Product Restructuring, Exports, Investment, and Growth Dynamics

(Job Market Paper)

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Abstract

This paper estimates a dynamic general equilibrium model of entry, exit, and endogenous productivity growth. Productivity is endogenous both at the industry level (firms enter and exit) and at the firm level (firms invest in productivity-enhancing activities). The focus of the paper is on two activities that make productivity-enhancing investments more attractive, namely, exporting and product-mix choices. A firm that increases its exports and/or its number of products will have higher sales – and this makes investing in productivity more attractive because there are more units (sales) across which the productivity gains can be applied. These insights are taken to firm-level Spanish data. We compute the Markov Perfect Equilibrium using a nested pseudo maximum likelihood estimator (NPL) with dynamic programming algorithms. Three key findings emerge. First, there is no evidence of learning by exporting: the observed positive correlation between exporting and productivity operates entirely via the impact of exporting on productivity-enhancing investments. Restated, exporting decision raises productivity, but only indirectly by making investing in productivity more attractive. Second, there is evidence of learning by producing multiple products: product-mix raises productivity directly in addition to the investment channel. Third, there are strong complementarities among the product-mix, exporting and investment decisions. Finally, we simulate the effects of reductions in foreign tariffs. This increases exporting, investing, and wages; and wage increases decrease the number of product produced per firm and force the least productive firms to exit. Productivity rises at the economy-wide level both because of the between firm reallocation effect and because of within firm increases in productivity.

Keywords: heterogeneous firms, endogenous product range, dynamic discrete games, continuous games, multiple equilibria, pseudo maximum likelihood estimation, entry, exit, and growth in monopolistic competition

JEL Classifications: F12, F13, F14, L11, O31, O33.

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Introduction

Gains from trade through both comparative advantage effects and intra-industry resource reallocations have been examined intensively in the past (Melitz (2003) and Bernard et al. (2003)). One element that is missing from this literature is the existence of multi-product firms that undertake investments that lead to higher productivity, a higher propensity to export, and more products produced. While multi-product firms that invest dominate international trade, comparatively little research examines their production and export decisions and their productivity trajectory or how these decisions are influenced by globalization.

A large empirical literature has emerged over the past decade trying to determine the causal relationship between productivity and exporting. Much of it documents the self-selection of more productive firms into the export market. The evidence that exporting raises productivity growth rates is less uniform, with some studies (Clerides, Lach, and Tybout (1998), Bernard and Jensen (1999), Bernard and Wagner (1997), Delgado, Fariñas, and Ruano (2002) and Bernard and Jensen (2004)) finding no such effect, and others finding varying degrees of support for a positive effect of exporting on productivity (Aw, Chung, and Roberts (2000),Baldwin and Gu (2003), Van Biesebroeck (2004), Lileeva (2004), Hallward-Driemeier, Iarossi, and Sokoloff (2005), Fernandes and Isgut (2006), Park et al. (2006), Aw, Roberts, and Winston (2007), De Loecker (2010) and Lileeva and Trefler (2007)). Two theoretical papers, Atkeson and Burstein (2007) andConstantini and Melitz (2008), have formalized how trade liberalizations can increase the rate of return to a firm's investment in new technology and thus lead to future endogenous productivity gains. Both papers share several common features: first, productivity is the underlying state variable that distinguishes heterogeneous producers; second, productivity evolution is endogenous, affected by the firm's investment decisions; and third, they each identify pathways through which export market size affects the firm's choice to export or invest.

Very little research has focused on the combination of investment, exporting and multi-product decision in a general equilibrium framework. This paper develops a tractable dynamic general equilibrium model of endogenous product selection and productivity growth that offers a natural and intuitive explanation for key features of the Spanish firm level data. Our model and empirical analysis demonstrate the importance of firm and industry endogenous productivity growth in response to trade liberalization. In every period, firms make decisions about entry and exit, investment, number of products, and exporting, and compete in a monopolistic competition product market. Following Bernard, Redding, and Schott (forthcoming) which builds on Melitz (2003), we allow firms to produce multiple products of varying profitability. We assume firm profitability in a particular product increases with two stochastic and independent draws in the first period the firm operates. The first is firm productivity, which is drawn stochastically after the firm enters and pays the sunk fixed entry cost. This governs the amount of labor that must be used to produce a unit of output. Firm productivity becomes a state variable in all subsequent periods and evolves over time based on firm investment and random shocks and/or depreciation. The second is firm-product consumer tastes drawn every period, which regulate the demand for a firm in a market. We assume both draws are revealed to firms after incurring a sunk cost of entry. If firms decide to enter after having observed these draws, they face fixed and variable costs for each good they choose to supply to a market as well as a fixed cost of serving each market that is independent of the number of goods supplied. We assume consumers possess constant elasticity of substitution preferences on the demand side as in Dixit and Stiglitz (1977). Demand for product variety depends on the own-variety price, the price index for the product, and the price indices for all other products. If a firm is active in a product market, it manufactures one of a continuum of varieties and so is unable to influence the price index for the product. This implies the price of a firm's variety in one product market influences only the demand for its varieties in other product markets through the price indices. Therefore, the firm's inability to influence the price indices implies that its profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety. The structure of our model eliminates strategic interaction within or between firms.

In this paper we develop an algorithm for computing the Markov Perfect Equilibrium (MPE) similar to C. Lanier Benkard and Weintroub (2007a)[1] and C. Lanier Benkard and Weintroub (2007b). A nice feature of the algorithm is that, unlike existing methods, there is no need to place a priori restrictions on the number of firms in the industry or the number of allowable states per

[1] C. Lanier Benkard and Weintroub (2008) define an oblivious equilibrium in which each firm is assumed to make decisions based only on its own state and knowledge of the long-run average industry state, but where firms ignore current information about competitors' states. They showed that as the market becomes large, if the equilibrium distribution of firm states obeys a certain "light-tail" condition, then the oblivious equilibrium closely approximates MPE.

firm. These are determined by the algorithm as part of the equilibrium solution. In the past, for Ericson and Pakes (1995) type models, MPE are usually computed using iterative dynamic programming algorithms (e.g. Pakes and McGuire (1995)).However, computational requirements grow exponentially with the number of firms and possible firm produtivity levels, making dynamic programming infeasible in many problems of practical interest. In this paper, we take a different tack and consider algorithms that can efficiently deal with any number of firms in a monopolistic competition setting. This is most closely related to Hopenhayn (1992) and Melitz (2003). As in Hopenhayn (1992), the analysis is restricted to stationary equilibria. Firms correctly anticipate this stable aggregate environment when making all relevant decisions. This becomes computationally feasible for MPE computation with common dynamic programming algorithms. We also use nested pseudo likelihood (NPL), a recursive extension of the two-step pseudo maximum likelihood (PML), that addresses inconsistent or very imprecise nonparametric estimates of choice probabilities to compute the MPE.

The reason to model the investment, multi-product and exporting decisions jointly is they are dependent on each other. Firms cannot export or produce multiple products, or have a lower probability of doing so, below a certain productivity cut-off. Therefore firms need to invest and increase their productivity in order to export and produce more products. The return to investment is higher for exporting and multi-product firms, which makes the probability that the firm will choose to invest dependent on the firm's export status and the number of products produced. This paper estimates a structural model of endogenous entry, exit, exports, product restructuring and investment to evaluate the role that fixed costs of operating, sunk entry costs, cost of investment and trade liberalization play in explaining the observed cross industry heterogeneity. Olley and Pakes (1996) show that ignoring endogenous market exit can generate significant biases in the estimation of production functions. The estimation of structural models of market entry is based on the Principle of Revealed Preference. In the context of these models, this principle establishes that if we observe a firm operating in a market it is because its value in that market is greater than the value of shutting down and putting its assets to alternative uses. Under this principle, firms' entry decisions reveal information about the firm's underlying latent profit (or value). The same for firms exit decisions.

Since our data do not provide firm-product-destination export information, we simplify the demand parameter in Bernard, Redding, and Schott (forthcoming) to firm-product level only. However, it is very simple to model the demand parameter in firm-product-destination level. The main results of this paper should still hold with data that support this kind of model.

The model yields a rich set of predictions on productivity, investment, product restructuring and exporting. First, firms self-select into exporting, investment, and range of products based on their current productivity. Productivity evolves over time and is endogenous and positively impacted by both investment and number of products produced. The direct positive impact on productivity from the number of products produced suggests learning by doing. However, there is no evidence of learning by exporting: the observed positive correlation between exporting and productivity operates entirely via the impact of exporting on productivity-enhancing investments. Past exporting is correlated with current productivity via past investing; that is, past exporting complements past investing which leads to current productivity gains. Second, there are strong complementarities among exporting, range of products and investment decisions. A rise in the number of products raises productivity by making investment more attractive. (There is also a direct impact of the number of products on productivity, which we conjecture captures unmeasured investments in new products). Finally, we simulate the effects of reductions in foreign tariffs. This increases exporting, investiment and wages; and wage increases cause a reduction in the number of products per firm and force the least productive firms to exit. Productivity rises at the economy-wide level both because of the between firm reallocation effect and because of within firm increases in productivity

The rest of paper is organized as follows. In Section 1 we outline the dynamic industry model. In Section 2 we define and solve for MPE. In Section 3 we discuss the data used and the limitations to the data. In Section 4 we provide our main result, namely, the role that product differentiation, fixed costs of operating, sunk entry costs, cost of investment and trade liberalization play in explaining the observed cross industry heterogeneity. In Section 5 we discuss the counterfactuals. Finally, Section 6 presents conclusions, policies and a discussion of future research directions. All proofs and mathematical arguments are provided in the Appendix.

1. The Model

Consider a world consisting of many countries and many products. Firms decide whether to produce, what products to make,and where to export these products. Products are imperfect substitutes of each other, and within each product firms supply horizontally differentiated varieties. For simplicity, we develop the model for symmetric products and n symmetric countries.

1.1. *Static Model*

1.1.1. *Consumers*

The world consists of a home country and a continuum of n foreign countries, each of which is endowed with L_n units of labor that are supplied inelastically with zero disutility.

Consumers prefer more varieties to less and consume all differentiated varieties in a continuum of products that we normalize to the interval [0,1]. The utility function of a representative consumer in country j is given by:

$$U = \left[\int_0^1 C_{jk}^{\nu} dk \right]^{1/\nu}, \qquad 0 < \upsilon < 1, \tag{1}$$

as in the standard Dixit and Stiglitz (1977) form, where k indexes products. Within each product, a continuum of firms produce horizontally differentiated varieties of the product. C_{jk} is a consumption index for a representative consumer in country j for product k and is of the form:

$$C_{jk} = \left[\int_0^{n+1} \int_{\omega \in \Omega_{ijk}} \left[\lambda_{jk}(\omega) \, c_{ijk}(\omega) \right]^{\rho} d\omega di \right]^{1/\rho}, \qquad 0 < \rho < 1, \tag{2}$$

where i and j index countries, ω indexes varieties of product k supplied from country i to j and Ω_{ijk} denotes the endogenous set of these varieties. Similar to Bernard, Redding, and Schott (forthcoming) the demand shifter $\lambda_{jk}(\omega)$ captures the strength of the representative consumer's tastes for firm variety ω and reflects demand heterogeneity. $\lambda_{jk}(\omega)$ can also be interpreted as the quality of variety ω. We assume $\sigma \equiv \frac{1}{1-\rho} > \kappa \equiv \frac{1}{1-\nu}$ or the elasticity of substitution across varieties within products is greater than the elasticity of substitution across products and σ is the same for all products. The corresponding price index for product k in country j is:

$$P_{jk} = \left[\int_0^{n \pm 1} \int_{\omega \in \Omega_{ijk}} \left(\frac{p_{ijk}(\omega)}{\lambda_{ijk}(\omega)} \right)^{1-\sigma} d\omega di \right]^{\frac{1}{1-\sigma}}. \tag{3}$$

Furthermore, countries are also symmetric, and the only difference between the domestic market and each export market is that a common value of trade costs has to be incurred for each export market. Therefore, instead of indexing variables in terms of country of production, i, and market of consumption, j, we distinguish between the domestic market, d, and each export market, x, except where otherwise indicated.

1.1.2. Production

The only factor of production is labor as in Melitz (2003). The potential entrants are identical prior to entry. A potential entrant who decides to stay out of the market gets zero profits. If the firm decides to enter, it must incur a sunk entry cost $f_{EN,i} > 0$ units of labor in country i. $f_{EN,i} = f_{EN,i} + \varepsilon_{EN,it}$ at time t, where $f_{EN,i}$ is the component of entry cost that is common to all the firms in the market in country i, and $\varepsilon_{EN,it}$ is a firm-specific component which is private information to the firm, has zero mean, and is i.i.d. over firms and over time. Similar to Bernard, Redding, and Schott (forthcoming) we augment the model to allow firms to manufacture multiple products and to allow for demand heterogeneity across products. The new entrant is not active until the next period. Furthermore, the initial quality and the product attributes that influence demand (consumer tastes λ) of a new entrant are uncertain when the firm makes its entry decision, and they are not realized until the next period. The initial productivity φ is common across all of a firms's products and is a random draw from the probability function $g(\varphi)$ with cumulative distribution function $G(\varphi)$.Consumer tastes for a firm's varieties, $\lambda_k \in [0, \infty)$, vary across products k and are drawn separately for each product from the probability function $z(\lambda)$ with cumulative distribution function $Z(\lambda)$. To make use of law of large number results, we make simplifying assumptions that productivity and consumer tastes distributions are independent across firms and products, respectively, and of one another.

Once the sunk entry cost has been incurred in period $t-1$, the potential entrant enters at the end of period $t-1$ and becomes an incumbent in period t. An incumbent in period t observes its sell-off value ϕ_t and makes exit and investment decisions. If the sell-off value (or the exit value) ϕ_t exceeds the value of continuing in the industry, then the firm chooses to exit, in which case it earns the sell-off value and then ceases operations permanently. If it decides to stay and invest, it faces fixed costs of supplying each market, which are $f_X > 0$ for foreign market and $f_D > 0$ for domestic market. These market specific fixed costs capture among other things the costs of building distribution networks. In addition, we assume that the incumbent must pay the fixed costs of supplying each product to a market, which are $f_x > 0$ for foreign market and $f_d > 0$ for domestic market. These product and market-specific fixed costs capture the costs of market research, advertising, and conforming to foreign regulatory standards for each product. As more products are supplied to a market, total fixed costs rise, but average fixed costs fall. The firm can invest to improve its productivity for next period. A detailed modelling of the investment decision is given under the Investment subsection.

In addition to fixed costs, there is also a constant marginal cost for each product that depends on firm productivity, such that $q_k(\varphi, \lambda_k)/\varphi$ units of labor are required to produce $q_k(\varphi, \lambda_k)$ units of output of product k. Finally, we allow for variable costs of trade, such as transportation costs, which take the standard iceberg cost form, where a fraction $\tau > 1$ of a variety must be shipped in order for one unit to arrive in a foreign country. We assume for simplicity that the fixed costs of serving each market are incurred in terms of labor in the country of production, although it is straightforward to instead consider the case where they are incurred in the market supplied.

1.1.3. Firm-Product Profitability

Demand for a product variety depends on the own-variety price, the price index for the product and the price indices for all other products. If a firm is active in a product market, it manufactures one of a continuum of varieties and so is unable to influence the price index for the product. At the same time, the price of firm's variety in one product market only influences the demand for its varieties in other product markets through the price indices. Therefore, the firm's inability to influence the price indices implies that its profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety. This optimization problem yields the standard result that the equilibrium price of a product variety is a constant mark-up over marginal cost:

$$p_d(\varphi, \lambda_d) = \frac{1}{\rho\varphi}, \qquad p_x(\varphi, \lambda_x) = \tau\frac{1}{\rho\varphi}, \tag{4}$$

where equilibrium prices in the export market are a constant multiple of those in the domestic market due to the trade costs; λ_d varies across products and λ_x varies across products and export markets. We choose wage in one country as the numeraire, which together with country symmetry implies $w = 1$ for all countries.

Demand for a variety is:

$$q_d(\varphi, \lambda_d) = Q_k\lambda_d^{\sigma-1}\left[\frac{p_d(\varphi, \lambda_d)}{P}\right]^{-\sigma}, \qquad q_x(\varphi, \lambda_x) = Q_k\lambda_x^{\sigma-1}\left[\frac{p_x(\varphi, \lambda_x)}{P}\right]^{-\sigma}. \tag{5}$$

Substituting for the pricing rule equation (4), the equilibrium revenue in each domestic and export market are respectively:

$$r_d(\varphi, \lambda_d) = E(\rho P\varphi\lambda_d)^{\sigma-1}, \qquad r_x(\varphi, \lambda_x) = \tau^{1-\sigma}\left(\frac{\lambda_x}{\lambda_d}\right)^{\sigma-1} r_d(\varphi, \lambda_d), \tag{6}$$

where E denotes aggregate expenditure on a product and P denotes the price index for a product (subscript product k is suppressed here). The equilibrium profits from a product in each domestic and export market are therefore:

$$\pi_d(\varphi, \lambda_d) = \frac{r_d(\varphi, \lambda_d)}{\sigma} - \theta_d, \qquad \pi_x(\varphi, \lambda_x) = \frac{r_x(\varphi, \lambda_x)}{\sigma} - \theta_x. \tag{7}$$

Firm productivity and consumer tastes enter the equilibrium revenue and profit functions in the same way, because prices are a constant mark-up over marginal costs and demand exhibits a constant elasticity of substitution.

Relative revenue from two varieties of the same product within a given market depends solely on relative productivity and consumer tastes:

$$r(\varphi', \lambda') = \left(\frac{\varphi'}{\varphi}\right)^{\sigma-1}\left(\frac{\lambda'}{\lambda}\right)^{\sigma-1} r(\varphi, \lambda). \tag{8}$$

Similarly, as countries are symmetric, equation (7) implies that the relative revenue derived from two varieties of the same product with the same values of productivity and consumer tastes in the export and domestic markets depends solely on variable trade costs: $r_x(\varphi,\lambda)/r_d(\varphi,\lambda) = \tau^{1-\sigma}$.

A firm with a given productivity φ and consumer taste draw λ decides whether or not to supply a product to a market based on a comparison of revenue and fixed costs for the product. For each firm productivity φ, there is a zero-profit cutoff for consumer tastes for the domestic market, $\lambda_d^*(\varphi)$, such that a firm supplies the product domestically if it draws a value of λ_d equal to or greater than $\lambda_d^*(\varphi)$. This value of $\lambda_d^*(\varphi)$ is defined by:

$$r_d(\varphi, \lambda_d^*(\varphi)) = \sigma\theta_d. \tag{9}$$

Similarly for the export market, $\lambda_x^*(\varphi)$ is given by:

$$r_x(\varphi, \lambda_x^*(\varphi)) = \sigma\theta_x. \tag{10}$$

We can write $\lambda_d^*(\varphi)$ and $\lambda_x^*(\varphi)$ as function of their lowest productivity supplier, $\lambda_j^*(\varphi_j)$ for $j \in \{d, x\}$, respectively:

$$\lambda_j^*(\varphi) = \left(\frac{\varphi_j^*}{\varphi}\right)\lambda_j^*\left(\varphi_j^*\right) \qquad j \in \{d, x\} \tag{11}$$

where φ_j^* for $j \in \{d, x\}$ is the lowest productivity at which a firm supplies the domestic and the export market, respectively. As a firm's own productivity increases, its zero-profit cutoff for consumer tastes falls because higher productivity ensures that sufficient revenue to cover product fixed costs is generated at a lower value of consumer tastes. In contrast, an increase in the lowest productivity at which a firm supplies the domestic market, φ_j^*, or an increase in the zero-profit consumer tastes cutoff for the lowest productivity supplier $\lambda_j^*\left(\varphi_j^*\right)$, raises a firm's own zero-profit consumer tastes cutoff. The reason is that an increase in either φ_j^* or $\lambda_j^*\left(\varphi_j^*\right)$ enhances the attractiveness of rival firms' products, which intensifies product market competition, and hence increases the value for consumer tastes at which sufficient revenue is generated to cover product fixed costs. Given $\tau^{\sigma-1}(\theta_x/\theta_d) > 1$, a firm is more likely to supply a product domestically than to export the product.

1.1.4. Firm Profitability

Having examined equilibrium revenue and profits from each product, we now turn to the firm's equilibrium revenue and profits across the continuum of products as a whole. As consumer tastes are independently distributed across the unit continuum of symmetric products, the law of large numbers implies that the fraction of products supplied to the domestic market by a firm with a given productivity φ equals the probability of drawing a consumer taste above $\lambda_d^*(\varphi)$, that is $[1 - Z(\lambda_d^*(\varphi))]$. As demand shocks are also independently and identically distributed across the continuum of countries, the law of large numbers implies that the fraction of foreign countries to which a given product is exported equals $[1 - Z(\lambda_x^*(\varphi))]$. A firm's expected revenue across the unit continuum of products equals its expected revenue for each product. Expected revenue for each product is a function of firm productivity φ and equals the probability of drawing a consumer taste above the cutoff times expected revenue conditional on supplying the product. Therefore total firm revenue across the unit continuum of products in the domestic and export markets is:

$$r_j(\varphi) = \int_{\lambda_j^*(\varphi)}^{\infty} r_j(\varphi, \lambda_j) z(\lambda_j) d\lambda_j \qquad j \in \{d, x\}\,. \tag{12}$$

total profits for domestic and export market is:

$$\pi_j(\varphi) = \int_{\lambda_j^*(\varphi)}^{\infty} \left[\frac{r_j(\varphi, \lambda_j)}{\sigma} - f_j\right] z(\lambda_j) d\lambda_j - F_j \qquad j \in \{d, x\} \tag{13}$$

Total profit is:

$$\pi(\varphi) = \pi_d(\varphi) + \pi_x(\varphi). \tag{14}$$

Equilibrium revenue from each product within the domestic market, $r_j(\varphi, \lambda_j)$, is increasing in firm productivity and consumer tastes. Hence the lower a firm's productivity, φ, the higher its zero-profit consumer tastes cutoff, $\lambda_d^*(\varphi)$, and the lower its probability of drawing a consumer tastes high enough for a product to be profitable. Therefore firms with lower productivities have lower expected profits from individual products and supply a smaller fraction of products to the domestic market, $[1 - Z(\lambda_d^*(\varphi))]$. For sufficiently low firm productivity, the excess of domestic market revenue over product fixed costs in the small range of profitable products falls short of the fixed cost of supplying the domestic market, F_d. The same is true for export market.

The profit function satisfies the following properties:

1. Total profit for domestic and export market is increasing in φ.
2. For all $\varphi \in R^+$ and t, $\pi(\varphi) > 0$ and $sup_\varphi \pi(\varphi) < \infty$.
3. $\ln \pi(\varphi)$ is continuously differentiable.
4. Strengthened competition cannot result in increased profit due to competition for labor.

The increased labor demand by the more productive firms and new entrants bids up the real wages and forces the least productive firms to exit. Work by Bernard and Jensen (1999) suggests that this channel substantially contributes to U.S. productivity increases within manufacturing industries.

1.1.5. Aggregation and Market Clearing

An equilibrium will be characterized by a mass M of firms and a distribution $g(\varphi)$ of productivity levels over a subset of $[0, \infty)$. The weighted average productivity in the domestic and export market, respectively, is:

$$\widetilde{\varphi}_j = \left[\int_0^\infty \left(\varphi \widetilde{\lambda}_j(\varphi) \right)^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}, \qquad j \in \{d, x\}, \tag{15}$$

where $\widetilde{\lambda}_d(\varphi)$ denotes weighted-average consumer tastes in the domestic market for a firm with productivity φ:

$$\widetilde{\lambda}_j(\varphi) = \left[\int_0^\infty (\lambda_j(\varphi))^{\sigma-1} z(\lambda_j) d\lambda_j \right]^{\frac{1}{\sigma-1}} \qquad j \in \{d, x\}. \tag{16}$$

The aggregate price index P is then given by:

$$\begin{aligned} P &= \left[M_d \int_0^\infty p_d(\varphi)^{1-\sigma} g(\varphi) d\varphi + n M_x \int_0^\infty p_x(\varphi)^{1-\sigma} g(\varphi) d\varphi \right]^{\frac{1}{1-\sigma}} \\ &= \left[M_d \left(\frac{1}{\rho \widetilde{\varphi}_d} \right)^{1-\sigma} + n M_x \left(\frac{1}{\rho \widetilde{\varphi}_x} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}. \end{aligned} \tag{17}$$

The weighted average productivity of all firms (domestic and foreign) competing in a single country is:

$$\widetilde{\varphi} = \left\{ \frac{1}{M} \left[M_d \widetilde{\varphi}_d^{1-\sigma} + n M_x \left(\tau^{-1} \widetilde{\varphi}_x \right)^{1-\sigma} \right] \right\}^{\frac{1}{1-\sigma}} . \tag{18}$$

where the productivity of exporters is adjusted by the trade cost τ. Thus the aggregate price index P and revenue R can be written as functions of only the productivity average $\widetilde{\varphi}$ and M:

$$P = M^{\frac{1}{1-\sigma}} \frac{1}{\rho \widetilde{\varphi}} \qquad R = M r_d(\widetilde{\varphi}). \tag{19}$$

1.2. Dynamic Model

In this section we formulate the static model discussed in the previous section into a dynamic model. The model evolves over discrete time periods and an infinite horizon. We index time periods with non-negative integers $t \in \mathbb{N}$ $(\mathbb{N} = \{0, 1, 2, ...\})$.

Firm heterogeneity is reflected through firm states. We refer to a firm's state as its productivity level. At time t, the productivity level of firm i is $\varphi_{it} \in \mathbb{R}_+$.We define the industry state s_t to be the number of incumbent firms M_t and the average productivity $\widetilde{\varphi}_t$ in period t. We define the state space $S = \{s \in \mathbb{R}_+^2 | M * \widetilde{\varphi} < \infty\}$In each period, each incumbent firm earns profits. As in the static model, a firm's single period profit $\pi_t(\varphi_t, s_t)$ depends on its productivity φ_t and the aggregate price index P_t, which can be written as a function of the productivity average $\widetilde{\varphi}_t$ and mass of firms M_t in period t.

The model also allows for entry and exit. In each period, each incumbent firm observes a positive real-valued sell-off value ϕ_{it} that is private information to the firm. If the sell-off value exceeds the value of continuing in the industry, then the firm may choose to exit, in which case it earns the sell-off value and then ceases operations permanently.

In each period potential entrants can enter the industry by paying a fixed entry cost f_{EN}. Entrants do not earn profits in the period that they enter. They appear in the following period with productivity and consumer tastes drawn from $g(\varphi)$ and $z(\lambda)$ and can earn profits thereafter. Each firm aims to maximize expected net present value. The interest rate is assumed to be positive and constant over time, resulting in a constant discount factor of $\beta \in (0, 1)$ per time period.

In each period, events occur in the following order:

1. Each incumbent firm observes its sell-off value ϕ_{it}, productivity at $t+1$, and demand shocks.

2. The number of entering firms is determined and each entrant pays an entry cost of f_{EN}.

3. Incumbent firms choose price and quantity to maximize profit.

4. Incumbent firms choose investment, exporting, and number of products to maximize the expected net present values.

4. Exiting firms exit and receive their sell-off values.

5. Productivity in $t+1$ is realized and new entrants enter.

We assume that there are an asymptotically large number of potential entrants who play a symmetric mixed entry strategy. This results in a Poisson-distributed number of entrants (see Weintraub, Benkard, and Van Roy (2008b) for a derivation of this result). Our associated modeling assumptions are as follows:

Assumption:

1. The number of firms entering during period t is a Poisson random variable that is conditionally independent of $\{\varphi_{it}, \lambda_{it,}\varepsilon_{it}|t > 0\}$, conditioned on s_t.

2. $f_{EN} < \beta\overline{\phi}$, where $\overline{\phi}$ is the expected net present value of entering the market, investing zero and earning zero profits each period, and then exiting at an optimal stopping time.

We denote the expected number of firms entering in period t, by $M_{EN,t}$. This state-dependent entry rate will be endogenously determined, and our solution concept will require that it satisfies a zero expected discounted profits condition. Modeling the number of entrants as a Poisson random variable has the advantage that it leads to simpler dynamics. However, other entry processes can be used as well. Assumption 2 ensures that the sell-off value by itself is not sufficient reason to enter the industry.

1.2.1. Evolution of Productivity

In order to model the firm's dynamic optimization problem for exporting, investment, and product restructuring decisions we begin with a description of the evolution of the process for firm productivity φ_{it}. We assume that productivity evolves over time as a Markov process that depends on firm's investment, its participation in the export market, the number of products firm produces, and a random shock:

$$\varphi_{it} = z(\varphi_{it-1}, I_{it-1}, X_{it-1,}N_{it-1}) + \xi_{it} \tag{20}$$

$I_{it-1}, X_{it-1}, N_{it-1}$ are, respectively, the firm's investment, export market participation, and number of products produced in the previous period. Note that this specification is very general as the function z may take on either positive or negative values (e.g., allowing for positive depreciation).The inclusion of I_{it-1} recognizes that the firm may affect the evolution of its productivity by investing. The inclusion of X_{it-1} allows for the possibility of learning-by-exporting, that participation in the export market is a source of knowledge and expertise that can improve future productivity. The inclusion of N_{it-1} allows for the possibility of learning-by-doing, that producing more products exposes the firm to a bigger pool of knowledge that can improve its future productivity. In the empirical section, we assess the strength of each of these decisions. The stochastic nature of productivity improvement is captured by ξ_{it} which is treated as an *iid* shock with zero mean and variance σ_ξ^2. This stochastic component represents the role that randomness plays in the evolution of a firm's productivity. Uncertainty may arise, for example, due to the risk associated with a research and development endeavor or a marketing campaign.

We also assume there exists a positive constant $\overline{z} \in \mathbb{R}_+$ s.t. $|z(I, X, N, \varepsilon)| \leq \overline{z}$, for all (I, X, N, ε).There exists a positive constant $\overline{I}$, $\overline{X}$, $\overline{N}$ s.t. $I_{it} < \overline{I}$, $X_{it} < \overline{X}$, $N_{it} < \overline{N}$, for all i, t. This assumption places a finite bound on how much progress can be made or lost in a single period. Under perfect capital market, firms cannot invest more than their expected net present value[2]. X_{it} is modeled as a discrete 0/1 variable in the empirical section. If modelled as a continuous variable, export volume is bounded by the consumer demand. Similarly, N_{it} is also bounded by the consumer demand.

1.2.2. Dynamic Decisions- Investment, Exporting, and Product Restructuring

In this section, we develop the firm's dynamic decision to export, invest and the number of products to produce. If the firm instead decides to remain in the industry, then it must choose the number

[2]We assume perfect capital market, firms investment decisions are constrained by the net present value of the firm, i.e. firms cannot borrow an infinite amount to increase their productivity. The role of imperfect capital market is left for future research.

of products to produce, whether to export, and how much to invest to improve its productivity level. We denote the unit cost of investment by d. We assume that the firm decides whether to stay in operation after observing the scrap value ϕ_{it}, and make the number of products and export decisions in year t after observing the fixed costs of operating in domestic market (F_d), export market (F_x),and fixed costs for each additional product produced in domestic and export market, (f_d, f_x) if it decides to remain in operation. We model these fixed costs as iid draws from a known join distribution G^f. The firm i's value function in year t if it chooses to continue, can be written as:

$$V^{stay}(\varphi_{it}, s_t) = \max_{X_{it}} \left\{ \int V^D_{\lambda_d}(\varphi_{it}, s_t) dG^f, \int V^E_{\lambda_d}(\varphi_{it}, s_t) dG^f \right\} \tag{21}$$

where X_{it} is a discrete 0/1 variable identifying the firm's export choice in period t, $V^D_{\lambda_d}(\varphi_{it}, s_t)$ is the current and expected future profit from producing products in domestic market only:

$$V^D_{\lambda_d}(\varphi_{it}, s_t) = \max_{\lambda_d^*} \int_{\lambda_d^*}^{\infty} \left[\frac{r_d(\varphi, \lambda_d)}{\sigma} - f_d \right] z(\lambda_d) d\lambda_d - F_d + V^D(\varphi_{it}, s_t)$$

where $V^D(\varphi_{it}, s_t)$ is the value of a non-exporting firm after it makes its optimal investment decision:

$$V^D(\varphi_{it}, s_t) = \int \left\{ \begin{array}{c} \max\limits_{I_{it}} \beta E_t V_{it+1}(\varphi_{it}, s_{t+1} | X_{it} = 0, N_{it} = [1 - Z(\lambda_d^*)], I_{it} = I_{it}) \\ -dI_{it} - 1_{(I_{it}>0)} f_I \end{array} \right\} dG^f$$

where if firm chooses to invest I_{it}, it incurs the cost of investment dI_{it} and a fixed cost component of investment f_I. It has an expected future return which depends on how investment affects future productivity. Similarly $V^E_{\lambda_d}(\varphi_{it}, s_t)$ is the current and expected future profit from producing products in both domestic and export market:

$$\begin{aligned} V^X_{\lambda_d}(\varphi_{it}, s_t) &= \max_{\lambda_d^*} \int_{\lambda_d^*}^{\infty} \left[\frac{r_d(\varphi, \lambda_d)}{\sigma} - f_d \right] z(\lambda_d) d\lambda_d - F_d \\ &\quad + \max_{\lambda_x^*} \int_{\lambda_x^*}^{\infty} \left[\frac{r_x(\varphi, \lambda_x)}{\sigma} - f_x \right] z(\lambda_x) d\lambda_x - F_x + V^E(\varphi_{it}, s_t) \end{aligned}$$

where $V^E(\varphi_{it}, s_t)$ is the value of an exporting firm after it makes its optimal investment decision:

$$V^X(\varphi_{it}, s_t) = \int \left\{ \begin{array}{c} \max_{I_{it}} \beta E_t V_{it+1}(\varphi_{it}, s_{t+1} | X_{it} = 1, N_{it} = [1 - Z(\lambda_d^*)], I_{it} = I_{it}) \\ -dI_{it} - 1_{(I_{it}>0)} f_I \end{array} \right\} dG^f$$

This shows that the firm chooses to export in year t when the current plus expected gain in future export profit exceeds the relevant fixed cost of exporting. Finally, to be specific, the expected future value conditional on different choices for X_{it}, N_{it},and I_{it} for firm staying in operation is:

$$E_t V^{stay}(\varphi_{it+1}, s_{t+1} | X_{it,} N_{it}, I_{it}) = \int_{s'} \int_{\varphi'} V^{stay}(\varphi', s') dF(\varphi' | X_{it,} N_{it}, I_{it}) dP(s' | s_t).$$

In this equation the evolution of productivity $dF(\varphi' | X_{it,} N_{it}, I_{it})$ is conditional on $X_{it,} N_{it}$, and I_{it} because of the assumption in equation (21). In this framework, the net benefit of product restructuring, exporting and investment are increasing in current productivity. This leads to the usual selection effect where high productivity firms are more likely to produce more products, export, and invest. By making future productivity endogenous this model recognizes that current choices lead to improvements in future productivity and thus more firms will self-select into, or remain in, multi-products, exporting and investment in the future.

After observing ϕ_{it}, if the firm chooses to exit, its exiting value function is current period profit with optimized $X_{it}(\varphi_{it}, s_t), N_{it}(\varphi_{it}, s_t), I_{it}(\varphi_{it}, s_t)$ decisions plus the scrap value of exit:

$$V^{exit}(\varphi_{it}, s_t) = \int [\pi(\varphi_{it}, s_t, N_{it}(\varphi_{it}, s_t), X_{it}(\varphi_{it}, s_t), I_{it}(\varphi_{it}, s_t))] + \phi_{it} dG^f$$

where

$$\begin{aligned} &\max_{I_{it}} \beta \pi(\varphi_{it}, s_t, N_{it}(\varphi_{it}, s_t), X_{it}(\varphi_{it}, s_t), I_{it}(\varphi_{it}, s_t)) = \\ &\quad \pi_d(\varphi_{it}, s_t, N_{it}(\varphi_{it}, s_t)) + 1_{(X_{it}(\varphi_{it}, s_t)=1)} \pi_x(\varphi_{it}, s_t, N_{it}(\varphi_{it}, s_t)) - dI_{it}(\varphi_{it}, s_t) - 1_{(I_{it}(\varphi_{it}, s_t)>0)} f_I \end{aligned}$$

Firm i stays in operation in period t if $V^{stay}(\varphi_{it}, s_t) \geqslant V^{exit}(\varphi_{it}, s_t)$.

2. Equilibrium

As a model of industry behavior we focus on pure strategy Markov perfect equilibrium (MPE), in the sense of Maskin and Tirole (1988). We further assume that equilibrium is symmetric, such that all firms use a common stationary investment, export, product restructuring and exit strategy. In particular, there is a function I, X, N such that at each time t, each incumbent firm i invests an amount $I_{it} = I(\varphi_{it}, s_t)$, exports amount $X_{it} = X(\varphi_{it}, s_t)$, and produce $N_{it} = N(\varphi_{it}, s_t)$ number of products. Similarly, each firm follows an exit strategy that takes the form of a cutoff rule: there is a real-valued function η such that an incumbent firm i exits at time t if and only if $\phi_{it} \geq \eta(\varphi_{it}, s_t)$. Weintraub, Benkard, and Van Roy (2008b) show that there always exists an optimal exit strategy of this form even among very general classes of exit strategies. Let Γ denote the set of investment, export, product restructuring and exit strategies such that an element $\mu \in \Gamma$ is a set of functions $\mu = (I, X, N, \eta)$, where $I : \mathbb{R}^+ \; x \; S- > \mathbb{R}_+$ is an investment strategy, $X : \mathbb{R}_+ \; x \; S- > \mathbb{R}_{\geqslant 0}$ is an export strategy, $N : \mathbb{R}_+ \; x \; S- > \mathbb{N}$ is a number of products to produce strategy, and $\eta : \mathbb{R}_+ \; x \; S- > \mathbb{R}_+$ is an exit strategy. Similarly we denote the set of entry rate functions by Ω, where an element of Ω is a function $\varpi : S- > \mathbb{R}_+$.

We define the value function $V(\varphi|\mu, \varpi)$ to be the expected net present value for a firm at state (productivity) φ when its competitors' state is s,given that its competitors each follows a common strategy $\mu \in \Gamma$, the entry rate function is $\varpi \in \Omega$, and the firm itself follows strategy $\mu \in \Gamma$. In particular,

$$V(\varphi, s|\mu, \varpi) = E_{\mu,\varpi}\left[\sum_{k=t}^{T_i} \beta^{k-t}\left(\pi(\varphi_{ik}, s_k, \mu\left(\varphi_{ik}, s_k\right))\right) + \beta^{T_i-t}\phi_{i,T_i}|\varphi_{it} = \varphi, s_t = s\right], \tag{22}$$

where T_i is a random variable representing the time at which firm i exits the industry, and the subscripts of the expectation indicate the strategy followed by firm i and its competitors, and the entry rate function.

An equilibrium to our model consists of an investment, export, product restructuring, and exit strategy $\mu = (I, X, N, \eta) \in \Gamma$,and an entry rate function $\varpi \in \Omega$ that satisfy the following conditions:

1. Incumbent firm strategies represent a MPE:

$$\sup_{\mu'} V(\varphi, s|\mu', \mu, \varpi) = V(\varphi, s|\mu, \varpi) \qquad \forall \varphi \in R^{+}, \forall s \in S. \tag{23}$$

2. At each state, either the entrants have zero expected discounted profits or the entry rate is zero (or both):

$$\sum_{s \in S} \varpi(s) \left(\beta E_{\mu} \left[V(\varphi, s_{t+1}|\mu, \varpi)|s_t = s\right] - f_{EN}\right) = 0$$
$$\beta E_{\mu,\varpi} \left[V(\varphi, s_{t+1}|\mu, \varpi)|s_t = s\right] - f_{EN} \leq 0 \qquad \forall s \in S$$
$$\varpi(s) \geq 0 \qquad \forall s \in S.$$

And the labor market clears in each period. Weintraub, Benkard, and Van Roy (2008b) showed that the supremum in part 1 of the definition above can always be attained simultaneously for all φ and s by a common strategy μ'.

Doraszelski and Satterthwaite (2007) establish existence of an equilibrium in pure strategies for a closely related model. We do not provide an existence proof here because it is long and cumbersome and would replicate this previous work. With respect to uniqueness, in general we presume that our model may have multiple equilibria.[3]

Dynamic programming algorithms can be used to optimize firm strategies and equilibria to our model can be computed via their iterative application without the curse of dimensionality problem commonly seen in the IO literature because s_t can be completely characterized by $\widetilde{\varphi}_t$. Stationary points of such iterations are MPE. An algorithm for computing the MPE is included under Empirical Analysis section.

2.0.3. Market Clearing:

The feasibility constraint on the investment and entry is: $M_{EN,t} f_{EN} = L_{EN,t}$, $M_t(d\overline{I}_t) = L_{I,t}$.Total payments to labor used in entry are equal to expected discounted profits $L_{EN,t} = M_t \overline{v}_t$, where $\overline{v}_t = \int V(\varphi, s_t) M_t(s_t) g(\varphi) d\varphi$. The evolution of the distribution of operating firms M_t over time is given by the optimal strategy μ consisting of I, X, N, η and entry rate ϖ. Total payments to labor used in production and investment, on the other hand, are equal to revenue minus expected

[3]Doraszelski and Satterthwaite (2007) also provide an example of multiple equilibria in their closely related model.

discounted profits, $L_{p,t} + L_{I,t} = R - M_t \overline{v}_t$ Combining these two expressions, $L = R$. Thus the labor market clears: $L_{EN,t} + L_{I,t} + L_{p,t} = L$.

3. Empirical Analysis

We begin with a description of the evolution of the process for firm productivity φ_{it}. We assume that productivity in period t evolves over time as a Markov process that depends on the firm's investments I_{it-1} in period t-1, its participation in the export market X_{it-1} in period t-1, the number of products firm produces N_{it-1} in $t-1$ and a random shock:

$$\begin{aligned} \ln \varphi_{it} &= \alpha_0 + \alpha_1 \ln \varphi_{it-1} + \alpha_2 \ln \varphi_{it-1}^2 + \alpha_3 \ln \varphi_{it-1}^3 \\ &\quad + \alpha_4 \ln I_{it-1} + \alpha_5 X_{it-1} + \alpha_6 N_{it-1} + \xi_{it}. \end{aligned} \tag{24}$$

Here we model investment as a continuous choice to allow for the possibility that firms can increase their productivity faster by investing more. The inclusion of X_{it-1} recognizes that the firm may affect the evolution of its productivity through learning-by-exporting. The inclusion of N_{it-1} allows the possibility of expanding into multiple products to have an effect on productivity. The stochastic nature of productivity improvement is captured by ξ_{it} which is treated as an iid shock with zero mean and variance σ_ξ^2. This stochastic component represents the role that randomness plays in the evolution of a firm's productivity. This is the change in the productivity process between$t-1$ and t that is not anticipated by the firm and by construction is not correlated with $\varphi_{it-1}, I_{it-1}, X_{it-1}$,and N_{it-1}.This allows the stochastic shocks in period t to be carried forward into productivity in future years.

3.1. Algorithm

To compute the MPE with the two-step-PML method, the beliefs about transition, entry, investment, export and exit strategies are computed non-parametrically. The second step is to construct a likelihood function using those beliefs and estimate the structural parameters of interest. When consistent nonparametric estimates of choice probabilities either are not available or are very imprecise, we can use k-step-PML or NPL algorithm to compute the MPE. NPL works as follows, start with any set of beliefs/strategies and compute the structural parameters of interest, update strategies with the estimated structural parameters non-parametrically, then construct likelihood function and update the structural parameters. Repeat this k times until the strategies converge.

3.1.1. Demand and Cost Parameters

We begin by estimating the domestic demand, marginal cost and productivity evolution pararmeters. The domestic revenue function for a single product firm in log form with an iid error term u_{it} that reflects measurement error in revenue or optimization errors in price choice is:

$$\begin{aligned} \ln r_{d,it} &= (\sigma_d - 1)\ln\left(\frac{\sigma_d - 1}{\sigma_d}\right) + (\sigma_d - 1)\ln\varphi_{it} \\ &\quad + \ln E_t + (\sigma_d - 1)\ln P_t + (\sigma_d - 1)\lambda_{it} + u_{it} \end{aligned} \tag{25}$$

where λ_{it} is the unobserved demand shock for firm i in domestic market in time t. The composite error term $(\sigma - 1)\ln(\varphi_{it}) + u_{it}$ contains firm productivity. Since the inputs are observed at firm level, using the product-level information requires an extra step of aggregating the data at the product level to the firm level. From equation (25) I can aggregate the production function to the firm level by assuming identical production functions across products produced which is a standard assumption in empirical work, see for instanceBernard and Jensen (2008) and De Loecker (2010). Under this assumption, and given that I observe the number of products each firm produces, I can relate the average production of a given product k of firm $\overline{Q}_{ikt}$ to its total input use and the number of products produced. The production function for product k of firm i is then given by:

$$\overline{Q}_{ikt} = N_{it}^{-1} Q_{it} \tag{26}$$

where N is the number of products produced. Introducing multi-product firms in this framework explicitly requires to control for the number of products product. Combining the production function and the expression for price from equation (4) leads to an expression for total revenue as a function of inputs, productivity, and the number of products.

$$\begin{aligned} \ln r_{d,it} &= \ln N_{it} + (\sigma_d - 1)\ln\left(\frac{\sigma_d - 1}{\sigma_d}\right) + (\sigma_d - 1)\ln\varphi_{it} \\ &\quad + \ln E_t + (\sigma_d - 1)\ln P_t + (\sigma_d - 1)\ln\overline{\lambda}_{it} + u_{it} \end{aligned} \tag{27}$$

where $\overline{\lambda}_{it}$ is the average unobserved demand shock across all products for firm i in time t and N is the number of products produced. For a single product firm, $\ln(1) = 0$, and therefore this extra term cancels out, whereas for multi-product firms an additional term is introduced.

We estimate firm productivity using Olley and Pakes (1996) and Levinsohn and Petrin (2003) approach to rewrite the unobserved productivity in terms of expenditure on intermediate goods for each firm. In general, the firm's choice of the variable input levels for materials, m_{it}, and electricity, e_{it}, will depend on the level of productivity and the demand shocks (which are both observable to the firm). Under our model setting, marginal cost is constant in output, the relative expenditures on all the variable inputs will not be a function of total output and thus not depend on the demand shocks. In addition, differences in productivity will lead to variation across firms and time in the mix of variable inputs used. Thus, material and energy expenditures by the firm will contain information on productivity level. We can write the level of productivity, conditional on the number of products produced, as a function of the variable input levels:

$$\varphi_{it} = \varphi_{it}(N_{it}, m_{it}, e_{it}) \tag{28}$$

We can rewrite (27) as follows:

$$\ln r_{d,it} = \gamma_0 + \sum_{t=1}^{T}\sum_{m=1}^{M} \gamma_{mt} D_m D_t + h(N_{it}, m_{it}, e_{it}) + v_{it} \tag{29}$$

where intercept γ_0 is the demand elasticity terms, D_t is the time varying aggregate demand shock, D_m is the market-level factor prices,m_{it} is expenditure on intermediate goods, and $h(.)$ captures the effect of productivity on domestic revenue. We specify $h(.)$ as a cubic function of its arguments and estimate (28) with OLS. The fitted value of the $h\,(.)$ function, which we denote $\widehat{h}_{it}$ is an estimate of $\ln N_{it} + (\sigma - 1)\ln\varphi_{it}$. Next, we can construct an estimate of productivity for each firm. Substituting $\ln\varphi_{it} = (\widehat{h} - \ln N_{it})/\,(\sigma - 1)$ into productivity evolution equation (24):

$$\begin{aligned} \widehat{h}_{it} - \ln N_{it} \;\; &= \;\; \alpha_0^* + \alpha_1(\widehat{h}_{it-1} - \ln N_{it-1}) + \alpha_2(\widehat{h}_{it-1} - \ln N_{it-1})^2 + \alpha_3(\widehat{h}_{it-1} - \ln N_{it-1})^3 \\ &\quad + \alpha_4^* \ln I_{it-1} + \alpha_5^* X_{it-1} + \alpha_6^* N_{it-1} + \xi_{it}^*. \end{aligned} \tag{30}$$

where the star represents that the α coefficients are multiplied by $(\sigma_d - 1)$. This equation can be estimated with nonlinear least squares and the underlying parameters α can be retrieved with an estimate of demand elasticities σ_d. We can estimate the demand elasticities using data on total variable cost. Total variable cost is an elasticity-weighted combination of total revenue in each market:

$$tvc_{it} = \rho_d * r_{d,it} + \rho_x * r_{x,it} + \varepsilon_{it} \tag{31}$$

where $\rho_j = 1 - 1/\sigma_j$ for $j = d, x$. Finally given estimate of $\widehat{\sigma}_d$, we can construct an estimate of productivity for each observation as:

$$\ln \widehat{\varphi}_{it} = (\widehat{h} - \ln N_{it}) / (\widehat{\sigma_d} - 1) . \tag{32}$$

Three aspects of this static empirical model are worth mentioning. First, because firm heterogeneity plays a crucial role in both the domestic and export market, we utilize data on firm revenue to estimate firm productivity. Second, we've also used total variable costs to estimate demand elasticities in both export and domestic market. Third, estimation of the process for productivity evolution is important for firm's dynamic investment equations because the parameters from equation (30) are used directly to construct the value functions that underlie firm's investment, export, and number of products choice.

The Melitz (2003) framework assumes the only factor of production is labor. For a Cobb-Douglas technology, the domestic revenue function becomes:

$$\begin{aligned} \ln r_{d,it} &= \ln N_{it} + (\sigma_d - 1) \ln \left(\frac{\sigma_d - 1}{\sigma_d} \right) + (\sigma_d - 1) \left(\beta_0 - \beta_k \ln k_{it} - \beta_\omega \ln \omega_t + \ln \varphi_{it} \right) \\ &\quad + \ln E_t + (\sigma_d - 1) \ln P_t + (\sigma_d - 1) \ln \overline{\lambda}_{it} + u_{it} \end{aligned} \tag{33}$$

where k_{it} is firm's capital stock and ω_t is a vector of variable input prices common to all firms. Productivity, conditional on the number of products produced and capital stock, can be written as a function of the variable input levels: $\varphi_{it} = \varphi_{it}(N_{it}, m_{it}, e_{it})$. Equation (29) becomes :

$$\ln r_{d,it} = \gamma_0 + \sum_{t=1}^{T} \sum_{m=1}^{M} \gamma_{mt} D_m D_t + h(N_{it}, k_{it}, m_{it}, e_{it}) + v_{it} \tag{34}$$

The fitted value of the $h(.)$ function, denoted $\widehat{h}_{it}$, is an estimate of $\ln N_{it} + (\sigma - 1)(-\beta_k \ln k_{it} + \ln \varphi_{it})$. Next, we can construct an estimate of productivity for each firm by substituting $\ln \varphi_{it} = (\widehat{h} - \ln N_{it}) / (\sigma - 1) + \beta_k \ln k_{it}$ into productivity evolution equation (24). The productivity evolution equation can be estimated with nonlinear least squares and the underlying β_k parameter can be retrieved given an estimate of σ_d. Finally, given estimates of $\widehat{\beta_k}$ and $\widehat{\sigma_d}$, we can construct an estimate of productivity for each firm as:

$$\ln \widehat{\varphi}_{it} = (\widehat{h} - \ln N_{it}) / (\widehat{\sigma_d} - 1) + \widehat{\beta_k} \ln k_{it}. \tag{35}$$

3.1.2. Dynamic Parameters

The algorithm below is designed to compute the beliefs about transition, entry, investment, export, exit strategies and the value function associated with these strategies with a positive entry rate given some values of structural parameters. It starts with two extreme entry rates: $\underline{\varpi} = 0$ and $\overline{\varpi} = \frac{1}{f_{EN}} \left(\frac{\sup_{\varphi,s} \pi(\varphi, s)}{1 - \beta} + \overline{\phi} \right)$. Any equilibrium entry rate must lie in between these two extremes. The algorithm searches over entry rates between these two extremes for one that leads to the MPE strategies and the value function associated with these strategies given a set of structural parameters. For each candidate entry rate, an inner loop (step6-10) computes an MPE firm strategy for that fixed entry rate. Strategies are updated smoothly (step 9).[4] If the termination condition is satisfied with $\varepsilon_1 = \varepsilon_2 = 0$, we have a set of MPE beliefs given structural parameters.

The algorithm is easy to program and computationally efficient. In each iteration of the inner loop, the optimization problem to be solved is a one dimensional dynamic program. The state space in this dynamic program is the set of productivity levels a firm can achieve. In principle, there could be an infinite number of them. However, beyond a certain productivity level the optimal strategy for a firm is not to invest, so its quality cannot increase to beyond that level.

[4] The parameters $\gamma$and N were set after some experimentation to speed up convergence.

4. Data

4.1. Spanish Firm Level Data

The model developed in the last section will be used to analyze the sources of productivity change of firms in Spain. The micro data used in estimation was collected by SEPI Foundation in Spain for the years 1999-2008. The products are classified into 20 manufacturing industries based on 3-figure CNAE-93 codes.

The data set we use is a collection of 3216 firms that operated in at least one of the five years between 1999–2008 and that reported necessary data on domestic and export revenue, investment, total variable costs, and number of products they are producing. Only 848 of those firms operated in all tenS years between 1999-2008.

Table 1 provides the summary measures of the size of the firms, measured in revenues and average employment. The top panel of the table provides the median firm size across operating firms in our sample in each year, while the bottom panel summarizes the average firm size. The first column shows that approximately 35 percent of the firms that do not export in a given year. The median firm's domestic revenue varies from 14.34 to 17.46 in hundred of thousands of Euros. Among the exporting firms, the median firm's domestic revenue is approximately eight times as large, 10.5 to 12.9 million Euros. The export revenue of the median firm ranges from 3.4 to 5.5 million Euros. The median number of products for both exporters and non-exporters are 1, while the average number of products produced by non-exporters ranges from 1.07 to 1.14 and the 1.13 to 1.15 for exporters.

The distribution of the firm revenue is highly skewed, particularly for firms that participate in the export market. The average domestic firm revenues are larger than the medians by a factor of approximately six for the exporting firms and the average export revenues are larger by a factor of approximately 10. The skewness in the revenue distributions can also be seen from the fact that the 100 largest firms in our sample in each year account for approximately 40 percent of the total domestic revenue and 75 percent of the export revenue. The skewness in revenues will lead to large differences in profits across firms and a heavy tail in the profit distribution. To fit the participation patterns of all the firms it is necessary to allow the possibility that a firm has large fixed and/or sunk costs. We allow for this in our empirical model by assuming exponential distributions for the fixed and sunk costs. Together this assumption allows for substantial heterogeneity in the costs across plants.

The other important variable in the data is the number of products the firms choose to produce. Number of products in the sample is defined as the number of products at 3 figures CNAE-93 each firm produces. Even though in the sample only five percent of the firms produce more than 1 product, they account for 20 percent of the total domestic revenue and 25 percent of the export revenue.

The last important variable in the data is the investment the firms make each year. Table 2 provides summary statistics for different measures of investment for exporters and non-exporters. We look at two measures of investment. The first one is investment on capital goods which includes the purchases of information processing equipment, technical facilities, machinery and tools, rolling stock and furniture, office equipment and other tangible fixed assets. The second one is total expenses on R&D which is the sum of the salaries of R&D personnel (researchers and scientists), material purchases for R&D, and R&D capital (equipment and buildings) expenses. The first column in Table 2 provides the percentage of firms with positive investment in capital in each year. In our sample, approximately 70 percent of the non-exporters invest in capital where close to 90 percent of the exporters invest in capital. Only 10 percent of non-exporters induce R&D expenses, whereas 50 percent of the exporters induce R&D expenses. The top panel of the second column provides the median of the capital investment given positive investment from operating firms, and the bottom panel provides the mean of the capital investment given positive investment. The average positive investment in capital is approximately ten times as large as the median positive capital investment for non-exporters and six times for exporters. All numbers in the table are expressed in tens of thousands of Euros. Median investment in capital for non-exporters ranges from 40 to 50 thousand Euros and 500-700 thousand Euros for exporters. Average investment in capital for exporters is approximately seven times that of the non-exporters. The average positive R&D expenses are approximately 5 times the median positive R&D expenses for non-exporters, and ten times of that for exporters. The difference in mean and median of the R&D expenses between non-exporters and exporters is also approximately ten folds.

4.2. Empirical Transition Patterns for Entry/Exit, Investment, and Export

In this section we summarize the patterns of entry, exit, R&D and exporting behavior in the sample, with a focus on the transition patterns that are important to estimating the fixed and sunk costs of entry, R&D, and exporting. Table 3 reports entry and exit rates over the years for firms that operated at least one year during 1999-2008. Operating firms are defined as firms with positive revenue. The first column reports the number of firms with positive revenue in each year. The second column reports the number of non-operating firms in the sample. Column 3 and 4 report the number of new entrants and exits in each year. New entrants are defined as firms generated positive revenue in time period t and zero revenue in time period t-1. Similarly for exits, firms that generated revenue in period t-1 but stopped operating in period t are defined as exits. In the year 2003, there are no new entrants and a high exit rate of 19 percent. This is the year following the technology bubble and 911 . Year 2004 has no entry and close to zero exits. In the year 2005, entry rate shot up to 33 percent with 7 percent of exit rate and in the year 2006 entry rate falls back to 15% and exit rate goes up to 10%. The average entry and exit rate for this time period is 14 and 9 percent. With significant entry and exit behaviors present, ignoring firms self selection into entry and exit will result in biased estimates of investment and export decisions.

Table 4 reports the proportion of firms that undertake each combination of the activities and the transition rates between pairs of activities over time. The top panel of Table 4 reports the average proportion of operating firms in both period t and t+1 that undertake neither investment nor export, investment only, export only, and both investment and export and the transition rates between pairs of activities over time. The middle panel reports those for new entrants in period t that continue to operate in t+1. The bottom panel reports those for firms that cease to produce in t+2, but operate in both t+1 and t. The first row of each panel reports the cross-sectional distribution of exporting and investment averaged over all years. It shows that in each year, the proportion of operating firms undertaking neither of these activities is .11. This number is higher for new entrants and firms that will cease to produce. The proportion that invest but do not export is .25 for operating firms. This number is higher for new entrants and lower for firms that will cease production in the sample, suggesting that new entrants are more likely to invest to improve their productivity due to a bad luck in the productivity draw and firms that have a higher probability of exit are the ones with lower productivity and therefore don't bother to invest. The proportion that export only and do not invest .07 for operating firms, .08 for new entrants and .11 for firms that will exit. The proportion that do both for operating firms is .57, which is higher than the number for new entrants and firms that will exit. Overall, 82% of the operating firms engage in investments and 64% of the operating firms export[5]. One explanation for difference in export and investment participation is that differences in productivity and the export demand shocks affect the return of each activity and the firms self-select into each activity based on the underlying profits.

The transition patterns among investment and exporting are important for the model estimation. The last four rows in each panel of the table report the transition rate from each activity in year t to each activity in year t+1. Several patterns are clear. First, there is significant persistence in the status over time for all three panels. This can reflect a high degree of persistence in the underlying sources of profit heterogeneity, which in our model, is productivity and the export market shocks. Of the operating firms that did neither activity in year t, .67 of them are in the same category in year t+1. This number is .88 for firms that will exit and only .49 for new entrants. This suggests that even though there is persistence in the status over time, different kind of firms will have different level of persistence. New entrants that did not invest nor export are more likely to invest than the incumbent firms and firms that will exit soon will be less likely to invest than the incumbent firms. The probability of remaining in the same category over adjacent years is .79, .49, and .92 for invest only, export only, and both for operating firms. These numbers are similar for new entrants and firms that will soon exit except for invest only. Firms that will soon exit with positive investment in period t are less likely to invest in t+1 when they decide to exit at the end of period t+1. This differences in persistence reflect the importance of modeling firms self-selection into entry and exit.

Second, firms that undertake one of the activities in year t are more likely to start the other activity than a firm that does neither. This is true for all firms. If the firm does neither activity in year t, it has a probability of .03 to enter the export market and .31 to invest in the next period for operating firms. These number are .07 and .85 for firms that only invest in period t, .93 and .47 for firms that only export in period t, and .98 and .94 for firms that do both in period t. Third, firms

[5] The export participation rate in Spain is comparable to those firms in France. The export participation rate of French firms (with 20 employees or above) is 69.4% in 1990 and 74.8% in 2004.

that conduct both activities in year t are less likely to abandon one of the activities than firms that only conduct one of them. Operating firms that conduct both activities have a .06 probability of abandoning investment and a .02 probability of leaving the export market. Operating firms that only do investment have a .15 probability of stopping in investment while firms that only export have a .07 probability of stopping in export. Exiting firms that only do investment have a .44 probability of stopping and those who only do export have a .03 probability of stopping. Fourth, export only firms are much more likely to do both (.44 probability) than investment only firms (only .06 probability).

The transition patterns reported in Table 4 illustrate the need to model the investment and exporting decision jointly. In our model, firms cannot export below a certain productivity cut-off. Therefore firms need to invest and increase their productivity in order to export. The return to investment can be higher or lower for exporting versus non-exporting firms, which makes the probability that the firm will choose to invest dependent on the firm's export status. Table 5 illustrates the average productivity constructed from equation 30 in each year for operating firms, new entrants, firms that exit, and firms that operated in all 5 years.

5. Results

5.1. Demand, Cost and Productivity Evolution

The parameter estimates from the first stage estimation of equations 31 and 32 are reported in Table 6. The first column reports the estimates using investment in capital, which we also use in the dynamic model. The second column reports estimates using investment in R&D.

Focusing on the first column, the implied value of the demand elasticity for domestic and export markets are 7.55 and 2.11. These elasticity estimates imply markups of price over marginal cost of 15.3 percent for domestic market sales and 89.7 percent for foreign sales. The coefficient on α_1 measures the effect of the lagged productivity on current productivity and it is positive and significant. The coefficient α_4 measures the effect of investment on capital on current productivity. Firms that increase their investment by 1% can increase their productivity by .03%. The effect of past exporting on current productivity is given by α_5. This a measure of learning-by-exporting's impact on productivity and in this case is not significant suggesting very little learning-exporting.

The last coefficient α_6 measures the impact of product restructuring on productivity. Producing one more product increases firm's productivity by 6%. This suggests learning by diversifying.[6] (more on this)?

Relative to a firm that never invests nor exports, a firm that invests an amount equal to the average investment in capital goods and export will have mean productivity that is 111% higher. A firm that does not export but able to invest the average investment is 104% higher in productivity. A firm that only exports is 8% higher in productivity. A firm that produces one additional product is 4.1% lower. While this provides a summary of the technology linkages between exporting, investing, diversifying, and productivity, it does not recognize the impact of this process on the firm's choice to enter into operation and exporting. This behavioral response is the focus of the second stage estimation. Given the estimates in Table 6 we construct estimate of firm productivity from equation 30. The mean of the productivity estimates is 2.36 among operating firms and the (.05, .95) percentiles of the distribution are (1.97, 2.78). The mean of productivity estimates including the 0's is 1.47 with (.05, .95) percentiles of the distribution (0, 2.7). The variation in productivity will be important in explaining which firms self-select into entry/exit, exporting and diversifying.

We can assess how well the productivity measure correlates with the firm's entry/exit, export and product restructuring choices. In the top panel of Table 7 we report estimates of a probit regression of exporting on the firm's productivity, lagged investment on capital goods, lagged export dummy, lagged number of products, and a set of time, industry, and time cross industry dummies. This regression is similar to the reduced form policy functions that come from our dynamic model. The export demand shocks are not included explicitly but rather captured in the error terms. In the probit model, all the coefficients are highly significant. Both productivity and lagged export status play a positive and significant role in determining the current export status. The coefficients on investment and product restructuring are negative and significant, suggesting crowding out effect from investment and product restructuring. In the second and third row of the top panel we report OLS regressions of export revenue for firms in the export market in equation (). The explanatory variables are productivity and a set of dummies (industry, time, and industry

[6] The second column repeats the estimation using log of R&D expenditure rather than log of investment on capital goods. This does not change any sign nor significance of the coefficients in the model.

cross time effects). The lagged export dummy and investment choice do not affect the volume once the firm is in the export market. Since the Spanish firm data does not provide information on how many products firms export, the number of products choice is also not included. The R square term for the regression without fixed effects is .66 and .69 for the regression without fixed effects, suggesting export demand heterogeneity is not a source of size and profit differences in the export market. (more on this)

The first row of the second panel in Table 7 reports estimates of a probit regression of firm exit. Firm is defined as exit in period t if it has zero total revenue (domestic plus export) in the period t+1. This definition of firm exit is consistent with the way we model firm entry/exit where we assumed in each period firms make their production decisions, produce, and decide if they want to exit and receive a scrap value at the end of the period. Firms with higher productivity are less likely to exit the market. This is both economically and statistically significant. The parameter on investment is insignificant. Multi-product firms are less likely to exit the market. Being an exporter in the past makes the firm more likely to exit. One explanation for this is firms involved in the export market are more likely to respond to the negative demand shocks in the export market. (more on this, maybe do an exit to export market from existing exporting firms).

The last panel in Table 6 reports estimates of an OLS regression of firm's product restructuring choice.[7] The dependent variable is the number of products firm chooses to produce. Firms with higher productivity will produce more products.(more on this)

Overall, it is clear from these reduced form regressions that the productivity variable we have constructed is measuring an important plant characteristic that is correlated with export and entry/exit decisions and the firm's export and domestic revenue once they choose to participate in the market. We report the estimates of the dynamic investment equations in the next section.

5.2. Dynamic Estimates

The remaining cost, export demand parameters, entry and exit rates are estimated in the second stage of the empirical model using the likelihood function that is the product over the firm-specific joint probability of the data given in equation (). The coefficients reported in Table 8 are the means and standard deviations of the parameters for the fixed and sunk cost of operation. The estimated

[7]Maybe do a multi-variate probit/logit or negative binomial regression later. but for now, OLS

fixed cost parameter is less than the sunk cost parameter, indicating that the firm entry cost is substantially larger than the per-period costs of maintaining operation.

5.3. In-sample Model Performance

To assess the overall fit of the model, we use the estimated parameters to simulate patterns of firm entry and survival, investment, exporting and product restructuring decision, transition patterns between the choices, and productivity trajectories for the firms in the sample and compare the simulated patterns with the actual data. Since each firm's productivity evolves according to equation (), we need to simulate each firm's trajectory of productivity jointly with its dynamic decisions. In Table 9 we report the actual and predicted percentage of entry, exit, investment, export, product restructuring and the mean productivity. Overall, the simulations do a good job of replicating these average data patterns for all three variables.

6. Counterfactuals

6.1. Within Firm Effect

In our model, the determinants of a firm's entry, exit, export, investment and product restructuring choices are its current productivity and cost draws. We will isolate the role of current productivity and the cost shocks on these activities. We do this by calculating the marginal benefit to each activity. Table 12 reports the partial equilibrium marginal benefits of exporting with different combinations of productivity and investment with entry and exit rate for calculated for firms optimal strategies. The first column in the top panel reports the logged values of $V(\varphi, X, I, N)$ with the optimal investment, export, product restructuring, entry and exit strategy for each productivity level. The second column reports the logged values of $V(\varphi, X, I = 0, N)$, forcing investment to be zero, allowing optimal export andproduct restructuring strategies every period, but take entry and exit rate as given when we calculated for $V(\varphi, X, I, N)$. The third column reports the logged values of $V(\varphi, X = 0, I, N)$, forcing profit to be consistent of domestic revenue only and the optimal investment and product restructuring strategy are recalculated based on domestic profit only. The fourth column reports the logged value of $V(\varphi, X = 0, I = 0, N)$ forcing both export and investment to be zero. All the values in the four columns are increasing, reflecting the increase in profits with higher productivity.

The fifth column reports the marginal benefit of exporting for a firm that is allowed to invest in its future productivity and choose number of products to produce with entry and exit rate as given. It is positive, reflecting the fact that a firm that does both activities has a higher future productivity trajectory, and is increasing in current productivity implying that a high productivity producer is more likely to self select into the export market. The benefit of exporting for a firm that is not allowed to invest in its future productivity is reported in the sixth column and it is also positive and increasing in the level of current productivity. Comparing the fifth and sixth column, we see that the difference between the marginal benefits of exporting with investment and without investment is positive, implying that the investment decision has important impact on the return to exporting. This is what we call the market size effect or complementarity in export and investment. The return to exporting is greatest for middle productivity firms because both low and high productivity firms investment rate (investment/profit) are less than the middle productivity investment rate.

Table 13 looks at the marginal benefit of exporting with different combinations of productivity and product restructuring strategy with entry and exit as given. The third column reports the logged values of $V(\varphi, X, I, N = 1)$, forcing all firms to produce only one product with optimal investment and export strategy but taking entry and exit as given before. The fourth column reports the logged values of $V(\varphi, X = 0, I, N = 1)$, not allowing firms to export nor to produce more than one product. Again, all the values in the first four columns are increasing reflecting higher profits with higher productivity. The fifth column is still the marginal benefit of exporting for firms with optimal strategies in investment and product restructuring. The sixth column is the marginal benefit of exporting for firms that are only allowed to produce one product. This is positive and increasing in productivity. The last column in the second panel is the difference between the marginal benefits of exporting for multi-product firms and firms that are allowed to produce only one product. This number is positive for middle productivity firms and negative for high productivity firms. Firms with higher productivity and higher profits, when opening up to trade, tend to reduce the number of products produced to better focus on their core competency groups to grab bigger market shares through exporting.

Table 14 looks at the marginal benefit of investment with different combinations of productivity and product restructuring strategy with entry and exit as given. Column five reports the marginal benefit of investment for firms with optimal product restructuring and export strategies. Column six reports the marginal benefit of investment for firms that are not allowed to produce more than one product. The last column reports the difference between the marginal benefits of investment for multi-product firms and firms that produce only one product. We see the market size effect or complementarity in investment and product restructuring present because the difference in marginal benefit is positive for all productivity levels but is greatest for the middle productivity firms.

6.2. Between Firm Effect

We looked at the above counterfactuals with entry and exit rate as given in the previous section, in this next section, we recompute entry and exit condition for each scenario and look at the general equilibrium marginal benefits of these activities.

7. Conclusion

This paper estimates a dynamic structural model that captures the relationship between investment, exporting and productivity for multi-product firms in the presence of endogenous entry and exit. It characterizes a firm's joint dynamic decision process for entry, exit, investment, exporting and number of products as depending on its productivity, and fixed and sunk costs. It also describes how a firm's decisions on investment, exporting and product restructuring endogenously affect its future productivity.

There are five broad conclusions we draw about the sources of productivity evolution among Spanish firms. First, firm productivity evolves endogenously in response to the firm's choice to invest and diversify, but not to the choice to export. An one percent increase in investment raises future productivity by three percent, and increasing the number of products produced by one raises future productivity by 6 percent. Second, the marginal benefits of exporting vs. non-exporting increase with firm's productivity. The marginal benefits of investment versus zero investment is positive; however it is greater for the middle productivity firms than for both low and high productivity firms. The marginal benefits of multi-product versus single product reveals a similar pattern. This leads to the self-selection of high productivity firms into exporting, investment, and multi-products. When combined with the fact that decisions to diversify and invest lead to endogenous productivity improvements, this further reinforces the importance of self-selection based on current productivity as the major factor driving the decision to export, invest and produce multiple products. Third, the cross-partials between exporting and investment, and investment and product restructuring are positive for all firms. This suggests that both exporting and investment and investment and product restructuring augment each other and further reinforce the self-selection through the complementarity effect. However, the cross partial between exporting and product restructuring is positive only for low and middle productivity firms and is negative for high productivity firms. This suggests that when opening up to trade, high productivity firms should decrease the number of products produced in order to focus on their core competency products and grab a greater market share through exporting. Fourth, the fixed cost of investment is smaller than the fixed costs of exporting, which results in a larger proportion of firms choosing to invest than to export. The larger proportion of firms choosing to invest is also a result of investment having a larger direct effect on future productivity. Finally, our counterfactual exercises show that a reduction in trade costs will have a significant positive effect on both the probability and the amount that a firm exports and invests, while the number of products produced is reduced. These three effects lead to an overall increase in mean productivity. The combination of larger export markets, and the firm's ability to invest and change the number of products they produce to take the advantage of larger export markets contributes to larger productivity gains.

Overall, our empirical findings emphasize the important role of heterogeneity in productivity as the driving force in determining a firm's total revenue and decision to export, invest and produce multiple products. This is further reinforced by the fact that investment and product restructuring decisions result in future productivity gains. The model can be extended in several ways. We can include the distinction between different types of investment and determine the return to each type of investment. The Spanish firm data includes investment expenditures on R&D, information computing technologies, industrial machinery, land, building and furniture. We will be able to look at whether one of the investment tools had a more substantial impact on the productivity. In addition, we assumed perfect capital markets in this paper. We may explore the role of imperfect capital market on the return to investment and therefore productivity for different sectors. Firms in some sectors need to finance a greater share of their costs externally and sectors differ in their endowment of tangible assets that can serve as collateral.

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Appendix

Sample Coverage and Data Collection

The ESEE's population of reference is made up by the firms with 10 or more employees and which belong to what is usually known as the manufacturing industry. The geographical scope of reference is all the Spanish territory, and the variables have a yearly temporal dimension.

One of the most relevant characteristics of the ESEE is its representativeness. The initial selection was carried out combining exhaustiveness and random sampling criteria. In the first category were included those firms which have over 200 employees, and whose participation was required. The second category was composed by the firms which employ between 10 and 200 workers, which were selected through a stratified, proportional, restricted and systematic sampling, with a random start. In the first year, 1990, 2,188 firms were interviewed along the above mentioned criteria. Later special care has been paid to maintaining its representativeness with regard to the population of reference. The effort has been oriented, on the one hand, to reducing as far as possible, the deterioration of the initial sample, trying to avoid the reduction of the firms' collaboration, and on the other, including each year into the sample all the newly incorporated firms which employ over 200 workers, as well as a randomly selected sample which represents around 5

Computation of the Firm's Dynamic Problem

Table 1
Domestic and Export Revenue (in 100,000 Euros) and Firm Size

Non-exporters

	Number of Firms	Median Domestic Revenue	Median Employment	Median Number of Products
2002	610	14.34	23	1
2003	495	15.96	24	1
2004	491	16.07	24	1
2005	716	17.46	22	1
2006	777	16.55	22	1
		Average Domestic Revenue	Average Employment	Average Number of Products
2002	610	89.31	55.77	1.07
2003	495	110.20	63.09	1.09
2004	491	122.17	65.07	1.09
2005	716	137.17	67.57	1.09
2006	777	144.28	66.27	1.14

Exporters

	Number of firms	Median Domestic Revenue	Median Employment	Median Number of Products	Median Export Revenue
2002	1097	118.10	169	1	47.55
2003	885	127.06	175	1	52.28
2004	883	129.60	175	1	55.82
2005	1195	120.76	156	1	40.04
2006	1246	105.59	124	1	34.24
		Average Domestic Revenue	Average Employment	Average Number of Products	Average Export Revenue
2002	1097	618.33	383.5	1.13	400.35
2003	885	671.04	400.3	1.14	455.22
2004	883	729.08	400.8	1.14	481.88
2005	1195	635.56	354.4	1.13	430.03
2006	1246	631.61	331.5	1.15	452.00

This table provides summary statistics for firm sizes measured in revenues (in 100,000 Euros), average employment and for number of products produced for exporters and non-exporters. The top panel consists of median and average measures for non-exporters and the bottom panel consists of median and average measures for exporters.

Table 2
Investment and R&D Expenses (in 10,000 Euros)

Non-exporters

Year	% of Nonzero Investment	Median Positive Investment	Average Positive Investment	% of Nonzero R&D	Median Positive R&D Expenses	Average Positive R&D Expenses
2002	69%	4.13	56.98	11%	4.18	22.39
2003	68%	5.37	53.55	9%	5.03	18.40
2004	71%	5.57	68.91	10%	4.68	28.59
2005	72%	5.31	65.66	11%	3.65	24.37
2006	67%	5.76	43.01	10%	6.00	10.47

Exporters

Year	% of Nonzero Investment	Median Positive Investment	Average Positive Investment	% of Nonzero R&D	Median Positive R&D Expenses	Average Positive R&D Expenses
2002	88%	57.54	391.25	51%	31.68	226.25
2003	86%	70.00	428.25	51%	36.90	316.13
2004	89%	61.75	393.10	52%	35.03	353.36
2005	90%	56.66	419.94	53%	34.14	314.86
2006	89%	48.46	416.41	50%	33.70	356.09

This table provides summary statistics for firm investment and R&D expenditures (in 10,000 Euros) for exporters and non-exporters. The top panel consists of median, and average investment and R&D expenditures for non-exporters and the bottom panel consists of median and average investment and R&D expenditures for exporters.

Table 3
Entry and Exit

Year	N of firms w/ positive Revenue	N of non-operating firms	N of New Entrants	N of Exits	Entry Rate	Exit Rate
2002	1707	941		327		0.19
2003	1380	1268	0	6	0.00	0.00
2004	1374	1274	0	97	0.00	0.07
2005	1911	737	634	195	0.33	0.10
2006	2023	625	307		0.15	
Average	1679	969	235.25	156.25	0.14	0.09

This table provides number of entry and exit and operating firms in each year.

Table 4

Transition Rates for Incumbents, New Entrants, and Exiting Firms

Status Year t	Neither	only Investment	only Export	Both
	Status Year t+1			
	Annual Transition Rates for Incumbents			
All Incumbents	0.11	0.25	0.07	0.57
Neither	0.67	0.29	0.01	0.02
only Investment	0.14	0.79	0.01	0.06
only Export	0.04	0.03	0.49	0.44
Both	0.00	0.02	0.06	0.92
	Annual Transition Rates for New Entrants			
All New Entrants	0.16	0.32	0.08	0.43
Neither	0.49	0.40	0.03	0.08
only Investment	0.20	0.72	0.00	0.08
only Export	0.03	0.00	0.45	0.52
Both	0.00	0.03	0.06	0.91
	Annual Transition Rates for Firms that Exit in t+2			
All Exiting Firms	0.27	0.17	0.11	0.45
Neither	0.88	0.10	0.02	0.00
only Investment	0.44	0.54	0.00	0.02
only Export	0.03	0.00	0.42	0.55
Both	0.00	0.01	0.04	0.95

This table provides the average annual transition rates for incumbent firms, new entrants and exiting firms in four possible activities: only invest, only export, both, and neither.

	Status Year t+1			
	Annual Transition Rates for Operating Firms			
Status Year t	Neither	Only Investment	Only Export	Both
All Firms	0.11	0.25	0.07	0.57
Neither	0.67	0.29	0.01	0.02
Only Investment	0.14	0.79	0.01	0.06
Only Export	0.04	0.03	0.49	0.44
Both	0.00	0.02	0.06	0.92

This table provides the average log productivity for new entrants, exiting firms, and incumbent firms in each year. The productivity measure is constructed using Levinsohn and Petrin (2003) approach.

Table 1
Variable Cost

Parameter	Coef		St. Error
ρ_d	0.87	*	(0.01)
ρ_x	0.53	*	(0.01)
R-Square	0.99		
sample size	4740		

Table 2
Productivity Evolution

	Investment measured as:					
	Capital			R&D		
intercept	0.04		(0.13)	0.50		(0.29)
productivity (t-1)	0.72	*	(0.15)	0.30		(0.41)
productivity $(t-1)^2$	0.03		(0.10)	0.33		(0.20)
productivity $(t-1)^3$	-0.01		(0.02)	-0.05		(0.03)
investment ($I_{i,t-1}$)	0.03	*	(0.01)	0.03	*	(0.01)
export ($X_{i,t-1}$)	0.00		(0.02)	0.03		(0.05)
product restr. ($N_{i,t-1}$)	0.06	*	(0.01)	0.06	*	(0.01)
R-Square	0.98			0.97		
sample size	4740			2121		

This table provides the estimated coefficients for equation () and (). The top panel provides the estimates for elasticity of substitution for domestic and export market, respectively. The bottom panel provides the estimates for the productivity evolution. *** indicates significance at 1%. ** at 5%, * at 10%.

Table 3
Dynamic Parameter Estimates

Parameter	Mean	St. Dev.	Thousands
Entry sunk cost	19.7	2.25	€ 359,000
Domestic fixed cost	12	0.41	€ 160
Product fixed cost	11	0.36	€ 59
Export fixed cost	13.5	0.45	€ 730
Investment Fixed Cost	10.4	0.32	€ 32
d (investment cost)	1	0.01	
Avg sell-off value	16	0.92	€ 8,900

This table provides the estimated coefficients for equation () and () in reduced form. The top panel provides the estimates for export decision in a probit model using productivity measures constructed from before. The middel panel provides the estimates for exit decision in a probit model. The last panel provides estimates from an OLS regression of number of products on firm productivities. *** indicates significance at 1%. ** at 5%, * at 10%.

Table 8
Dynamic Parameter Estimates

Parameter	Mean	St. Dev.	In Euros
Entry sunk cost	19.7	2.25	359 million
Domestic fixed cost	12	0.41	.16 million
Product fixed cost	11	0.36	59 thousand
Export fixed cost	13.5	0.45	.73 million
Investment Fixed Cost	10.4	0.32	32 thousand
d (investment cost)	1	0.01	
avg sell-off value	16	0.92	8.9 million

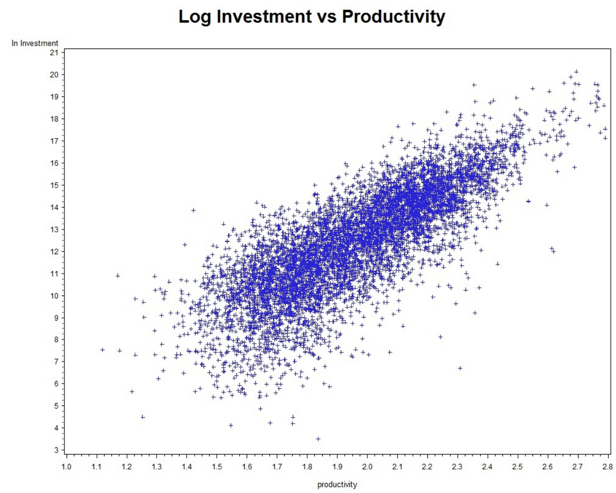
This table provides the estimated coefficients for the dynamic parameters in the model and the correponding dollar values for these estimates.

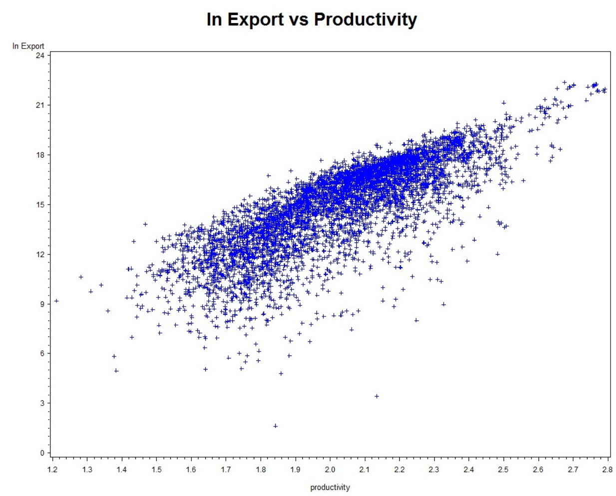
Table 9

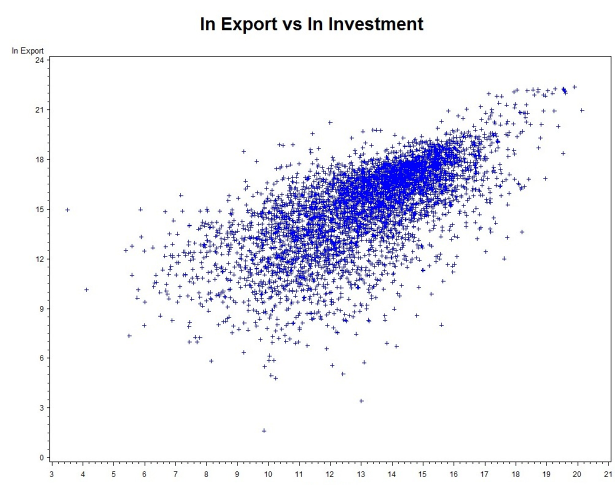
In-Sample Performance

export participation		Average Productivity	
actual	0.63	Actual	2.22
predicted	0.62	Predicted	2.36
Investment		Exit rate	
Actual	15.41	Actual	0.09
Predicted	15.75	Predicted	0.06
Entry rate			
Actual	0.15		
Predicted	0.07		

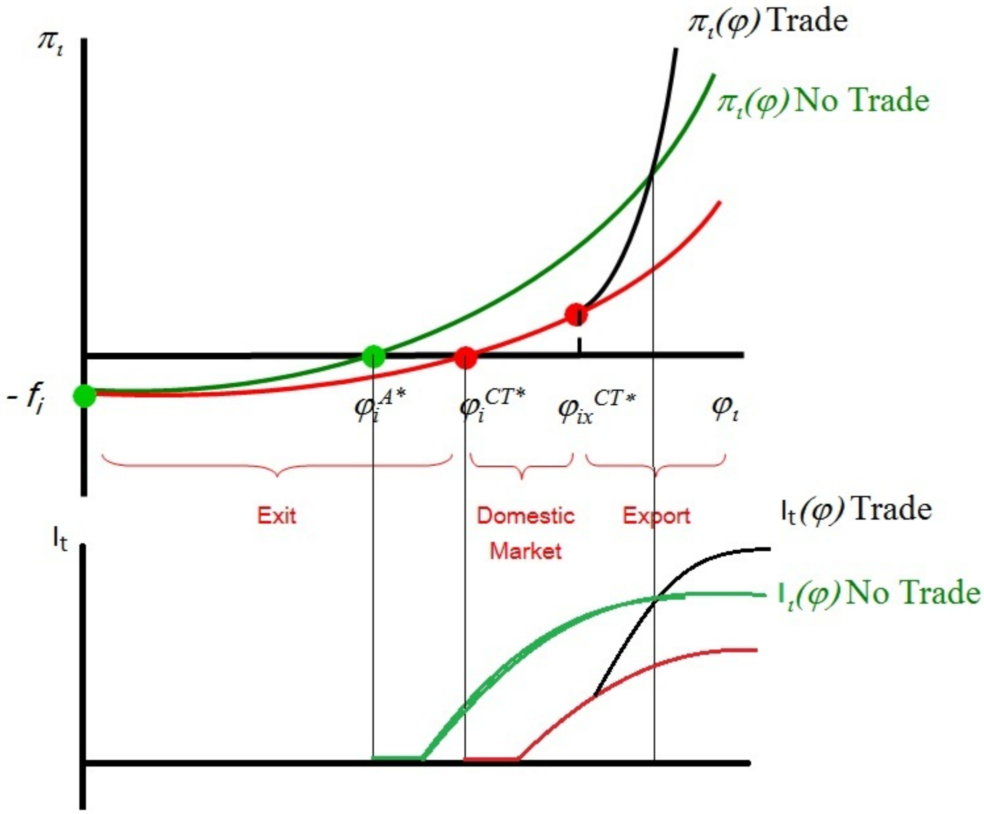
This table shows the in-sample performance for estimated static and dynamic parameters.







This graphs shows that investment, export and productivity are all correlated.

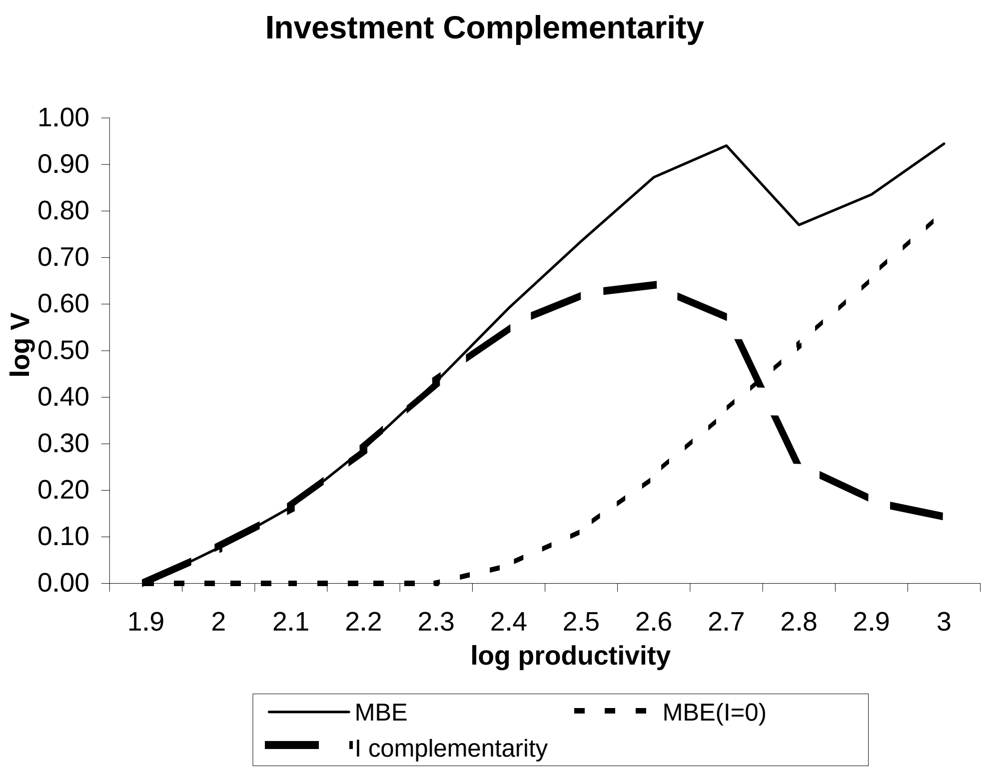


This graphs shows how the investment, export, and profits change when a country is opening up to trade. The top panel shows the change in profits when a country is opening up to trade. The bottom panel shows the change in investment profile when a country is opening up to trade. φ_{it} is the productivity. $\varphi_i^{A^*}$ is the productivity cut-off in autarky below which firms cannot operate. $\varphi_i^{CT^*}$ is the productivity cut-ff when a country goes from autarky to trading, below which firms cannot operate. $\varphi_{ix}^{CT^*}$ is the productivity cut-off for exprting firms, below which firms cannot export.

Table 12 Investment and Export Complementarity

productivity	V(X,I,N)	V(X,I=0,N)	V(X=0,I,N)	V(X=0,I=0,N)	MBE	MBE (I=0)	MBE - MBE(I=0)
1.9	16.44	16.44	16.44	16.44	0.00	0.00	0.00
2	16.73	16.48	16.66	16.48	0.08	0.00	0.08
2.1	16.96	16.53	16.80	16.53	0.16	0.00	0.16
2.2	17.27	16.57	16.98	16.57	0.29	0.00	0.29
2.3	17.65	16.66	17.21	16.66	0.43	0.00	0.43
2.4	18.08	16.84	17.49	16.80	0.59	0.04	0.55
2.5	18.53	17.14	17.79	17.02	0.74	0.11	0.62
2.6	18.98	17.55	18.11	17.32	0.87	0.23	0.64
2.7	19.43	18.13	18.49	17.76	0.94	0.37	0.57
2.8	19.77	18.81	19.00	18.29	0.77	0.51	0.26
2.9	20.40	19.53	19.57	18.87	0.84	0.66	0.18
3	21.10	20.30	20.16	19.50	0.94	0.80	0.14

This table shows the investment and export complementarity. All values are in logs. MBE is marginal benefit of exporting and is defined as V(X,I,N)-V(X=0,I,N), where V(X,I,N) is the value function when all three choices, X, I, and N are chosen optimally and the dynamic parameters are computed using maximum likelihood estimator. V(X=0,I,N) is the value function when export is restricted to 0 and I and N are chosen optimally with entry and exit rate fixed as in V(X,I,N). MBE (I=0) is the marginal benefit of exporting when investment is restricted to 0 and is defined as V(X,I=0,N)- V(X=0, I=0, N). The cross partial between I and X is defined as MBE - MBE (I=0).

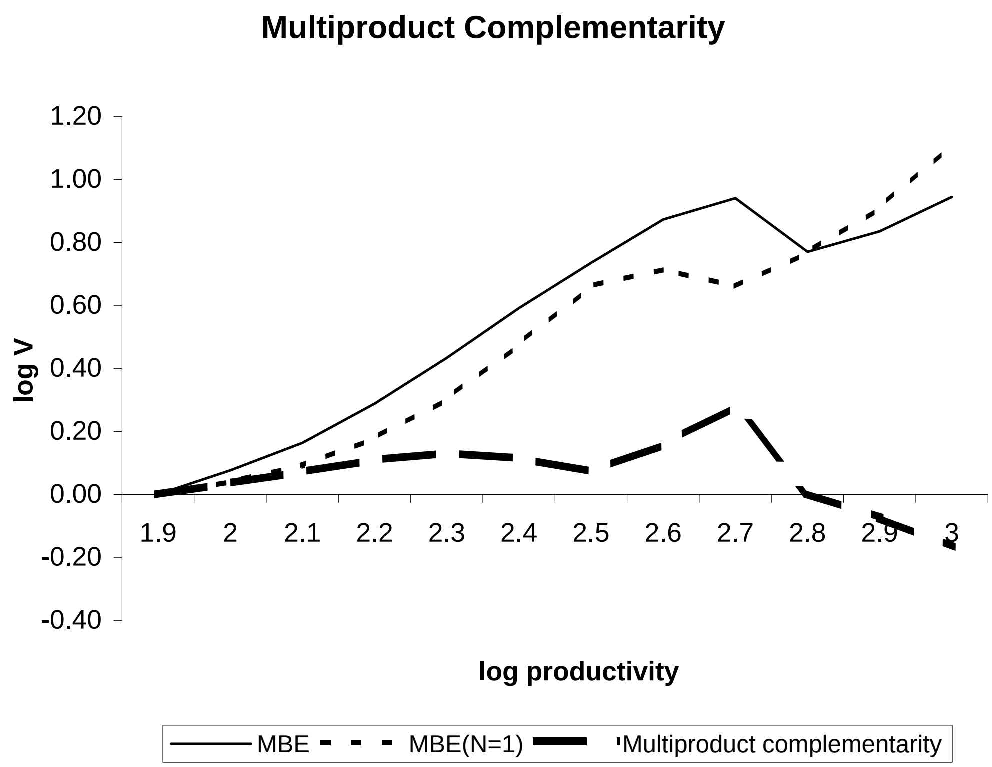


This graph shows the investment and export complementarity. The thin continuous line plots the MBE defined before, the dotted line plots the MBE (I=0), and the dashed line plots the cross partial between investment and export, or what we call investment complimentarity.

Table 13 Multiproduct and Export Complementarity

productivity	V(X,I,N)	V(X=0,I,N)	V(X,I,N=1)	V(X=0,I,N=1)	MBE	MBE (N=0)	MBE-MBE(N=0)
1.9	16.44	16.44	16.44	16.44	0.00	0.00	0.00
2	16.73	16.66	16.67	16.63	0.08	0.04	0.04
2.1	16.96	16.80	16.83	16.73	0.16	0.09	0.07
2.2	17.27	16.98	17.04	16.86	0.29	0.18	0.11
2.3	17.65	17.21	17.32	17.01	0.43	0.30	0.13
2.4	18.08	17.49	17.67	17.19	0.59	0.48	0.11
2.5	18.53	17.79	18.06	17.39	0.74	0.66	0.07
2.6	18.98	18.11	18.47	17.75	0.87	0.71	0.16
2.7	19.43	18.49	18.86	18.20	0.94	0.66	0.28
2.8	19.77	19.00	19.49	18.72	0.77	0.77	0.00
2.9	20.40	19.57	20.18	19.27	0.84	0.91	-0.07
3	21.10	20.16	20.93	19.82	0.94	1.11	-0.17

This table shows the multi-product and export complementarity. All values are in logs. MBE is marginal benefit of exporting and is defined as V(X,I,N)-V(X=0,I,N), where V(X,I,N) is the value function when all three choices, X, I, and N are chosen optimally and the dynamic parameters are computed using maximum likelihood estimator. V(X=0,I,N) is the value function when export is restricted to 0 and I and N are chosen optimally with entry and exit rate fixed as in V(X,I,N). MBE (N=0) is the marginal benefit of exporting when number of products is restricted to 1 and is defined as V(X,I,N=1)-V(X=0, I, N=1). The cross partial between N and X is defined as MBE - MBE (N=1).

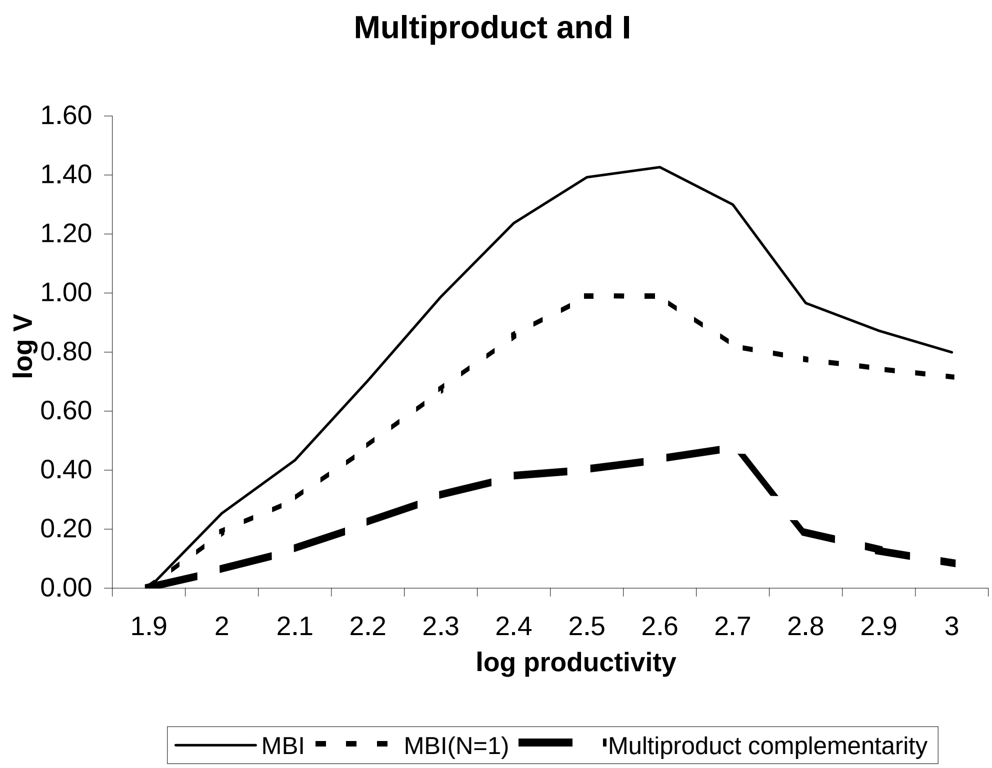


This graph shows the multi-product and export complementarity. The thin continuous line plots the MBE defined before, the dotted line plots the MBE (N=1), and the dashed line plots the cross partial between multi-product and export, or what we call multi-product complimentarity.

Table 14 Investment and Multiproduct Complementarity

productivity	V(X,I,N)	V(X,I=0,N)	V(X,I,N=1)	V(X,I=0,N=1)	MBI	MBI (N=1)	MBI - MBI(N=1)
1.9	16.44	16.44	16.44	16.44	0.00	0.00	0.00
2.0	16.73	16.48	16.67	16.48	0.25	0.19	0.06
2.1	16.96	16.53	16.83	16.52	0.43	0.30	0.13
2.2	17.27	16.57	17.04	16.56	0.70	0.48	0.22
2.3	17.65	16.66	17.32	16.64	0.99	0.67	0.31
2.4	18.08	16.84	17.67	16.81	1.24	0.86	0.38
2.5	18.53	17.14	18.06	17.07	1.39	0.99	0.40
2.6	18.98	17.55	18.47	17.48	1.43	0.99	0.44
2.7	19.43	18.13	18.86	18.04	1.30	0.82	0.48
2.8	19.77	18.81	19.49	18.71	0.97	0.78	0.19
2.9	20.40	19.53	20.18	19.44	0.87	0.74	0.13
3.0	21.10	20.30	20.93	20.22	0.80	0.72	0.08

This table shows the investment and multi-product complementarity. All values are in logs. MBI is marginal benefit of investment and is defined as V(X,I,N)-V(X,I=0,N), where V(X,I,N) is the value function when all three choices, X, I, and N are chosen optimally and the dynamic parameters are computed using maximum likelihood estimator. V(X,I=0,N) is the value function when investment is restricted to 0 and X and N are chosen optimally with entry and exit rate fixed as in V(X,I,N). MBI (N=1) is the marginal benefit of investment when number of products is restricted to 1 and is defined as V(X,I,N=1) - V(X, I=0, N=1). The cross partial between I and N is defined as MBI - MBI (N=1).



This graph shows the investment and multi-prodcut complementarity. The thin continuous line plots the MBI defined before, the dotted line plots the MBI (N=1), and the dashed line plots the cross partial between investment and multi-product, or what we call investment-multi-product complimentarity.

Table

Summary of the sample's evolution in the years 1990-2008

	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008
1. Current sample	2188	2059	1977	1869	1876	1703*	1716	1920	1776	1754	1870	1724	1708	1380	1374	1911	2023	2013
1.1 Firms which answer	1888	1898	1768	1721	1693	1584	1596	1764	1631	1634	1693	1635	1380	1374	1277	1716	1892	1853
1.2 Disappear[1]	62	52	72	53	51	28	35	18	45	38	20	18	51	4	17	35	30	57
1.3 Do not collaborate	187	62	124	45	55	33	54	22	35	24	0	12	88	0	12	14	18	10
1.4 No access[2]	51	47	13	50	77	58	31	116	65	58	157	59	189	2	68	146	83	93
2. Recovered[3]	129			99								73			46	0	3	2
3. Entries in the current year	42	79	101	56	9	132	324	12	123	236	31	0	0	0	588	307	118	154
Number of record on file	2359	2438	2539	2595	2604	2736	3060	3072	3195	3431	3462	3462	3462	3462	4050	4357	4475	4629

Notas:
1. Closures, firms in liquidation, change to a non-manufacturing activity, disappearance through merger or acquisition.
2. Cannot be found, provisional close-up.
3. In 1991 they are large-sized firms to which the questionnaire was already submitted in 1990, but which did not answer it. In 1994 it is made up of large-sized firms which had filled the questionnaire before but which at some time they stopped doing it.

* Includes a company which did not collaborated in 1995 but which collaborated in 1996.

The table shows the evolution of the sampled firms during the period 1990-2007

SAMPLE COVERAGE ESEE 2005*

	Less than 20	21 to 50	51 to 100	101 to 200	More than 200
Meat-processing industry	1.54%	3.03%	2.04%	14.29%	32.00%
Foodstuffs and tobacco	2.42%	3.10%	3.97%	5.32%	38.75%
Drinks	3.32%	3.40%	7.41%	23.81%	41.03%
Textiles	3.06%	3.75%	6.98%	16.04%	42.62%
Leather and footware	3.03%	4.67%	5.63%	14.81%	14.29%
Wood industry	1.37%	3.25%	7.69%	16.67%	50.00%
Paper	3.53%	3.38%	6.82%	14.29%	60.53%
Editing and printing	1.98%	2.96%	6.67%	7.08%	40.32%
Chemical industry	2.25%	4.28%	6.61%	11.03%	39.16%
Rubber and plastics	3.09%	2.95%	5.79%	12.90%	46.43%
Non-metallic minerals products	2.35%	3.34%	2.60%	10.18%	43.40%
Iron and steel	2.12%	3.02%	5.65%	14.29%	48.57%
Metallic products	2.01%	2.88%	5.31%	7.45%	41.67%
Machinery and mechanical goods	2.41%	3.27%	5.42%	12.84%	51.95%
Office machinery, computers, processing, optical and similar	1.54%	5.63%	13.64%	4.55%	45.45%
Electrical and electronic machinery and material	3.06%	3.95%	4.81%	11.70%	45.83%
Motor vehicles	2.74%	3.85%	6.19%	12.66%	44.85%
Other transport material	1.57%	6.15%	13.04%	44.00%	31.58%
Furniture	2.45%	3.31%	4.86%	12.00%	50.00%
Other manufacturing	3.06%	2.73%	2.70%	9.38%	40.00%
Total	2.36%	3.37%	5.44%	11.40%	42.95%

* Sample coverage, calculated with respect to the Spanish Social Security Census

The table shows the sample coverage.