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Committed Versus Contingent Pricing Under Competition

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Should capacitated firms set prices responsively to uncertain market conditions in a competitive environment? We study a duopoly selling differentiated substitutable products with fixed capacities under demand uncertainty, where firms can either *commit* to a fixed price *ex ante*, or elect to price *contingently ex post*, e.g., to charge high prices in booming markets, and low prices in slack markets. Interestingly, we analytically show that even for completely symmetric model primitives, *asymmetric* equilibria of strategic pricing decisions may arise, in which one firm commits statically and the other firm prices contingently; in this case, there also exists a unique mixed strategy equilibrium. Such equilibrium behavior tends to emerge, when capacity is ampler, and products are less differentiated or demand uncertainty is lower. With *asymmetric* fixed capacities, if demand uncertainty is low, a unique asymmetric equilibrium emerges, in which the firm with more capacity chooses committed pricing and the firm with less capacity chooses contingent pricing. We identify two countervailing profit effects of contingent pricing under competition: gains from responsively charging high price under high demand, and losses from *intensified* price competition under low demand. It is the latter detrimental effect that may prevent both firms from choosing a contingent pricing strategy in equilibrium. We show that the insights remain valid when capacity decisions are endogenized. We caution that responsive price changes under aggressive competition of less differentiated products can result in profit-killing discounting.

Key words: competition; contingent pricing; committed pricing; revenue management; demand uncertainty
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1. Introduction

Growing levels of demand uncertainty imply that firms should benefit from keeping pricing responsive. With capacity decisions made before the sales season, contingent pricing during the sales horizon allows firms to adjust pricing decisions in response to market conditions, so they can set higher prices in booming markets and charge less if demand is low. Such pricing responsiveness seems to provide firms with competitive advantage in battling with competitors. However, in the fiercely competitive marketplaces, many firms do not adjust prices in response to market conditions. For example, it is well known that Wal-Mart commits to bring customers products at "every day low prices" (EDLP). Take another example, on February 1, 2012, J.C. Penney, an apparel retailer, rolled out a "fair and square every day" pricing strategy that includes everyday, always great regular prices for a large set of clothing products (Businessweek 2012, Penney 2012). The mismatch between supply and demand can be huge for short lifecycle products. By committing themselves to fixed prices, EDLP firms inadvertently curb their abilities to respond contingently to changes in the competitive environment.

EDLP is reminiscent of the "value pricing" strategy that American Airlines implemented in 1992. At that time, the airline sought to roll out a fixed pricing strategy that eliminated possible rock-bottom discounting, when selling fixed amounts of aircraft seats before planes take off. Then what happened? The rest became history as rivals responded by offering contingent pricing and American Airlines quickly abandoned value pricing. Nowadays every single airline implements contingent pricing (it is more often called dynamic pricing in the airline industry, but here we emphasize the feature that prices are set contingent on the demand realization). Whether EDLP firms can sustain their committed pricing strategy seems puzzling. It makes one wonder under what conditions committed or contingent pricing strategies at the strategic level may emerge in equilibrium under competition. In this work, we provide some clues to this puzzle by building a stylized model with no externalities other than competition.

While EDLP firms may be at a competitive disadvantage when competitors run contingent sales promotions, there is certainly a rationale for this strategy. For instance, simple pricing relieves firms from the effort involved in filling Sunday circulars, simplifies consumers' decision making, generates customers loyalty, increases supply chain efficiency, and more importantly, weans shoppers away from expecting deep discounts. While admitting all of these reasons remain valid, we provide an alternative explanation from a competitive perspective. Wal-Mart and J.C. Penney's products are national brands that are fairly homogeneous. When selling a relatively undifferentiated product with capacity fixed before the sales season, the race-to-the-bottom price competition, should the market be sluggish, can be brutal. This *destructive* effect of joint contingent pricing under competition, should demand be low, may counteract the positive effect of profitably reacting to the market, should demand be high. In expectation, the firms may be better off by committing to a fixed price. This argument leaves alone the aspect that strategic consumers in anticipation of the rock-bottom prices may further intensify the price competition.

In particular, we consider a multi-stage duopoly game, where firms sell symmetrically *differentiated* products with given fixed capacities under demand uncertainty. In the first stage, firms can either choose to pre-commit to a fixed price, or elect to postpone pricing decisions in response to market conditions. In the subsequent stages, the following game plays out according to the strategic pricing decisions determined in the first stage: If both firms select the same committed or contingent pricing strategy, they simultaneously make pricing decisions, *ex ante* or *ex post*, respectively; if the two firms select different strategies, they play a sequential game of setting prices, with one *ex ante* and the other *ex post*.

We show that the endogenized strategic pricing decisions in equilibrium depend on product differentiation, supply capacity and demand uncertainty. Interestingly, even for completely symmetric model primitives, *asymmetric* equilibria of strategic pricing decisions may arise, in which one firm commits to a fixed price and the other sets prices contingently. This equilibrium outcome is more likely to occur, when capacity is *ampler*, products are *less differentiated* (including in the limiting case, almost homogeneous) or demand uncertainty is *lower*. However, with sufficiently *limited* symmetric capacities, regardless of product differentiability, contingent pricing always arises in equilibrium for *both* firms, though a prisoner's dilemma may occur, namely, both firms could have been better off by implementing committed pricing. The phenomena of asymmetric equilibrium and prisoner's dilemma are two sides of the same coin, resulted from the destructive effect of joint contingent pricing. These insights are further confirmed when the primitives are relaxed to be asymmetric, when capacities are endogenized and when there are more than one selling period. The case of ampler capacity and less differentiated products is consistent with J.C. Penney's current situation where national brands are carried with ample supply. Our result seems to support J.C. Penney's strategy of "fair and square pricing," if the marketplace has not dramatically changed after its switch from "high-low pricing" to EDLP.[1] Our result for the limited capacity case seems to predict well the prevalent practice of contingent pricing in the airline industry, in which firms have relatively scarce supply.

As mentioned, we identify that the asymmetric equilibrium behavior of strategic pricing decisions for symmetric capacities is driven by the detrimental effect of intensified price competition under joint contingent pricing, should demand be slack. If demand turns out to be high, firms always benefit from contingent pricing, which allows them to profitably respond to booming markets. The detrimental effect becomes unequal when the capacities of the firms are asymmetric. With asymmetric fixed capacities, if demand uncertainty is low, a unique equilibrium emerges in which the firm with more capacity chooses committed pricing and the firm with less capacity chooses contingent pricing. This prediction seems consistent with the observation that Wal-Mart, the behemoth in the retail sector with ample supply, practices the EDLP strategy, and other smaller competitive retailers such as Kmart, with less capital investment and more stringent supply conditions, are more likely to implement the contingent sales promotion strategy for seasonal products.

Though our model is stylized, the analytically delivered results are nontrivial, and the implied messages should be of interests to both academic researchers and practitioners. The rise of e-commerce, along with an explosion in data and the technology for analyzing it, has made it possible for price changes to be done more accurately, responsively, and faster than ever. For consumers, that could mean access to the best bargains. For firms, however, the risk is that the profit-killing discounting could expand, as they are forced to be more aggressive in responding to market conditions when battling with each other. To fight the competition, at the strategic level, firms should focus on shifting as much as possible to exclusive products that cannot easily be substituted. At the operational level, with more differentiated products, firms then can better enjoy contingent pricing by dynamically responding to market conditions.

2. Literature Review

Our setting can be viewed as a stylized revenue management (RM) model. RM studies how a firm can optimally sell a fixed amount of capacities over a finite horizon. If demand is exogenous, it is obvious that a monopolist is better equipped with contingent pricing to cope with demand uncertainty. However, committed pricing can be indeed advantageous over contingent pricing, if the firm sells to forward-looking customers who strategically time their purchases in anticipation of future discounts (Aviv and Pazgal 2008). On the other hand, Cachon and Swinney (2009) show that contingent pricing (pricing flexibility) can recover the advantage if coupled with quick response (inventory flexibility). Moreover, Cachon and Feldman (2013) identify another type of strategic consumer behavior that may give committed pricing an edge: given costly visits, uncertain prices under contingent pricing may cause consumers to avoid visiting the firm altogether. Isolated from forward-looking consumer behavior, we identify a detrimental effect of joint contingent pricing under competition, which may result in asymmetric strategic pricing decisions.

In the literature of RM under competition, Netessine and Shumsky (2005) examine one-shot quantity-based games of booking limit control. For an RM game of selling differentiated products, Lin and Sibdari (2009) study contingent pricing strategies in discrete time, and Gallego and Hu (2013) propose both committed (open-loop) and contingent (closed-loop) strategies in continuous time. Levin et al. (2009) present a unified stochastic dynamic pricing game of multiple firms where differentiated goods are sold to finite segments of forward-looking customers. Talluri and Martínez de Albéniz (2011) study perfect competition of a homogeneous product under demand uncertainty and derive a closed-form solution to the equilibrium price paths. In the literature of joint pricing and inventory control under competition, Van Mieghem and Dada (1999) study the benefits of production and price postponement strategies, with limited analysis of competitive models under quantity competition. Bernstein and Federgruen (2005) and Zhao and Atkins (2008) study the single period problem in the classic newsvendor framework where pricing and inventory decisions are determined before demand is realized. Bernstein and Federgruen (2004a, b) study periodic-review infinite-horizon oligopoly problems, but under such conditions they reduce to myopic single period problems where decisions in each period are made before demand is realized. Adida and Perakis (2010) study a make-to-stock manufacturing system where two firms compete through pre-commitment to a price path and an inventory control policy. None of these studies compare performances of committed and contingent pricing under competition.

In the operations management literature, some works do look into the co-existence of pros and cons of pricing or inventory flexibility under competition. Afèche et al. (2013) show that quick response may intensify price competition, should demand be *high*, and yield a net negative value. While Afèche et al. (2013) focus on the detrimental effect of volume flexibility under competition, we concentrate on pricing flexibility under competition with a fixed capacity. The identified detrimental effect of joint contingent pricing is caused by an intensified price competition, should demand be *low*. There are two papers that are closely related to ours: Xu and Hopp (2006) and Liu and Zhang (2013). Xu and Hopp (2006) study a two-stage capacity-pricing oligopoly game of selling a *homogeneous* product, where in the first stage firms build up capacities, and in the second stage they either pre-commit to a fixed price *ex ante*, or set prices contingent on demand realization in continuous time over a finite horizon. The authors argue that the downside of contingent pricing comes from overstocking in the initial ordering decisions. We complement their results by studying the differentiated product competition and identifying that the detrimental effect comes from the profit-killing competition should demand be low. More importantly, we characterize how *endogenized* strategic pricing decisions emerge, depending on fixed capacities, product differentiation and demand uncertainty.

Liu and Zhang (2013) study a duopoly selling asymmetrically differentiated products to a group of strategic customers with heterogeneous valuations on product quality, without considering capacity constraints and demand uncertainty. The authors numerically compare performances of a fixed-pricing strategy and a time-varying price-skimming strategy, given their closed-form expressions. They identify a detrimental effect of the joint price-skimming strategy, which comes from an on-going battle of competition over time (vs. only a one-shot competition if one commits to a fixed price for the whole horizon), and from induced strategic consumer waiting given a decreasing price path. There are several fundamental differences between Liu and Zhang (2013) and our paper. First, due to lack of demand uncertainty, both fixed-pricing strategy and time-varying price-skimming strategy in Liu and Zhang (2013) are pre-committed strategies, namely, the price path in these strategies is set before the game starts, whereas only under demand uncertainty can one compare pre-commitment and contingent strategy. Second, the mechanism behind the detrimental effect of the joint time-varying price-skimming strategy is different from that of joint contingent pricing under competition with demand uncertainty. We isolate the competition effect from other externalities such as forward-looking consumers, and analytically pinpoint the cause of the detrimental effect. Lastly, we further analytically investigate endogenized strategic pricing decisions.

There are many economic theories on price rigidity, see Kauffman and Lee (2004) for an extensive survey. As far as we know, none of them studies the endogenized strategic pricing choices between committed and contingent pricing from the competitive perspective under demand uncertainty. There is also an extensive literature in marketing on EDLP strategy – a strategy in which the retailer charges constant prices over time. In contrast, "Hi-Lo" strategy is a strategy in which higher prices are charged for most of time but frequent promotions are run with lower prices than the EDLP price. There are some studies on the co-existence of the two retailing strategies. For example, Lal and Rao (1997) explain it using the theory of market segmentation. In our work, we focus on the effect of demand uncertainty and competition and argue that one reason that a firm chooses a committed pricing strategy is to avoid the detrimental effect of joint contingent pricing under competition. On the empirical side of the marketing literature, Hoch et al. (1994) question about the sheer existence of EDLP. From a set of field experiments, the authors find that consumer demand does not respond much to changes in everyday prices and hence question about the profitability of EDLP as compared to Hi-Lo pricing. In our model, we assume the same demand function for both the committed and contingent pricing strategy. In contrast to our theory that supports asymmetric pricing strategy in equilibrium, Ellickson and Misra (2008) discover that firms may tend to choose EDLP or Hi-Lo pricing that agrees with their rivals. These works imply that experimental and empirical studies need to be carefully carried out to identify in specific settings what forces would emerge from competing theories as dominant effects.

3. Model

In the base model, we consider a symmetric duopoly selling differentiated substitutable products. Each firm i, $i = 1, 2$, has the same amount of fixed capacity x. (We will consider asymmetric capacities and endogenized capacity decisions in section 5.) The firms can only supply the market up to the given capacity level.

On the demand side, we adopt a symmetric linear demand structure and introduce demand uncertainty in the form of a binary additive shock, applied to both firms. That is, given prices p_1, p_2 of each firm, the random demands D_1 and D_2 are defined as follows (we use x^+ to denote $\max\{x,0\}$):

$$\begin{aligned} D_1(p_1, p_2) &= (c - p_1 + \gamma p_2 + \epsilon)^+, \\ D_2(p_1, p_2) &= (c - p_2 + \gamma p_1 + \epsilon)^+, \end{aligned} \tag{1}$$

where

$$\epsilon = \begin{cases} t & \text{(High demand)} \quad \text{with probability } \frac{1}{2} \\ -t & \text{(Low demand)} \quad \text{with probability } \frac{1}{2} \end{cases}. \tag{2}$$

The linear demand structure with additive shocks is widely used in the economics literature (see, e.g., Vives 1999). The parameter $\gamma \in [0,1)$ represents the degree of product differentiation, with $\gamma = 0$ meaning that products are perfectly differentiated, and γ approaching 1 meaning that products are *almost* homogeneous. We assume that γ is *ex ante* known to the firms, based on the notion that brand loyalty and price sensitivity are well understood. The demand shock ε can be interpreted as the market size uncertainty, due to a range of factors that equally affect differentiated products in the same product category, e.g., weather in the case of seasonal products or trend in the case of fashion apparels. The assumption of a two-point distribution with equal likely high and low demand realizations, is mainly made for the ease of analysis. A relaxation of this assumption does not change our main qualitative insights: It will be shown in Proposition 8 that joint contingent pricing under competition is still beneficial should demand be higher than expectation, and is detrimental should demand be lower than expectation; the distribution of demand uncertainty and the likelihood of high and low scenarios simply affect weights of the positive and negative side of the contingent pricing strategy in a competitive environment.

We assume that demand uncertainty will realize immediately after the start of the selling season. Taken literally, this captures a situation where firms gain demand information through factors other than their own early season sales, such as weather, market news and fashion trends. However, the model can also be viewed as a reasonable approximation of the settings in which sales that materialize before the prevalent pricing decisions only make up a small fraction of the capacity, but are still of significant value for demand forecasting. It is quite common that the forecast accuracy for the total season demand increases dramatically after observing a few days of early season sales (see, e.g., Fisher and Raman 1996).

Before the uncertainty unfolds, each firm has two choices for its strategic pricing decision: it could either commit to a price, which we call a *committed pricing* strategy or wait until the demand realizes, which we call a *contingent pricing* strategy. If a firm commits to a price, it can no longer change its price no matter which demand scenario realizes, while a firm adopting the contingent pricing strategy could set its price contingent on the demand realization. We assume that there is no salvage value for the unsold capacity, and the unmet demand for one firm will not spill over to the other. Both firms are risk neutral and each one aims to maximize the expected revenue during the selling season.

More specifically, we consider a multi-stage non-cooperative game between the two firms with the following sequence of events (see Figure 1). At stage 0, both firms simultaneously choose whether to use committed or contingent pricing strategy. At stage 1, any firm that chooses committed pricing strategy makes a price commitment. At stage 2, demand uncertainty unfolds; any firm that chooses contingent pricing strategy sets their prices. If both firms make decisions at the same stage, they do so simultaneously.

In the remainder of this paper, we denote the stage 0 actions by "S" and "C," corresponding to committed (S stands for committing to a "static" price) and contingent pricing, respectively. If firm i chooses action S in stage 0, its subsequent price decision is denoted by p_i^S; while if firm i chooses action C in stage 0, its subsequent price decisions are denoted by p_i^H when high demand realizes and p_i^L when low demand realizes. We are interested in the equilibrium strategy of the firms in this multi-stage game. In particular, we are interested in the equilibrium outcome in stage 0, that is, whether firms would prefer committed or contingent pricing strategies in equilibrium, and under what conditions they do so. In the next subsection, we formally define what an "equilibrium" means in this game. Then we study the equilibrium outcome in section 4. All proofs are relegated to the Online Appendix.

3.1. Subgame Perfect Equilibrium

We adopt the concept of *subgame perfect equilibrium* (SPE). A SPE is a strategy profile in which it is simultaneously a Nash equilibrium for every subgame of the initial game. In our model, there are three subgames that need to be analyzed. In the following, we define the SPE for each subgame. Conditional on whether the demand realizes as high or low, the revenue functions of the first firm when it uses price p_1 and the other firm uses price p_2 are:

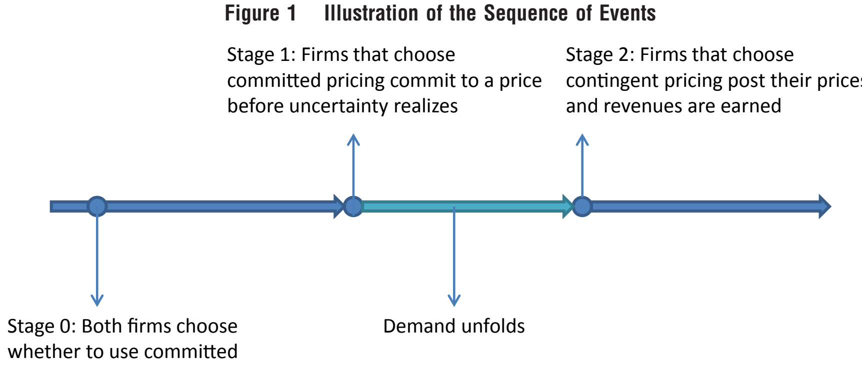
$$r^H(p_1, p_2) \equiv p_1 \min(x, (c - p_1 + \gamma p_2 + t)^+),$$
$$r^L(p_1, p_2) \equiv p_1 \min(x, (c - p_1 + \gamma p_2 - t)^+).$$

We first consider the subgame when both firms choose to use contingent pricing strategies in stage 0.There are two scenarios in this case (high demand and low demand). In each scenario, we can show that the equilibrium is symmetric and unique (see Proposition 1). We define p^H and p^L to be the equilibrium prices when high and low demand realizes, i.e., $p^H \in \arg\max_p r^H(p,p^H)$, $p^L \in \arg\max_p r^L(p,p^L)$. We also define $V^C \equiv V_1(C,C) = V_2(C,C) = \frac{1}{2}\left(r^H(p^H,p^H) + r^L(p^L,p^L)\right)$ to be the equilibrium revenue for each firm in this subgame.

Next, we consider the subgame when both firms choose to use committed pricing strategies in stage 0. For this subgame, we focus on symmetric equilibrium, and under some assumptions we show that the equilibrium is indeed unique and symmetric (see Proposition 2). By definition, (p^S, p^S) is a symmetric SPE for this subgame if and only if $p^S \in \arg\max_p \frac{1}{2}\left(r^H(p,p^S) + r^L(p,p^S)\right)$. We define $V^S \equiv V_1(S,\ S) = V_2(S,\ S) = \frac{1}{2}\left(r^H(p^S,p^S) + r^L(p^S,p^S)\right)$ to be the equilibrium revenue for each firm in this subgame.

Finally, we consider the subgame when one firm chooses to use committed pricing strategy, while the other chooses to use contingent pricing strategy.

Figure 1 Illustration of the Sequence of Events



Without loss of generality, we assume that firm 1 chooses committed pricing strategy, because the equilibrium prices and revenues in the other case would be the same only with the indices switched. By definition, a tuple $(p_1^*, p_2^{H^*}, p_2^{L^*})$ is a SPE if and only if

$$p_1^* = \arg\max_{p_1} \frac{1}{2}(r^H(p_1, p_2^H(p_1)) + r^L(p_1, p_2^L(p_1))), \tag{3}$$

$$\begin{aligned} p_2^H(p_1) &= \arg\max_{p_2^H} r^H(p_2^H, p_1), \\ p_2^L(p_1) &= \arg\max_{p_2^L} r^L(p_2^L, p_1), \end{aligned} \tag{4}$$

$$p_2^{H^*} = p_2^H(p_1^*), \quad p_2^{L^*} = p_2^L(p_1^*). \tag{5}$$

The last set of equations requires that $p_2^{H^*}$ and $p_2^{L^*}$ are the optimal responses of firm 2, given the committed price p_1^* chosen by the first firm, when high and low demand realizes, respectively; and the first equation requires that p_1^* is the optimal committed price, given that firm 1 anticipates firm 2 to optimally react to its price. We show in section 4.1.3 that there exists a unique SPE under some assumptions. The equilibrium revenues of this subgame are $V_1(S,\ C) = \frac{1}{2}\left(r^H(p_1^*, p_2^{H^*}) + r^L(p_1^*, p_2^{L^*})\right)$ for firm 1 and $V_2(S,\ C) = \frac{1}{2}\left(r^H(p_2^{H^*}, p_1^*) + r^L(p_2^{L^*}, p_1^*)\right)$ for firm 2. Similarly, we can define $V_1(C, S)$ and $V_2(C, S)$, and by symmetry, we have $V_1(C, S) = V_2(S, C)$ and $V_2(C, S) = V_1(S, C)$.

Now we have defined the equilibrium prices and revenues for each of the subgames after stage 0. At stage 0, both firms face a two-strategy game with the payoff matrix shown in Table 1. From Table 1, we could find the Nash equilibrium for the stage 0 game. We explore all possible equilibrium strategies, including mixed strategy equilibria. The following three situations may happen. First, if $V^S \geq V_1(C, S) = V_2(S, C)$, then (S, S) is an equilibrium. Second, if $V_1(C, S) = V_2(S, C) \geq V^S$ and $V_1(S, C) = V_2(C, S) \geq V^C$, then (S, C) and (C, S) are pure strategy equilibria, and moreover, there exists a unique mixed strategy equilibrium. Third, if $V^C \geq V_1(S, C) = V_2(C, S)$, then (C, C) is an equilibrium. The goal of the subsequent analysis is to identify conditions under which each outcome arises as a Nash equilibrium at stage 0, and to draw managerial insights from these results.

Table 1 Payoff Matrix of the Stage 0 Game

		Firm 2	
		Committed (S)	Contingent (C)
Firm 1	Committed (S)	V^S, V^S	$V_1(S, C)$, $V_2(S, C)$
	Contingent (C)	$V_1(C, S)$, $V_2(C, S)$	V^C, V^C

4. Equilibrium Behavior

In this section, we examine the equilibrium behavior of the strategic pricing game defined in section 3. We proceed by studying the equilibrium prices and revenues in each subgame, then we investigate the stage 0 equilibrium and discuss our findings.

4.1. Stage 1 Equilibrium

4.1.1 When Both Firms Choose Contingent Pricing. It suffices to study the subgame equilibrium under each demand realization in stage 2. The equilibrium expected revenue in stage 1 will be the average of the revenues from the two states of demand realization. To compute the equilibrium revenue under each demand realization, we first define a term that we will frequently use in our later discussions.

Definition 1. (Revenue-Maximizing and Capacity-Depleting Prices). Consider a firm selling a single product with capacity x. Given all competitors' prices fixed, suppose the demand function for this firm is $d(p)$ when this firm chooses price p. Denote the optimal price by p^* given the fixed capacity x, i.e., $p^* \in \arg\max_p p \cdot \min\{d(p), x\}$. We call p^* a *revenue-maximizing price* if $d(p^*) < x$ and p^* a *capacity-depleting price* if $d(p^*) \geq x$.

We have the following lemma showing the equilibrium prices and revenues when the demand function is deterministic.

Lemma 1. *Assume both firms have capacity x and compete on prices with demand functions: $d_1(p_1,p_2) = (c - p_1 + \gamma p_2)^+$, $d_2(p_1,p_2) = (c - p_2 + \gamma p_1)^+$. Then there is a unique Nash equilibrium given as follows:*

(i) (Ample Capacity) If $x \geq \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = p_2^* = \frac{c}{2-\gamma}$, which are revenue-maximizing prices. The equilibrium revenues are $\frac{c^2}{(2-\gamma)^2}$ for both firms.

(ii) (Limited Capacity) If $x < \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = p_2^* = \frac{c-x}{1-\gamma}$, which are capacity-depleting prices. The equilibrium revenues are $\frac{(c-x)x}{1-\gamma}$ for both firms.

As an immediate result of Lemma 1, we can obtain the expected equilibrium revenue in stage 1 after both firms choose contingent pricing strategy in stage 0, by considering whether revenue-maximizing prices or capacity-depleting prices are contingently used in equilibrium for each demand realization.

Proposition 1. *When both firms choose contingent pricing strategy at stage 0, there is a unique equilibrium at stage 1 that is symmetric, with expected revenues,*

Table 2 Equilibrium Prices and Revenues When Both Firms Choose Contingent Pricing

	Equilibrium revenue	High demand	Low demand
$x > \frac{c+t}{2-\gamma}$	$\frac{c^2+t^2}{(2-\gamma)^2}$	$p^H = \frac{c+t}{2-\gamma},\ R^H = \frac{(c+t)^2}{(2-\gamma)^2}$	$p^L = \frac{c-t}{2-\gamma},\ R^L = \frac{(c-t)^2}{(2-\gamma)^2}$
$\frac{c-t}{2-\gamma} < x \le \frac{c+t}{2-\gamma}$	$\frac{(c-t)^2}{2(2-\gamma)^2} + \frac{(c+t-x)x}{2(1-\gamma)}$	$p^H = \frac{c+t-x}{1-\gamma},\ R^H = \frac{(c+t-x)x}{1-\gamma}$	$p^L = \frac{c-t}{2-\gamma},\ R^L = \frac{(c-t)^2}{(2-\gamma)^2}$
$x \le \frac{c-t}{2-\gamma}$	$\frac{(c-x)x}{1-\gamma}$	$p^H = \frac{c+t-x}{1-\gamma},\ R^H = \frac{(c+t-x)x}{1-\gamma}$	$p^L = \frac{c-t-x}{1-\gamma},\ R^L = \frac{(c-t-x)x}{1-\gamma}$

conditional equilibrium prices and revenues on demand realization, shown in Table 2.

4.1.2 When Both Firms Choose Committed Pricing.

We have the following result.

PROPOSITION 2. *Assume $c \ge 3t$. When both firms choose committed pricing strategy at stage* 0*, there is a unique symmetric equilibrium at stage* 1 *with equilibrium prices and expected revenues shown in Table* 3*. Under a further assumption that $x \ge 2t$, all equilibria for this problem must be symmetric.*

Proposition 2 solves the Bertrand-Edgeworth pricing game for differentiated products under demand uncertainty, given symmetric linear demand structures. To our best knowledge, this is the first time such a game is solved, which may be of independent interest. In Proposition 2, the assumption $c \ge 3t$ guarantees that the demand is non-zero in equilibrium. This assumption that the potential market size (i.e., the market size when both firms set price at zero) is at least three times of the size of the demand shock, is usually satisfied in practical settings when the demand shock is moderate compared to the potential market size.

The four cases in Table 3 correspond to whether the capacity is cleared when demand turns out to be either high or low. Specifically, the first case is when there are excess capacities in both demand scenarios, the second case is when the capacity is exactly cleared when high demand realizes but has extra when low demand realizes, the third case is when the capacity is less than the demand when high demand realizes but has extra when low demand realizes, and the last case is when the capacity is less than the demand in both demand scenarios.

Table 3 Equilibrium Prices and Revenues When Both Firms Choose Committed Pricing

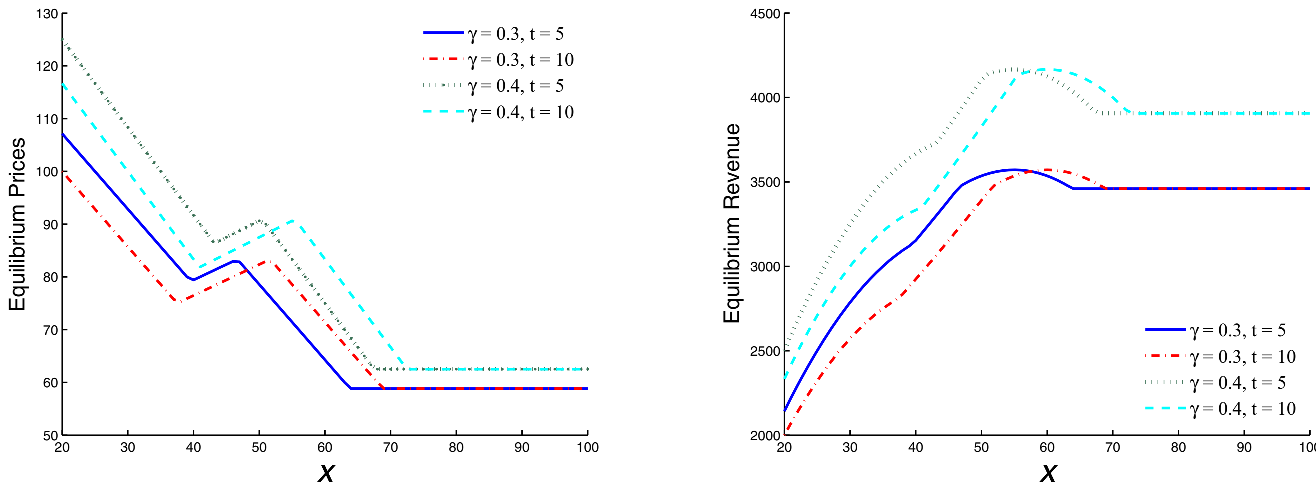
	Equilibrium price	Equilibrium revenue
$x > \frac{c}{2-\gamma} + t$	$\frac{c}{2-\gamma}$	$\frac{c^2}{(2-\gamma)^2}$
$\frac{c}{3-2\gamma} + t \le x \le \frac{c}{2-\gamma} + t$	$\frac{c+t-x}{1-\gamma}$	$\frac{(c+t-x)(x-t)}{1-\gamma}$
$\frac{c-t}{3-2\gamma} < x < \frac{c}{3-2\gamma} + t$	$\frac{c-t+x}{2-\gamma}$	$\frac{(c-t+x)^2}{2(2-\gamma)^2}$
$x \le \frac{c-t}{3-2\gamma}$	$\frac{c-t-x}{1-\gamma}$	$\frac{(c-t-x)x}{(1-\gamma)}$

There are several interesting observations from Table 3. Most notably, the equilibrium prices and expected revenues are *not monotone* in the capacity level, unlike in the monopoly case. The equilibrium price decreases in the capacity level x, except in an intermediate range $\frac{c-t}{3-2\gamma} < x < \frac{c}{3-2\gamma} + t$ (see Figure 2(a)). In this range, the firms could sell all its capacity at the equilibrium committed price under high demand but not under low demand; and the equilibrium price is the maximizer of the expected revenue $p \cdot (c-p-t+x+\gamma p)$, thus increases in x. Note that this behavior is unique due to the existence of demand uncertainty, as one can verify that when there is no demand uncertainty, the price would always decrease in the capacity level. Another observation is that the equilibrium expected revenue may be increasing as the capacity reduces (see Figure 2(b)). This is due to the competition. Indeed, one can verify that in a monopoly case, the optimal revenue always decreases as the capacity reduces. However, under competition, a decrease in the capacity would have two countervailing effects on profit: it forces the firm's own price up directly due to the stringent capacity, which incurs a loss; meanwhile, it alleviates the price competition, which incurs a gain. The gain outweighs the loss when both firms' equal capacity is just falling short of satisfying the uncapacitated equilibrium demand.

4.1.3. When Firm 1 Chooses Committed Pricing and Firm 2 Chooses Contingent Pricing.

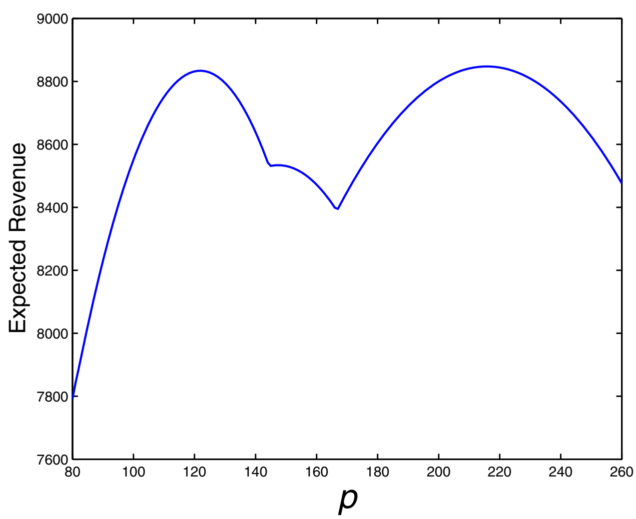
The conditions for a tuple $(p_1^*, p_2^{H*}, p_2^{L*})$ to be SPE prices are given by Equations (3)–(5). Unfortunately, solving Equations (3)–(5) analytically for all possible capacity

Figure 2 **Equilibrium Price and Revenue Curve When Both Firms Use Committed Pricing (c = 100)**



(a) Price Curve **(b)** Revenue Curve

Figure 3 **Revenue as a Function of the First Mover's Price (c = 100, t = 10, x = 100 and γ = 0.9)**



levels is difficult. For example, the optimization problem (3) is not necessarily concave in p_1. This non-concavity is inherent in price competition, coupled with the sequential nature of one firm's pre-commitment followed by the contingent policy of the other. Solving the optimal value requires discussions and comparisons piece-wisely (see Figure 3 for an example). In the following, we solve two cases of this problem, i.e., when the capacity is either sufficiently high or sufficiently low, and leave other cases to numerical studies.

Proposition 3. (*Ample Capacity*). *Assume $c \geq 3t$. If $x \geq h(\gamma)c+2t$, then when firm 1 chooses committed pricing strategy and firm 2 chooses contingent pricing strategy, the unique equilibrium prices are*

$$p_1^* = \frac{2+\gamma}{2(2-\gamma^2)}c,$$
$$p_2^{H*} = \frac{c+\gamma p_1^* + t}{2} = \frac{4+2\gamma-\gamma^2}{4(2-\gamma^2)}c + \frac{t}{2},$$
$$p_2^{L*} = \frac{c+\gamma p_1^* - t}{2} = \frac{4+2\gamma-\gamma^2}{4(2-\gamma^2)}c - \frac{t}{2},$$

with equilibrium revenues

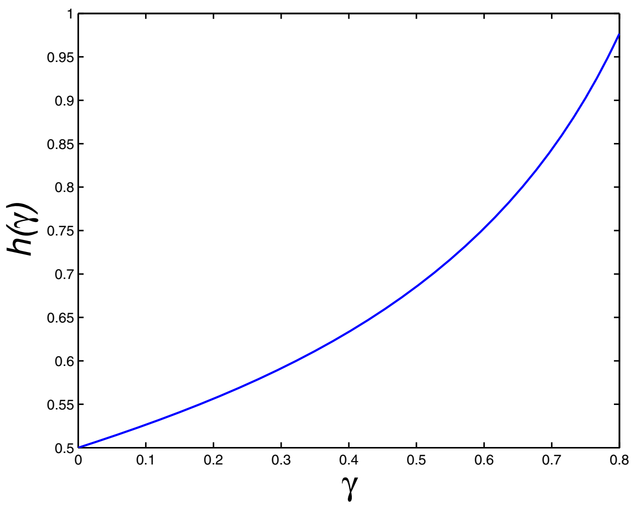
$$V_1(S,\ C) = \frac{(2+\gamma)^2}{8(2-\gamma^2)}c^2,$$
$$V_2(S,\ C) = \left(\frac{4+2\gamma-\gamma^2}{4(2-\gamma^2)}\right)^2 c^2 + \frac{t^2}{4}.$$

Here $h(\gamma) \equiv \max\{\gamma_1, \gamma_2, \gamma_3\}$ where

$$\gamma_1 \equiv \frac{1}{2} + \frac{\gamma(2+\gamma)}{4(2-\gamma^2)}, \qquad \gamma_2 \equiv \frac{1+\gamma}{\gamma} - \frac{(2+\gamma)\sqrt{1-\gamma^2}}{\gamma\sqrt{4-2\gamma^2}}$$
$$\text{and} \quad \gamma_3 \equiv \frac{8+4\gamma-3\gamma^2}{16-10\gamma^2}.$$

Figure 4 shows a plot of $h(\gamma)$. We can see that $h(\gamma)$ increases in γ, though the rate of increase is quite mild when γ is not too large. Proposition 3 considers a case in which the symmetric capacity is sufficiently high such that it is never optimal to clear the capacity. In this case, the firm that chooses contingent pricing strategy always uses revenue-maximizing prices, given either demand realization. The firm that chooses committed pricing strategy sets its committed price higher than the revenue-maximizing price of the other firm with contingent pricing, should

Figure 4 Plot of Function $h(\gamma)$



demand be low, but not necessarily lower than the other firm's revenue-maximizing price should demand be high.

Now we consider another case when the capacity is scarce.

Proposition 4. (Limited Capacity). *Assume $c \geq 3t$. If $x \leq \bar{x} \equiv \frac{1+\gamma}{3+\gamma}(c-t)$, then when firm 1 chooses committed pricing strategy and firm 2 chooses contingent pricing strategy, the unique equilibrium prices are*

$$p_1^* = \frac{c-x-t}{1-\gamma},$$
$$p_2^{H*} = c + \gamma p_1^* + t - x = \frac{c-x+(1-2\gamma)t}{1-\gamma},$$
$$p_2^{L*} = c + \gamma p_1^* - t - x = \frac{c-x-t}{1-\gamma},$$

with equilibrium revenues

$$V_1(S,\ C) = \frac{(c-x-t)x}{1-\gamma},$$
$$V_2(S,\ C) = \frac{(c-x-\gamma t)x}{1-\gamma}.$$

Proposition 4 considers a case in which the symmetric capacity is sufficiently low such that it is always optimal to clear the capacity, regardless of whether a firm chooses the committed or continent pricing strategy. In this case, the firm that chooses contingent pricing strategy always uses capacity-depleting prices, given either demand realization. The firm that chooses committed pricing strategy sets the committed price exactly equal to the capacity-depleting price when the demand is low and the other firm sets the optimal price. It can be seen from Proposition 4 that the expected revenue of the firm with contingent pricing is higher than that of the firm with precommitment.

4.2. Stage 0 Equilibrium

We have discussed the equilibrium of each subgame, and are ready to derive the stage 0 equilibrium. A precise statement can be made when the capacity is either sufficiently high or sufficiently low.

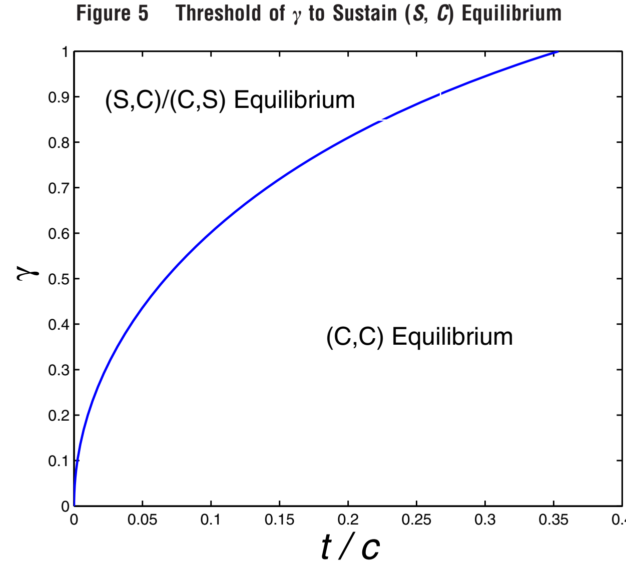
Proposition 5. (Ample Capacity). *Suppose $c \geq 3t$, $\gamma > 0$, $x \geq h(\gamma)c + 2t$, where $h(\gamma)$ is given in Proposition 3.*

(i) If $\frac{t}{c} \leq \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}$, then (S, C) and (C, S) are Nash equilibria in the stage 0 game. The firm that chooses C has a higher revenue than the firm that chooses S. At stage 2, conditional on the demand realization, the firm that chooses C uses a revenue-maximizing price. Moreover, there exists a unique mixed strategy equilibrium where firms randomize between S and C.

(ii) If $\frac{t}{c} > \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}$, then (C, C) is the unique Nash equilibrium in the stage 0 game. At stage 2, conditional on the demand realization, firms use revenue-maximizing prices. Moreover, there exists no mixed strategy equilibrium.

It is interesting to see that asymmetric equilibrium (namely, one firm chooses committed pricing strategy and the other firm adjusts its price depending on demand realization) naturally arises for completely symmetric model primitives. Such outcomes arise when product substitution is sufficiently high (i.e., γ is large, including "almost homogeneous" as a special case), or demand uncertainty is sufficiently low (i.e., t is small). The exact condition is given by Proposition 5 (see Figure 5 for the threshold on γ to sustain

Figure 5 Threshold of γ to Sustain (*S*, *C*) Equilibrium



asymmetric equilibria with respect to t/c). The intuition behind is as follows: When competition is intense with low demand uncertainty, one firm can optimally commit to a committed high price, to avoid potential fierce price competition under joint contingent pricing that otherwise might slash the profit of both firms should demand be low, while the loss from not being able to dynamically react to high demand is relatively small due to the low demand uncertainty. Meanwhile, the other firm, who sets prices contingently, also benefits from its competitor's pre-commitment. Thus, (S, C) and (C, S) are sustained as equilibria.

To take a closer examination of the asymmetric equilibria, $V_2(S, C)$ is always greater than $V_1(S, C)$. Therefore, the firms, if possible, always prefer to be the "follower" of choosing the contingent pricing strategy, but may be threatened by a looming price war such that eventually one firm would settle on making a price commitment. It is worth noting that such unfairness for the "leader" in a pure strategy equilibrium is not uncommon (e.g., see Osborne 1994 for the well-known Battle of the Sexes game and the game of Chicken). A unique mixed strategy equilibrium exists in this case, however, the resulting payoff is inefficient. To resolve this dilemma, one theoretic solution is to adopt the notion of *correlated equilibrium*, in which firms make their decisions based on some commonly observed signal and correlate their strategies based on the signal (see Fudenberg and Tirole 1991, section 2.2). The correlated equilibrium can maintain the efficiency of the outcome yet make the game relatively fair. Moreover, in practice, firms have costs of price adjustment, e.g., menu costs, managerial and customer costs (Zbaracki et al. 2004). With cost of price adjustment built into the model, the payoff matrix in Table 1 needs to be updated with the payoffs associated with the C action to be undercut by the cost. Even with a symmetric price-adjustment cost, (C, S) and (S, C) can still sustain as equilibria, with a property that the firm choosing C may have a lower revenue than the firm choosing S. This happens when the cost of price adjustment falls into an intermediate range (if the price-adjustment cost is less than $V_1(C, S)-V_2(C, S)$, we still have the asymmetric equilibrium outcomes with the C firm earning higher revenue than the S firm; if the cost is more than $V_1(C, S)-V^S$, the joint strategy (S, S) will become the unique equilibrium). In this case, the dilemma can be resolved because there is an incentive for firms to move "earlier" by announcing price commitment. Lastly, as we shall see in section 5.1, this dilemma of who has an incentive to move first can also be resolved when the firms start with asymmetric capacities.

On the other hand, if the competition is not intense or the demand uncertainty is high, then both firms will prefer to use the contingent pricing strategy which enables them to better react to demand shocks, while competition, even if demand is realized as low, will be mild. In this case, there will be no mixed strategy equilibrium.

When the capacity is sufficiently low, we find a slightly different result. That is, when γ, $t>0$ and $x \le \bar{x} \equiv \frac{1+\gamma}{3+\gamma}(c-t)$,

$$V_2(S,\ C) = \frac{(c-x-\gamma t)x}{1-\gamma} > \frac{(c-x-t)x}{1-\gamma} = V^S \quad \text{and}$$
$$V^C = \frac{(c-x)x}{1-\gamma} > \frac{(c-x-t)x}{1-\gamma} = V_1(S,\ C),$$

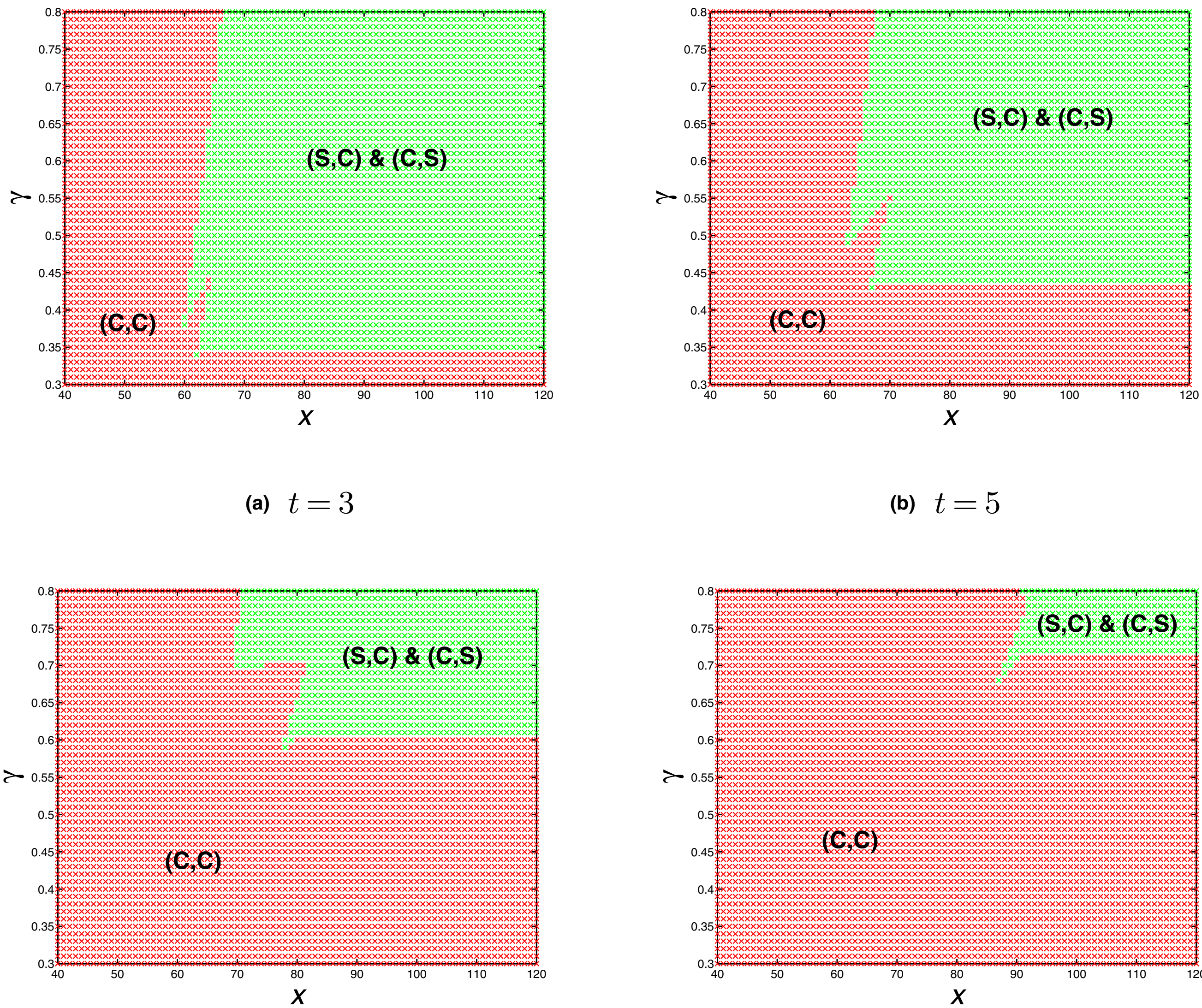
thus we have the following result.

Proposition 6. (*Limited Capacity*). *Assume $c \ge 3t$. If $x \le \bar{x} = \frac{1+\gamma}{3+\gamma}(c-t)$, then (C, C) is the unique Nash equilibrium in the stage 0 game. At stage 2, conditional on the demand realization, firms use capacity-depleting prices. Moreover, there exists no mixed strategy equilibrium.*

Proposition 6 says that when the capacity is scarce, it is always more critical to stay nimble to adjust to market conditions, in order to more profitably utilize limited capacity. Making price commitment to "preempt" intense competition becomes a secondary concern.

For the cases when the capacity x is neither high nor low, we are not able to obtain analytical results. Instead, we conduct numerical experiments to study the equilibrium outcomes in those cases. A representative experiment is illustrated in Figure 6 , where we fix $c = 100$ and vary demand uncertainty $t \in \{3, 5, 10, 15\}$. For each t, we test the degree of product differentiation γ ranging from 0.3 to 0.8 and the capacity level x from 40 to 120, and study the stage 0 equilibrium for each combination. In Figure 6, we identify regions where (S, C) and (C, S) are stage 0 equilibria. Consistent with Propositions 5 and 6, such equilibria arise when the capacity is ample and the product differentiability is low (i.e., γ is high); in all other cases, (C, C) is the unique Nash equilibrium. It never sustains in the equilibrium that both firms pre-commit to the committed pricing strategy. By observing Figure 6, we can see that the higher the demand uncertainty is, the higher capacity and higher product homogeneity it requires to sustain an asymmetric equilibrium. Furthermore, the results align well with the analytic threshold in Proposition 5, namely, the threshold on γ for (S, C) to be an equilibrium, when capacity x is ample, indeed satisfies $\frac{\gamma^2}{2\sqrt{4-2\gamma^2}} = \frac{t}{c}$. Another thing we observe in Figure 6 is that the region where (S, C) and (C, S) sustain as equilibrium is not convex, there is some irregular shape when γ is low and x is in the

Figure 6 Stage 0 Equilibrium Under Different Inputs



(a) $t = 3$ (b) $t = 5$ (c) $t = 10$ (d) $t = 15$

intermediate range. This is because when both firms choose committed pricing, the equilibrium revenue is not always increasing in x, as shown in Proposition 2. We also perform more extensive experiments with different primitives, and the results are similar. In particular, we see no instances where (S, S) could sustain in a stage 0 equilibrium. We conjecture this is always true, however, we are not able to prove it analytically.

4.3. Pareto Efficiency of the Equilibrium

In previous sections, we have derived the Nash equilibrium of the stage 0 game for the base model. In this section, we study the efficiency of such equilibria. We first investigate whether the equilibrium outcomes are Pareto efficient, and by doing so, we can exactly pinpoint the detrimental effect of joint contingent pricing under competition. We identify situations in which the firms are involved in a prisoner's dilemma: The Pareto optimal solution is for both firms to use the committed pricing strategy, however, the contingent pricing strategy is a dominant strategy. Finally, we briefly discuss the value of information in this competitive situation.

To study the Pareto efficiency of the stage 0 equilibrium, we note that if the equilibrium outcome is (S, C) or (C, S), then it must be Pareto efficient, due to the definition of the problem and the symmetry between joint strategy pairs (S, C) and (C, S). Therefore, we focus on the cases in which (C, C) is the equilibrium. In such cases, the attention is on whether the equilibrium revenue V^S when both firms choose the committed pricing strategy is higher than the equilibrium revenue V^C when both firms choose the contingent pricing strategy. We answer this question in the following proposition.

PROPOSITION 7. *We have the following comparison between V^C and V^S:*

(i) If $\frac{t}{c} \geq \frac{\gamma}{2-\gamma}$, then $V^C \geq V^S$.

(ii) If $\frac{t}{c} < \frac{\gamma}{2-\gamma}$, then there exists $x_l = x(\gamma, c, t) < x_u \equiv \frac{c}{2} + t + \frac{\sqrt{\gamma^2c^2 - 4t^2(1-\gamma)}}{2(2-\gamma)}$ such that when $x_l < x < x_u$, $V^S > V^C$. The definition of x_l is rather complicated, and we leave it to the proof of Proposition 7 in the Online Appendix.

Proposition 7 characterizes the situations when V^S is higher than V^C, or the other way around. When products are relatively highly differentiated (i.e., γ is small) or the demand uncertainty is relatively high (i.e., $\frac{t}{c} \geq \frac{\gamma}{2-\gamma}$), both firms are always better off using the contingent pricing strategy, which is in accordance with our results for the stage 0 equilibrium (see Proposition 5). In this case, the benefit of being able to react to demand shocks is dominant. However, if the homogeneity of the products is relatively high (i.e., γ is large) and the demand uncertainty is relatively low (i.e., $\frac{t}{c} < \frac{\gamma}{2-\gamma}$), then there exists an *intermediate* range of capacity levels such that the equilibrium revenues when both firms choose the committed pricing strategy are higher.

Since the mathematical formulas for the boundaries of the intervals are quite complicated, we conduct several numerical experiments to illustrate these intervals. The results are shown in Figure 7, where we fix $c = 100$ and vary demand uncertainty $t \in \{3, 5, 10, 15\}$. For each t, we draw the ranges of capacities such that $V^S > V^C$ for different γ's.

For γ's such that this range is empty, V^C is always greater than V^S, regardless of the capacity levels. We see that given demand uncertainty t, the competition parameter γ has to be greater than a certain threshold so that there exists some intermediate capacity level x that results in $V^S > V^C$. It can be verified that this threshold on γ's satisfies $\frac{\gamma}{2-\gamma} = \frac{t}{c}$, consistent with the result in Proposition 7. Moreover, the threshold increases as t increases, meaning that as demand uncertainty grows, it requires higher product homogeneity to justify the benefit of the joint committed pricing strategy.

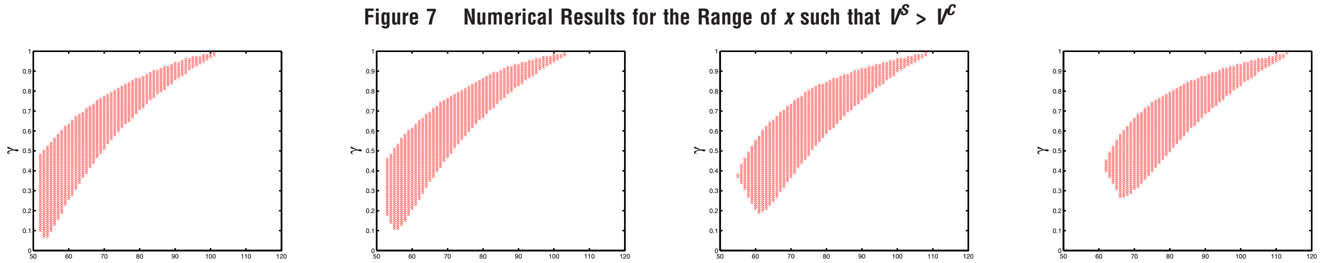
Combining the comparison of V^S and V^C with the next proposition, we can exactly pinpoint the cause of the detrimental effect of joint contingent pricing under competition.

PROPOSITION 8. *If demand realizes as high, the equilibrium revenue when both firms choose contingent pricing is always larger than that when both firms choose committed pricing.*

As Proposition 8 implies, the gain of the committed pricing strategy can only derive from the scenario when the demand realization is low. When low demand realizes, the competition between firms under joint contingent pricing can be cutthroat. Price commitment of both firms before the realization of the demand forces them to stay in a non-equilibrium price, which may alleviate the competition, should demand be low. On the other hand, when high demand realizes, the firms under pre-commitment cannot adjust prices to meet the market condition and thus suffer a loss. The former effect tends to outweigh the latter when the demand uncertainty or product differentiability is low, and vice versa.

Now we go back to the original question that whether the stage 0 equilibrium is Pareto efficient. The answer is "not always." For cases when the stage 0 equilibrium is (C, C) but $V^S > V^C$, the equilibrium is not Pareto efficient. Indeed, we identify such cases in our numerical experiments. Table 4 shows one example of such a situation where $c = 100$, $t = 5$, $x = 60$ and $\gamma = 0.3$. The unique pure strategy equilibrium is for both firms to choose the contingent pricing strategy, however, the revenues are strictly worse than those when both firms choose the committed pricing strategy. One may immediately identify that this is

Figure 7 Numerical Results for the Range of x such that $V^S > V^C$



(a) $t = 3$ **(b)** $t = 5$ **(c)** $t = 10$ **(d)** $t = 15$

Table 4. Payoff Matrix of the Stage 0 Game

		Firm 2	
		Committed	Contingent
Firm 1	Committed	(4125, 4125)	(3975, 4200)
	Contingent	(4200, 3975)	(4013, 4013)

exactly the same situation as the classic prisoner's dilemma. The firms that only play the game once may not want to cooperate (i.e., pre-commit to a price), although it is in their best interests to do so. However, if the stage 0 game is played repeatedly, a cooperative solution may arise. The analysis of the repeated game follows the standard game theory approach. We refer the readers to Fudenberg and Tirole (1991) for the related discussions.

Finally, we comment on the value of demand information in the stage 0 game. The concept of the *value of information* is used in decision sciences to measure the expected gain by the decision maker given certain information, and it can also be interpreted as the maximal amount one would be willing to pay for the information. It is well known that if there is only a single decision maker, the value of information cannot be less than zero, since the decision maker can always ignore the additional information and make decisions as if such information is not available (Ponssard 1976). However, in a competitive environment, this might not be true. Our stage 0 game is such an example. The expected equilibrium revenue when both firms know the demand information and competitively make decisions based on it may be less than that when both firms do not know the demand information.

5. Extensions

We next examine several extensions to our base model. In section 5.1, we relax the assumption that firms are symmetric in their capacities, and show that the firm with more capacity is more likely to engage in a price commitment. In section 5.2, we further explore the situation where the capacity decisions are endogenized rather than exogenously given. We show that our insights still hold. Then in section 5.3, we extend our model setup to a two-period competition model, and again, we show that our main insights remain valid.

5.1. Asymmetric Capacities

First, we study the case in which the firms have asymmetric capacity levels. We denote the capacity for firms 1 and 2 by x_1 and x_2, respectively. Without loss of generality, we assume $x_1 > x_2$. We investigate the stage 0 equilibrium and obtain the following result.

PROPOSITION 9.

(i) *Assume $c \geq 3t$. If $x_1 > x_2 \geq h(\gamma)c + 2t$, where $h(\gamma)$ is defined in Proposition 3, then (S, C) and (C, S) are Nash equilibria in the stage 0 game if $\frac{t}{c} \leq \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}$ and (C, C) is the unique Nash equilibrium in the stage 0 game if $\frac{t}{c} > \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}$.*

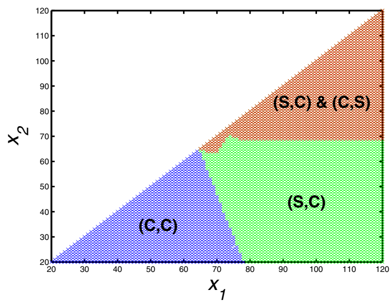
(ii) *Assume $c \geq 3t$. If $x_2 < x_1 \leq \frac{1+\gamma}{3+\gamma}(c-t)$, then (C, C) is the unique Nash equilibrium in the stage 0 game.*

(iii) *If $x_1 \geq \frac{c+t}{2(1-\gamma)}$ and $x_2 \leq \frac{2+\gamma}{6-2\gamma^2}(c-2t)$, then (S, C) is the unique Nash equilibrium in the stage 0 game if $t < \frac{((1+\gamma)c-\gamma x_2)\gamma^2}{2(1+\gamma)\sqrt{1-\gamma^2}}$, and (C, C) is the unique Nash equilibrium if $t > \frac{((1+\gamma)c-\gamma x_2)\gamma^2}{2(1+\gamma)\sqrt{1-\gamma^2}}$.*

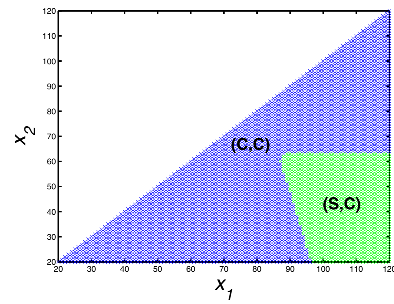
The first and second parts of Proposition 9 are extensions of Propositions 5 and 6, respectively. They show that the results in the ample/limited capacity cases can be extended to the asymmetric setting. The most interesting result in Proposition 9 is part (iii). It shows that a *unique* Nash equilibrium tends to emerge in the settings where one firm has ample capacity and the other has limited capacity. In particular, in this case, the firm with more capacity prefers to act first by committing to a price upfront, while the other firm with less capacity chooses to price contingently. This result seems to support the observation that large retailers with ample supply, e.g., Wal-Mart, tend to practice the EDLP strategy, while smaller retailers with stringent supply, e.g., Kmart, are more likely to run seasonal promotions. The rationale behind this result is that the firm with less capacity is more concerned about profitably utilizing its scarce capacity when random demand realizes, thus prefers to retain the flexibility to adjust its price based on the demand realization. On the other hand, the firm with higher capacity level is more concerned about the equilibrium pricing, should demand be low, while less concerned about matching supply with demand due to its excess capacity. By making a price commitment, the firm with more capacity could induce a relatively higher price from its competitor and thus save itself from brutal competition of undercutting prices, if the market turns out to be sluggish.

Similar to the difficulty we have for symmetric capacities, we are not able to obtain analytical results for other cases besides those shown in Proposition 9. Instead, we perform numerical experiments for those cases. We illustrate one set of numerical results in Figure 8, where we choose $c = 100$, $\gamma = 0.6$ and $t = 5$ as the base case, and vary one parameter between γ and t in each experiment. For each experiment, we study how the stage 0 equilibrium changes in the

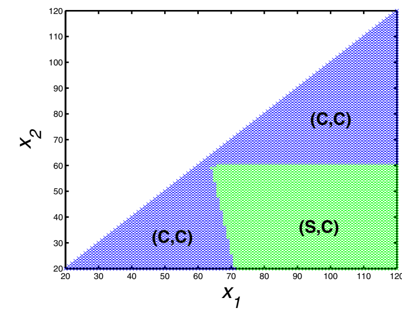
Figure 8 Stage 0 Equilibrium for Asymmetric Capacities



(a) $t = 5,\ \gamma = 0.6$



(b) $t = 15,\ \gamma = 0.6$



(c) $t = 5,\ \gamma = 0.4$

capacity levels (x_1,x_2). In the base case (see Figure 8(a)) where demand uncertainty and product differentiation are low, we can see that if the capacities of both firms are low, (*C*, *C*) is the unique equilibrium in stage 0 and if the capacities of both firms are high, both (*S*, *C*) and (*C*, *S*) are stage 0 equilibria. These results are similar to those with symmetric capacities. And when the capacity level of one firm is much higher than the other (the right lower region of the figure), the larger firm will choose to make a price commitment. Moreover, from Figure 8(b) and (c), we see that the (*C*, *C*) equilibrium is more likely to occur if t is relatively large or γ is relatively small. These observations are consistent with the analytical results shown in Proposition 9.

5.2. Endogenized Capacity Decision

In our base model, the capacity of each firm is assumed to be exogenously given, with only pricing-related decisions being made. In this section, we consider the situations where the capacity of each firm is also a decision variable. The main purpose of this section is to verify whether the main results in our base model can sustain when capacity decisions are endogenized.

In this section, we assume that there is a preceding stage before stage 0 in which the firms simultaneously make their capacity decisions. We denote the unit capacity cost of firm i, i = 1, 2, by c_i. After the capacities are determined, the pricing game described in section 3 follows. An illustration of this game is shown in Figure 9.

We are interested in the equilibrium behavior of the capacity-pricing game. In particular, we are interested in whether different equilibrium outcomes described in section 4 would emerge. One complication when we consider the endogenized capacity game is that the subsequent subgame may not necessarily have a unique equilibrium when (*S*, *C*) and (*C*, *S*) are both equilibria, thus the payoff of the subsequent game cannot be uniquely defined. The way we choose to deal with this issue is to consider a correlated equilibrium in which either equilibrium outcome is chosen with probability $\frac{1}{2}$ (see Fudenberg and Tirole 1991 for a discussion on the notion of correlated equilibrium). Indeed, this choice may be somewhat arbitrary, however, our task is to investigate whether our main insights for the subsequent pricing game still hold under some plausible assumptions. Moreover, we also conduct the same analysis under other equilibrium choices such as the mixed strategy equilibrium and obtain similar results.

Since the analysis of the endogenized capacity game is quite complicated, we adopt a numerical

Figure 9 Illustration of the Sequence of Events

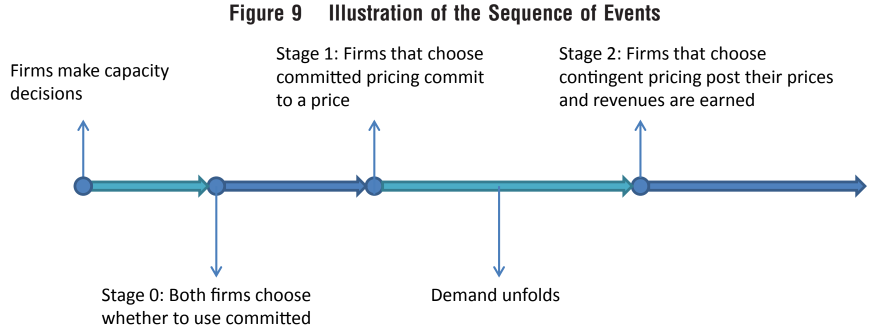
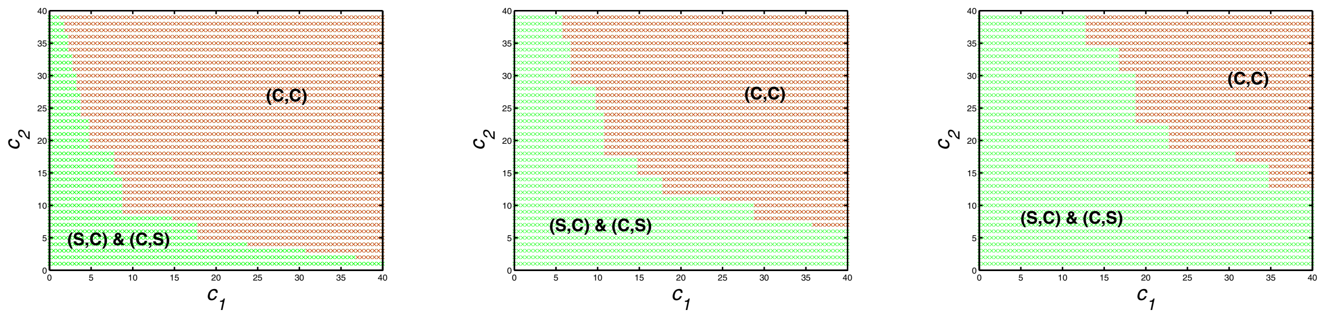


Figure 10 The Equilibrium Outcomes When the Capacity is Endogenized



(a) $\gamma = 0.7$ **(b)** $\gamma = 0.75$ **(c)** $\gamma = 0.8$

approach. In the following, we fix $c = 100$, $t = 5$ and vary $\gamma \in \{0.7,0.75,0.8\}$. For each γ, we consider different capacity costs, by varying each of c_1 and c_2 from 0 to 40. Then, for each combination of the primitives, we compute the equilibrium of the capacity game. We mark the scenarios by the induced equilibrium behavior in the subsequent pricing subgame. The results are shown in Figure 10.

In Figure 10, the lower left region corresponds to the cases when asymmetric equilibria arise in the pricing subgame, whereas the upper right region corresponds to the cases when (C, C) is the unique equilibrium in the pricing subgame. We can observe that for each choice of γ, when the capacity cost for at least one firm is low, asymmetric equilibria could arise in the subsequent pricing game. Intuitively, this is because when the capacity cost is low, a firm has an incentive to build more capacity, thus it is more likely to result in the ample capacity case of Proposition 9, where asymmetric equilibria of strategic pricing decisions arise. On the other hand, when the capacity costs for both firms are relatively high, a unique equilibrium (C, C) arises in the subsequent pricing game. This is because, in this case both firms will not build up a lot of capacity due to high capacity costs, thus it is more likely to fall into the limited capacity case of Proposition 9, where (C, C) arises as a unique equilibrium.

Finally, we observe from Figure 10 that as γ increases, it is easier to sustain asymmetric equilibria in the strategic pricing game. This is consistent with our discussion in section 4, where we conclude that asymmetric equilibria tend to emerge when capacity is ampler, products are less differentiated or demand uncertainty is lower. When γ is large, price competition when low demand realizes becomes fiercer, forcing firms to avoid joint contingent pricing.

5.3. Two-Period Model

In this section, we extend our base model to one with two selling periods. In this extension, after each firm selects whether to adopt the committed or contingent pricing strategy, the firm that chooses committed pricing must apply the same price for both selling periods, while the firm that chooses contingent pricing could change its price in the second period depending on the demand realization. An illustration of this model is shown in Figure 11.

In this model, we assume that the demand in the ith period, $i = 1, 2$, is: $D_1^i(p_1,p_2) = (c - p_1 + \gamma p_2 + \epsilon_i)^+$, $D_2^i(p_1,p_2) = (c - p_2 + \gamma p_1 + \epsilon_i)^+$, where ε_i is the demand shock in period i. The main goal of this

Figure 11 Illustration of the Sequence of Events in the Two-Period Model

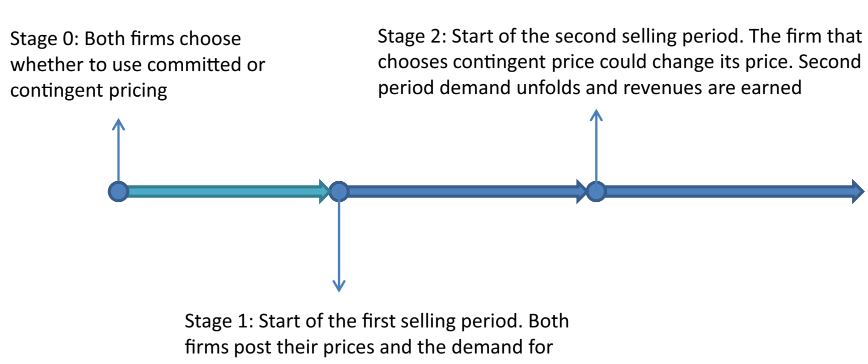
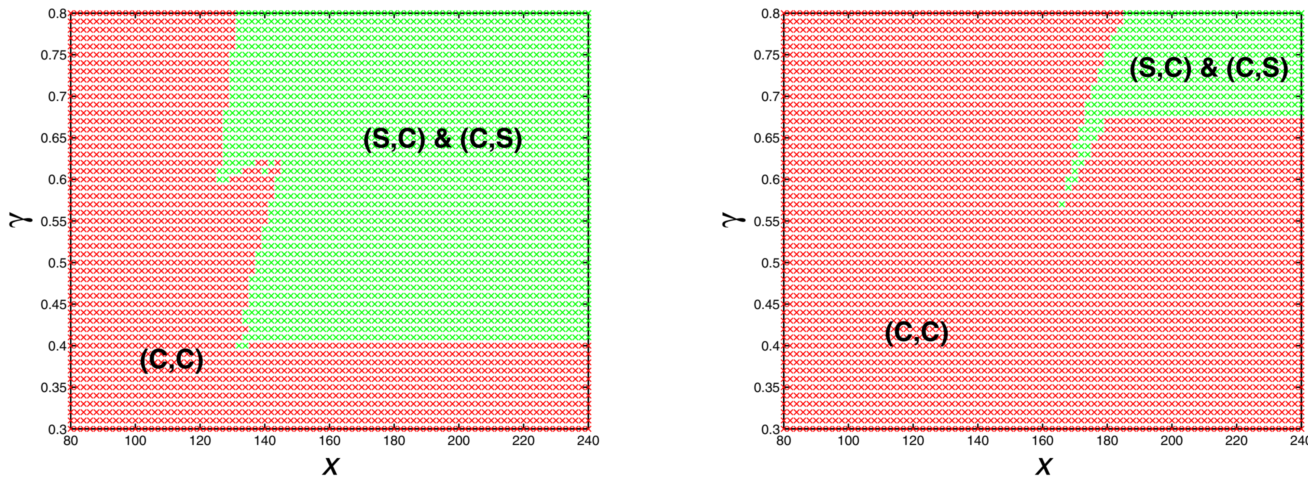


Figure 12 Stage 0 Equilibrium for the Two-Period Model



(a) $t = 5$

(b) $t = 15$

extension is to verify whether the insights of our base model still hold.

We obtain some analytical results for $\varepsilon_2 = 0$, i.e., when there is only demand shock in the first period (included in the Online Appendix). We further conduct numerical experiments to study the stage 0 equilibrium in the two-period model. In the numerical experiments, we assume $\varepsilon_1 = \varepsilon_2$. This means that the demand uncertainty for the second period can be inferred from that in the first time period. The results are shown in Figure 12, where we fix $c = 100$ and vary demand uncertainty $t \in \{5,15\}$. For each t, we test the degree of product differentiation factor γ ranging from 0.3 to 0.8 and the capacity level x ranging from 80 to 240. We then identify the stage 0 equilibrium in each combination of the primitives. We can see from Figure 12 that the results are consistent with those of our base model (see, e.g., Figure 6). That is, when the capacity level is high and the product differentiation is low (i.e., γ is large), the stage 0 equilibrium is asymmetric with one firm committing to a static price and the other choosing contingent pricing. In the other cases, both firms choosing contingent pricing is the unique Nash equilibrium.These results suggest that our stylized one-period model is quite robust in terms of capturing the key tradeoffs in similar strategic pricing games that may have more complicated sequence of events.

6. Conclusion

In this paper, we consider a duopoly price competition of selling differentiated products with capacity constraints and under demand uncertainty. We analyze firms' strategic pricing decisions: whether to commit to a price *ex ante* or delay pricing decisions *ex post*. We show that even for completely symmetric primitives, asymmetric equilibria, in which one firm pre-commits to a price and the other firm prices contingently, may arise in the equilibrium. The driving force behind such an endogenous price commitment is the detrimental effect of fierce price competition under joint contingent pricing, should demand be low. Such a detrimental effect tends to be more significant if capacity is higher, and product differentiation or demand uncertainty is lower, hence under these circumstances asymmetric equilibria are more likely to emerge. On the other hand, if capacity is more limited, and product differentiation or demand uncertainty is higher, a joint contingent pricing strategy is more likely to arise in equilibrium. Our results seem to be consistent with many industry practices.

The model can be extended to gain additional insights. First, one can extend our study to more than two firms carrying multiple products (see, e.g., Federgruen and Hu 2013). It is expected that there are possibly more than one firm choosing committed pricing strategy in equilibrium. As the number of firms increases, it is likely that more firms would choose contingent pricing in equilibrium. It is expected that the inter-firm cross-product substitution has a larger impact on firms' strategic pricing decisions than the intra-firm cross-product substitution.The exact dynamics of a multi-firm competition game with each firm carrying multiple products is worth of future study. Second, one can enrich the two-period model in section 5.3 by imposing practical price-trend constraints (see, e.g., Pang et al. 2013 and the references therein). In the apparel industry, the price trend over time is typically downward, with retailers tending to markdown unsold inventory towards the end of the sales horizon. If we impose such a constraint that the second-period price cannot be more than the first-period price, then one may expect that the first-period price competition would be alleviated. This is because a firm who adopts contingent pricing has an incentive to set a higher first-period price, to leave room for the possible high demand scenario in the second period, while it still can choose to markdown should the demand be low. This extension can serve as a good example to illustrate how practical constraints, other than precommitments, can help firms to alleviate price competition. Finally, Hu and Wang (2013) show that contingent pricing is more profitable than committed pricing for a monopoly who sells network goods under demand uncertainty. It would be interesting to compare the two strategies under competition of selling network goods.

There are several limitations of our model. For example, to simplify analysis, we assume that the two firms, each with one product, are symmetrically differentiated. In practice, firms usually carry multiproducts and these products can be vertically or horizontally differentiated. We also use a linear demand structure for tractability, but in the empirical studies, multinomial logit demand structures seem more prevalent.Despite these limitations, our stylized model captures the core tensions of how capacity, demand uncertainty and product differentiation may interplay in influencing strategic pricing decisions and thus may provide useful managerial insights for practitioners.

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Note

[1]J.C. Penney abandoned the strategy of "fair and square pricing" in April 2013 because this strategy appeared to alienate its core shoppers; in other words, its market structure, e.g., market size and price elasticity, was drastically different before and after the strategy shift (Time 2013). This cautions that the takeaways of the paper are applicable under certain assumptions, because our stylized model does not capture all the relevant aspects of real market dynamics.

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Supporting Information

Additional Supporting Information may be found in the online version of this article:

Appendix S1. Proofs