected through sensing information are evenly spaced on the path.

In our setup, though each vehicle may sense several neighbor vehicles, we use only the information of one neighbor (called pre-neighbor) to synthesize control so that coordinated path following can be achieved with minimum information. For vehicle i in \mathcal{Z}_1 , another vehicle j is called vehicle i 's *pre-neighbor* if (1) it is in the sensing range of vehicle i ; (2) it is also in \mathcal{Z}_1 ; (3) its nearest point (say p_j) on Γ is in front of vehicle i 's nearest point p_i in the direction of the path; (4) no other vehicle's nearest point lies between them on the path Γ . Next, for notation simplicity, we use ζ_i to denote the arc distance between its pre-neighbor and itself, i.e., $\zeta_i = l_{ij}$ where vehicle j is vehicle i 's pre-neighbor.

Thus, for the coordinated path following problem, in final steady configuration, the path following error $\phi_k = 0$ for all k and the arc distance ζ_i between any vehicle i and its pre-neighbor is L (a desired value) when its pre-neighbor is still in its sensing range. In the paper, it is assumed that $R > L + 2R_0$ for technical reasons. Fig. 3 is a schematic illustration for coordinated path following.

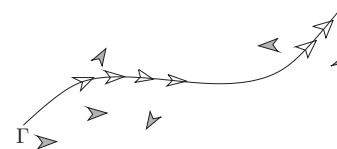


Fig. 3. Coordinated path following.

III. USING REACHABILITY TO SPECIFY COORDINATED PATH FOLLOWING

In this section, we focus on constructing reachability specifications for the coordinated path following problem.

Intuitively, when a vehicle is far away from the path, there is no need for the vehicle to coordinate with other vehicles right away so that they can achieve spontaneously ordered motion to follow the path. Instead, its primary objective should be getting close to the path. On the other hand, for vehicles that are already close to the path (e.g., in \mathcal{Z}_1), they can sense the path, so their primary objective is to follow the path and coordinate their motion with other vehicles that are also in \mathcal{Z}_1 in order to attain a desired inter-vehicle formation. Moreover, it is expected that no vehicle will leave \mathcal{Z}_1 as otherwise it may lose the vision of the path and can not achieve path following thereafter. In terms of this natural observation, we partition the state space \mathcal{Z} into two disjoint parts: \mathcal{Z}_1 and the remaining part (that is denoted by $\mathcal{Z}_2 := \mathcal{Z} \setminus \mathcal{Z}_1$). For vehicles in \mathcal{Z}_2 , it means that the

distances from them to the path are greater than R_0 . Thus, in order to solve the coordinated path following problem, we propose the following specifications: (1) navigate each vehicle that is originally in \mathcal{Z}_2 into \mathcal{Z}_1 in finite time; (2) ensure the vehicles remain in \mathcal{Z}_1 and achieve coordinated path following. In the paper, we use reachability to describe the specifications. The notation $\mathcal{X} \rightarrow \mathcal{Y}$ will be used to represent the reachability specification from one state set \mathcal{X} to its adjacent state set \mathcal{Y} , which means, for all initial states $x(0) \in \mathcal{X}$, there exists $T > 0$ satisfying (a) $x(t) \in \mathcal{X}$ for all $0 \leq t < T$; (b) $x(t) \in \mathcal{Y}$ when $t = T$. Using this rigorous formula, specification (1) becomes $\mathcal{Z}_2 \rightarrow \mathcal{Z}_1$. In addition, a part of specification (2) can be interpreted as making \mathcal{Z}_1 positively invariant.

However, even though the vehicles are already in \mathcal{Z}_1 , it may still not be easy to have a unified control law for each vehicle such that it not only solves the coordinated path following problem but also keeps them in \mathcal{Z}_1 . As a result, we propose to partition \mathcal{Z}_1 again into two disjoint sets: \mathcal{Z}_{11} and \mathcal{Z}_{12} . It is expected that \mathcal{Z}_{11} is as large as possible so that for vehicles in \mathcal{Z}_{11} , they retain in it and are able to solve the coordinated path following problem. Then for the set \mathcal{Z}_{12} , it is expected that $\mathcal{Z}_{12} \rightarrow \mathcal{Z}_{11}$ for any vehicle, i.e., vehicles in \mathcal{Z}_{12} can be steered into \mathcal{Z}_{11} in finite time so that eventually the coordinated path following problem can be solved.

Based on the aforementioned ideas, the state space \mathcal{Z} of each vehicle is partitioned into three disjoint sets \mathcal{Z}_{11} , \mathcal{Z}_{12} , and \mathcal{Z}_2 . Then find control strategies for each vehicle such that (1) $\mathcal{Z}_2 \rightarrow \mathcal{Z}_1 = \mathcal{Z}_{11} \cup \mathcal{Z}_{12}$, (2) $\mathcal{Z}_{12} \rightarrow \mathcal{Z}_{11}$, and (3) \mathcal{Z}_{11} is positively invariant. A schematic state transition graph is depicted in Fig. 4. When a group of n vehicles are considered, the transition graph is the Cartesian product of n copies of such graph. Therefore, the following three

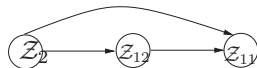


Fig. 4. Reachability specification for every vehicle.

subproblems should be addressed.

Problem 3.1: Find a set $\mathcal{Z}_{11} \subset \mathcal{Z}_1$ as large as possible and devise a control law for each vehicle so that

- (i) $q_i(0) \in \mathcal{Z}_{11} \implies q_i(t) \in \mathcal{Z}_{11}$ for all $t \geq 0$;
- (ii) if $q_i(0) \in \mathcal{Z}_{11}$ for all i , then
 - a) $\phi_i(t) \rightarrow 0$ as $t \rightarrow \infty$;
 - b) $\lim_{t \rightarrow \infty} \zeta_i(t) = L$ if its pre-neighbor is still in its sensing range.

Problem 3.2: Devise a control law for each vehicle such that $\mathcal{Z}_{12} \rightarrow \mathcal{Z}_{11}$.

Problem 3.3: Devise a control law for each vehicle such that $\mathcal{Z}_2 \rightarrow \mathcal{Z}_{11} \cup \mathcal{Z}_{12}$.

IV. SYNTHESIS OF HYBRID CONTROL

In this section, we synthesize control laws for each subproblem. Then, a hybrid control is constructed to solve the coordinated path following problem.

A. Coordinate Transformation

First, we introduce a coordinate transformation that is originally developed for single vehicle path following [1], [15]. For each vehicle $i = 1, \dots, n$, we define the corresponding virtual vehicle i on the path Γ , whose position (x_i^r, y_i^r) is the

orthogonal projection of the moving vehicle i 's position onto Γ at each time and whose orientation θ_i^r is tangent to Γ at the current location (x_i^r, y_i^r) and in the direction of motion (see Fig. 5). In other words, the position of the virtual vehicle

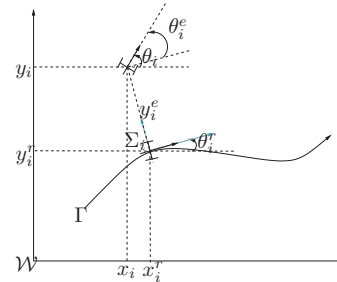


Fig. 5. Virtual vehicle and new coordinate variables.

i is the nearest point p_i on Γ to vehicle i . Recall that it is unique when vehicle i is in \mathcal{Z}_1 . Denote $q_i^r = (x_i^r, y_i^r, \theta_i^r)$ the posture of the virtual vehicle i in \mathcal{W} . Its kinematic model has the same form as (1) with control inputs v_i^r and ω_i^r .

We now construct the Frenet-Serret frame Σ_i that is fixed on the virtual vehicle i with the origin at (x_i^r, y_i^r) and x -axis tangent to the path in the direction of motion. Then we are able to define the posture of vehicle i in Σ_i as

$$q_i^e := \begin{pmatrix} \cos \theta_i^r & \sin \theta_i^r & 0 \\ -\sin \theta_i^r & \cos \theta_i^r & 0 \\ 0 & 0 & 1 \end{pmatrix} (q_i - q_i^r).$$

Thus, the dynamics for q_i^e is

$$\dot{q}_i^e = \begin{pmatrix} \dot{x}_i^e \\ \dot{y}_i^e \\ \dot{\theta}_i^e \end{pmatrix} = \begin{pmatrix} \omega_i^r y_i^e - v_i^r + v_i \cos \theta_i^e \\ -\omega_i^r x_i^e + v_i \sin \theta_i^e \\ \omega_i - \omega_i^r \end{pmatrix}. \quad (2)$$

By the definitions of virtual vehicle i and its corresponding Frenet-Serret frame Σ_i , one obtains that x_i^e and \dot{x}_i^e remain equal to zero. Moreover, note that $\chi(p_i) = -\frac{\omega_i^r}{v_i^r}$. From the first equation of (2), for any vehicle i in \mathcal{Z}_1 , it follows that

$$v_i^r = \frac{v_i \cos \theta_i^e}{1 + \chi(p_i) y_i^e},$$

$$\omega_i^r = -\chi(p_i) v_i^r = -\frac{\chi(p_i) v_i \cos \theta_i^e}{1 + \chi(p_i) y_i^e},$$

where $1 + \chi(p_i) y_i^e > 0$ ensures that Σ_i is uniquely defined. On the other hand, notice that $\phi_i = (\rho_i, \psi_i)$ is exactly the vector (y_i^e, θ_i^e) . Hence, it is obtained from (2) that

$$\begin{cases} \dot{\rho}_i = v_i \sin \psi_i, \\ \dot{\psi}_i = \omega_i + \frac{\chi(p_i) v_i \cos \psi_i}{1 + \chi(p_i) \rho_i}, \end{cases} \quad i = 1, \dots, n, \quad (3)$$

which describes the evolution of the path following error.

Define $\mathcal{S}_1 := \{(\rho, \psi) \mid \rho \in [-R_0, R_0], \psi \in [-\pi, \pi]\}$. It is clear that for any state $q \in \mathcal{Z}_1$, there is a correspondent $\phi \in \mathcal{S}_1$ and for any $\phi \in \mathcal{S}_1$ all the correspondent states are in \mathcal{Z}_1 . We denote this correspondence by $h : \mathcal{Z}_1 \rightarrow \mathcal{S}_1$.

B. Solving Problem 3.1

In this subsection, we focus on Problem 3.1. First, we are going to find $\mathcal{Z}_{11} \subset \mathcal{Z}_1$ as large as possible. Observe that when a vehicle is oriented to the same direction as the path (i.e. $|\psi_i| < \frac{\pi}{2}$) and in addition it is heading towards the path,

then the trajectory may remain in \mathcal{Z}_1 , but if it is heading away from the path, with no backward motion, it may leave \mathcal{Z}_1 depending on how close to the boundary. Hence, this intuition can be used to partition \mathcal{S}_1 as in Fig. 6, where

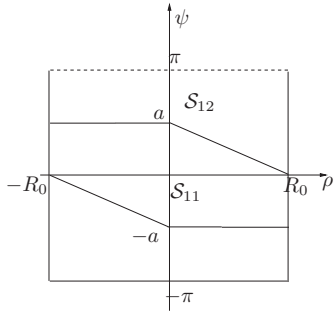


Fig. 6. A partition of \mathcal{S}_1 .

$\mathcal{S}_{11} = \{(\rho, \psi) \in \mathcal{S}_1 \mid |\rho| \leq R_0, |\psi| \leq a, |a\rho + R_0\psi| \leq aR_0\}$ and $\mathcal{S}_{12} = \mathcal{S}_1 \setminus \mathcal{S}_{11}$. The parameter a satisfies $0 < a < \min\{\frac{\pi}{2}, R_0\}$. The reason to partition it into polytopes is for the convenience of analysis. Then, we define $\mathcal{Z}_{11} := \{q \mid h(q) \in \mathcal{S}_{11}\}$ and $\mathcal{Z}_{12} := \{q \mid h(q) \in \mathcal{S}_{12}\}$. Clearly, $\mathcal{Z}_{11} \cup \mathcal{Z}_{12} = \mathcal{Z}_1$.

We now construct a distributed control law for each vehicle when its state is currently in \mathcal{Z}_{11} . That is, for vehicle i , as long as $\phi_i \in \mathcal{S}_{11}$, we let

$$\begin{cases} v_i = [c + s(\zeta_i - L)] \left(\frac{1 + \chi(p_i)\rho_i}{\cos \psi_i} \right), \\ \omega_i = v_i \left[-\frac{1}{k}(\rho_i + k\psi_i + 2 \sin \psi_i) - \frac{\chi(p_i) \cos \psi_i}{1 + \chi(p_i)\rho_i} \right], \end{cases} \quad (4)$$

where $c > 1$, $k \geq \frac{R_0}{a}$ are control parameters, $s(\cdot)$ is the sign function (with $s(0) = 0$). In the above control law, ζ_i is the arc distance between its pre-neighbor and itself. In a special case when the vehicle has no pre-neighbor, we artificially let $\zeta_i = L$ in order for the control law to have a unified form.

With this distributed control law, we first show that if a vehicle initially has its state in \mathcal{Z}_{11} , then its trajectory remains in \mathcal{Z}_{11} no matter where its pre-neighbor is. In other words, the set \mathcal{Z}_{11} is robust positively invariant for the dynamics of vehicle i .

Theorem 4.1: For any vehicle i under the control law (4), if $q_i(0) \in \mathcal{Z}_{11}$, then $q_i(t) \in \mathcal{Z}_{11}$ for all $t \geq 0$.

Suppose now that all the vehicles are initially in \mathcal{Z}_{11} . From the above theorem, we know that they remain in \mathcal{Z}_{11} forever under distributed control law (4). Next, we will show that if the arc distance of any two neighbor vehicles is nonzero, then the arc distance can never be zero as the system evolves under control law (4). Moreover, if the arc distance of any two neighbor vehicles is less than $R - 2R_0$, then these two neighbor vehicles are kept with a distance less than R all the time (i.e., they will never become disconnected in the sense of visibility.). Thus, it implies that the pre-neighbor of each vehicle will never change.

Theorem 4.2: Suppose that $q_i(0) \in \mathcal{Z}_{11}$ for all $i = 1, \dots, n$. Then for any i the following holds: 1) If $\zeta_i(0) > 0$, then $\zeta_i(t) > 0$ for all $t \geq 0$; 2) If $\zeta_i(0) \leq R - 2R_0$, then $d_{ij}(t) \leq R$ for all $t \geq 0$, where j is its pre-neighbor and d_{ij} is the distance between them.

Finally, we show that distributed control law (4) solves the coordinated path following problem for initial states in

\mathcal{Z}_{11} . That is, $\phi_i(t) \rightarrow 0$ as $t \rightarrow \infty$ (each vehicle converges to the path Γ) and $\lim_{t \rightarrow \infty} \zeta_1(t) = \dots = \lim_{t \rightarrow \infty} \zeta_n(t) = L$ (they are evenly spaced with the same arc distance) if they are still connected in the sense of pre-neighbor relationship.

Theorem 4.3: Suppose for all i , $q_i(0) \in \mathcal{Z}_{11}$ and $0 < \zeta_i(0) \leq R - 2R_0$. Then under distributed control law (4), 1) $\lim_{t \rightarrow \infty} \phi_i(t) = 0$ for all i ; 2) $\lim_{t \rightarrow \infty} \zeta_i(t) = L$ for all i .

Remark 4.1: If initially not all $\zeta_i(0)$ is less than $R - 2R_0$, the group of vehicles may lose connectivity, and in the final steady state they may be separated into several subgroups (namely, the distance between any two robots from different subgroups is larger than R). However, for each subgroup, the members are evenly spaced along the path with desired arc distance L . This can be inferred from Theorem 4.3 easily.

C. Solving Problem 3.2

In this subsection, we consider to devise a control law for each vehicle such that $\mathcal{Z}_{12} \rightarrow \mathcal{Z}_{11}$. This is equivalent to $\mathcal{S}_{12} \rightarrow \mathcal{S}_{11}$. Based on feedback-linearization technique, we are going to construct a continuous feedback such that the resulting closed-loop system is a linear system with its phase portrait like the one depicted in Fig. 7 and its equilibrium is at the origin.

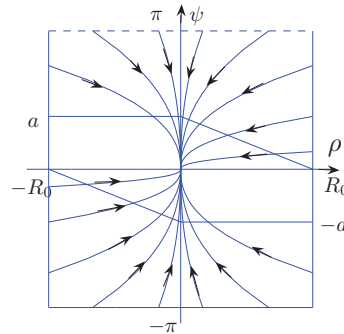


Fig. 7. Phase portrait of the resulting linear closed-loop system.

Thus, we consider the following control law

$$\begin{cases} v_i = -k_1 \frac{\rho_i}{\text{Sat}_\delta(\sin \psi_i)} \\ \omega_i = -k_2 \text{Sat}_\epsilon(\psi_i) - \frac{\chi(p_i)v_i \cos \psi_i}{1 + \chi(p_i)\rho_i} \end{cases} \quad (5)$$

where $k_1 > k_2 > 0$, $\epsilon > 0$ is a small number, $\delta = \sin(\epsilon)$, and $\text{Sat}_a(\cdot)$ is a saturation function defined as

$$\text{Sat}_\delta(x) = \begin{cases} x, & |x| \geq a, \\ a, & 0 \leq x < a, \\ -a, & -a < x < 0. \end{cases}$$

The presence of the saturation function is to avoid singularity.

Theorem 4.4: For any vehicle i under control law (5), $\mathcal{Z}_{12} \rightarrow \mathcal{Z}_{11}$.

D. Solving Problem 3.3

In this subsection, we study the problem of $\mathcal{Z}_2 \rightarrow \mathcal{Z}_1$. Consider now a vehicle i in \mathcal{Z}_2 . By our assumption, it can obtain the position information (distance and bearing angle) of at least one vehicle that is in \mathcal{Z}_1 or a landmark on the path Γ . If there are more than one, arbitrarily select a vehicle or a landmark (saying j) as its tracking target using the following control law

$$\begin{cases} v_i = k_3 d_{ij} \cos \alpha_{ij}, \\ \omega_i = k_3 (\sin \alpha_{ij} \cos \alpha_{ij} + \alpha_{ij}), \end{cases} \quad (6)$$

where k_3 is a constant. Notice that for any vehicle in \mathcal{Z}_1 , control law (4) or (5) is used so that it remains in \mathcal{Z}_1 forever. So from (4) and (5), we know that its linear speed is upper bounded. Let \bar{v} denote its upper bound. In the paper, we set $k_3 > \frac{3\bar{v}}{R_0}$. If the tracking target is a moving vehicle in \mathcal{Z}_1 , the dynamics of distance d_{ij} and bearing angle α_{ij} can be

$$\begin{cases} \dot{d}_{ij} = -v_i \cos \alpha_{ij} - v_j \cos \alpha_{ji} \\ \dot{\alpha}_{ij} = -\omega_i + \frac{v_i}{d_{ij}} \sin \alpha_{ij} + \frac{v_j}{d_{ij}} \sin \alpha_{ji} \end{cases}$$

where v_j is the linear speed of vehicle j and α_{ji} is the bearing angle of vehicle i in the local frame of vehicle j . Instead, if the tracking target is a landmark on Γ , we can still use the above equation to describe its dynamics by treating the landmark as a virtual vehicle with $v_j = 0$ and $\alpha_{ji} = 0$.

Thus, considering control law (6), it is obtained that

$$\begin{cases} \dot{d}_{ij} = -k_3 d_{ij} \cos^2 \alpha_{ij} - v_j \cos \alpha_{ji}, \\ \dot{\alpha}_{ij} = -k_3 \alpha_{ij} + \frac{v_j}{d_{ij}} \sin \alpha_{ji}. \end{cases} \quad (7)$$

Hence, we know that $|v_j| \leq \bar{v}$. Next, we show that vehicle i will be steered into \mathcal{Z}_1 in finite time with control law (6).

Theorem 4.5: Suppose vehicle i in \mathcal{Z}_2 has local knowledge of relative distance and bearing angle to a vehicle in \mathcal{Z}_1 or a landmark on Γ . Then under control law (6), $\mathcal{Z}_2 \rightarrow \mathcal{Z}_1$.

Proof: Let j denote the vehicle in \mathcal{Z}_1 (or the landmark on Γ) whose position information is available to vehicle i . Then for control law (6), its closed-loop dynamics is (7). We first show that for the dynamic system (7) there exists a $T > 0$ such that $d_{ij}(T) \leq \frac{2}{3}R_0$. Suppose by contradiction that $d_{ij}(t) > \frac{2}{3}R_0$ for all $t \geq 0$. Consider now a nonnegative function $V = \frac{1}{2}\alpha_{ij}^2$, and then we obtain

$$\begin{aligned} \dot{V} &= -k_3 \alpha_{ij}^2 + \frac{v_j \alpha_{ij}}{d_{ij}} \sin \alpha_{ji} \\ &\leq -k_3 \alpha_{ij}^2 + \frac{\bar{v}}{d_{ij}} |\alpha_{ij}| \\ &= -k_3 \left(1 - \frac{\sqrt{3}}{2}\right) \alpha_{ij}^2 - k_3 \frac{\sqrt{3}}{2} \alpha_{ij}^2 + \frac{\bar{v}}{d_{ij}} |\alpha_{ij}|. \end{aligned}$$

So it follows that

$$\dot{V} \leq -k_3 \left(1 - \frac{\sqrt{3}}{2}\right) \alpha_{ij}^2 < 0 \text{ for all } |\alpha_{ij}| \geq \frac{2\bar{v}}{\sqrt{3}k_3 d_{ij}}.$$

Thus, we know α_{ij} is uniformly ultimately bounded ([10]) and its ultimate bound is $\frac{2\bar{v}}{\sqrt{3}k_3 d_{ij}}$. In other words, there exists a $T_1 > 0$ such that $|\alpha_{ij}(t)| \leq \frac{2\bar{v}}{\sqrt{3}k_3 d_{ij}}$ for all $t \geq T_1$. Recall that by our assumption $k_3 > \frac{3\bar{v}}{R_0}$ and $d_{ij}(t) > \frac{2}{3}R_0$ for all $t \geq 0$. So $|\alpha_{ij}(t)| \leq \frac{2\bar{v}}{\sqrt{3}k_3 d_{ij}} \leq \frac{1}{\sqrt{3}}$ for all $t \geq T_1$. Due to the above assumptions, for $t \geq T_1$

$$\begin{aligned} \dot{d}_{ij}(t) &= -k_3 d_{ij} \cos^2 \alpha_{ij} - v_j \cos \alpha_{ji} \\ &< -\left(\frac{3\bar{v}}{R_0}\right) \left(\frac{2}{3}R_0\right) \cos^2 \left(\frac{1}{\sqrt{3}}\right) + \bar{v} \\ &= -\bar{v} \left[2 \cos^2 \left(\frac{1}{\sqrt{3}}\right) - 1\right] < 0, \end{aligned}$$

which contradicts to the assumption that $d_{ij}(t) > \frac{2}{3}R_0$ for all t . Thus, there exists a $T > 0$ satisfying $d_{ij}(T) \leq \frac{2}{3}R_0$.

If j is a vehicle in \mathcal{Z}_1 , we know that it will eventually converge to the path Γ . Instead, if j is a landmark, it is on the path Γ . So for both cases $d_{ij}(T) \leq \frac{2}{3}R_0$ implies that vehicle i will be in \mathcal{Z}_1 from the definition of \mathcal{Z}_1 . ■

Remark 4.2: In the paper, we assume that each vehicle in \mathcal{Z}_2 can have the position information of at least one vehicle in \mathcal{Z}_1 or a landmark on the path. Actually, if a vehicle i in

\mathcal{Z}_2 can only see a vehicle j that is also in \mathcal{Z}_2 and moreover vehicle j can see a vehicle in \mathcal{Z}_1 or a landmark on the path, then we let vehicle i track j using control law (6) and let vehicle j track the vehicle (or the landmark) in \mathcal{Z}_1 , which will drive both vehicles to \mathcal{Z}_1 in finite time. In general, if the group of vehicles has a tree-like topology connection with a root moving in \mathcal{Z}_1 , they can all be steered into \mathcal{Z}_1 in finite time using the tracking control law (6).

E. Hybrid Control Synthesis

Combining the results above, we are now able to construct a distributed hybrid control to solve the coordinated path following problem globally. Select a control law for each vehicle depending on its state according the following table.

Controller (4)	Controller (5)	Controller (6)
\mathcal{Z}_{11}	\mathcal{Z}_{12}	\mathcal{Z}_2

Thus, a state-dependent switching controller is obtained.

From the analysis in previous subsections, the states of all the vehicles will get into \mathcal{Z}_{11} in finite time under the hybrid control, and moreover, the vehicles eventually achieve coordinated path following. It is worth pointing out that for our coordinated path following control strategy in \mathcal{Z}_{11} , the arc distance between any two vehicles is assumed to be nonzero initially as otherwise the vehicles are not able to determine autonomously which of them is going to move in front of the other. However, the set of states such that two vehicles have zero arc distance is of low dimension. In practice, due to the presence of measurement error and/or other perturbations, it is quite reasonable that no two vehicles can be in that configuration.

V. SIMULATION RESULTS

In this section, we present simulation results of five unicycles using the hybrid control to solve the coordinated path following problem. We consider five unicycles (labeled 1 through 5) with initial states $(-5, 1, 0.15\pi)$, $(-10, 5, -0.15\pi)$, $(-15, 15, 0.65\pi)$, $(5, -4, 0.1\pi)$ and $(-10, -10, 0.75\pi)$, respectively. The maximal curvature of Γ is $\chi_0 = 0.2$ (i.e., $R_0 = 5$). The sensing radius of each vehicle is $R = 15.5$ and the desired arc distance L is set to be 5. The parameter a used to partition \mathcal{S}_1 in Fig. 6 is selected to be 1.3. With respect to the condition of each theorem, the control parameters are chosen as follows: $c = 1.5$, $k = 4$, $\epsilon = 0.1$, $k_1 = 5$, $k_2 = 3$, $k_3 = 15.45$. The simulated trajectories of five unicycles under the hybrid control are shown in Fig. 8 - Fig. 10.

In the figures, the dashed curve is the desired path to be followed, and two real lines are the boundary of the zonal region corresponding to \mathcal{Z}_1 , in which the distance from a location to the path is less than R_0 . From Fig. 8, we can see that vehicles 1, 4, 5 are in \mathcal{Z}_1 and vehicles 2, 3 are in \mathcal{Z}_2 initially. As a result, before vehicle 2 and 3 enter the zonal region \mathcal{Z}_1 , vehicle 4 is the pre-neighbor of vehicle 1, and vehicle 1 is the pre-neighbor of vehicle 5. These three vehicles coordinate their motion first without getting far away from the path. For vehicles 2 and 3 that are outside of \mathcal{Z}_1 initially, it is assumed that vehicle 3 has the relative position information of vehicle 2 and vehicle 2 has the relative position information of vehicle 1, so vehicle 3 tracks vehicle 2 and vehicle 2 tracks vehicle 1 using control law (6). As soon as vehicle 2 enters \mathcal{Z}_1 , it switches the control law depending on its current state. Moreover, the

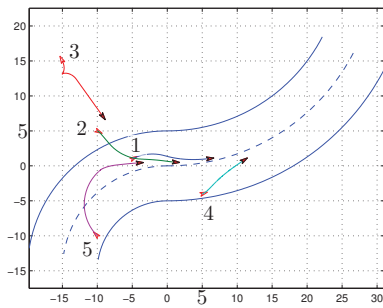


Fig. 8. Trajectories of five unicycles in the plane at $t = 5s$.

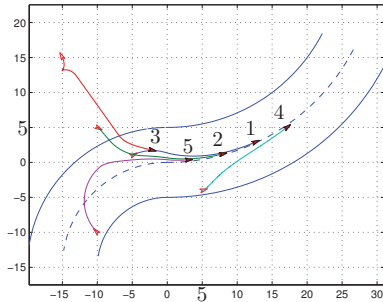


Fig. 9. Trajectories of five unicycles in the plane at $t = 10s$.

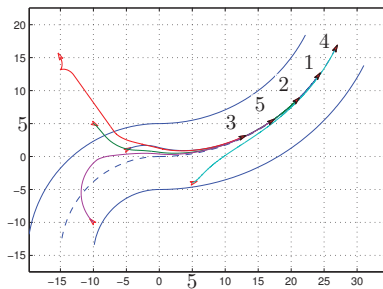


Fig. 10. Trajectories of five unicycles in the plane at $t = 20s$.

pre-neighbor of vehicle 5 changes to vehicle 2, too. Vehicle 3 keeps tracking vehicle 2 until it gets into \mathcal{Z}_1 , and then switches the control law where its pre-neighbor is vehicle 5. In the simulation, the five vehicles are kept within the sensing range when they are \mathcal{Z}_1 . So finally they are evenly spaced along the path with the desired arc distance $L = 5$ and move as a whole.

VI. CONCLUSION

In this paper, we focus on coordinated path following problems, namely, steering a fleet of unicycles along a given path while achieving a desired inter-vehicle formation pattern. A novel approach based on reachability specifications and hybrid control is proposed to solve the problem. First, coordinated path following problems are specified using reachability specifications and the state space is partitioned into several regions. Second, control laws are synthesized on each region to meet the reachability specifications so that all the vehicles are steered into a region close to the path with certain orientation conditions satisfied in finite time. Then,

starting in this region, all the vehicles are able to achieve coordinated path following. That is, the path following error for each vehicle is reduced to zero eventually and the arc distance used to describe the inter-vehicle formation converges to a desired constant. When vehicles are far away from the path, only brief information (such as relative distance and bearing angle) of a fixed neighbor is necessary to navigate them close to the path. When vehicles are already close to the path, more information about the path and their neighbors is assumed to be measured by onboard sensors with a limited sensing range. As a result, the neighboring relationship may be dynamically changed. In our control strategy, only the local measurement of one neighbor called pre-neighbor is used for the coordinated control purpose, which makes the information measurement at a minimum. The pre-neighbor relationship may also be dynamically changed as more and more vehicles are getting close to the path.

REFERENCES

- [1] L. Aguilar, P. Soueres, M. Courdesses, and S. Fleury, "Robust path-following control with exponential stability for mobile robots," in *Proceedings of the 1998 IEEE International Conference on Robotics and Automation*, Leuven, Belgium, 1998, pp. 3279–3284.
- [2] R. Beard, L. Lawton, and F. Hadaegh, "A coordination architecture for spacecraft formation control," *IEEE Transactions on Control Systems Technology*, vol. 9, no. 6, pp. 777–790, 2001.
- [3] N. Ceccarelli, M. D. Marco, A. Garulli, and A. Giannitrapani, "Collective circular motion of multi-vehicle systems," *Automatica*, vol. 44, no. 12, pp. 3025–3035, 2008.
- [4] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transaction on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, 2004.
- [5] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1465–1476, 2004.
- [6] R. Ghabelloo, A. P. Aguiar, A. Pascoal, C. Silvestre, I. Kaminer, and J. Hespanha, "Coordinated path-following in the presence of communication losses and time delays," *SIAM Journal on Control and Optimization* (in press), 2009.
- [7] R. Ghabelloo, A. Pascoal, C. Silvestre, and I. Kaminer, "Nonlinear coordinated path-following control of multiple wheeled robots with bi-directional communication constraints," *International Journal of Adaptive Control and Signal Processing*, vol. 21, pp. 133–157, 2007.
- [8] I. F. Ihle, M. Arcaç, and T. I. Fossen, "Passivity-based designs for synchronized path following," *Automatica*, vol. 43, no. 9, pp. 1508–1518, 2007.
- [9] I. Kaminer, O. Yakimenko, A. Pascoal, and R. Ghabelloo, "Path generation, path following and coordinated control for time-critical missions of multiple UAVs," in *Proceedings of American Control Conference*, Minneapolis, Minnesota, USA, 2006, pp. 4906–4913.
- [10] H. K. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.
- [11] Y. Lan, G. Yan, and Z. Lin, "Synthesis of distributed control of coordinated path following based on hybrid approach," *submitted to IEEE Transactions on Automatic Control*, 2009.
- [12] L. Lapierre, D. Soetanto, and A. Pascoal, "Coordinated motion control of marine robots," in *Proceedings of the 6th IFAC Conference on Manoeuvring and Control of Marine Craft (MCMC)*, Girona, Spain, 2003, pp. 250–255.
- [13] N. E. Leonard, D. Paley, F. Lekien, R. Sepulchre, D. Frantantoni, and R. Davis, "Collective motion, sensor networks, and ocean sampling," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 48–74, 2007.
- [14] J. Lin, A. S. Morse, and B. D. O. Anderson, "The multi-agent rendezvous problem," in *Proceedings of the 42th IEEE Conference on Decision and Control*, New Haven, CT, USA, 2003, pp. 1508–1513.
- [15] A. Micaelli and C. Samson, "Trajectory tracking for unicycle-type and two-steering-wheels mobile robots," Institut National de Recherche en Informatique et en Automatique (INRIA), France, Tech. Rep., 1993.
- [16] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.
- [17] R. Skjetne, I. A. Flakstad, and T. I. Fossen, "Formation control by synchronizing multiple maneuvering systems," in *Proceedings of the 6th IFAC Conference on Manoeuvring and Control of Marine Craft*, Girona, Spain, 2003.
- [18] F. Zhang, D. M. Fratantoni, D. Paley, J. Lund, and N. E. Leonard, "Control of coordinated patterns for ocean sampling," *International Journal of Control*, vol. 80, no. 7, pp. 1186–1199, 2007.