Online Appendix to
"Committed versus Contingent Pricing under Competition"

Proof of Lemma 1. (i) Without capacity constraints, price competition has equilibrium prices $\frac{c}{2-\gamma}$ and sales $\frac{c}{2-\gamma}$ for both firms. If $x \geq \frac{c}{2-\gamma}$, indeed $p_1^* = p_2^* = \frac{c}{2-\gamma}$ can be sustained as equilibrium prices. (ii) If $x < \frac{c}{2-\gamma}$, both firms set price to clear the market in equilibrium, i.e., $c - p_1 + \gamma p_2 = x$ and $c - p_2 + \gamma p_1 = x$, which we solve for $p_1^* = p_2^* = \frac{c-x}{1-\gamma}$. Equilibrium sales and revenues follow immediately. □

Proof of Proposition 1. The proposition follows by applying Lemma 1 to both high and low demand scenarios, and then computing the expected revenues. □

Proof of Proposition 2. We discuss different ranges of capacity levels. The four cases in Table 3 correspond to the cases of whether the capacity is cleared when demand turns out to be either high or low. For each case, we derive the Nash equilibrium. For (p^S, p^S) to be a symmetric equilibrium, we must have

$$p^S = \arg\max_p \frac{1}{2} p \left(\min(x, (c - p + \gamma p^S - t)^+) + \min(x, (c - p + \gamma p^S + t)^+)\right). \quad (0.1)$$

We first argue that we can remove the $(\cdot)^+$ in the discussion. It can be easily seen that at equilibrium the latter $(\cdot)^+$ cannot be active, because then the revenue will be 0 and cannot be optimal. If the first $(\cdot)^+$ is active, then we know that $p^S > c + \gamma p^S - t$, i.e., $p^S > \frac{c-t}{1-\gamma}$. However, in this case, p^S will not be the optimal response in (0.1) unless $p^S \leq \frac{c+t}{2+\gamma}$. Given that $c \geq 3t$, we have $\frac{c-t}{1-\gamma} \geq \frac{c+t}{2+\gamma}$, thus the $(\cdot)^+$ can not be active at equilibrium and we can safely remove the $(\cdot)^+$ operator in our following discussion. We consider several cases as follows:

- Case 1: $c - p^S + \gamma p^S + t < x$. In this case, the optimality condition is $p^S = \frac{c+\gamma p^S}{2}$, thus $p^S = \frac{c}{2-\gamma}$. To make the condition hold, x has to be greater than $\frac{c}{2-\gamma} + t$. Note that the right hand side of (0.1) (with $(\cdot)^+$ removed, the same applied to later discussions) is a concave function of p, therefore p^S is indeed the best response when the other firm chooses p^S. We can further compute the expected revenue from (0.1). Therefore, $p^S = \frac{c}{2-\gamma}$ is an equilibrium when $x > \frac{c}{2-\gamma} + t$ with equilibrium revenue $\frac{c^2}{(2-\gamma)^2}$.
- Case 2: $c - p^S + \gamma p^S - t \geq x$. In this case, the only possible equilibrium price is $p^S = \frac{c-t-x}{1-\gamma}$ (and this will make the equality hold). To show that this is indeed an equilibrium, we need to show that $p = p^S$ is the best response when the other firm chooses p^S, in particular, it is worse off to choose a larger p (there is clearly no benefit for choosing a smaller p). We consider the right gradient of the function $\left(r^H(p, p^S) + r^L(p, p^S)\right)$ at p^S. We have

$$\frac{\partial}{\partial p}\left(r^H(p, p^S) + r^L(p, p^S)\right)|_{p^S+} = c - 2p^S + \gamma p^S - t + x = \frac{3-2\gamma}{1-\gamma} x - \frac{c-t}{1-\gamma}.$$

Since $\left(r^H(p,p^S)+r^L(p,p^s)\right)$ is concave, p^S is the maximum if and only if $\frac{3-2\gamma}{1-\gamma}x - \frac{c-t}{1-\gamma} \le 0$. The revenue can be computed following (0.1). Therefore, $p^S = \frac{c-x-t}{1-\gamma}$ is the equilibrium price when $x \le \frac{c-t}{3-2\gamma}$ with revenue $\frac{(c-t-x)x}{1-\gamma}$.

- Case 3: $c-p^S+\gamma p^S - t < x < c - p^S + \gamma p^S + t$. In this case, the only possible symmetric equilibrium is $p^S = \frac{c-t+x}{2-\gamma}$. In order to make the condition hold, we must have $\frac{c-t}{3-2\gamma} < x < \frac{c}{3-2\gamma} + t$. Also since (0.1) is concave, p^S must be the best response when the other firm chooses p^S. Therefore, $p^S = \frac{c-t+x}{2-\gamma}$ is the equilibrium price when $\frac{c-t}{3-2\gamma} < x < \frac{c}{3-2\gamma} + t$ with revenue $\frac{(c-t+x)^2}{2(2-\gamma)^2}$.
- Case 4: $c-p^S+\gamma p^S + t = x$. In this case, $p^S = \frac{c+t-x}{1-\gamma}$. For this to be the equilibrium, we require

$$\frac{\partial}{\partial p}\left(r^H(p,p^S)+r^L(p,p^S)\right)|_{p^S-} \ge 0 \quad \text{and} \quad \frac{\partial}{\partial p}\left(r^H(p,p^S)+r^L(p,p^s)\right)|_{p^S+} \le 0.$$

We have

$$\frac{\partial}{\partial p}\left(r^H(p,p^S)+r^L(p,p^S)\right)|_{p^S-} = c - 2p^S + \gamma p^S - t + x = \frac{3-2\gamma}{1-\gamma}(x-t) - \frac{c}{1-\gamma}$$

and

$$\frac{\partial}{\partial p}\left(r^H(p,p^S)+r^L(p,p^S)\right)|_{p^S+} = c - 2p^S + \gamma p^S = \frac{2-\gamma}{1-\gamma}(x-t) - \frac{c}{1-\gamma}.$$

Therefore, $p^S = \frac{c+t-x}{1-\gamma}$ is an equilibrium when $\frac{c}{3-2\gamma} + t \le x \le \frac{c}{2-\gamma} + t$. The equilibrium revenue can be also computed following (0.1) which is $\frac{(c+t-x)(x-t)}{1-\gamma}$.

Next we provide a condition under which the equilibrium must be symmetric.

Claim: When $c \ge 3t$ and $x \ge 2t$, the pure strategy equilibrium of the game defined by

$$p_1 = \arg\max_p \frac{1}{2}\left(r^H(p,p_2)+r^L(p,p_2)\right) \tag{0.2}$$

$$p_2 = \arg\max_p \frac{1}{2}\left(r^H(p,p_1)+r^L(p,p_1)\right) \tag{0.3}$$

must be symmetric.

Proof. We show that p_1 defined in (0.2) is increasing in p_2 and p_2 defined in (0.3) is increasing in p_1. If this holds, then we can easily show that at equilibrium, $p_1 = p_2$. Otherwise, if $p_1 > p_2$, by the monotonicity of (0.2) and (0.3), we have

$$p_2 = \arg\max_p \frac{1}{2}\left(r^H(p,p_1)+r^L(p,p_1)\right) \ge \arg\max_p \frac{1}{2}\left(r^H(p,p_2)+r^L(p,p_2)\right) = p_1$$

which is a contradiction.

Now we show that p_1 defined in (0.2) increases in p_2 (the other part is the same). To this end, we show that $r^H(p,p')$ and $r^L(p,p')$ are supermodular in (p,p'). If this holds, then by Topkis's Theorem, the result holds.

To show that $r^H(p,p')$ and $r^L(p,p')$ are supermodular in (p,p'), we first show that under the assumption that $c \geq 3t$ and $x \geq 2t$,

$$\begin{aligned} &\arg\max_p \frac{1}{2}\left(r^H(p,p') + r^L(p,p')\right) \\ =& \arg\max_p \frac{1}{2}\left(p\min(x, c-p+\gamma p'+t)^+ + p\min(x, c-p+\gamma p'-t)^+\right) \\ =& \arg\max_p \frac{1}{2}\left(p\min(x, c-p+\gamma p'+t) + p\min(x, c-p+\gamma p-t)\right), \end{aligned} \tag{0.4}$$

that is, the $(\cdot)^+$ operator can be removed without changing the optimal solution. The first part is easy, since the firm will never choose a price such that the demand is negative under high demand. To show the second part, note that when the firm chooses price $p \geq c+\gamma p'-t$ (i.e., the second $(\cdot)^+$ operator is active), the demand of the first part is $\min(x, 2t)$. Under the assumption that $x \geq 2t$, the revenue function under the high demand is $p(c-p+\gamma p'+t)$. When $c \geq 3t$, the derivative of $p(c-p+\gamma p'+t)$ with respect to p at $p = c+\gamma p'-t$ is negative, meaning that reduce the price will always increase the overall revenue. Therefore, one can remove the second $(\cdot)^+$ in (0.4).

Lastly we show the function $p\min(x, c-p+\gamma p'+t) + p\min(x, c-p+\gamma p'-t)$ is supermodular in (p,p') on the positive orthant. We prove the supermodularity for the first term. The proof for the second term is similar and the supermodularity of the sum follows immediately.

To show $p\min(x, c-p+\gamma p'+t)$ is supermodular, it suffices to show that $\min(0, p(c-p+\gamma p'+t-x))$ is supermodular. We define $f(p,p') = p(c-p+\gamma p'+t-x)$. It is easy to see that $f(p,p')$ is supermodular. For $p_1 \geq p_2 \geq 0$ and $p'_1 \geq p'_2 \geq 0$, we consider

$$\min(0, f(p_1,p'_1)) + \min(0, f(p_2,p'_2)) - \min(0, f(p_1,p'_2)) - \min(0, f(p_2,p'_1)). \tag{0.5}$$

We consider several cases:

- If $f(p_1,p'_1) \leq 0$ and $f(p_2,p'_1) \leq 0$, then since f is increasing in p', we have $f(p_1,p'_2) \leq 0$ and $f(p_2,p'_2) \leq 0$, therefore (0.5) is nonnegative due to the supermodularity of f.
- If $f(p_1,p'_1) \leq 0$ and $f(p_2,p'_1) \geq 0$, then again since f is increasing in p', we must have $f(p_1,p'_2) \leq 0$. If $f(p_2,p'_2) \leq 0$, then the nonnegativity of (0.5) follows from the supermodularity of f and the fact that the last term is truncated at 0. And if $f(p_2,p'_2) \geq 0$, then the nonnegativity of (0.5) follows from the monotonicity of f in p'.

- If $f(p_1, p'_1) \geq 0$. then by the form of f, we must have $f(p_2, p'_1) \geq 0$. Therefore, (0.5) reduces to $\min(0, f(p_2, p'_2)) - \min(0, f(p_1, p'_2))$. If $f(p_1, p'_2) \geq 0$, then by the form of f, we must have $f(p_2, p'_2) \geq 0$, therefore (0.5) is nonnegative; if $f(p_1, p'_2) \leq 0$, we must have $f(p_2, p'_2) \geq f(p_1, p'_2)$.

Therefore (0.5) is also nonnegative and the claim is proved. □

Proof of Proposition 3. First, by using the same argument as in the proof of Proposition 1, we can safely remove the $(\cdot)^+$ operator in our discussions. Next we show that $p_1^* \in \arg\max_{p_1} \frac{1}{2}(r^H(p_1, p_2^H(p_1)) + r^L(p_1, p_2^L(p_1)))$ when $x \geq h(\gamma)c + 2t$. We first note that p_1^* is a local maximizer. This is because when $p_1 = p_1^*$, and $x \geq \gamma_1 c + 2t$, we have $p_2^H(p_1^*) = \frac{c+\gamma p_1^* + t}{2}$, $p_2^L(p_1^*) = \frac{c+\gamma p_1^* - t}{2}$ and $x \geq c - p_1^* + \gamma p_2^H(p_1^*) + t \geq c - p_1^* + \gamma p_2^L(p_1^*) - t$. Therefore, locally, the objective value function is $p(c - p + \gamma \frac{c+\gamma p}{2})$ and p_1^* is exactly its maximizer.

Now we want to show that it is also a global optimum. First we note that for all p such that

$$p_2^H(p) = \frac{c + \gamma p + t}{2}, \qquad p_2^L(p) = \frac{c + \gamma p - t}{2}, \tag{0.6}$$

the objective is smaller than that achieved by p_1^*. This is because that when the above equations hold, the objective function can be written as

$$p\left(\min(x, c - p + \gamma\frac{c + \gamma p + t}{2} + t) + \min(x, c - p + \gamma\frac{c + \gamma p - t}{2} - t)\right),$$

which is concave. And as we have argued, p_1^* is a local maximizer of the concave function thus achieves a greater value than all other p's.

Next we consider p that doesn't satisfy (0.6). We first consider the case when

$$p_2^H(p) = c + \gamma p + t - x, \qquad p_2^L(p) = c + \gamma p - t - x. \tag{0.7}$$

Note that in this case, $p \geq p_1^*$. Since $c - p + \gamma p_2^H(p)$ and $c - p + \gamma p_2^L(p)$ are both decreasing in p, we must still have $x \geq c - p + \gamma p_2^H(p) + t \geq c - p + \gamma p_2^L(p) - t$. Therefore, the objective function in this case is $p(c - p + \gamma(c + \gamma p - x))$, which achieves maximum at $p = \frac{(1+\gamma)c - \gamma x}{2(1-\gamma^2)}$ with an objective value of $\frac{((1+\gamma)c-\gamma x)^2}{4(1-\gamma^2)}$. When $x \geq \gamma_2 c$, we can verify that $\frac{((1+\gamma)c-\gamma x)^2}{4(1-\gamma^2)} \leq \frac{(2+\gamma)^2 c^2}{8(2-\gamma^2)}$. Thus, when $x \geq \gamma_2 c$, there is no p satisfying (0.7) that achieves a higher objective value than p_1^*.

Last, we consider p such that

$$p_2^H(p) = c + \gamma p + t - x, \qquad p_2^L(p) = \frac{c + \gamma p - t}{2}. \tag{0.8}$$

Note that if $p_2^H(p) = \frac{c+\gamma p+t}{2}$, then we must have $p_2^L(p) = \frac{c+\gamma p-t}{2}$. Therefore, (0.8) is the only remaining case to be discussed. Now we consider the objective in this case. By the same argument, we still have $x \geq c - p + \gamma p_2^H(p) + t \geq c - p + \gamma p_2^L(p) - t$ for all p in this range. Therefore, the objective function is $p(c-p+\frac{\gamma}{2}(c+\gamma p+t-x+\frac{1}{2}(c+\gamma p-t)))$, which achieves its maximum at $\tilde{p} = \frac{(1+\frac{3}{4}\gamma)c-\frac{\gamma}{2}x}{2(1-\frac{3}{4}\gamma^2)}$. However, when $x \geq \gamma_3 c + 2t$, $\tilde{p} < \frac{2x-c-t}{\gamma}$. This means that $\tilde{p}$ does not satisfy (0.8). Therefore, when $x \geq \gamma_3 c + 2t$, there is no local maximum in the range of (0.8). The maximum values obtained in this range is no higher than the maximum values in (0.6) and (0.7). Therefore, we proved that p_1^* is indeed the optimal solution, and $p_2^{H^*}$, $p_2^{L^*}$ follow as maximizers as well. And note that essentially the above argument showed that p_1^* is the unique maximizer to (3), and it is a sequential game, therefore, the equilibrium is also unique. □

Proof of Proposition 4. To show this, we need to show that

$$p_1^* = \arg\max_{p_1} \frac{1}{2}(r^H(p_1, p_2^H(p_1)) + r^L(p_1, p_2^L(p_1))) \tag{0.9}$$

when $x \leq \frac{1+\gamma}{3+\gamma}(c-t)$. Note that when $p_1 = p_1^*$ and $x \leq \frac{1+\gamma}{3+\gamma}(c-t)$, $p_2^H(p_1^*) = c + \gamma p_1^* + t - x, p_2^L(p_1^*) = c + \gamma p_1^* - t - x$ and $x = c - p_1^* + \gamma p_2^L(p_1^*) - t \leq c - p_1^* + \gamma p_2^H(p_1^*) + t$. For all $p < p_1^*$, the objective value is still $px < p_1^* x$. Therefore, p_1^* achieves higher revenue than all $p < p_1^*$. Now we consider $p > p_1^*$. Note that for all $p > p_1^*$, we still have $p_2^H(p) = c+\gamma p+t-x, p_2^L(p) = c+\gamma p-t-x$. Therefore the objective is $p \cdot (\min(x, c-p+\gamma(c+\gamma p+t-x)+t)+\min(x, c-p+\gamma(c+\gamma p-t-x)-t))$. Note that this is a concave function. And the right gradient at p_1^* is $(1+\gamma)(c-t)+(1-\gamma)x-2(1-\gamma^2)p_1^* = (3+\gamma)x-(1+\gamma)(c-t) \leq 0$ when $x \leq \frac{1+\gamma}{3+\gamma}(c-t)$. Therefore, p_1^* must be the maximizer of (0.9) and $p_2^{H^*}$, $p_2^{L^*}$ follows as optimal point as well. Again, by the above argument, p_1^* is the unique maximizer to (3), and it is a sequential game, therefore, the equilibrium is also unique. □

Proof of Proposition 5. Note $\gamma_1 > \frac{1}{2-\gamma}$. Therefore when $x \geq h(\gamma)c + 2t$, by Propositions 1, 2 and 3, we have

$$V_1(S,S) = V_2(S,S) = \frac{c^2}{(2-\gamma)^2}, \qquad V_1(C,C) = V_2(C,C) = \frac{c^2+t^2}{(2-\gamma)^2},$$

and

$$V_1(S,C) = V_2(C,S) = \frac{(2+\gamma)^2}{8(2-\gamma^2)}c^2, \qquad V_1(C,S) = V_2(S,C) = \left(\frac{4+2\gamma-\gamma^2}{4(2-\gamma^2)}\right)^2 c^2 + \frac{t^2}{4}.$$

By simple algebra, we have $V_2(S,C) > V_2(S,S)$ for all $\gamma > 0$, or equivalently $V_1(C,S) > V_1(S,S)$. We also have

$$V_1(S,C) - V_1(C,C) = \frac{\gamma^4}{8(2-\gamma^2)(2-\gamma)^2}c^2 - \frac{t^2}{(2-\gamma)^2}.$$

Therefore, $V_1(S,C) > V_1(C,C)$ if and only if $t < \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}c$. Equivalently, $V_2(C,S) \leq V_2(C,C)$ if and only if $t \geq \frac{\gamma^2}{2\sqrt{4-2\gamma^2}}c$. Therefore, the claims on the pure-strategy equilibria hold. Moreover, suppose firm 2 assigns probability weight q to S and probability weight $(1-q)$ to C. If a mixed strategy is a best response then each of the pure strategies involved in the mix must itself be a best response. In particular, each must yield the same expected payoff. Hence, in equilibrium, for firm 1, the payoffs of firm 1 against firm 2's mixed strategy must be the same regardless which strategy firm 1 plays, i.e.,

$$qV_1(S,S) + (1-q)V_1(S,C) = qV_1(C,S) + (1-q)V_1(C,C). \tag{0.10}$$

If $V_1(S,C) > V_1(C,C)$, then by $V_1(C,S) > V_1(S,S)$, the weight $q = 1/(1 + (V_1(C,S) - V_1(S,S))/(V_1(S,C) - V_1(C,C)))$ indeed belongs to the range $[0,1]$. Similarly, we can solve the weight in firm 1's mixed strategy in equilibrium. Since the weights are unique in each firm's strategy in equilibrium, there exists a unique mixed strategy equilibrium. If $V_1(S,C) \leq V_1(C,C)$, the weight satisfying (0.10) falls outside of $[0,1]$ and hence there exists no mixed strategy equilibrium. □

Proof of Propositions 7 and 8. We first define x_l as in Proposition 7: If $\frac{1}{3-2\gamma} > \frac{t}{c} \geq \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$ then

1. If $\frac{t}{c} \geq \frac{-2\gamma^2+4\gamma-1}{3-2\gamma}$, then $x_l = \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t$.
2. If $\frac{-2\gamma^2+4\gamma-1}{3-2\gamma} > \frac{t}{c} \geq \frac{(1-\gamma)(-\gamma^2+2\gamma+1)}{-\gamma^3+9\gamma^2-23\gamma+17}$, then $x_l = \frac{\gamma^2-2\gamma+2}{\gamma^2-5\gamma+5}c + \frac{\gamma^2-6\gamma+6}{\gamma^2-5\gamma+5}t$.
3. If $\frac{t}{c} < \frac{(1-\gamma)(-\gamma^2+2\gamma+1)}{-\gamma^3+9\gamma^2-23\gamma+17}$, then x_l is the larger root to the equation:

$$(2\gamma^2 - 9\gamma + 9)x^2 - (2\gamma^2 - 6\gamma + 6)cx + (1-\gamma)c^2 - 2(1-\gamma)ct - 2(1-\gamma)xt + (1-\gamma)t^2 = 0. \tag{0.11}$$

If $\frac{t}{c} < \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$, then

1. If $\frac{t}{c} \geq \frac{\gamma}{4-\gamma}$, then $x_l = \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t$.
2. If $\frac{t}{c} < \min\{\frac{1-2\gamma}{3-2\gamma}, \frac{\gamma}{4-\gamma}\}$, then $x_l = \frac{c+t}{2}$.
3. If $\frac{1-2\gamma}{3-2\gamma} < \frac{t}{c} < \frac{\gamma}{4-\gamma}$,then x_1 is the larger root to (0.11).

We note that the equilibrium revenues shown in Table 3 and Table 2 have different breakpoints. We first establish the following lemma that orders these breakpoints.

LEMMA 1. *For all input c, t and x, we have $\frac{c+t}{2-\gamma} \leq \frac{c}{2-\gamma} + t$ and $\frac{c-t}{3-2\gamma} \leq \frac{c-t}{2-\gamma}$. Furthermore,*

- *When $\frac{t}{c} \geq \frac{1}{3-2\gamma}$, $\frac{c-t}{2-\gamma} < \frac{c+t}{2-\gamma} \leq \frac{c}{3-2\gamma} + t$.*
- *When $\frac{1}{3-2\gamma} > \frac{t}{c} \geq \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$, $\frac{c-t}{2-\gamma} \leq \frac{c}{3-2\gamma} + t < \frac{c+t}{2-\gamma}$.*
- *When $\frac{t}{c} < \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$, $\frac{c}{3-2\gamma} + t < \frac{c-t}{2-\gamma} < \frac{c+t}{2-\gamma}$.*

We prove each case in Lemma 1 and then summarize the results in the end. We have the following claims.

Claim 1. *If* $\frac{t}{c} \geq \frac{1}{3-2\gamma}$, *then* $V^C \geq V^S$.

Claim 2. *If* $\frac{1}{3-2\gamma} > \frac{t}{c} \geq \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$, *then*

- *If* $\frac{t}{c} \geq \frac{\gamma}{2-\gamma}$, *then* $V^C \geq V^S$.
- *If* $\frac{t}{c} < \frac{\gamma}{2-\gamma}$, *then* $V^S > V^C$ *when* $x^* < x < \frac{c}{2} + t + \frac{\sqrt{\gamma^2c^2 - 4t^2(1-\gamma)}}{2(2-\gamma)}$ *and* $V^S \leq V^C$ *otherwise.*

Here x^* *is defined as follows:*

1. *If* $\frac{t}{c} \geq \frac{-2\gamma^2+4\gamma-1}{3-2\gamma}$, *then* $x^* = \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t$.
2. *If* $\frac{-2\gamma^2+4\gamma-1}{3-2\gamma} > \frac{t}{c} \geq \frac{(1-\gamma)(-\gamma^2+2\gamma+1)}{-\gamma^3+9\gamma^2-23\gamma+17}$, *then* $x^* = \frac{\gamma^2-2\gamma+2}{\gamma^2-5\gamma+5}c + \frac{\gamma^2-6\gamma+6}{\gamma^2-5\gamma+5}t$.
3. *If* $\frac{t}{c} < \frac{(1-\gamma)(-\gamma^2+2\gamma+1)}{-\gamma^3+9\gamma^2-23\gamma+17}$, *then* x^* *is the larger root to (0.11).*

Claim 3. *If* $\frac{t}{c} < \frac{1-\gamma}{3-\gamma} \cdot \frac{1}{3-2\gamma}$, *then*

- *If* $\frac{t}{c} \geq \frac{\gamma}{2-\gamma}$, *then* $V^C \geq V^S$.
- *If* $\frac{t}{c} < \frac{\gamma}{2-\gamma}$, *then* $V^S > V^C$ *when* $x^* < x < \frac{c}{2} + t + \frac{\sqrt{\gamma^2c^2 - 4t^2(1-\gamma)}}{2(2-\gamma)}$ *and* $V^S \leq V^C$ *otherwise.*

Here x^* *is defined as follows:*

1. *If* $\frac{t}{c} \geq \frac{\gamma}{4-\gamma}$, *then* $x^* = \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t$.
2. *If* $\frac{t}{c} < \min\{\frac{1-2\gamma}{3-2\gamma}, \frac{\gamma}{4-\gamma}\}$, *then* $x^* = \frac{c+t}{2}$.
3. *If* $\frac{1-2\gamma}{3-2\gamma} < \frac{t}{c} < \frac{\gamma}{4-\gamma}$ *,then* x^* *is the larger root to (0.11).*

Proof of Claim 1. We consider different cases for x.

- $x > \frac{c}{2-\gamma} + t$. In this case, $V^C = \frac{c^2+t^2}{(2-\gamma)^2} > \frac{c^2}{(2-\gamma)^2} = V^S$. And when high demand realizes, V^C is always higher than V^S. When low demand realizes, $V^C \geq V^S$ if and only if $t \geq \gamma c$.
- $\frac{c}{3-2\gamma} + t \leq x \leq \frac{c}{2-\gamma} + t$. In this case, we have:

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{c^2+t^2}{(2-\gamma)^2} = -\frac{(x-t-\frac{c}{2})^2}{1-\gamma} + \frac{\gamma^2c^2 - 4t^2(1-\gamma)}{4(1-\gamma)(2-\gamma)^2}. \quad (0.12)$$

Note that if $\gamma^2c^2 \leq 4t^2(1-\gamma)$, then $V^S \leq V^C$. Now we consider the case when $\gamma^2c^2 > 4t^2(1-\gamma)$. First we claim that $\gamma \geq 1/2$ in this case. Otherwise, $\gamma^2c^2 \leq \gamma^2(3-2\gamma)^2t^2 = (3\gamma-2\gamma^2)^2t^2 \leq t^2$ when $0 \leq \gamma \leq 1/2$, but $4(1-\gamma)t^2 > 2t^2$, which contradicts with the case assumption $\gamma^2c^2 > 4t^2(1-\gamma)$. Therefore, $\gamma \geq 1/2$ and thus $\frac{c}{3-2\gamma} + t \geq \frac{c}{2} + t$. Therefore, the maximum value of (0.12) must be obtained at $\frac{c}{3-2\gamma} + t$ given x in this case. We plug in $x = \frac{c}{3-2\gamma} + t$ in to (0.12) and get:

$$V^S - V^C = -\frac{(x-t-\frac{c}{2})^2}{1-\gamma} + \frac{\gamma^2c^2 - 4t^2(1-\gamma)}{4(1-\gamma)(2-\gamma)^2} = \frac{1}{(2-\gamma)^2}\left(\frac{4\gamma-2\gamma^2-1}{(3-2\gamma)^2}c^2 - t^2\right) \leq 0.$$

The last inequality is because $c \leq (3-2\gamma)t$ and $4\gamma - 2\gamma^2 - 1 \leq 1$. Therefore, in this interval, V^C is always greater than V^S. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c+t-x)x}{1-\gamma} - \frac{(c+t)^2}{(2-\gamma)^2} = -\frac{-(x-\frac{c+t}{2})^2}{1-\gamma} + \frac{\gamma^2(c+t)^2}{4(1-\gamma)(2-\gamma)^2} \geq 0. \quad (0.13)$$

And when the demand is low, we have

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t)^2}{(2-\gamma)^2} = -\frac{(x-\frac{c+3t}{2})^2}{1-\gamma} + \frac{\gamma^2(c-t)^2}{4(1-\gamma)(2-\gamma)^2}. \quad (0.14)$$

It can be shown that (0.14) is positive when $x \in [\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}, \frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}]$. If $t > \gamma c$, then R_C^L is always higher, however, if $\gamma c > t$, then R_S^L is greater for $x \in [\max\{\frac{c}{3-2\gamma} + t, \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t\}, \frac{c}{2-\gamma} + t]$.

- $\frac{c+t}{2-\gamma} < x < \frac{c}{3-2\gamma} + t$. We have

$$V^S - V^C = \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{c^2+t^2}{(2-\gamma)^2}. \quad (0.15)$$

Note that (0.15) is a convex function. And by the continuity of the revenue function in Table 3 and 2, we know that at $x = \frac{c}{3-2\gamma} + t$, (0.15) is smaller than 0. And at $x = 0$, (0.15) is also less than 0, therefore, by convexity, (0.15) is less than 0 for any x in this case. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t)^2}{(2-\gamma)^2} = \frac{(x-\frac{c-t}{2})^2}{2-\gamma} - \frac{(c-t)^2}{4(2-\gamma)} - \frac{(c+t)^2}{(2-\gamma)^2}, \quad (0.16)$$

which is a convex function and the minimizer is $\frac{c-t}{2} \leq \frac{c+t}{2-\gamma}$, therefore it is maximized at $\frac{c}{3-2\gamma} + t$ within this interval. However, (0.16) is negative at $x = \frac{c}{3-2\gamma} + t$. Therefore, R_C^H is always greater.

When the demand is low, we have

$$R_S^L - R_C^L = \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t)^2}{(2-\gamma)^2} = \frac{x}{(2-\gamma)^2}(-(1-\gamma)x + \gamma(c-t)). \quad (0.17)$$

Therefore, (0.17) is positive when $x \leq \frac{\gamma(c-t)}{1-\gamma}$. And it can be shown that $\frac{\gamma(c-t)}{1-\gamma} \leq \frac{c}{3-2\gamma} + t$ in the range we are considering. Therefore, R_S^L is larger if $x \in [\frac{c+t}{2-\gamma}, \frac{\gamma(c-t)}{1-\gamma}]$.

- $\frac{c-t}{2-\gamma} < x < \frac{c+t}{2-\gamma}$. We have:

$$V^S - V^C = \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{(c-t)^2}{2(2-\gamma)^2} - \frac{(c+t-x)x}{1-\gamma} = x\left\{\left(\frac{1}{(2-\gamma)^2} + \frac{1}{1-\gamma}\right)x - \frac{c+t}{1-\gamma} + \frac{2c-2t}{(2-\gamma)^2}\right\}.$$

This is maximized at $x = \frac{c+t}{2-\gamma}$. However, by the results in the previous part, we know that at $x = \frac{c+t}{2-\gamma}$, $V^S < V^C$. Therefore, we have $V^S < V^C$ for any x in this case. To decompose it into each sub-case, when the demand is high, we have

$$R^H_{PC} - R^H_{AD} = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t-x)x}{1-\gamma} = \frac{((3-2\gamma)x - c - (3-2\gamma)t)x}{(1-\gamma)(2-\gamma)} < 0.$$

Therefore when high demand realizes, V^C is always higher. When low demand realizes,

$$R^L_S - R^L_C = \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t)^2}{(2-\gamma)^2} = \frac{x}{(2-\gamma)^2}(-(1-\gamma)x + \gamma(c-t)).$$

Therefore, R^L_S is higher when $x \in [\frac{c-t}{2-\gamma}, \min(\frac{c+t}{2-\gamma}, \frac{\gamma(c-t)}{1-\gamma})]$.

- $\frac{c-t}{3-2\gamma} \le x < \frac{c-t}{2-\gamma}$. We have

$$V^S - V^C = \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{(c-x)x}{1-\gamma}. \tag{0.18}$$

Note that (0.18) is a convex function, and it is negative at $\frac{c-t}{2-\gamma}$ (by the argument in the previous part). We can also verify that it is negative at $\frac{c-t}{3-2\gamma}$. Therefore, $V^S < V^C$ for x in this range. To decompose it into each sub-case, when the demand is high, we have

$$R^H_S - R^H_C = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t-x)x}{1-\gamma} = x\left[\frac{(3-2\gamma)x - c - (3-2\gamma)t}{(2-\gamma)(1-\gamma)}\right], \tag{0.19}$$

which is always negative. Therefore, R^H_S is always less than R^H_C. When low demand realizes, we have

$$\begin{aligned} R^L_S - R^L_C &= \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t-x)x}{1-\gamma} \\ &= \frac{1}{(1-\gamma)(2-\gamma)^2}((3-2\gamma)x - (c-t))(x - (1-\gamma)(x-t)). \end{aligned} \tag{0.20}$$

When $\gamma > 1/2$, R^L_S is always greater than R^L_C in this range. When $\gamma \le 1/2$, R^L_S is greater than R^L_C if $x \in [(1-\gamma)(c-t), \frac{c-t}{2-\gamma}]$.

- $x < \frac{c-t}{3-2\gamma}$. It is obvious that in this case $V^S = \frac{(c-x-t)x}{1-\gamma} < \frac{(c-x)x}{1-\gamma} = V^C$, R^H_C is always greater than R^H_S, and R^L_C is always the same as R^L_S.

To summarize the results, when $c < (3-2\gamma)t$, V^C is always greater than V^S. And when high demand realizes, V^C is always higher. When low demand realizes, V^S is sometimes higher depending on the relationship between c, t and γ. □

Proof of Claim 2. We consider different cases for x.

- $x > \frac{c}{2-\gamma} + t$. In this case, $V^S = \frac{c^2}{(2-\gamma)^2} < \frac{c^2+t^2}{(2-\gamma)^2} = V^C$. And when high demand realizes, V^C is always higher; when low demand realizes, V^C is higher if and only if $t > \gamma c$.

- $\frac{c+t}{2-\gamma} \le x \le \frac{c}{2-\gamma} + t$. In this case, we have

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{c^2+t^2}{(2-\gamma)^2} = -\frac{(x-t-\frac{c}{2})^2}{1-\gamma} + \frac{\gamma^2 c^2 - 4t^2(1-\gamma)}{4(1-\gamma)(2-\gamma)^2}. \quad (0.21)$$

Only if $\gamma^2 c^2 > 4(1-\gamma)t^2$ can this be positive. Assume $\gamma^2 c^2 > 4(1-\gamma)t^2$, it is easy to see that $\gamma c > 2(1-\gamma)t$, which implies $\frac{c+t}{2-\gamma} > \frac{c}{2} + t$. Thus (0.21) is maximized at $x = \frac{c+t}{2-\gamma}$. Plugging it in, we have

$$V^S - V^C = -\frac{(x-t-\frac{c}{2})^2}{1-\gamma} + \frac{\gamma^2 c^2 - 4t^2(1-\gamma)}{4(1-\gamma)(2-\gamma)^2} = \frac{\gamma ct - (2-\gamma)t^2}{(2-\gamma)^2}.$$

which is positive only if $\gamma c > (2-\gamma)t$ (which implies $\gamma^2 c^2 > 4(1-\gamma)t^2$). Therefore we can conclude that in this case, when $\gamma c > (2-\gamma)t$ and $\frac{c+t}{2-\gamma} < x < t + \frac{c}{2} + \frac{\sqrt{\gamma^2 c^2 - 4t^2(1-\gamma)}}{2(2-\gamma)}$, then $V^S > V^C$. Otherwise, $V^C \ge V^S$. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c+t-x)x}{1-\gamma} - \frac{(c+t)^2}{(2-\gamma)^2} = -\frac{-(x-\frac{c+t}{2})^2}{1-\gamma} + \frac{\gamma^2(c+t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is always negative in this range. When the demand is low, we have

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t)^2}{(2-\gamma)^2} = -\frac{(x-\frac{c+3t}{2})^2}{1-\gamma} + \frac{\gamma^2(c-t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is positive when $x \in [\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}, \frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}]$. And this implies that R_S^L is higher when $x \in [\max\{\frac{c+t}{2-\gamma}, \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t\}, \frac{c}{2-\gamma} + t]$.

- $\frac{c}{3-2\gamma} < x \le \frac{c+t}{2-\gamma}$. In this case, we have

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{(c+t-x)x}{2(2-\gamma)} - \frac{(c-t)^2}{2(2-\gamma)^2} = \frac{(c-t)^2\gamma^2}{8(1-\gamma)(2-\gamma)^2} - \frac{(x-\frac{c+3t}{2})^2}{2(1-\gamma)},$$

which is positive when x is between $\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}$ and $\frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}$. Note that the positive root is always greater than $\frac{c+t}{2-\gamma}$. However, the negative root $\frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t$ is less than $\frac{c+t}{2-\gamma}$ only if $\gamma c > (2-\gamma)t$. Therefore, when $\gamma c > (2-\gamma)t$ and $\max\left\{\frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t, \frac{c}{3-2\gamma} + t\right\} < x < \frac{c+t}{2-\gamma}$, then $V^S > V^C$, otherwise $V^S \le V^C$. To decompose it into each sub-case, it is easy to see that when high demand realizes, $R_S^H = R_C^H$. When low demand realizes,

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t)^2}{(2-\gamma)^2} = -\frac{(x-\frac{c+3t}{2})^2}{1-\gamma} + \frac{\gamma^2(c-t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is positive when $x \in [\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}, \frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}]$. Therefore, R_S^L is higher when $x \in [\max\{\frac{c}{3-2\gamma} + t, \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t\}, \frac{c+t}{2-\gamma}]$.

- $\frac{c-t}{2-\gamma} < x < \frac{c}{3-2\gamma} + t$. In this case, we have

$$\begin{aligned} V^S - V^C &= \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{(c-t)^2}{2(2-\gamma)^2} - \frac{(c+t-x)x}{2(1-\gamma)} \\ &= \frac{x}{2(2-\gamma)^2(1-\gamma)} \left\{ (\gamma^2 - 5\gamma + 5)x - (\gamma^2 - 2\gamma + 2)c - (\gamma^2 - 6\gamma + 6)t \right\}. \end{aligned}$$

Therefore $V^S > V^C$ if and only if

$$x \geq x^* = \frac{\gamma^2 - 2\gamma + 2}{\gamma^2 - 5\gamma + 5} c + \frac{\gamma^2 - 6\gamma + 6}{\gamma^2 - 5\gamma + 5} t. \tag{0.22}$$

Now we want to consider several cases. First we compare the right hand side of (0.22) to $\frac{c}{3-2\gamma} + t$. We have that $x^* < \frac{c}{3-2\gamma} + t$ if and only if $(-2\gamma^2 + 4\gamma - 1)c > (3 - 2\gamma)t$. This is consistent with the previous result, i.e., it is also if and only if $\frac{1-\gamma}{2-\gamma} c + \frac{3-\gamma}{2-\gamma} t < \frac{c}{3-2\gamma}$. Therefore, the range for $V^S > V^C$ will be $(\max\{x^*, \frac{c-t}{2-\gamma}\}, \frac{c}{3-2\gamma} + t]$. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t-x)x}{1-\gamma} = \frac{((3-2\gamma)x - c - (3-2\gamma)t)x}{(1-\gamma)(2-\gamma)} < 0.$$

When low demand realizes, we have

$$R_S^L - R_C^L = \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t)^2}{(2-\gamma)^2} = \frac{x}{(2-\gamma)^2}(-(1-\gamma)x + \gamma(c-t)).$$

Therefore, R_S^L is higher when $x \in [\frac{c-t}{2-\gamma}, \min(\frac{c}{3-2\gamma} + t, \frac{\gamma(c-t)}{1-\gamma})]$.

- $\frac{c-t}{3-2\gamma} < x < \frac{c-t}{2-\gamma}$. In this case, we have

$$\begin{aligned} V^S - V^C &= \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{(c-x)x}{1-\gamma} \\ &= \frac{(2\gamma^2 - 9\gamma + 9)x^2 - (2\gamma^2 - 6\gamma + 6)cx + (1-\gamma)(c^2 - 2ct - 2xt + t^2)}{2(2-\gamma)^2(1-\gamma)}. \end{aligned} \tag{0.23}$$

Note (0.23) is a convex function and it is less than 0 at $x = \frac{c-t}{3-2\gamma}$ (because the continuity of the revenue function and the result in next part). And it is positive at $x = \frac{c-t}{2-\gamma}$ if and only if $c > \frac{-\gamma^3 + 9\gamma^2 - 23\gamma + 17}{(1-\gamma)(-\gamma^2 + 2\gamma + 1)} t$. Therefore, if $c > \frac{-\gamma^3 + 9\gamma^2 - 23\gamma + 17}{(1-\gamma)(-\gamma^2 + 2\gamma + 1)} t$, there is a unique root $x^* \in [\frac{c-t}{3-2\gamma}, \frac{c-t}{2-\gamma}]$ of (0.23). And $V^S > V^C$ if $x \in (x^*, \frac{c-t}{2-\gamma}]$. And if $c \leq \frac{-\gamma^3 + 9\gamma^2 - 23\gamma + 17}{(1-\gamma)(-\gamma^2 + 2\gamma + 1)} t$, then $V^S \leq V^C$ in this range.

To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t-x)x}{1-\gamma} = \frac{((3-2\gamma)x - c - (3-2\gamma)t)\,x}{(2-\gamma)(1-\gamma)},$$

which is always negative in this range. When the low demand realizes, we have

$$R_S^L - R_C^L = \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t-x)x}{1-\gamma} = \frac{((3-2\gamma)x-(c-t))(x-(1-\gamma)(x-t))}{(1-\gamma)(2-\gamma)^2}.$$

When $\gamma > 1/2$, R_S^L is always greater than R_C^L in this range. When $\gamma \le 1/2$, R_S^L is greater than R_C^L if $x \in [(1-\gamma)(c-t), \frac{c-t}{2-\gamma}]$ (could be empty set depending on the value of γ).

- $x \le \frac{c-t}{3-2\gamma}$. It is easy to see that $V^S < V^C$ in this range. And for this case, it is easy to see that R_C^H is always greater than R_S^H, and R_C^L is always the same as R_S^L. □

Proof of Claim 3. We consider different cases for x.

- $x \ge \frac{c}{2-\gamma} + t$. In this case, it is easy to see that $V^S \le V^C$. And when high demand realizes, V^C is always higher; when low demand realizes, V^C is higher if and only if $t > \gamma c$.
- $\frac{c+t}{2-\gamma} \le x \le \frac{c}{2-\gamma} + t$. In this case, we have

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{c^2+t^2}{(2-\gamma)^2} = -\frac{(x-t-\frac{c}{2})^2}{1-\gamma} + \frac{\gamma^2c^2 - 4t^2(1-\gamma)}{4(1-\gamma)(2-\gamma)^2}. \quad (0.24)$$

Similar to the proof of Claim 2, we must ensure that $\gamma^2c^2 \ge 4t^2(1-\gamma)$ in order for $V^S > V^C$, which also guarantees that $\frac{c+t}{2-\gamma} > \frac{c}{2} + t$. When $\frac{c+t}{2-\gamma} > \frac{c}{2} + t$, the maximum of (0.24) in this range is obtained at $x = \frac{c+t}{2-\gamma}$. We plug in $x = \frac{c+t}{2-\gamma}$ and find out that $V^S > V^C$ if and only if $\gamma c > (2-\gamma)t$ (which implies $\gamma^2c^2 > 4(1-\gamma)t$). Therefore, we can conclude that in this case, when

$$\gamma c > (2-\gamma)t \quad \text{and} \quad \frac{c+t}{2-\gamma} < x < t + \frac{c}{2} + \frac{\sqrt{\gamma^2c^2 - 4t^2(1-\gamma)}}{2(2-\gamma)},$$

$V^S > V^C$. Otherwise, $V^C \ge V^S$. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c+t-x)x}{1-\gamma} - \frac{(c+t)^2}{(2-\gamma)^2} = -\frac{-(x-\frac{c+t}{2})^2}{1-\gamma} + \frac{\gamma^2(c+t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is always negative in this range. When the demand is low, we have

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t)^2}{(2-\gamma)^2} = -\frac{(x-\frac{c+3t}{2})^2}{1-\gamma} + \frac{\gamma^2(c-t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is positive when $x \in [\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}, \frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}]$. Therefore, R_S^H is higher when $x \in [\max\{\frac{c+t}{2-\gamma}, \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t\}, \frac{c}{2-\gamma} + t]$.

- $\frac{c-t}{2-\gamma} \le x \le \frac{c+t}{2-\gamma}$. In this case, we have

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{(c-t)^2}{2(2-\gamma)^2} - \frac{(c+t-x)x}{2(1-\gamma)} = \frac{(c-t)^2\gamma^2}{8(1-\gamma)(2-\gamma)^2} - \frac{(x-\frac{c+3t}{2})^2}{2(1-\gamma)},$$

which is positive when x is between $\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}$ and $\frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}$. Note that the positive root is always greater than $\frac{c+t}{2-\gamma}$. And the negative root is less than $\frac{c+t}{2-\gamma}$ if $\gamma c > (2-\gamma)t$. Moreover, when $\gamma c < (4-\gamma)t$, we have the negative root is greater than $\frac{c-t}{2-\gamma}$ and when $\gamma c > (4-\gamma)t$, the negative root is less than $\frac{c-t}{2-\gamma}$. Therefore, when $c > \frac{4-\gamma}{\gamma}t$, $V^S > V^C$ for any capacity level x. When $\frac{2-\gamma}{\gamma} < c < \frac{4-\gamma}{\gamma}t$, $V^S > V^C$ if and only if $\frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t < x \leq \frac{c+t}{2-\gamma}$. And when $c < \frac{2-\gamma}{\gamma}t$, $V^C \geq V^S$ for all x in this range. To decompose it into each sub-case, it is easy to see that when high demand realizes, $R_S^H = R_C^H$. When low demand realizes, we have

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t)^2}{(2-\gamma)^2} = -\frac{(x-\frac{c+3t}{2})^2}{1-\gamma} + \frac{\gamma^2(c-t)^2}{4(1-\gamma)(2-\gamma)^2},$$

which is positive when $x \in [\frac{c+3t}{2} - \frac{\gamma(c-t)}{2(2-\gamma)}, \frac{c+3t}{2} + \frac{\gamma(c-t)}{2(2-\gamma)}]$. Therefore, R_S^L is higher when $x \in [\max\{\frac{c-t}{2-\gamma} + t, \frac{1-\gamma}{2-\gamma}c + \frac{3-\gamma}{2-\gamma}t\}, \frac{c+t}{2-\gamma}]$.

- $\frac{c}{3-2\gamma} + t < x < \frac{c-t}{2-\gamma}$. In this case, we have

$$V^S - V^C = \frac{(c+t-x)(x-t)}{1-\gamma} - \frac{(c-x)x}{1-\gamma} = \frac{2tx - t^2 - ct}{1-\gamma} = \frac{t}{1-\gamma}(2x-t-c),$$

which is positive if $x > \frac{c+t}{2}$. One can show that $\frac{c+t}{2} < \frac{c-t}{2-\gamma}$ if and only if $(4-\gamma)t < \gamma c$. And $\frac{c+t}{2} > \frac{c}{3-2\gamma} + t$ if and only if $(1-2\gamma)c > (3-2\gamma)t$. Therefore, if $(1-2\gamma)c \leq (3-2\gamma)t$, then $V^S > V^C$ for all x in this range, and if $(1-2\gamma)c > (3-2\gamma)t$, then $V^S > V^C$ when $x > \frac{t+c}{2}$ and $V^S \leq V^C$ otherwise.

To decompose it into each sub-case, it is easy to see that when high demand realizes, $R_S^H = R_C^H$. When low demand realizes,

$$R_S^L - R_C^L = \frac{(c+t-x)(x-2t)}{1-\gamma} - \frac{(c-t-x)x}{1-\gamma} = \frac{2t(2x-c-t)}{1-\gamma}.$$

Therefore, R_S^L is higher if $x > \frac{c+t}{2}$.

- $\frac{c-t}{3-2\gamma} < x < \frac{c}{3-2\gamma} + t$. In this case, we have:

$$\begin{aligned} V^S - V^C &= \frac{(c-t+x)^2}{2(2-\gamma)^2} - \frac{(c-x)x}{1-\gamma} \\ &= \frac{(2\gamma^2 - 9\gamma + 9)x^2 - (2\gamma^2 - 6\gamma - 6)cx + (1-\gamma)(c^2 - 2ct - 2xt + t^2)}{2(2-\gamma)^2(1-\gamma)}, \end{aligned} \quad (0.25)$$

which is convex and less than 0 at $x = \frac{c-t}{3-2\gamma}$. And it is positive at $\frac{c}{3-2\gamma} + t$ if and only if $(1-2\gamma)c < (3-2\gamma)t$. Denote the larger root of (0.25) by x^*. Therefore, when $(1-2\gamma)c \geq$

$(3-2\gamma)t$, $V^C \geq V^S$ for all x in this range and when $(1-2\gamma)c < (3-2\gamma)t$, $V^S > V^C$ if $x \in (x^*, \frac{c}{3-2\gamma}+t]$. To decompose it into each sub-case, when the demand is high, we have

$$R_S^H - R_C^H = \frac{(c-t+x)x}{2-\gamma} - \frac{(c+t-x)x}{1-\gamma} = x\left[\frac{(3-2\gamma)x - c - (3-2\gamma)t}{(2-\gamma)(1-\gamma)}\right],$$

which is always negative in this range. When low demand realizes,

$$\begin{aligned} R_S^L - R_C^L &= \frac{(c-t+x)(c-t-(1-\gamma)x)}{(2-\gamma)^2} - \frac{(c-t-x)x}{1-\gamma} \\ &= \frac{1}{(1-\gamma)(2-\gamma)^2}((3-2\gamma)x-(c-t))(x-(1-\gamma)(x-t)). \end{aligned}$$

When $\gamma > 1/2$, R_S^L is always greater than R_C^L in this range. When $\gamma \leq 1/2$, R_S^L is greater than R_C^L if $x \in [(1-\gamma)(c-t), \frac{c}{3-2\gamma}+T]$ (could be empty set depending on the value of γ).

- $x \leq \frac{c-t}{3-2\gamma}$. It is easy to see that $V^S < V^C$ in this range. And for this case, it is easy to see that R_C^H is always greater than R_S^H, and R_C^L is always the same as R_S^L. □

Proof of Proposition 9. Part (i): The proof of this part is similar to the proof of Proposition 5. We first show the following lemma:

LEMMA 2. *Assume the capacities for the firms are x_1 and x_2, and the demand functions are*

$$D_1(p_1,p_2) = (c - ap_1 + a\gamma p_2)^+ \text{ and } D_2(p_1,p_2) = (c - ap_2 + a\gamma p_1)^+.$$

Then in a one-period game, the unique Nash equilibrium pricing and equilibrium revenue are as follows:

- *Case 1: If $x_1 \geq \frac{c}{2-\gamma}$ and $x_2 \geq \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = p_2^* = \frac{c}{(2-\gamma)}$ and the equilibrium revenue is $\frac{c^2}{(2-\gamma)^2}$ for both firms. In this case, both firms use the revenue-maximizing price.*
- *Case 2: If $x_1 < \frac{c}{2-\gamma}$ and $x_2 \geq \frac{c+\gamma(c-x_1)}{2-\gamma^2}$, then the equilibrium prices are $p_1^* = \frac{c\gamma+2(c-x_1)}{(2-\gamma^2)}$ and $p_2^* = \frac{c+\gamma(c-x_1)}{(2-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(c\gamma+2(c-x_1))x_1}{(2-\gamma^2)}$ and $v_2^* = \frac{(c+\gamma(c-x_1))^2}{(2-\gamma^2)^2}$. In this case, the first firm uses the capacity-depleting price and the second firm uses revenue-maximizing price.*
- *Case 3: If $x_1 \geq \frac{c+\gamma(c-x_2)}{2-\gamma^2}$ and $x_2 < \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = \frac{c+\gamma(c-x_2)}{(2-\gamma^2)}$ and $p_2^* = \frac{c\gamma+2(c-x_2)}{(2-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(c+\gamma(c-x_2))^2}{(2-\gamma^2)^2}$ and $v_2^* = \frac{(c\gamma+2(c-x_2))x_2}{(2-\gamma^2)}$. In this case, the second firm uses the capacity-depleting price and the first firm uses revenue-maximizing price.*
- *Case 4: Lastly, if $\gamma x_1 + (2-\gamma^2)x_2 < (1+\gamma)c$ and $\gamma x_2 + (2-\gamma^2)x_1 < (1+\gamma)c$, then the equilibrium prices are $p_1^* = \frac{\gamma(c-x_2)+c-x_1}{(1-\gamma^2)}$ and $p_2^* = \frac{\gamma(c-x_1)+c-x_2}{(1-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(\gamma(c-x_2)+c-x_1)x_1}{(1-\gamma^2)}$ and $v_2^* = \frac{(\gamma(c-x_1)+c-x_2)x_2}{(1-\gamma^2)}$. In this case, both firms are using the capacity-depleting policy.*

The proof of Lemma 2 is very similar to that of Proposition 1 and is omitted for the sake of space.

Therefore, we know that when $x_1 > x_2 \geq \frac{c}{2-\gamma} + t$, the equilibrium revenue for both firms are $\frac{c^2}{(2-\gamma)^2}$ when both of them choose contingent pricing. And furthermore, by Proposition 3, when $x_1 > x_2 \geq h(\gamma)c + 2t$, and one firm chooses committed pricing and the other one chooses contingent pricing, we must have that the capacity is not binding under either demand realizations, and thus the equilibrium is as stated in Proposition 3. Therefore, by Proposition 5, we have the first part of Proposition 5.

Part (ii): We first study the equilibrium revenues for both firms when both firms choose contingent pricing strategy. By Lemma 2, we know that when

$$\gamma x_1 + (2-\gamma^2)x_2 < (1+\gamma)(c-\epsilon) \text{ and } \gamma x_2 + (2-\gamma^2)x_1 < (1+\gamma)(c-\epsilon), \qquad (0.26)$$

the expected revenues are

$$V_1(C,C) = \frac{(\gamma(c-x_2)+c-x_1)x_1}{1-\gamma^2} \text{ and } V_2(C,C) = \frac{(\gamma(c-x_1)+c-x_2)x_2}{1-\gamma^2}.$$

And if $x_2 < x_1 \leq \frac{1+\gamma}{3+\gamma}(c-\epsilon)$, we know that condition (0.26) holds. Thus the equilibrium revenues are as specified.

Next, we study the equilibrium revenues for both firms when firm 1 chooses committed pricing and firm 2 chooses contingent pricing. We show that when $x_2 < x_1 \leq \frac{1+\gamma}{3+\gamma}(c-\epsilon)$, the equilibrium revenues for firm 1 is $V_1(S,C) = \frac{(\gamma(c-x_2)+c-x_1-(1+\gamma)\epsilon)x_1}{1-\gamma^2} < V_1(C,C)$, and thus (C,C) is the Nash equilibrium. To show this, we first show that at equilibrium, all firms will use capacity-depleting price. We find that in order to show that the firms are using capacity-depleting prices at equilibrium, it is equivalent as showing that at optimal prices p_1^*, p_2^H and p_2^L, we have

$$x_2 \leq \frac{c-\epsilon+\gamma p_1^*}{2} \text{ and } x_1 \leq \frac{c+\gamma p_2^L-\epsilon}{3}.$$

To find the optimal prices, we obtain the following conditions:

$$p_1 = c + \gamma p_2^L - \epsilon - x_1 \text{ and } p_2 = c + \gamma p_1 - \epsilon - x_2.$$

Solving the above conditions and combining with the condition that $x_1, x_2 \leq \frac{1+\gamma}{3+\gamma}(c-\epsilon)$, we can verify that the optimality conditions of p_1 and p_2 indeed hold. Similarly, we can also show that $V_2(C,S) = \frac{(\gamma(c-x_1)+c-x_2-(1+\gamma)\epsilon)x_2}{1-\gamma^2} < V_2(C,C)$. Thus, under the conditions of part 2, (C,C) is the Nash equilibrium.

Part (iii): We study the subgame equilibrium when the stage 0 decision are (S,S), (S,C), (C,S) and (C,C), respectively. Note that since firms are asymmetric, the subgame equilibrium of (S,C) and (C,S) will no longer be symmetric. For the sake of space, we only present the main steps of the proof.

Case 1. Both firms make price commitment. In this case, when $x_1 \geq \frac{c+\gamma c+t-\gamma^2 t}{2-\gamma^2}$, $x_2 \leq \frac{(2+\gamma)(c-2t)}{6-2\gamma^2}$, the equilibrium prices are: $p_1^S = \frac{1}{2-\gamma^2}(c+\gamma c-\gamma t-\gamma x_2)$, $p_2^S = \frac{1}{2-\gamma^2}(2c+\gamma c-2t-2x_2)$ and the equilibrium revenues are: $R_1^S = \frac{1}{(2-\gamma^2)^2}(c+\gamma c-\gamma t-\gamma x_2)^2$, $R_2^S = \frac{1}{2-\gamma^2}(2c+\gamma c-2t-2x_2)x_2$.

Case 2. Both firms use contingent pricing. In this case, when $x_1 \geq \frac{1+\gamma}{2-\gamma^2}c+t$, $x_2 \leq \frac{c-t}{2-\gamma}$, the equilibrium revenues are (based on Lemma 1): $R_1^C = \frac{(c+\gamma c-\gamma x_2)^2}{(2-\gamma^2)^2} + \frac{t^2(1+\gamma)^2}{(2-\gamma^2)^2}$, $R_2^C = \frac{1}{2-\gamma^2}(2c+\gamma c-2t-2x_2)x_2$.

Case 3. Firm 1 moves first, firm 2 follows. In this case, when $x_1 \geq \frac{c}{2(1-\gamma)}$, $x_2 \leq \frac{(2-\gamma)(1+\gamma)}{(4-3\gamma^2)}c$, The equilibrium prices are $p_1 = \frac{(1+\gamma)c-\gamma x_2}{2(1-\gamma^2)}$, $p_2^H = c+\gamma p_1+t-x_2$, $p_2^L = c+\gamma p_1-t-x_2$ and the equilibrium revenues are $R_1^{1F} = \frac{((1+\gamma)c-\gamma x_2)^2}{4(1-\gamma^2)}$, $R_2^{1F} = \frac{(2+\gamma-\gamma^2)c-(2-\gamma^2)x_2}{2(1-\gamma^2)}x_2$.

Case 4. Firm 2 moves first, firm 1 follows. In this case, when $x_1 \geq \frac{1+\gamma}{2-\gamma^2}c$, $x_2 \leq \frac{2+\gamma}{6-2\gamma^2}(c-t)$, the equilibrium prices are $p_2 = \frac{(2+\gamma)(c-t)-2x_2}{2-\gamma^2}$, $p_1^H = \frac{c+\gamma p_2+t}{2}$, $p_1^L = \frac{c+\gamma p_2-t}{2}$ and the equilibrium revenues are $R_1^{2F} = \frac{(2+2\gamma c-2\gamma x_2-(2+\gamma)\gamma t)^2}{4(2-\gamma^2)^2} + \frac{t^2}{4}$, $R_2^{2F} = \frac{(2+\gamma)(c-t)-2x_2}{2-\gamma^2}x_2$. Summarizing the results in the four parts and performing simple algebraic comparisons, we have the desired result. □

Proposition 1 (Two-Period Model: $\epsilon_2 = 0$). *The equilibrium prices and revenues when both firms use committed pricing strategy or contingent pricing strategy are given in Tables 1 and 2.*

	Equilibrium Price	Equilibrium Revenue
$x \geq \frac{2c}{2-\gamma}+t$	$\frac{c}{(2-\gamma)}$	$\frac{2c^2}{(2-\gamma)^2}$
$\frac{2c}{3-2\gamma}+t \leq x < \frac{2c}{2-\gamma}+t$	$\frac{2c+t-x}{2(1-\gamma)}$	$\frac{(2c+t-x)(x-t)}{2(1-\gamma)}$
$\frac{2c-t}{3-2\gamma} \leq x < \frac{2c}{3-2\gamma}+t$	$\frac{2c-t+x}{2(2-\gamma)}$	$\frac{(2c-t+x)^2}{4(2-\gamma)^2}$
$x < \frac{2c-t}{3-2\gamma}$	$\frac{2c-t-x}{2(1-\gamma)}$	$\frac{(2c-x-t)x}{2(1-\gamma)}$

Table 1 **Equilibrium prices and revenues when both firms use committed pricing**

Proof of Proposition 1. First, we consider the second period problem with general capacity levels. Assume there are x_1 and x_2 capacity left in the beginning of the second period, then the unique Nash equilibrium are given as follows (an extension of Lemma 1):

	Equilibrium Price	Equilibrium Revenue
$x \geq \frac{2c}{2-\gamma} + t$	$\frac{c}{(2-\gamma)}$	$\frac{2c^2}{(2-\gamma)^2}$
$\frac{2c}{2-\gamma} - 2t < x < \frac{2c}{2-\gamma} + t$	$\frac{1}{3}\frac{c}{(2-\gamma)} + \frac{1}{3}\frac{2c-x+t}{(1-\gamma)}$	$\frac{1}{3}\frac{(2c)^2}{2(2-\gamma)^2} + \frac{2}{3}\frac{(x-t)(2c-x+t)}{2(1-\gamma)}$
$2t \leq x \leq \frac{2c}{2-\gamma} - 2t$	$\frac{2c-x}{2(1-\gamma)}$	$\frac{(2c-x)x-2t^2}{2(1-\gamma)}$

Table 2 Equilibrium prices and revenues when both firms use contingent pricing

- Case 1: If $x_1 \geq \frac{c}{2-\gamma}$ and $x_2 \geq \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = p_2^* = \frac{c}{(2-\gamma)}$ and the equilibrium revenues are $\frac{c^2}{(2-\gamma)^2}$ for both firms. In this case, both firms use the revenue-maximizing price.
- Case 2: If $x_1 < \frac{c}{2-\gamma}$ and $x_2 \geq \frac{c+\gamma(c-x_1)}{2-\gamma^2}$, then the equilibrium prices are $p_1^* = \frac{c\gamma+2(c-x_1)}{(2-\gamma^2)}$ and $p_2^* = \frac{c+\gamma(c-x_1)}{(2-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(c\gamma+2(c-x_1))x_1}{(2-\gamma^2)}$ and $v_2^* = \frac{(c+\gamma(c-x_1))^2}{(2-\gamma^2)^2}$. In this case, the first firm uses the capacity-depleting price and the second firm uses the revenue-maximizing price.
- Case 3: If $x_1 \geq \frac{c+\gamma(c-x_2)}{2-\gamma^2}$ and $x_2 < \frac{c}{2-\gamma}$, then the equilibrium prices are $p_1^* = \frac{c+\gamma(c-x_2)}{(2-\gamma^2)}$ and $p_2^* = \frac{c\gamma+2(c-x_2)}{(2-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(c+\gamma(c-x_2))^2}{(2-\gamma^2)^2}$ and $v_2^* = \frac{(c\gamma+2(c-x_2))x_2}{(2-\gamma^2)}$. In this case, the second firm uses the capacity-depleting price and the first firm uses the revenue-maximizing price.
- Case 4: Lastly, if $\gamma x_1 + (2-\gamma^2)x_2 < (1+\gamma)c$ and $\gamma x_2 + (2-\gamma^2)x_1 < (1+\gamma)c$, then the equilibrium prices are $p_1^* = \frac{\gamma(c-x_2)+c-x_1}{(1-\gamma^2)}$ and $p_2^* = \frac{\gamma(c-x_1)+c-x_2}{(1-\gamma^2)}$. The equilibrium revenues are $v_1^* = \frac{(\gamma(c-x_2)+c-x_1)x_1}{(1-\gamma^2)}$ and $v_2^* = \frac{(\gamma(c-x_1)+c-x_2)x_2}{(1-\gamma^2)}$. In this case, both firms use the capacity-depleting price.

Now we consider the first period problem. We denote $V_1(x_1,x_2)$ and $V_2(x_1,x_2)$ as the equilibrium revenues of the second stage for firms 1 and 2 respectively, if there are (x_1,x_2) capacity left. The formulas of $V_1(x_1,x_2)$ and $V_2(x_1,x_2)$ have been given above. Now we want to study the equilibrium prices p_1^* and p_2^* in the first period for this two stage game. For any Nash equilibrium, we must have:

$$\begin{aligned} p_1^* &= \arg\max_{p_1} E\left\{p_1 \cdot \min\{x, D_1\} + V_1((x-D_1)^+, (x-D_2)^+)\right\}, \\ p_2^* &= \arg\max_{p_2} E\left\{p_2 \cdot \min\{x, D_2\} + V_2((x-D_1)^+, (x-D_2)^+)\right\}. \end{aligned} \tag{0.27}$$

Note that $\min\{x, D\} = x - (x-D)^+$ and define

$$\begin{aligned} R_1 &= (x-D_1)^+ = (x-c+p_1-\gamma p_2-\epsilon)^+, \\ R_2 &= (x-D_2)^+ = (x-c+p_2-\gamma p_1-\epsilon)^+. \end{aligned} \tag{0.28}$$

We can transform (0.27) to

$$p_1^* = \arg\max_{p_1}\{p_1 x - p_1 E R_1 + E V_1(R_1, R_2)\},$$

$$p_2^* = \arg\max_{p_2}\{p_2 x - p_2 ER_2 + EV_2(R_1, R_2)\}. \tag{0.29}$$

Now we solve this problem. We focus on symmetric Nash equilibria. We will study several cases based on the remaining capacity (at equilibrium) after the first period and solve for (possible) equilibria that satisfy the remaining capacity constraint in each case. Then we characterize the equilibrium prices and revenues (and the corresponding capacity conditions).

- Case 1. We study the situation where

$$x - c + p^* - \gamma p^* - t \geq \frac{c}{2-\gamma}. \tag{0.30}$$

This is the case where the remaining capacity after the first period is still sufficiently high so that the optimal pricing in the second period is to use the revenue-maximizing pricing (and note under condition (0.30), the remaining capacity must be strictly positive after the first period). The equilibrium pricing should satisfy

$$\frac{\partial\{p_1 x - p_1 ER_1 + EV_1(R_1, R_2)\}}{\partial p_1}(p_1^*, p_2^*) = 0.$$

And since $\frac{\partial EV_1(R_1,R_2)}{R_1} = \frac{\partial EV_1(R_1,R_2)}{\partial R_2} = 0$, we have the equilibrium condition as: $c - 2p^* + \gamma p^* = 0$, or equivalently, $p_1^* = p_2^* = \frac{c}{(2-\gamma)}$. And to make (0.30) hold, we need to have $x \geq \frac{2c}{2-\gamma} + t$.

To summarize, when $x \geq \frac{2c}{2-\gamma} + t$, the equilibrium price is $p_1^* = p_2^* = \frac{c}{(2-\gamma)}$, and the firm will use the revenue-maximizing price throughout the horizon. And the equilibrium revenue is $v_1^* = v_2^* = \frac{2c^2}{(2-\gamma)^2}$.

- Case 2: Next we study the equilibrium when

$$\begin{aligned} &x - c + p^* - \gamma p^* + t < \frac{c}{2-\gamma}, \\ &x - c + p^* - \gamma p^* - t > 0. \end{aligned} \tag{0.31}$$

The first condition says that even if the first period demand realizes to be low, the remaining capacity is still not sufficiently high and a capacity depleting pricing is optimal for the second stage. The second condition says that even if the first period demand realizes to be high, we still have positive remaining capacities for the second stage. The second condition removes the $(\cdot)^+$ in R_1 and R_2 and allows us to take a neat derivative for (0.29).

We consider the equilibrium equation (0.29), we consider the first-order optimality condition. We have given (0.31), $\frac{\partial ER_1}{\partial p_1} = 1$, and

$$\frac{\partial EV_1}{\partial p_1} = E\left[\frac{\partial V_1}{\partial R_1} \cdot \frac{\partial R_1}{\partial p_1} + \frac{\partial V_1}{\partial R_2} \cdot \frac{\partial R_2}{\partial p_1}\right] = E\left[\frac{-2R_1 + c + \gamma(c - R_2)}{1-\gamma^2} + \frac{R_1\gamma^2}{1-\gamma^2}\right],$$

where the second equality is because of (0.31) and case 4 in the beginning of the proof. Setting the derivative to zero and set $p_1 = p_2 = p^*$, we have $p^* = \frac{2c-x}{2(1-\gamma)}$. And to make (0.31) to hold, we need to have $2t < x < \frac{2c}{2-\gamma} - 2t$.

To summarize, when $2t < x < \frac{2c}{2-\gamma} - 2t$, the equilibrium pricing is $p_1^* = p_2^* = \frac{2c-x}{2(1-\gamma)}$. Now we compute the equilibrium revenue. We just need to summarize the revenues under two scenarios. Under the high-demand scenario, the revenue is

$$\frac{2c-x}{2(1-\gamma)}(\frac{x}{2}+t) + V_1(\frac{x}{2}-t, \frac{x}{2}-t) = \frac{2c-x}{2(1-\gamma)}(\frac{x}{2}+t) + \frac{(c-\frac{x}{2}+t)(\frac{x}{2}-t)}{(1-\gamma)}.$$

And under the low-demand scenario, the revenue is

$$\frac{2c-x}{2(1-\gamma)}(\frac{x}{2}-t) + V_1(\frac{x}{2}+t, \frac{x}{2}+t) = \frac{2c-x}{2(1-\gamma)}(\frac{x}{2}-t) + \frac{(c-\frac{x}{2}-t)(\frac{x}{2}+t)}{(1-\gamma)}.$$

Therefore the total expected revenue is

$$v_1^* = v_2^* = \frac{(2c-x)x - 2t^2}{2(1-\gamma)}.$$

- Case 3: Lastly, we consider the case when

$$\begin{aligned} x - c + p^* - \gamma p^* + t &> \frac{x}{2-\gamma}, \\ x - c + p^* - \gamma p^* - t &< \frac{x}{2-\gamma}. \end{aligned} \tag{0.32}$$

This is the case when the first period demand is high, the remaining capacity is relatively low so that a capacity depleting pricing should be used, and when the first period demand is low, the remaining capacity is relatively high so that a revenue-maximizing price should be used. We again consider the first order optimality condition of the equilibrium condition (0.29). In this case, the optimality condition is

$$c - 2p_1^* + \gamma p_2^* + \frac{1}{2}\left(\frac{-2R_1^H + c - \gamma(c - R_2^H)}{1-\gamma^2} + \frac{R_1\gamma^2}{1-\gamma^2}\right) = 0,$$

where $R_1^H = x - c + p_1 - \gamma p_2 - t$ and $R_2^H = x - c + p_2 - \gamma p_1 - t$. Then by setting $p_1^* = p_2^* = p^*$, we have the equilibrium price in this case is:

$$p^* = \frac{c(5-3\gamma) - (x-t)(2-\gamma)}{3(1-\gamma)(2-\gamma)}.$$

And in order to make (0.32) hold, we need to have:

$$\frac{2c}{2-\gamma} - 2t < x < \frac{2c}{2-\gamma} + t.$$

This is actually exactly filling the gap of x ranges in the previous two cases. To summarize, when

$$\frac{2c}{2-\gamma} - 2t < x < \frac{2c}{2-\gamma} + t,$$

the equilibrium price is

$$p^* = \frac{c(5-3\gamma) - (x-t)(2-\gamma)}{3(1-\gamma)(2-\gamma)}.$$

And it is easy to show that this price is strictly between the equilibrium price in case 1 and case 2, i.e.,

$$\frac{c}{(2-\gamma)} < p^* < \frac{2c-x}{2(1-\gamma)}.$$

And one can compute the equilibrium revenue in this case is

$$v_1^* = v_2^* = \frac{2}{3}\frac{c^2}{(2-\gamma)^2} + \frac{1}{3}\frac{(x-t)(2c-x+t)}{(1-\gamma)} = \frac{1}{3}\frac{(2c)^2}{2(2-\gamma)^2} + \frac{2}{3}\frac{(x-t)(2c-x+t)}{2(1-\gamma)}.$$

This is actually a very interesting results, it says that the equilibrium revenue in this setting is a mix of the revenues of the maximizing pricing and the capacity depleting pricing, with a weight of $(1/3, 2/3)$. $\square$