An Optimized Space Vector Modulation Sequence for Improved Harmonic Performance

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***Abstract*—This paper proposes a method for optimization of the harmonic performance of converters under space vector modulation (SVM) control. It is shown that SVM provides a number of degrees of freedom, which make it suitable for optimization, subject to desired constraints. An objective function is defined, which seeks to minimize the most significant harmonic components of the generated waveform while keeping other harmonic components within the acceptable range outlined in the available standards. Since the formulation of the problem involves both real and integer variables, specialized optimization methods capable of handling mixed-integer variables need to be employed, and hence, a genetic algorithm optimizer is used to solve the optimization problem. Optimized SVM sequences obtained for various operating points under different sampling frequencies are shown to result in significant reduction of dominant harmonics while maintaining the waveform quality within prescribed limits.**

***Index Terms*—Genetic algorithms (GAs), harmonic analysis, optimization, space vector modulation (SVM), voltage-sourced converter (VSC).**

I. INTRODUCTION

HIGH POWER electronic devices are being used increasingly to control and facilitate flow of electric power while meeting stringent operating conditions of today's heavily loaded networks. These devices are able to enhance the voltage profile of the power system, control the flow of real and reactive power, and improve the dynamic performance and stability of the system.

One of such devices is voltage-sourced converter (VSC) that acts as a controlled voltage source, converting a dc voltage to an ac voltage with desired frequency, phase, and magnitude. Fig. 1 shows the schematic diagram of a two-level VSC. Pulsewidth modulation (PWM) methods are normally used for synthesis and control of the output voltage of VSCs [1], [2].

The following two aspects of operation of a VSC are of primary importance: 1) The operation under PWM typically results in large switching losses and 2) the resulting output waveform contains harmonics, largest of which occurs around the switching frequency. Using a high switching frequency improves the harmonic spectrum by drifting the harmonics toward higher orders, which would have less adverse impact on the network. However, it also increases the switching losses. Filters are used at the output of a VSC to trim down the generated harmonics; however, they are costly.

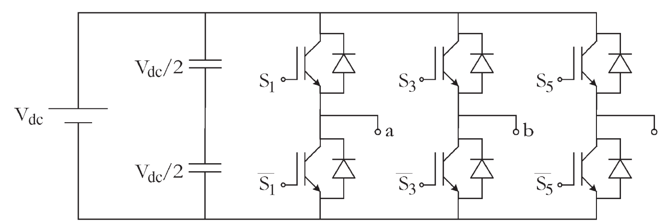

Fig. 1. Schematic diagram of a two-level VSC.

Sinusoidal PWM (SPWM) has a relatively robust harmonic spectrum, i.e., the harmonic spectrum of the resulting waveform is tied to the selected switching frequency. In contrast, space vector modulation (SVM), which is a relatively new approach to waveform synthesis using a VSC [3]–[5], offers several degrees of freedom that can be used effectively to design an improved harmonic spectrum and obtain the desired waveform quality.

SVM has found numerous applications in both power system schemes, such as static compensator (STATCOM) [6] and high-voltage direct current converter systems [7], and electrical drive applications [8]. Similar to SPWM, SVM is capable of controlling multilevel VSCs. Several three-level SVM approaches, which provide a higher number of degrees of freedom, have been presented in the literature [9], [10].

In [6], an SVM switching sequence with inherent voltage balancing properties for multilevel converters is presented, and its application for a STATCOM is demonstrated. Various SVM sequences for current-sourced inverters are investigated in [11], and two SVM sequences are proposed, each providing the lowest losses and line current total harmonic distortion (THD) for a certain range of modulation index values. A similar idea of using different vector sequences based on the value of the modulation index is employed in [12], and sets of three, five, or seven sequences are suggested. In each set, the sequence to be used is determined based on the location of the space vector such that the line current ripple is minimized. The use of multiple SVM sequences, however, increases the complexity of the logics of the controller. The application of SVM to a variable speed electric drive is studied in [13], and a switching sequence is proposed for a multilevel multiphase converter such that it minimizes the number of switchings. The manipulation of the placement of zero vectors in a cycle and their share with

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the goal of minimization of the zero-sequence voltage for a two-inverter system is shown in [14].

This paper aims to optimize space vector sequences through exploiting the degrees of freedom available in SVM. The objective of the optimization is to relieve the filtering requirement [15] by distributing the energy of the waveform more uniformly among harmonic components. This is constrained by limits on individual harmonics and overall quality of the waveform as imposed by applicable industry standards. Since the same sequence is used throughout the entire linear range of operation of the converter, it features a simplified electronic implementation.

As will be described later, the problem involves both real and integer variables. Therefore, genetic algorithms (GAs) [16], which can handle such mixed-integer problems, are used for its optimization. GAs have been previously applied to find the filter parameters in electrical drives [17], as well as to a two-level SPWM converter to minimize the harmonic content of its output waveform [18].

An overview of the SVM methods is provided in the next section, which also describes the parameters that are used in the optimization. GAs are briefly reviewed afterwards. Methods for the presentation of the problem in a GA-ready form are then proposed. Finally, the optimization results for different values of sampling frequency and modulation index are discussed.

II. Review of SVM Methods

In PWM methods, the reference voltage is approximated by a number of voltage pulses at the converter output. The approach to determine the duration of such pulses is what constitutes the difference among various PWM methods [19]–[21]. For example, in conventional SPWM, which is an analog domain method, the duration of each pulse is found through *comparison* of a sinusoidal reference waveform and a triangular carrier waveform. A digital domain variation of PWM, which is the SVM, on the other hand, directly *computes* the duration of voltage pulses using the amplitude and angular location of the reference vector.

To generate firing pulses, SVM employs the concept of converter states. The states are determined by the status of switches of the three legs of the converter. In a three-phase two-level VSC, the output voltage of a leg can be either $+V_{\mathrm{dc}}/2$ or $-V_{\mathrm{dc}}/2$ (see Fig. 1), and as such, the VSC can be placed in eight states, depending on the ON/OFF status of its six switches (switches on the same leg have complementary states). The voltages corresponding to each state can be transformed to a space vector using

$$\mathbf{V} = \frac{2}{3}\left(\nu_a + \nu_b e^{j2\pi/3} + \nu_c e^{-j2\pi/3}\right). \tag{1}$$

Six of these space vectors, denoted by $\mathbf{V}_1$ to $\mathbf{V}_6$ in Fig. 2, point to the vertices of a hexagon; two of the states translate to space vectors with zero length and are denoted by zero vectors $\mathbf{V}_0$ and $\mathbf{V}_7$ located at the origin.

Similarly, any set of balanced reference phase voltages ν_a, ν_b, and ν_c can be represented as a corresponding reference space vector $\mathbf{V}$ in two dimensions using (1). Any reference

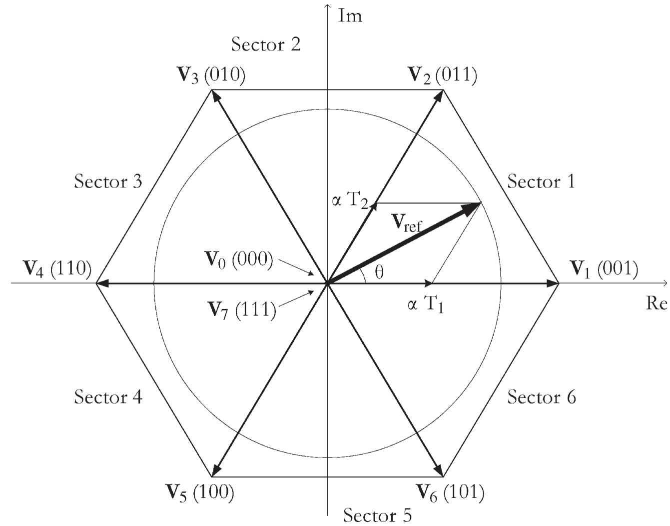

Fig. 2. Space vectors and decomposition of a reference vector.

vector that lies entirely within the hexagon can be decomposed to space vectors $\mathbf{V}_0$ to $\mathbf{V}_7$, as shown in Fig. 2, for a $\mathbf{V}_{\mathrm{ref}}$ in the first sector of the hexagon. This is achieved by constructing voltages that average to the reference vector [22], which is sampled at a rate determined by a given sampling frequency F_s.

The voltages are synthesized by placing the converter in respective states for designated time shares within each sampling period. The time shares are proportional to the length of the projected vectors. The rest of the sampling period is filled with zero space vectors $\mathbf{V}_0$ and/or $\mathbf{V}_7$ ($\mathbf{Z}_0$ and $\mathbf{Z}_7$, respectively, hereinafter). Time shares are calculated as

$$T_1 = \sqrt{3}/2 \times T_s m \sin(\pi/3 - \theta) \tag{2}$$

$$T_2 = \sqrt{3}/2 \times T_s m \sin(\theta) \tag{3}$$

$$T_0 + T_7 = T_s - T_1 - T_2 \tag{4}$$

where T_1, T_2, T_0, and T_7 are the time shares of the respective voltage vectors (T_1 for the first active vector $\mathbf{A}_1$, T_2 for $\mathbf{A}_2$, T_0 for $\mathbf{Z}_0$, and T_7 for $\mathbf{Z}_7$), T_s is the sampling period and is equal to $1/F_s$, θ is the angle between the reference vector and the space vector $\mathbf{A}_1$, and m is the modulation index, which is defined as

$$m = V_{\mathrm{ref}}/(V_{\mathrm{dc}}/2). \tag{5}$$

Note that the SVM strategy only determines the time shares of space vectors and does not identify the order in which they are applied. Moreover, the individual time shares of $\mathbf{Z}_0$ and $\mathbf{Z}_7$ are not specified. These parameters can be used to obtain the desired performance, as discussed in the next section.

Fig. 3 shows an example of a space vector sequence conventionally used for SVM, in which the arrangement $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$ is used, and each of $\mathbf{Z}_0$ and $\mathbf{Z}_7$ is applied for half of the total zero-vector period ($\mathbf{A}_1$ and $\mathbf{A}_2$ are the two active vectors currently adjacent to the reference vector). The harmonic spectrum corresponding to this sequence is shown in Fig. 4 for a sampling frequency of 36 and a modulation index of 0.8.

Once a sequence format is chosen, it can be applied in two ways: either the same for consecutive sampling periods

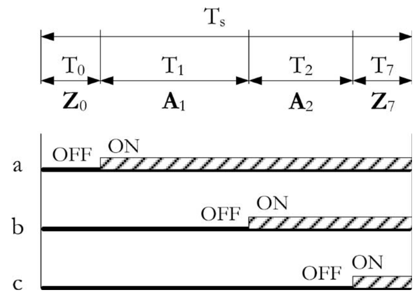

Fig. 3. Conventional SVM sequence.

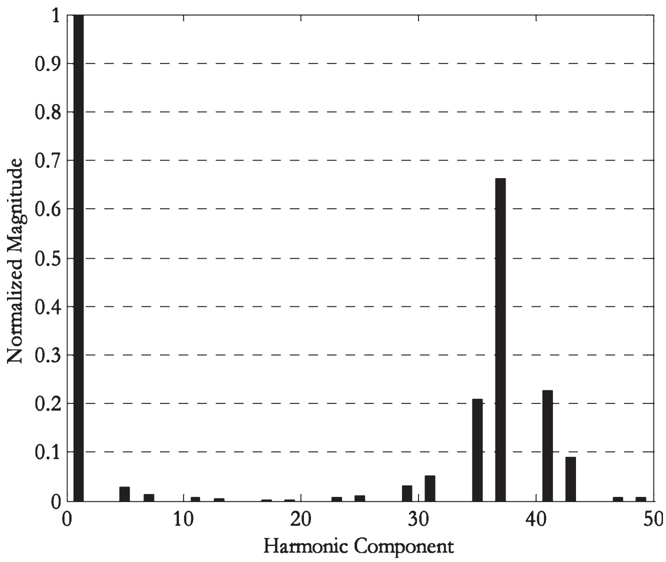

Fig. 4. Harmonic spectrum of the conventional SVM for a normalized sampling frequency of 36 and a modulation index of 0.8.

or alternatively forward and backward. For example, if the sequence $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$ is chosen, it can be applied as $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7|\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$ (only forward sequence) or forward sequence for one sampling period and backward for the next period, i.e., $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7|\mathbf{Z}_7\mathbf{A}_2\mathbf{A}_1\mathbf{Z}_0$. This is another degree of freedom offered by the SVM.

It is worth noting that the effective switching frequency of the converter under the SVM method, as described earlier, equals the sampling frequency of the reference vector. Alternatively, in some implementations, the sampling period is divided into two consecutive and equal segments; active and zero vectors are combined in the first segment to approximate the reference vector, and either the same sequence or its mirror image is used for the second segment as well. In this case, the effective switching frequency is two times larger than the sampling frequency. The optimization presented in the following sections is based on the former implementation approach, which considers the entire sampling period as one segment.

III. SVM Optimization Setup

As shown earlier, SVM provides certain degrees of freedom in controlling the switches of a converter. This feature of the SVM can be exploited to enhance certain performance characteristics such as its harmonic spectrum.

A. Potential for Optimization

The flexibility of SVM comes from the seemingly unlimited number of ways for its implementation. This is due to the degrees of freedom inherent to the SVM method and can be used as parameters for optimizing the modulation [23]. These parameters are as follows.

1) Decomposition of Reference Vector: A reference vector can be decomposed using virtually any subset of the eight space vectors. Nonetheless, decomposition is done typically using only the two adjacent active vectors. For example, the reference vector shown in Fig. 2 lies in the first sector of the hexagon (for the given angle θ) and hence is decomposed onto $\mathbf{V}_1$ and $\mathbf{V}_2$. This allows for computational simplicity and lower number of switchings per cycle.

2) Sequence of Active Space Vectors: Once constituent active space vectors for decomposition of the reference vector and their durations are found [as in (2) and (3)], they generate the same average output voltage regardless of the *order* in which they are applied within the current sampling period. SVM does not recommend any specific order, and this can be exploited as a degree of freedom for crafting waveforms with given properties.

3) Placement and Share of Zero Space Vectors: SVM also offers flexibility with zero vectors. Not only can they be used in any order within the sampling period and relative to active vectors but one can also divide the total share of zero vectors determined by the SVM algorithm in any proportion between them. In the extreme case, one may use only one of the two available vectors, i.e., exclusively $\mathbf{Z}_0$ or $\mathbf{Z}_7$ and not the other vector. The share of individual zero vectors during the inactive time is another degree of freedom used in the proposed optimization.

4) Selection of Sampling Frequency: The selection of sampling frequency is a compromise between harmonic content and switching losses. An excessively high sampling frequency results in significant losses in the power electronic elements. On the other hand, a low sampling frequency results in poor sampling of the reference waveform, which further exacerbates the harmonic performance. Since the first group of significant harmonics clusters around the sampling frequency, care should be exercised in choosing the frequency to achieve an acceptable tradeoff between switching losses and harmonic spectrum.

B. Posing as Optimization Problem

As previously stated, the decomposition of the reference vector does not determine the zero vectors ($\mathbf{Z}_0$ and $\mathbf{Z}_7$) to be used nor does it indicate the share of each to fill the inactive part of the sampling period. By exploiting these degrees of freedom built-in to SVM, appropriate switching sequences can be devised to obtain the desired waveform quality, through the definition of an appropriate objective function (OF).

The available degrees of freedom have been previously used to devise sequences that feature lower switching losses or harmonic content [24]. Although this paper focuses on two-level SVM, the developed method can be extended to multilevel schemes by devising proper formulation.

The definition of the OF depends on the specific application of interest. For example, if it is desired to lower the harmonic content of the output current, one can use weighted THD (WTHD) as the OF. The WTHD, defined as follows, is calculated in the same way as THD, but each harmonic component is divided by its order, so that higher order harmonics receive lower weight and contribute less in this figure of merit. An upper limit of 50 is often recommended for the calculation of the WTHD [25]

$$\mathrm{WTHD} = \frac{\sqrt{\sum_{h=2}^{n} \left(\frac{V_h}{h}\right)^2}}{V_1}. \tag{6}$$

A more insightful OF can be devised by considering the industry standards concerning the harmonic content of voltage and current waveforms. The IEEE Standard 519 [25] provides guidelines for harmonic control in electrical power systems and is used as the reference for this paper. It limits the magnitude of individual harmonic components of a system operating below 69 kV to 3% of the fundamental.

As can be seen in the harmonic spectrum of the conventional SVM shown in Fig. 4 for a normalized sampling frequency of 36, harmonic components 35 and 37 are the largest ones, and hence, they determine the parameters of the filter required for the elimination of harmonics. The goal of this paper is to reduce these two adjacent harmonic components grouped around the sampling frequency to relieve the filtering requirement of the VSC. However, this is possible only at the expense of increasing other harmonics, which are tolerable as long as they are below the 3% limit imposed by the IEEE Standard 519. Therefore, the compromise is between lowering the high-order harmonics to relieve filtering and keeping the low-order harmonics within their maximum permissible values. At the same time, the overall quality of the waveform indicated by its WTHD should not exceed its maximum allowable value of 5%, according to the IEEE Standard 519.

It should be noted that any alterations to the sequence of active and zero vectors will lead to different numbers of switchings within each fundamental period of the reference vector, thus leading to different switching losses. Since the aim of this paper is to optimize the harmonic performance of the SVM-based voltage waveform, the associated switching losses are not included.

C. Development of OF

The OF needs to embed the aforementioned requirements and also reflect the allowable harmonic magnitudes. The following conditions must be considered in the OF.

1) The two harmonic components adjacent to the sampling frequency are minimized.
2) Individual harmonic components do not exceed their 3% limit.
3) The waveform WTHD is below 5%.

This constrained optimization problem can be posed mathematically as follows. This is shown for a normalized sampling frequency of 36; for other frequencies, the two adjacent harmonic components are adjusted accordingly

$$\begin{aligned} &\text{Minimize} \\ &\quad \mathrm{OF}(\text{seq, share } 0) = \nu_{35} + \nu_{37} \\ &\text{Subject to} \\ &\qquad\qquad \nu_i \le 3\%, \qquad i \le 50; \quad i \ne 35, 37 \\ &\qquad \mathrm{WTHD} \le 5\% \end{aligned} \tag{7}$$

where seq and share 0 are the SVM sequence of vectors and share of the zero vector $\mathbf{Z}_0$, respectively, and ν_i is the magnitude of the ith harmonic.

Certain aspects of the aforementioned OF deserve further attention. By lowering the two harmonic components adjacent to the switching frequency, the energy of harmonics will be transferred to other harmonic components, i.e., some harmonic components may experience an increase. In order to prevent an increase in the lower order harmonics, a stringent limit of 3% is imposed along with a 5% limit on the overall quality of the waveform indicated by its WTHD. By disallowing an increase in the low-order harmonics, the aforementioned OF effectively aims to direct any increase in harmonic spectrum of the waveform toward higher frequency ranges. The natural damping of ac systems at high frequency bands aids to further eliminate the adverse impact of high-order harmonics.

It is often easier to optimize an unconstrained OF than one with constraints as in (7). Optimization theory suggests the use of techniques commonly referred to as transformation methods to remove constraints by augmenting them onto the original OF [26]. The augmented OF developed to eliminate constraints in (7) is given in (8).

The least pth (see Appendix A) interior point penalty functions (with large even exponents p and p^*) are used to ensure that the WTHD and individual harmonic components are limited to their permissible ranges of 5% and 3%, respectively. As long as WTHD and harmonic components meet these criteria, their contribution to the OF is negligibly small, whereas if they exceed these limits, they sharply increase the OF to prompt the optimization algorithm that it has violated a vital condition

$$\begin{aligned} \mathrm{OF}(\text{seq, share } 0) = \alpha_1 &\left[\left(\frac{\mathrm{WTHD}}{0.05}\right)^p + \nu_{35} + \nu_{37}\right] \\ &+ \alpha_2 \left(\frac{\nu_5}{0.03}\right)^{p^*} + \alpha_3 \sum_{\substack{i=7,11,13,\\17,19,23,25}} \left(\frac{\nu_i}{0.03}\right)^p \\ &+ \alpha_4 \left(\frac{\nu_{29}}{0.03}\right)^p + \alpha_5 \left(\frac{\nu_{31}}{0.03}\right)^p. \end{aligned} \tag{8}$$

Weighting factors (α_i) and exponents p and p^* are listed in Table I. These values do affect the optimization process and are normally obtained through preliminary sample runs of the optimization algorithm. Formal procedures, such as Pareto frontiers [27], for the determination of weighting factors also exist; however, they have not been used in this paper.

The assignment of the weighting factors is a formal way to determining the relative importance of the competing subobjectives. As a general rule, subobjectives with higher weighting factors are given more significance than the ones with lower

TABLE I
PARAMETERS OF OF

PARAMETER	α_1	α_2	α_3	α_4	α_5	p	p^*
VALUE	1000	2000	100	400	200	8	16

weighting factors. On the other hand, the exponents used in the least pth function determine the strength of the penalty given to any violation outside the permissible range. During the preliminary development of the OF in (8), it was noticed that, compared to other harmonics, the 5th harmonic has a strong tendency to rise far beyond its permissible 3% limit. Therefore, it was necessary to assign a higher weighting function and a higher penalty exponent to its corresponding term in the OF.

It should be noted that the optimization of the harmonic spectrum of an SVM-based waveform is not limited to the case described and developed earlier. Depending on the specific features of the system in which a converter with SVM output is to be installed, one can develop other OFs to enhance other aspects of the resulting harmonic spectrum. For example, specific harmonic components may need to be targeted for reduction or possibly elimination due to concerns such as resonance. The formulation of an OF should include mechanisms that prevent excessive increase in the lower harmonic bands. Since such cases are dependent on a specific system, they are not considered hereinafter.

IV. Adaptation of Problem for Solution With GA

GAs belong to a group of evolutionary algorithms that are inspired by natural processes [16]. Such methods seek to maximize the fitness of a population or, as in the common notion, minimize the associated cost (or objective) function. An important feature of GAs is their ability to deal with real and integer (or binary-coded) optimization variables simultaneously. This proves to be essential in SVM harmonic spectrum optimization, as it involves both such parameters (the order of space vectors is specified as an integer variable, and the share of zero vectors is a real variable).

In GAs, the solution space is represented by a generation of candidate solutions, called chromosomes. Each chromosome, in turn, consists of a number of genes, representing the possible values of optimization parameters. An OF is calculated to reflect the fitness of each chromosome, i.e., its conformity to the desired objectives and serves as a quantitative measure of its suitability as a potential solution. In the particular case of SVM optimization, the calculation of the OF for a chromosome in each generation involves the following: 1) construction of a waveform using the SVM method for the sequence and $\mathbf{Z}_0$-share specified in the chromosome; 2) calculation of the harmonic spectrum; and 3) evaluation of the OF in (8). This figure of merit is then returned to the GA optimizer. Akin to what happens in the natural process of evolution, such chromosomes evolve, and since good traits tend to pass to next generations, the possibility of survival of unfit chromosomes is low.

The first stage in implementing GA is to find a suitable representation of the optimization parameters and, hence, the chromosomes. Such a representation seeks to facilitate the application of GA operators during mating and mutation operations.

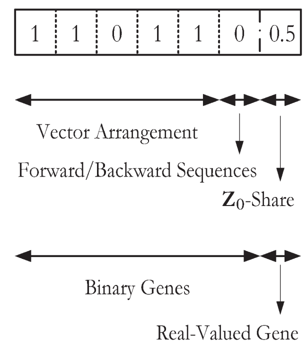

Fig. 5. Structure of the proposed hybrid chromosome.

A. Chromosome Structure

Each chromosome needs to fully describe the SVM sequences, i.e., a chromosome should have information about the order of the space vectors, plus the share of zero space vectors (the shares of active space vectors are determined by the SVM technique itself and are not part of the optimization). As mentioned earlier, this leads to a mixture of real (the share of zero vectors) and integer variables (the arrangement of active and zero space vectors and whether forward or forward-and-backward sequences are used), which necessitates the definition of hybrid chromosomes that consist of both real and integer genes, as shown in Fig. 5. The chromosome structure shown in Fig. 5 uses binary values to represent integers.

The real part of the chromosome does not need manipulation to become fit for the GA application. A real value ranging between zero and one represents the share of the zero space vector $\mathbf{Z}_0$ in the inactive part of the sampling period. A $\mathbf{Z}_0$-share of zero means that $\mathbf{Z}_0$ is not used at all and $\mathbf{Z}_7$ is applied for the whole inactive period, leading to a sequence such as $\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$. On the other hand, a $\mathbf{Z}_0$-share of one means that only $\mathbf{Z}_0$ is used, as in $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2$.

The part that bears information about the arrangement of vectors, however, needs to be properly represented in the binary part of the chromosome. Therefore, a method to translate the sequences into a string of binary genes is suggested.

Since the number of valid SVM sequences (taking into account only the order of space vectors) for a two-level converter is $4! = 24$, one simple approach is to number the sequences from 1 to 24 (or 0 to 23) and encode them in binary format. This method, while being easy to implement, introduces some problems. The binary coding of the 24 sequences requires 5 b; however, the number of possible combinations represented by 5 b is 32. Since GA operators, i.e., mating and mutation, combine and alter the bits in a random manner, the resulting binary-coded combinations may correspond to invalid sequences (numbers between 25 and 32).

To remedy these problems, a method for coding SVM sequences has been developed. The proposed method has the advantage that it defines the position of individual vectors relative to each other. This makes passing the traits of each chromosome from one generation to the next ones more meaningful and hence is tailored to how GAs work. Moreover, this method inherently avoids the problem of getting invalid combinations. Such benefits are not present in the simple numbering method.

In the proposed method, the sequences are defined based on how active and zero vectors are located with respect to each other. Stating the active vectors as $\mathbf{A}_1$ and $\mathbf{A}_2$ and zero vectors as $\mathbf{Z}_0$ and $\mathbf{Z}_7$, one can describe a sequence based on factors such as whether it starts with an active vector ($\mathbf{A}_1$ or $\mathbf{A}_2$) or a zero vector ($\mathbf{Z}_0$ or $\mathbf{Z}_7$) and whether $\mathbf{A}_1$ appears before $\mathbf{A}_2$ or vice versa. Such factors are described in more detail in the next section.

B. Proposed Coding Approach

The arrangement of vectors shows the order in which they appear in each sampling period. Examples of possible arrangements are $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$, $\mathbf{A}_1\mathbf{Z}_0\mathbf{A}_2\mathbf{Z}_7$, and $\mathbf{A}_2\mathbf{Z}_7\mathbf{A}_1\mathbf{Z}_0$. In the proposed method, each arrangement is described by a finite string of binary digits.

In the proposed method, there are five binary genes that represent answers to five YES/NO questions, which translate to a series of 1/0's in the binary domain, respectively. These five questions are as follows.

1) Does $\mathbf{A}_1$ appear before $\mathbf{A}_2$ in the sequence?
2) Does $\mathbf{Z}_0$ appear before $\mathbf{Z}_7$ in the sequence?
3) Is the first vector in the sequence an active one?
4) Is the second vector in the sequence an active one?
5) Is the third vector in the sequence an active one?

To demonstrate how sequences are encoded, consider the conventionally used sequence $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$. Encoding results in a string of 1 ($\mathbf{A}_1$ is before $\mathbf{A}_2$), 1 ($\mathbf{Z}_0$ is before $\mathbf{Z}_7$), 0 (the first vector in the sequence is a zero vector), 1 (the second vector is an active one), and 1 (the third vector is an active one) or in the condensed form: $1\,1\,0\,1\,1$. Other sequences are also encoded in a similar way.

Decoding is done similarly by deciphering the answers into a valid sequence. For example, a string of $1\,0\,1\,0\,1$ is decoded as $\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0$. In some cases, decoding does not need answers to all five questions, which is the reason why this 5-b representation only decodes into 24 vector arrangements rather than the original 32. As such, the GA operators of mating and mutation can be directly used to manipulate chromosomes without any further adjustments.

V. Optimal Solution and Discussion of Results

A. Harmonic Spectrum

The GA optimizer is run with the aforementioned OF. Although mutation is implemented in the GA solver as a means to reduce the likelihood of converging to a local minimum, the stochastic nature of GAs necessitates running the optimizer several times to further reduce this possibility. For each run, the best sequence of vectors and the share of $\mathbf{Z}_0$ are recorded.

Fig. 6 shows the sequence and share of space vectors for the conventional [Fig. 6(a)] and optimized [Fig. 6(b)] SVM arrangements. Both sequences are forward; however, the optimized sequence positions the vectors in a different order and allots 85% of the inactive time interval to the zero vector $\mathbf{Z}_0$. The sequence of constituent vectors has been changed from $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$ (1 1 0 1 1) to $\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0$ (1 0 1 0 1).

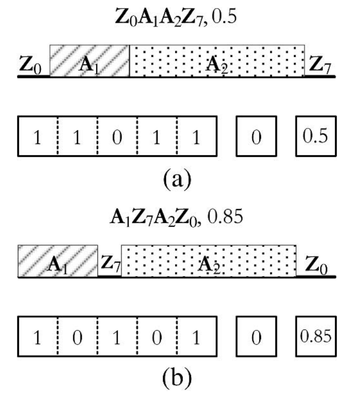

Fig. 6. SVM sequences. (a) Conventional. (b) Optimized.

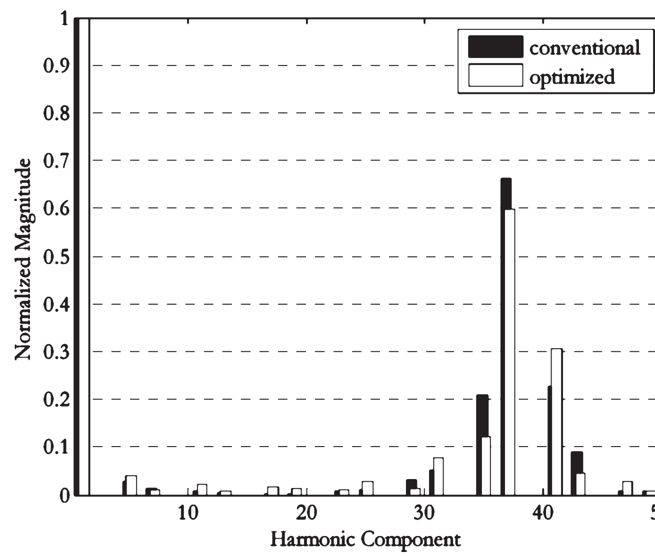

Fig. 7. Harmonic spectra of conventional and optimized SVMs.

Fig. 7 shows the harmonic spectrum of the voltage waveform for both conventional and optimized SVM sequences for a modulation index of 0.8 and a normalized sampling frequency of 36. For both cases, harmonic components 35 and 37 are the largest contributors to the harmonic distortion. However, optimization has reduced the 35th and 37th harmonics from 21.6% and 65.6% for the conventional SVM sequence to 12.1% and 59.9% for the optimized sequence, respectively. This is a reduction of 44% and 9% for the 35th and 37th harmonics, respectively. Such significant reductions will result in smaller harmonic currents in the filter and thus allow the use of smaller filters.

Note that the implementation of the SVM method, as described in the earlier sections, ensures that the waveform of the output voltage retains half-wave symmetry; therefore, even harmonic components are absent in its harmonic spectrum. This is also the reason why even harmonics are not included in the development of the OF in (7).

As shown in Table II, as a result of lowering harmonics 35 and 37, some of the lower order harmonics have been increased;

TABLE II
HARMONICS OF TWO SVM METHODS FOR NORMALIZED SAMPLING FREQUENCY OF 36 AND MODULATION INDEX OF 0.8

ORDER	CONVENTIONAL SVM	OPTIMIZED SVM
1	100	100
5	3.02	3.86
7	1.71	1.06
11	1.29	2.06
13	0.99	0.73
17	1.17	1.46
19	1.17	1.45
23	1.54	1.06
25	1.90	2.79
29	3.75	1.41
31	5.71	7.79
35	21.59	12.10
37	65.58	59.88
41	23.24	30.42
43	9.43	4.57
47	1.04	2.84
49	1.26	0.75
WTHD	**2.10**	**2.01**

TABLE III
HARMONIC COMPONENTS OF OPTIMIZED SVM SEQUENCE AS FUNCTION OF $\mathbf{Z}_0$-SHARE

Order	$\mathbf{Z}_0$-share		
	0.80	0.85	0.90
1	100.00	100.00	100.00
5	4.30	3.86	3.60
7	0.98	1.06	1.28
11	2.44	2.06	1.67
13	1.18	0.73	0.46
17	1.84	1.46	1.05
19	1.93	1.45	0.96
23	1.58	1.06	0.59
25	3.26	2.79	2.25
29	1.03	1.41	1.96
31	8.27	7.79	7.21
35	9.11	12.10	14.99
37	57.41	59.88	62.02
41	32.80	30.42	27.90
43	3.09	4.57	6.18
47	3.10	2.84	2.44
49	1.15	0.75	0.20
WTHD	**2.01**	**2.01**	**2.03**

however, the reduction in the two adjacent harmonics has been so significant that the WTHD of the optimized waveform (2.01%) is 5% lower than that of the original waveform obtained using the conventional sequence (2.10%). Note that the harmonic components that experienced an increase (e.g., 11 and 25) are still within their permissible 3% range.

The 5th harmonic component of the original waveform is 3.02%, which is slightly larger than the recommended 3% limit. The optimized sequence shows a 3.86% value for the 5th harmonic. However, the converter is connected to the power system through a converter transformer, with a leakage reactance of 10%–15%. This converter transformer acts as a low-pass filter, which further reduces the harmonics. Therefore, a slight increase in the 5th harmonic is not expected to adversely affect the overall performance.

As can be seen, the optimized SVM sequence does not reduce all individual harmonic components. Instead, compared to the conventional SVM, it distributes the energy of the waveform more evenly among the harmonic components. This results in a smaller WTHD, which helps decrease the current distortion and hence lowers the propagation of harmonics through the network.

B. Sensitivity to Optimized Parameters

In practice, it is not always possible to produce the switching signals at the exact instants of time prescribed by the SVM algorithm. This could be due to a number of factors, such as the limited resolution of the digital implementation and numerical round offs (the number of bits of the microcontroller) and also device delays. Therefore, it is important to study the sensitivity of the results to small changes in the optimized SVM parameters. In the following, the impact of the variations of the time share of the zero vector $\mathbf{Z}_0$ on the harmonic performance of the SVM is studied. The analysis is done for a normalized sampling frequency of 36.

The magnitudes of harmonic components of the waveforms generated by the optimized sequence of $\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0$ for $\mathbf{Z}_0$-shares of 0.80, 0.85 (the optimized value), and 0.90 are shown in Table III. While WTHD in all three cases remains practically identical (at just above 2%), harmonic components slightly change with variations in the share of $\mathbf{Z}_0$ space vector. As observed in Fig. 8 and indicated in Table III, the magnitudes of harmonic components vary as the time share of $\mathbf{Z}_0$ deviates from its optimal value of 0.85.

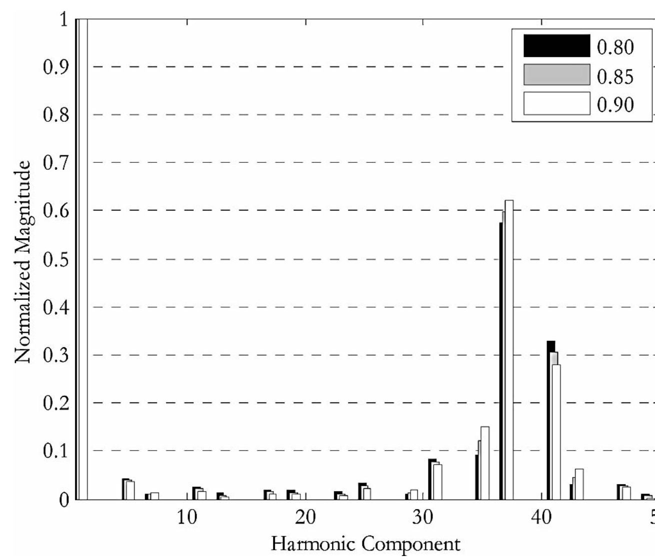

Fig. 8. Harmonic spectrum of the optimized SVM for the three values of $\mathbf{Z}_0$-share of 0.80, 0.85 (the optimized value), and 0.90.

For some midorder harmonics (e.g., 11 to 25), a decrease in the share results in a decrease in the magnitude of respective harmonic components, while some other components (e.g., 29, 35, and 37) see an increase with decreasing the value of $\mathbf{Z}_0$-share. This dual behavior results in the WTHD remaining essentially constant and maintains the balance between harmonic components. In other words, even with a slight change in $\mathbf{Z}_0$-share, the optimized SVM sequence is capable of distributing the energy of the waveform in such a way that individual changes in harmonic magnitudes cancel each other

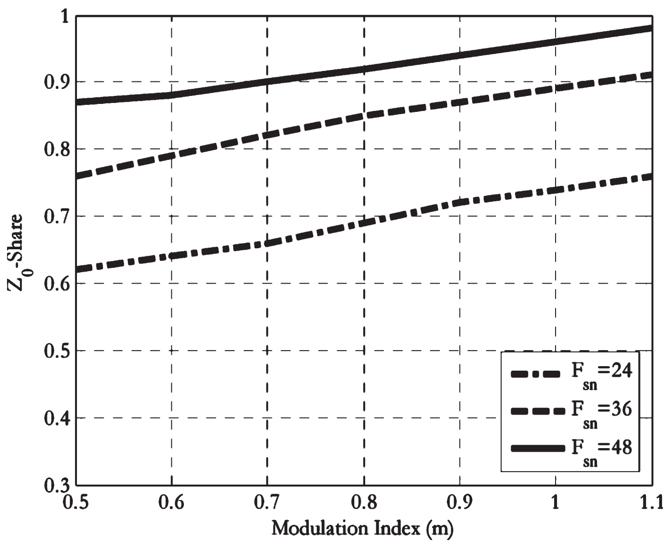

Fig. 9. Dependence of $\mathbf{Z}_0$-share on modulation index and normalized sampling frequency.

and result in an essentially constant WTHD. This relieves the implementation requirements of the optimized SVM scheme, as it demonstrates that the harmonics have limited sensitivity to potential variations of the time share that may result from its digital implementation. Note that insignificant sensitivity is valid only for small deviations in the $\mathbf{Z}_0$-share from its optimized values and large deviations do cause deterioration in the optimized harmonic spectrum.

C. Dependence on SVM Parameters (m and $F_{\rm sn}$)

The preceding results are reported for the specific case of a normalized sampling frequency $(F_{\rm sn})$ of 36 and a modulation index (m) of 0.8. However, it is important to determine whether the optimized sequence varies with the modulation index or the normalized sampling frequency. In this section, the dependence of results on various modulation indices and sampling frequencies is studied. Normalized sampling frequencies of 24, 36, and 48 are used for optimization, and best performing switching patterns for modulation indices of 0.5 to 1.1 are recorded.

The optimization results suggest that, in all cases, the optimized sequences are the same as the base case of $F_{\rm sn} = 36$ and $m = 0.8$, i.e., the sequence $\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0$ yields the optimal harmonic spectrum according to the OF defined in (8). The $\mathbf{Z}_0$-share, however, changes with a change in either sampling frequency or modulation index.

Fig. 9 shows the trend of change in the optimized value of the $\mathbf{Z}_0$-share as a function of the modulation index for the three values of normalized sampling frequency ($F_{\rm sn} = 24, 36,$ and 48). As shown, the $\mathbf{Z}_0$-share increases almost linearly (with a relatively small slope) with the modulation index. Considering that the optimized SVM sequence is not highly sensitive to small variations in the optimal value of the $\mathbf{Z}_0$-share (as shown in the preceding section for $F_{\rm sn} = 36$ and $m = 0.8$), it is possible to use average values for the $\mathbf{Z}_0$-share for various normalized sampling frequencies, as shown in Table IV.

TABLE IV
OPTIMIZED $\mathbf{Z}_0$-SHARE VALUES FOR DIFFERENT MODULATION INDICES AND SAMPLING FREQUENCIES OF SVM AND SUGGESTED VALUES

M	$\mathbf{Z}_0$-share		
	F_{SN}= 24	F_{SN}= 36	F_{SN}= 48
0.5	0.62	0.76	0.87
0.6	0.64	0.79	0.88
0.7	0.66	0.82	0.90
0.8	0.69	0.85	0.92
0.9	0.72	0.87	0.94
1.0	0.74	0.89	0.96
1.1	0.76	0.91	0.98
Suggested Value	**0.70**	**0.85**	**0.92**

D. Computational Aspects

The performance of the optimizer depends on GA parameters. In this optimization study, an initial population of 100 chromosomes is used, which is reduced to 50 after the first round of fitness evaluation. In each generation, top 30 best performing chromosomes are kept as parents, which are paired according to rank-weighted method and mated using one crossover point to generate 20 offspring. The maximum number of generations is set to 20. Chromosomes are subject to mutation with a probability of 10%, with the exception of the top two chromosomes that are kept as elite ones.

The optimizer is run using MATLAB (v.7.3) on a computer with a 2.41-GHz dual-core processor and 1 GB of RAM. Each run of the optimizer takes between 6 to 10 min, depending on the sampling frequency. Note that a GA optimizer normally evaluates a relatively large number of generations of potential solutions before converging to the optimum; this leads to typically long CPU times. The presence of real-valued real-coded (as opposed to discrete-coded) parameters, such as $\mathbf{Z}_0$-share, is often a contributor to a prolonged solution time.

It is worth noting that the GA optimization of the SVM sequence for multilevel converters is likely to be more time consuming than the present case of a two-level converter. This is due to the fact that the number of switch combinations in multilevel converters rapidly grows as the number of output voltage levels increases. This, however, does not have drastic implications, as the GA optimization is carried out as an offline procedure (see Appendix B for implementation details).

E. Note on Comparability to Other Specialized Sequences

The objective of determining the optimal sequence of vectors in this paper has been to distribute the energy of the voltage waveform more evenly among harmonic components such that it meets the requirements set out by the IEEE Standard 519. Other studies on specialized SVM sequences have considered the following: 1) current-sourced rectifiers [11]; 2) minimization of switching losses [11], [13]; 3) minimization of the zero-sequence component of the voltage waveform [14]; and 4) multilevel or cascaded converters [6], [7], [14]. Since different objectives are sought after in these studies, making a fair comparison between the performance of the vector sequence proposed in this paper and those of the other specialized switching sequences is not straightforward. The converter topology is another variable that affects the performance of a given SVM sequence.

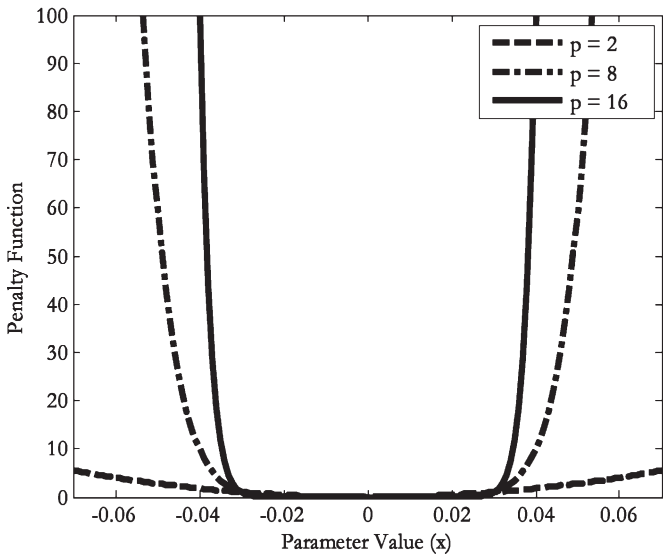

Fig. 10. Penalty function for the three exponents of 2, 8, and 16 ($d = 0.03$).

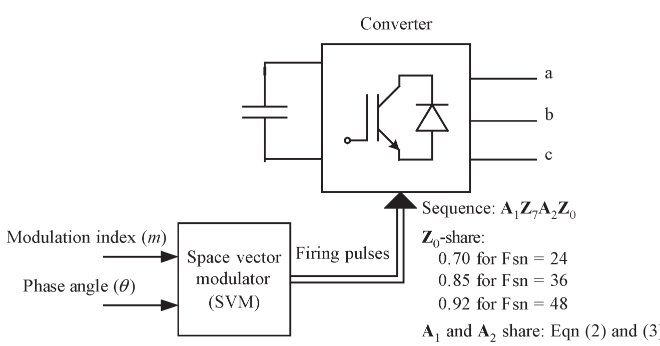

Fig. 11. Schematic diagram of the implementation of the optimized SVM.

Moreover, the majority of specialized vector sequences available in the literature are based on the manipulations of the switching sequences only; however, this paper also takes advantage of the $\mathbf{Z}_0$-share as an added degree of freedom. This realization adds to the flexibility of the algorithm and provides the optimizer with an expanded solution space.

VI. Conclusion

The possibility of applying GAs to obtain optimized SVM sequences has been investigated in this paper. The OF has been defined with the goal of minimizing the filtering requirement by lowering most significant harmonics while conforming to the available standards for voltage waveform quality.

The conventional SVM sequence is $\mathbf{Z}_0\mathbf{A}_1\mathbf{A}_2\mathbf{Z}_7$ with the duration of the zero vector shared equally between $\mathbf{Z}_0$ and $\mathbf{Z}_7$ (resulting a $\mathbf{Z}_0$-share of 0.5); the optimized sequence obtained using GA is $\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0$, for which the share of $\mathbf{Z}_0$ depends on the values of the normalized sampling frequency and modulation index. This sequence distributes the waveform energy more evenly throughout the spectrum, resulting in less harmonic pollution. By reducing the dominant harmonic components, the filtering requirements of the resulting waveform will be less. This can lead to more compact and potentially less expensive filters.

For operation at a normalized sampling frequency of 36 and a modulation index of 0.8, the optimizer resulted in a reduction of 44% for the 35th harmonic and 9% for the 37th harmonic. This paper also demonstrated that the optimized SVM sequence has insignificant sensitivity to small deviations in the $\mathbf{Z}_0$-share (due to such factors as digital implementation and discretization), which simplifies its digital implementation. Optimization was carried out for the three values of the normalized sampling frequency and for a range of modulation index values. It was shown that the same optimized sequence is valid for all sampling frequencies, while the $\mathbf{Z}_0$-share varies slightly with the modulation index, which makes use of average $\mathbf{Z}_0$-share values possible.

Appendix A
Penalty Function

The least pth interior point penalty functions are used to transform optimization problems with inequality constraints to unconstrained problems. This is done by augmenting the original OF and penalizing the points that do not comply with the constraints.

The general form of a least pth function is $\alpha(x/d)^p$. For optimization purposes, even p values are used. The value of such a function is small for $-d < x < d$ and is large otherwise. Therefore, although such a function allows x values larger than d (or less than $-d$), it penalizes them. The exponent p determines the level of penalization of the variable x. Fig. 10 shows the graphs of the function for the three values of p. As shown, large p values tend to more severely penalize deviations of the variable x outside the range specified by d. Note that, for $p = 2$, the least pth interior point penalty function reduces to the well-known least square method.

Appendix B
Implementation of Optimized SVM

The optimization of the SVM sequence using a GA is carried out offline, and results are obtained, as listed in Table IV. The actual implementation of the optimized SVM method is essentially identical to the conventional SVM except that a new sequence $(\mathbf{A}_1\mathbf{Z}_7\mathbf{A}_2\mathbf{Z}_0)$ is used along with a new $\mathbf{Z}_0$-share that depends on the normalized sampling frequency (Table IV). Fig. 11 shows a schematic diagram of the digital implementation of the optimized SVM.

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