Online Appendix to "Socially Beneficial Rationality: The Value of Strategic Farmers, Social Entrepreneurs and For-Profit Firms in Crop Planting Decisions"

A. Continuous Decisions

We allow a farmer's planting quantity q to be any fraction between 0 and 1. In period t, as a strategic farmer bases his planting decision q_t^s on the market price p_t in the current period and a naïve farmer bases his decision q_t^n on the market price p_{t-1} in the last period, their individual objectives are, respectively,

$$\max_{q_t^s \in [0,1]} (p_t - c) q_t^s \text{ and } \max_{q_t^n \in [0,1]} (p_{t-1} - c) q_t^n.$$

Note that the market price here in a period is determined by the total output of all farmers in that period as in (3).

Lemma A.1. *In any period t,*

$$q_t^s = \begin{cases} 0 & \textit{if } p_t < c, \\ \textit{any } q \in [0,1] & \textit{if } p_t = c, \\ 1 & \textit{if } p_t > c. \end{cases} \tag{A.1}$$

$$q_t^n = \begin{cases} 0 & \textit{if } p_{t-1} < c, \\ \textit{any } q \in [0,1] & \textit{if } p_{t-1} = c, \\ 1 & \textit{if } p_{t-1} > c. \end{cases} \tag{A.2}$$

The market dynamics is the same as specified in Proposition 1.

Proof of Lemma A.1. A naïve farmer's planting decision as (A.2) can be directly derived according to the definition of a naïve farmer. For a strategic farmer, although he is aware of the fact that decisions from all strategic farmers can collectively affect the near-future market price p_t, his individual decision q_t^s has no impact on p_t, because his land size is infinitesimal relative to the entire population. Hence, a strategic farmer's planting decision follows (A.1).
Since the set of strategic farmers with $p_t = c$ and naïve farmers with $p_{t-1} = c$ is of measure zero, the total amount of crop produced by all farmers as specified in (2) does not change. Therefore, the market dynamics still follows Proposition 1 even when continuous decisions are considered. $\square$

B. Backward-Looking Farmers

In this extension, we assume farmers lack full information (e.g., Ω, α and $F(\cdot)$) and thus cannot use (3) to rationally anticipate the near-future market price p_t. Instead, in each period strategic farmers predict the market price on the basis of the average price in the last two periods, i.e., $\hat{p}_t = \frac{p_{t-1}+p_{t-2}}{2}$. This is a simple but widely used adaptive forecasting method (see Chopra and Meindl 2013, Chapter 7), as it is easier to obtain the necessary information about historical prices than future prices. The strategic farmers' perceived utility becomes

$$u_t^s = \hat{p}_t - c = \frac{p_{t-1}+p_{t-2}}{2} - c. \tag{B.1}$$

We adopt the convention that $p_i = p_0$ for any $i<0$. By the same decision and price processes as in (2)-(3) with p_t replaced by $\hat{p}_t$, for any $t \geq 1$, we have

$$p_t = \Omega - b(1-\alpha)F(p_{t-1}) - b\alpha F\Big(\frac{p_{t-1}+p_{t-2}}{2}\Big). \tag{B.2}$$

Proposition B.1. *Suppose Assumptions 1-3 hold and there exist backward-looking strategic farmers ($\alpha>0$). If $b \leq \bar{c}$, then the market price converges to the limiting market price, i.e.,*

$$\lim_{t\to\infty} p_t = \bar{p}.$$

Proposition B.1 tells us that as long as there are backward-looking strategic farmers and the market price is not sensitive to the total supply (i.e., $b \leq \bar{c}$), the market price converges to the same value as if the strategic farmers had full information and were forward-looking. This implies that looking into the past can be as efficient as looking into the future in a stationary market.

C. Impact of Bankruptcy

In this subsection, we consider the scenario that farmers may go bankrupt and quit the farming business if their loss exceeds the capital or savings. Let L denote the maximal cumulative loss that farmers can afford. To be self-contained, we restate Proposition 4 in Appendix II as follows and prove it in Online Appendix K.

Proposition C.1 (Impact of Bankruptcy). *Suppose Assumptions 1-4 hold and those farmers who exit the market due to bankruptcy never come back. Then the market price still converges, but the limiting market price is (weakly) higher than $\bar{p}$.*

Next, in Section C.1 we will describe the market dynamics if we allow farmers to exit the market due to bankruptcy, and then in Section C.2 we will present some numerical experiment results from which we draw managerial insights.

C.1. Market Dynamics

In this subsection, we explicitly show what happened in each cycle when we incorporate farmer exiting to the model. For ease of exposure, we just show the case when $g(\alpha) < 1$.

If no farmers exit in cycle i and all previous cycles, then the price dynamics until cycle i is the same as (5). That is,

$$p_{2i-1} = \bar{p} + [-g(\alpha)]^{2i-1}(p_0 - \bar{p}) > \bar{p},$$
$$p_{2i} = \bar{p} + [-g(\alpha)]^{2i}(p_0 - \bar{p}) < \bar{p}.$$

Note that p_{2i-1} is decreasing and p_{2i} is increasing. Observe also that the naïve farmers with production cost $p_{2i} < c < p_{2i-1}$ will suffer a loss $c - p_{2i}$ in period $2i$. Moreover, for a naïve farmer with production cost $p_{2i} < c < p_{2i-1}$, the total loss until cycle i he has suffered is $\sum_{l=1}^{i}(c - p_{2l})$. For a naïve farmer with production cost $p_{2i-1} < c < p_{2i-3}$, the total loss he has suffered is $\sum_{l=1}^{i-1}(c - p_{2l})$ and he does not suffer any loss in cycle i because he did not plant in cycle i. For a naïve farmer with production cost $p_{2i-2} < c < p_{2i}$, the total loss he has suffered is $\sum_{l=1}^{i-1}(c - p_{2l})$ and he does not suffer any loss in cycle i because he earns surplus in cycle i.

Suppose cycle j is the first cycle that naïve farmers begin to exit the market. That is, $\sum_{l=1}^{j}(p_{2j-1} - p_{2l}) \geq L$. In period $2j$, by the definition of j, we have $p_{2j} = \bar{p} + [-g(\alpha)]^{2j}(p_0 - \bar{p})$. Solving $\sum_{l=1}^{j}(c - p_{2l}) = L$ yields $c \geq \frac{1}{j}(L + \sum_{l=1}^{j} p_{2l})$, indicating that the farmers with $\frac{1}{j}(L + \sum_{l=1}^{j} p_{2l}) \leq c < p_{2j-1}$ suffer an overall loss exceeding L and thus exit the market. Let $c_{2j} = \frac{1}{j}(L + \sum_{l=1}^{j} p_{2l})$. Next we have two cases.

Case 1: Suppose $c_{2j} < \bar{p}$. Then as Part 2 in the proof of Proposition C.1, we could show $c_{2j} = c_{2j+1} = c_{\infty}$ and the market price remains higher than $\bar{p}$.

Case 2: Suppose $c_{2j} > \bar{p}$.

In period $2j+1$, the farmers with $c < p_{2j}$ plant the crop. Since $p_{2j} < \bar{p} < c_{2j}$, the exited farmers do not affect this period's production quantity. Hence, $p_{2j+1} = \bar{p} + [-g(\alpha)]^{2j+1}(p_0 - \bar{p}) > \bar{p}$. No farmers exit in this period. Hence, $c_{2j+1} = c_{2j}$.

In period $2j+2$, the farmers with $c < p_{2j+1}$ are supposed to plant the crop. Note that the ones with $c > c_{2j+1}$ have exited the market, so $p_{2j+2} = \Omega - \alpha b F(p_{2j+2}) - (1-\alpha) b F(\min\{p_{2j+1}, c_{2j+1}\})$. The naïve farmers with $p_{2j+2} < c < \min\{p_{2j+1}, c_{2j+1}\}$ suffer a loss $c - p_{2j+2}$. Solving $\sum_{l=1}^{j+1}(c - p_{2l}) \geq L$ yields $c \geq \frac{1}{j+1}(L + \sum_{l=1}^{j+1} p_{2l})$, indicating that the farmers with $\frac{1}{j+1}(L + \sum_{l=1}^{j+1} p_{2l}) \leq c < c_{2j+1}$ suffer an overall loss exceeding L and thus exit the market. Let $c_{2j+2} = \frac{1}{j+1}(L + \sum_{l=1}^{j+1} p_{2l}) = \frac{1}{j+1}(j \cdot c_{2j} + p_{2j+2})$. Again, one needs to compare c_{2j+2} with $\bar{p}$. If $c_{2j+2} < \bar{p}$, then the price converges and we are done. Otherwise, the dynamics is as follows.

In period $2j+3$, the farmers with $c < p_{2j+2}$ will plant the crop. Since $p_{2j+2} < \bar{p} < c_{2j+2}$, the exited farmers do not affect this period's production quantity. Hence,
$p_{2j+3} = \Omega - \alpha b F(p_{2j+3}) - (1-\alpha) b F(p_{2j+2}) = \Omega - \alpha b F(p_{2j+3}) - (1-\alpha)\frac{b}{\bar{c}} p_{2j+2} > \bar{p}$. No farmers exit in this period. Hence, $c_{2j+3} = c_{2j+2}$.
Continuing in this fashion, we could show for any $k \geq 0$, if $c_{2j+2(k-1)} > \bar{p}$, then $c_{2j+2k-1} = c_{2j+2(k-1)}$ and in period $2j+2k$, the farmers with $c_{2j+2k} < c < c_{2j+2k-1}$ will exit the market, where $c_{2j+2k} = \frac{1}{j+k}\left(L + \sum_{l=1}^{j+k} p_{2l}\right)$. Furthermore,

$$p_{2j+2k} = \Omega - \alpha b F(p_{2j+2k}) - (1-\alpha) b F(\min\{p_{2j+2k-1}, c_{2j+2k-1}\}),$$
$$p_{2j+2k+1} = \Omega - \alpha b F(p_{2j+2k+1}) - (1-\alpha)\frac{b}{\bar{c}} p_{2j+2k}.$$

As it can be seen, the price dynamics is quite complicate when the farmer exit is incorporated. In fact, when $g(\alpha) < 1$, there are two forces that make the price converge. One is due to the existence of strategic farmers who can predict the near-future price and alleviate naïve farmers' herding behavior. The other is due to farmers' exit which reduces the number of naïve farmers and thus reduces thee impact of their irrational behavior. These two forces are interwined with each other, and it becomes hard to explicitly track how many naïve farmers are left in the market in each period (e.g., c_t needs to be updated in each cycle) and thus explicitly write down the price dynamics equation (e.g., c_t needs to be compared with the last period price to see whether the exited farmers affect the production quantity).

C.2. Numerical Study

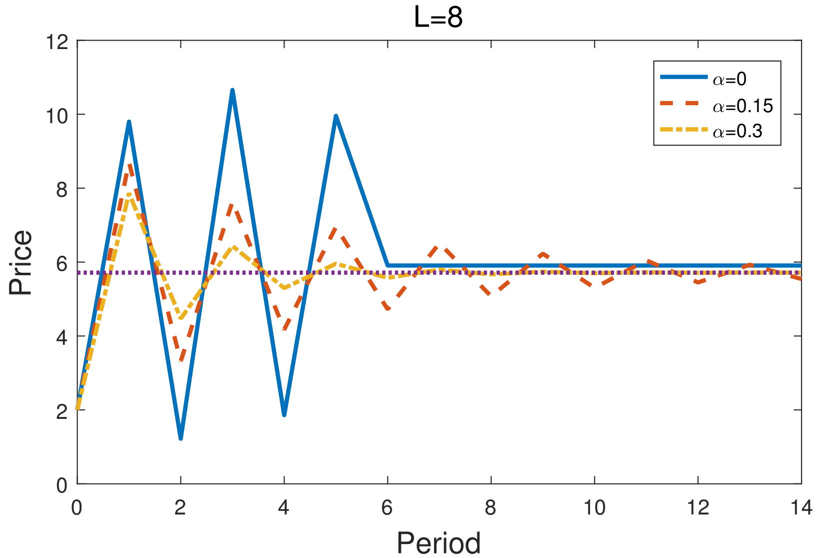
Given the complexity of the market dynamics described above, in this subsection we resort to numerical experiments to draw managerial insights when we allow farmers to exit due to bankruptcy. We numerically find that when the fraction of strategic farmers α or a farmer's budget L is not very small, the farmers' exit behavior has very limited impact on the market price dynamics, and thus the model with farmers' exit tends not to alter the long-term welfare results derived for our basic model. We summarize our findings into the following observations and discussions.

Observation C.1. When the fraction of strategic farmers α is not very small, the market price converges in a way that farmers who exit the market (if any) are all high-cost farmers (whose cost is above $\bar{p}$) and the market price converges to $\bar{p}$.

To give an example, Figure 1 shows the market dynamics across different α when farmers have limited budget.

Figure C.1 Numerical comparison of different scenarios across various α.

Parameters: $b = 11, \bar{c} = 10, \Omega = 12$.



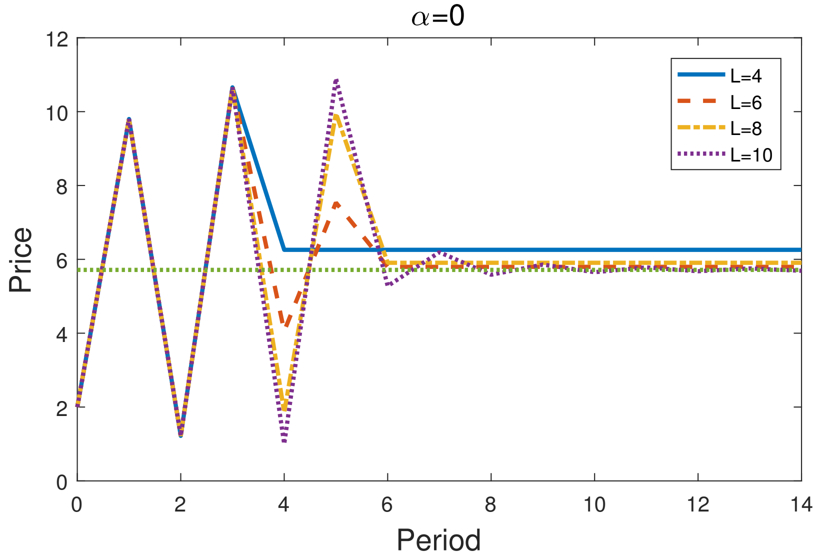
As shown in Figure C.1, with $\alpha = 0.15$ the market converges slowly, and naïve farmers gradually go bankrupt (from cycle 3 to cycle 6) after running out of their budgets. But in this case only high-cost farmers, whose cost is above $\bar{p}$, exit the market due to bankruptcy. Therefore, farmers' exit behavior does not change the limiting market price. When $\alpha = 0.3$, the market converges even faster, and no farmers exit the market due to bankruptcy. However, when $\alpha = 0$ (i.e., in the absence of strategic farmers), some low-cost farmers (whose cost is below $\bar{p}$ but not very low) go bankrupt in periods 2 and 4 in the first two cycles, due to the limited budget and highly fluctuating market price. As fewer and fewer farmers remain in the market, the supply in even periods decreases since period 2, resulting in an increasing market price in periods 4 and 6. This process leads to a permanent supply shortage and a market price that is higher than $\bar{p}$ starting from the 6th period in the 3rd cycle. This also explains why the market price may converge to a value higher than $\bar{p}$ as stated in Proposition C.1 above.

Observation C.2. When the fraction of strategic farmers α is very small, the limiting market price is higher than $\bar{p}$ and the limiting market price depends on L in a way that is very difficult to derive explicit expressions. In addition, the limiting market price is not monotone in L and is close to or the same as $\bar{p}$ for L that is not very small.

In Figure C.2 we show the market price dynamics under various budget levels with $\alpha = 0$ (which removes the effect of α). We find the limiting price under budget $L = 6$ is smaller than that under $L = 4$ and $L = 8$.

Figure C.2 **Numerical comparison of different scenarios across various L.**

Parameters: $b = 11, \bar{c} = 10, \Omega = 12.$



When L is getting larger, the limiting price cannot be far away from $\bar{p}$. In particular, it can be proved that when $b = \bar{c}$ the market price converges to $\bar{p}$ as long as the budget L is greater than $(\bar{p} - p_0)$. That is, the farmer exit does not affect the long-term welfare results as long as the budget is enough to support farmers with cost $c \leq \bar{p}$ for just the *first* cycle.

D. Other Forms of Contracts

In the main model, we assume the SE offers a fixed buyout price contract and the farmers accept the contract only if the fixed price p^o is strictly higher than the market price in the last period. In this section, we first relax the fixed buyout price assumption and consider that the SE offers a time-varying buyout price contract, and then consider a long-term contract in which case the type-S farmers commit to cultivate the crop and sell it to SE at p^o in every period once they accept the contract.

D.1. Time-varying Contract

Let $\boldsymbol{p}^o := \{p_1^o, p_2^o, \ldots, p_n^o, \ldots\}$ be the time-varying pricing policy. For any given sufficiently small $\epsilon > 0$, define $T(\boldsymbol{p}^o; \epsilon) := \min\{t' : |p_t(\boldsymbol{p}^o) - \bar{p}| \leq \epsilon, t \geq t'\}$. The objective is to find a time-varying price contract that achieves the fastest convergence to $\bar{p}$ while the SE does not incur any loss at any time. For any given sufficiently small ϵ, the SE aims at solving:

$$\begin{aligned} \min_{\boldsymbol{p}^o} \quad & T(\boldsymbol{p}^o; \epsilon) \\ \text{s.t.} \quad & \pi_t(\boldsymbol{p}^o) \geq 0 \text{ for all } t, \end{aligned} \tag{D.1}$$

where π_t is defined in (9).

Proposition D.1. *Suppose Assumptions 1-6 hold.*[2] *The optimal solution to SE's problem (D.1) for any sufficiently small ϵ is*

$$(p^o_{2i-1}, p^o_{2i}) = \left(\bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - p_{2i-2}), 0\right) \text{ for any } i \geq 1. \tag{D.2}$$

As implied by Proposition D.1, SE's time-varying price contract is accepted by farmers only in odd periods (i.e., period $2i-1$ for any i), when the latest market price p_{2i-2} is less than $\bar{p}$ and thus less than p^o_{2i-1}. Compared to the optimal price $\bar{p}$ in the static contract (Proposition 6), the optimal price p^o_{2i-1} in the time-varying contract is higher and thus attracts more type-S farmers to plant the crops to offset the shortsighted behavior by naïve farmers. Hence, it accelerates the price convergence. To avoid any loss, the SE has to reduce the price p^o_{2i-1} over time to ensure that it is no more than the corresponding realized market price. In the limit, as i approaches ∞, the price p^o_{2i-1} converges to $\bar{p}$ from above. Moreover, if the constraint $\pi_t \geq 0$ can be relaxed, i.e., SE is able to endure losses, the market limiting price can be achieved in one period when α is sufficiently large.

Proposition D.2. *Suppose Assumptions 1-4 hold and the SE has a sufficiently large budget and can endure losses. At any odd period t, if $\alpha \geq \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$, then the SE can set $p^o_t = \frac{\bar{p}-p_{t-1}}{\alpha} + p_{t-1}$ to stabilize the price in one period; if $\alpha < \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$, then the SE sets $p^o_t = \bar{c}$ to achieve the highest rate of the price convergence.*

Besides its complexity and inconvenience in revising the contract terms, another potential risk associated with a time-varying contract, as opposed to committing to a fixed buyout price, is that the SE could be more speculative and abuse the contract when offering it, in order to earn more profits in the short term. For instance, the SE could contract with farmers at a price even lower than $\bar{p}$ but strictly higher than p_{t-1} at the beginning of an odd period t and sell at a higher market price in the end of that period. This could dramatically slow down the market convergence and result in lower farmer welfare. Therefore, monitoring and regulating SE's behavior can be necessary.

D.2. Long-Term Contract

Consider that the SE offers type-S farmers a long-term contract in which a constant buyout price p^o is specified. If a type-S farmer accepts the contract, he commits to cultivate the crop and sell it at p^o to the SE in *every* period. It is reasonable to assume that a type-S farmer with production

[2] For this result, Assumption 6 can be relaxed to $\alpha > \alpha_1$.

cost c will accept the contract if $p^o \geq c$, and do not otherwise. (We will verify shortly it is indeed incentive compatible for type-S farmers to do so.) Then the market price evolves as follows:

$$p_t = \Omega - b\alpha F(p^o) - b(1-\alpha)F(p_{t-1}). \tag{D.3}$$

We replace the definition of p_t by (D.3) and find that Proposition 6 still holds if we relax the individual welfare constraint in the SE's problem (10) to that the total individual welfare in each cycle (as opposed to each period) is nonnegative. As a result, the optimal long-term contract is also beneficial to both the SE and farmers. However, promoting this contract could be difficult. First, naïve farmers may hesitate to make any long-term commitment, as they likely find it difficult to assess its value over the long run. Furthermore, this contract may not seem incentive compatible: In view of conceivably higher future market prices than the offered contract price, the farmers may not be willing to be locked in at the contract price.
The following proposition shows that the extent, to which SE's long-term buyout contract can help reduce the price fluctuation, varies, depending on the buyout price p^o and the size of type-S farmers α.

PROPOSITION D.3. *Suppose Assumptions 1-5 hold. Define* $\hat{\alpha} = \frac{b+\bar{c}-\Omega}{b(1-\frac{p^o}{\bar{c}})}$.
(i) If $\alpha \geq \hat{\alpha}$*, then* $p_t = \Omega - \alpha\frac{b}{\bar{c}}p^o - b(1-\alpha)$ *for any* $t \geq 1$.
(ii) If $\alpha_1 < \alpha < \hat{\alpha}$*, then* $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c}+(1-\alpha)b}$.
(iii) If $\alpha \leq \min\{\alpha_1, \hat{\alpha}\}$*, then the market price process does not converge.*

Similar to Proposition 6, we could show under some conditions, $p^{o*} = \bar{p}$ is the unique optimal solution to SE's problem (10). In that case, the market price converges to $\bar{p}$ and the SE receives a positive profit in each cycle over any finite horizon and the profit per cycle diminishes to 0 in the limit. In the meantime, both type-S and type-N farmers, with production cost $c > \underline{c}$, receive (weakly) higher welfare due to the market stabilization in the presence of SE, while the ones with $c < \underline{c}$ may receive a lower welfare, compared to the surplus that farmers would have received in the absence of SE.

E. Proof of Propositions 1, 2, and 3

Before the proof of Proposition 1, we first introduce two lemmas.

LEMMA E.1. *Suppose Assumptions 1-3 hold and* $g(\alpha) \leq 1$*. For any* $i \geq 0$*, if* $p_i \leq \bar{c}$*, then* $p_{i+1} \leq \bar{c}$.

Proof of Lemma E.1. If $p_i \leq \bar{c}$, then by (3) we have
$p_i = \Omega - b\alpha \frac{p_i}{\bar{c}} - b(1-\alpha)F(p_{i-1}) \geq \Omega - b\alpha \frac{p_i}{\bar{c}} - b(1-\alpha)$, by which we obtain

$$p_i \geq \frac{\Omega - (1-\alpha)b}{1+\alpha\frac{b}{\bar{c}}}. \tag{E.1}$$

Now we show $p_{i+1} \leq \bar{c}$. Suppose for a contradiction that $p_{i+1} > \bar{c}$, then it follows by $p_i \leq \bar{c}$ and (3) that $p_{i+1} = \Omega - \alpha b - (1-\alpha)\frac{b}{\bar{c}} p_i \leq \Omega - \alpha b - (1-\alpha)\frac{b}{\bar{c}} \frac{\Omega-(1-\alpha)b}{1+\alpha\frac{b}{\bar{c}}}$, where the last inequality holds because of (E.1). To end the proof, it suffices to show $\Omega - \alpha b - (1-\alpha)\frac{b}{\bar{c}} \frac{\Omega-(1-\alpha)b}{1+\alpha\frac{b}{\bar{c}}} \leq \bar{c}$, which will arrive a contradiction. We have

$$\Omega - \alpha b - (1-\alpha)\frac{b}{\bar{c}}\frac{\Omega-(1-\alpha)b}{1+\alpha\frac{b}{\bar{c}}} - \bar{c} = (\Omega - b - \bar{c})\left\{1 - \frac{(1-\alpha)b}{\bar{c}+\alpha b}\right\} \leq 0,$$

where the last inequality holds because $\Omega \leq b + \bar{c}$ and $g(\alpha) = \frac{(1-\alpha)b}{\bar{c}+\alpha b} \leq 1$. $\square$

Lemma E.2. *Suppose Assumptions 1-3 hold and $g(\alpha) > 1$. If $p_{2i-1} \geq \bar{c}$, then for any $j \geq 0$, $p_{2i+2j} = \bar{p} - g(\alpha)(\bar{c} - \bar{p}) \leq \bar{c}$ and $p_{2i+2j+1} = \Omega - b\Big(\alpha + (1-\alpha)\frac{p_{2i+2j}}{\bar{c}}\Big) \geq \bar{c}$.*

Proof of Lemma E.2. We first derive the expression of p_{2i}. It follows by $p_{2i-1} \geq \bar{c}$ and (3) that $p_{2i} = \Omega - b(1-\alpha) - b\alpha F(p_{2i})$. Suppose $p_{2i} > \bar{c}$ for a contradiction. Then $p_{2i} = \Omega - b \leq \bar{c}$, which contradicts our supposition. Hence it must be the case that $p_{2i} \leq \bar{c}$. Therefore,
$p_{2i} = \Omega - b(1-\alpha) - b\alpha\frac{p_{2i}}{\bar{c}}$, by which we obtain $p_{2i} = \frac{\Omega-(1-\alpha)b}{1+\alpha\frac{b}{\bar{c}}} = \bar{p} - g(\alpha)(\bar{c} - \bar{p}) \leq \bar{c}$.
Next we derive the expression of p_{2i+1}. It follows by $p_{2i} \leq \bar{c}$ and (3) that

$$p_{2i+1} = \Omega - b\alpha F(p_{2i+1}) - b(1-\alpha)\frac{p_{2i}}{\bar{c}}. \tag{E.2}$$

Suppose for a contradiction that $p_{2i+1} < \bar{c}$. Then (E.2) reduces to $p_{2i+1} = \Omega - b\alpha\frac{p_{2i+1}}{\bar{c}} - b(1-\alpha)\frac{p_{2i}}{\bar{c}}$, by which we obtain $p_{2i+1} = \frac{\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i}}{1+\alpha\frac{b}{\bar{c}}} = \frac{\Omega\Big(1-(1-2\alpha)\frac{b}{\bar{c}}\Big)+(1-\alpha)^2\frac{b^2}{\bar{c}}}{(1+\alpha\frac{b}{\bar{c}})^2}$. However,

$$p_{2i+1} - \bar{c} = \frac{\Omega\big(1-(1-2\alpha)\frac{b}{\bar{c}}\big) + \frac{(1-\alpha)^2 b^2}{\bar{c}}}{(1+\alpha\frac{b}{\bar{c}})^2} - \bar{c} = \frac{1}{1+\alpha\frac{b}{\bar{c}}}(\Omega - b - \bar{c})\big(1 - g(\alpha)\big) \geq 0,$$

where the last inequality holds because $\Omega \leq b + \bar{c}$ and $g(\alpha) > 1$. This contradicts our supposition that $p_{2i+1} < \bar{c}$. Therefore, it must be the case that $p_{2i+1} \geq \bar{c}$. Hence (E.2) reduces to
$p_{2i+1} = \Omega - b\alpha - b(1-\alpha)\frac{p_{2i}}{\bar{c}}$.
Continuing in this fashion, we could show that for any $j \geq 0$, $p_{2i+2j} = \bar{p} - g(\alpha)(\bar{c} - \bar{p}) \leq \bar{c}$ and
$p_{2i+2j+1} = \Omega - b\alpha - b(1-\alpha)\frac{p_{2i+2j}}{\bar{c}} \geq \bar{c}$. $\square$

Proof of Proposition 1. Part 1: We study the case when $g(\alpha) \leq 1$.
Given $p_0 \leq \bar{c}$ in Assumption 2, Lemma E.1 implies that $p_i \leq \bar{c}$ for any $i \geq 0$. Thus (3) reduces to $p_t = \Omega - \alpha \frac{b}{\bar{c}} p_t - (1-\alpha)\frac{b}{\bar{c}} p_{t-1}$. Some algebras show that

$$p_t = \frac{\Omega}{1+\alpha\frac{b}{\bar{c}}} - \frac{(1-\alpha)\frac{b}{\bar{c}}}{1+\alpha\frac{b}{\bar{c}}} p_{t-1} = \bar{p} + \left\{ - \frac{(1-\alpha)\frac{b}{\bar{c}}}{1+\alpha\frac{b}{\bar{c}}} \right\}^t (p_0 - \bar{p}) = \bar{p} + [-g(\alpha)]^t (p_0 - \bar{p}).$$

If $g(\alpha) < 1$, then $\lim_{t\to\infty} p_t = \bar{p}$. If $g(\alpha) = 1$, then p_t alternates between p_0 and $2\bar{p} - p_0$.
Part 2: We study the case when $g(\alpha) > 1$.
We first show there exists an $i' \geq 0$ such that $p_{2i-1} \geq \bar{c}$. Suppose for a contradiction that there does not exists such an i', then for any t we have $p_t \leq \bar{c}$. Thus as in Part 1, we have for any $t < j$, $p_t = \bar{p} + [-g(\alpha)]^t (p_0 - \bar{p})$. Note that $g(\alpha) > 1$, so there must exist an $i' \geq 0$ such that $p_{2i'-1} \geq \bar{c}$ because $p_0 < \bar{p}$, leading to a contradiction.
Therefore, it follows by Lemma E.2 that the market price alternates between two constant prices. $\square$

Proof of Proposition 2. Note that $p_{2i-1} > p_{2i}$. We divide the farmers based on their production cost endowment. (a) Suppose $c \leq p_{2i}$. By (1), we have $u^n_{2i-1} = p_{2i-2} - c = p_{2i} - c \geq 0$ and $u^n_{2i} = p_{2i-1} - c > p_{2i} - c \geq 0$. Therefore, the farmer plants the crop in both periods. Thus $w^n(c) = \frac{1}{2}(p_{2i-1} - c) + \frac{1}{2}(p_{2i} - c) = \frac{1}{2}(p_{2i-1} + p_{2i}) - c > 0$. One can check that $u^s_{2i-1} \geq 0$ and $u^s_{2i} \geq 0$. Hence, the strategic farmer plants in both periods too and thus $w^s(c) = \frac{1}{2}(p_{2i-1} + p_{2i}) - c > 0$. (b) Suppose $p_{2i} < c \leq p_{2i-1}$. We have $u^n_{2i-1} = p_{2i-2} - c = p_{2i} - c < 0$ and $u^n_{2i} = p_{2i-1} - c \geq 0$. Therefore, the farmer plants the crop only in period $2i$. Thus $w^n(c) = \frac{1}{2}(p_{2i} - c) < 0$. It is easy to check $u^s_{2i-1} = p_{2i-1} - c > 0$ and $u^s_{2i} = p_{2i} - c < 0$. Hence, the strategic farmer plants only in period $2i-1$ and thus $w^s(c) = \frac{1}{2}(p_{2i-1} - c) > 0$. (c) Suppose $c > p_{2i-1}$. We have $u^n_{2i-1} = p_{2i-2} - c = p_{2i} - c < 0$ and $u^n_{2i} = p_{2i-1} - c < 0$. Therefore, the farmer does not plant the crop in any period. Thus $w^n(c) = 0$. It is easy to check $w^s(c) = 0$. $\square$

Proof of Proposition 3. Part (i)(a): In the presence of strategic farmers, the naïve and strategic farmers enjoy the same price in each period. Since strategic farmers can predict future prices based on which their planting decisions are made, it is easy to see they always obtain a (weakly) higher surplus than the naïve farmers with the same production cost.
Part (i)(b): In the model with naïve farmers only, we use p^n_t and $w^n_i(c)$ to denote the price in period t and the welfare of naïve farmers with cost c in cycle i, while in the model with strategic farmers we use p_t and $w^{ns}_i(c)$.
If $\alpha = 0$, by Assumption 4, we have $g(\alpha) \geq 1$. According to Proposition 2, the naïve farmers' welfare in the absence of strategic farmers is as follows: (a) If $c \leq p^n_{2i-2}$, then $w^n_i(c) = \frac{p^n_{2i-1} + p^n_{2i}}{2} - c$. (b) If $p^n_{2i-2} < c \leq p^n_{2i-1}$, then $w^n_i(c) = \frac{1}{2}(p^n_{2i} - c) < 0$. (c) If $c > p^n_{2i-1}$, then $w^n_i(c) = 0$.

When $g(\alpha) < 1$, the prices in cycle i are (p_{2i-1}, p_{2i}). In this case, the naïve farmers' welfare is as follows. (a) If $c \le p_{2i-2}$, then $w_i^{ns}(c) = \frac{p_{2i-1}+p_{2i}}{2} - c > 0$. (b) If $p_{2i-2} < c \le p_{2i-1}$, then $w_i^{ns}(c) = \frac{1}{2}(p_{2i} - c) < 0$. (c) If $c > p_{2i-1}$, then $w_i^{ns}(c) = 0$.

Next we compare $w_i^n(c)$ with $w_i^{ns}(c)$. Note that $p_{2i-2}^n < p_{2i-2} < p_{2i-1} < p_{2i-1}^n$ and $p_{2i}^n < p_{2i}$ for each cycle i. (i) If $p_{2i-2}^n < c \le p_{2i-2}$, then $w_i^n(c) = \frac{1}{2}(p_{2i}^n - c)$ and $w_i^{ns}(c) = \frac{1}{2}(p_{2i-1} - c) + \frac{1}{2}(p_{2i} - c)$. Since $p_{2i} > p_{2i}^n$ and $p_{2i-1} > c$, it is easy to see $w_i^{ns}(c) > w_i^n(c)$. (ii) If $p_{2i-2} < c \le p_{2i-1}$, then $w_i^n(c) = \frac{1}{2}(p_{2i}^n - c)$ and $w_i^{ns}(c) = \frac{1}{2}(p_{2i} - c)$. We have $w_i^{ns}(c) > w_i^n(c)$ because $p_{2i} > p_{2i}^n$. (iii) If $p_{2i-1} < c \le p_{2i-1}^n$, then $w_i^n(c) = \frac{1}{2}(p_{2i}^n - c) < 0$ and $w_i^{ns}(c) = 0$. Clearly, $w_i^{ns}(c) > w_i^n(c)$. (iv) If $c > p_{2i-1}^n$, then $w_i^n(c) = w_i^{ns}(c) = 0$. Therefore, the naïve farmers with production cost not too low (to be specific, $c > p_{2i-2}^n$) obtain a higher surplus than that without strategic farmers.

Part (i)(c): We first derive $w_i^n(c)$. If $\alpha = 0$, by Proposition 1, we have the alternating prices are $(2\bar{p} - p_0, p_0)$ if $b = \bar{c}$ and $(\Omega - \frac{b}{\bar{c}}(\Omega - b), \Omega - b)$ if $b > \bar{c}$ and $i > i'$. By Part (i)(b), we know if $c \le p_{2i-2}^n$, then

$$w_i^n(c) = \frac{p_{2i-1}^n + p_{2i}^n}{2} - c = \begin{cases} \bar{p} - c & \text{if } b = \bar{c}, \\ \frac{1}{2}\Big(\Omega - \frac{b}{\bar{c}}(\Omega - b) + \Omega - b\Big) - c & \text{if } b > \bar{c}. \end{cases} \tag{E.3}$$

Next we show $w_i^{ns}(c)$. If $g(\alpha) < 1$, by Proposition 1, we have $p_t = \bar{p} + [-g(\alpha)]^t (p_0 - \bar{p})$. By Part 1b, we know if $c \le p_{2i-2}^n < p_{2i-2}$, then

$$w_i^{ns}(c) = \frac{p_{2i-1} + p_{2i}}{2} - c = \frac{1}{2}\Big\{2\bar{p} + [-g(\alpha)]^{2i-1}(p_0 - \bar{p})[1 - g(\alpha)]\Big\} - c > \bar{p} - c.$$

Hence, $w_i^{ns}(c) > w_i^n(c)$ if $b = \bar{c}$. As for $b > \bar{c}$, it is hard to compare.

Part (ii)(a): Since the price is stabilized at $\bar{p}$ in the long run, each strategic farmer obtains the same surplus as the naïve farmer with the same production cost.

Part (ii)(b): When $g(\alpha) < 1$, the price is stabilized at $\bar{p}$. In this case, the naïve farmers with $c < \bar{p}$ cultivate in both periods and thus obtain $\lim_{i\to\infty} w_i^{ns}(c) = \bar{p} - c$, while the rest do not cultivate and thus obtain $\lim_{i\to\infty} w_i^{ns}(c) = 0$.

In the absence of strategic farmers, the price eventually alternates between two constant prices $\lim_{i\to\infty} p_{2i-1}^n$ and $\lim_{i\to\infty} p_{2i}^n$. By Proposition 2, we have (a) If $c \le \lim_{i\to\infty} p_{2i}^n$, then $\lim_{i\to\infty} w_i^n(c) = \frac{\lim_{i\to\infty} p_{2i-1}^n + \lim_{i\to\infty} p_{2i}^n}{2} - c$; (b) If $\lim_{i\to\infty} p_{2i}^n \le c \le \lim_{i\to\infty} p_{2i-1}^n$, then $\lim_{i\to\infty} w_i^n(c) = \frac{\lim_{i\to\infty} p_{2i}^n - c}{2}$; (c) If $c > \lim_{i\to\infty} p_{2i-1}^n$, then $\lim_{i\to\infty} w_i^n(c) = 0$.

Note that $\lim_{i\to\infty} p_{2i}^n < \bar{p} < \lim_{i\to\infty} p_{2i-1}^n$, so one can check $\lim_{i\to\infty} w_i^n(c) \le \lim_{i\to\infty} w_i^{ns}(c)$ for any $c > \lim_{i\to\infty} p_{2i}^n$. Therefore, the naïve farmers with production cost not too low (i.e., $c > \lim_{i\to\infty} p_{2i}^n$) obtain a (weakly) higher surplus than that without strategic farmers.

Part (ii)(c): Suppose $c < \lim_{i\to\infty} p_{2i}^n$. We obtain $\lim_{i\to\infty} w_i^{ns}(c) = \bar{p} - c$, while $\lim_{i\to\infty} w_i^n(c)$ is given by (E.3). Note that

$$\Omega - \frac{b}{\bar{c}}(\Omega - b) + \Omega - b - 2\bar{p} = \Omega + (1 - \frac{b}{\bar{c}})(\Omega - b) - \frac{2\bar{c}}{b + \bar{c}}\Omega = \frac{b(\bar{c} - b)}{\bar{c}}(\frac{\Omega}{\bar{c} + b} - 1) > 0,$$

so $\lim_{i\to\infty} w_i^{ns}(c) \le \lim_{i\to\infty} w_i^n(c)$ where the equality holds only when $b = \bar{c}$.

Part (iii): We show under some conditions the total welfare of all farmers is improved.

If $\alpha = 0$, then by Assumption 4, we have $g(\alpha) \ge 1$. Proposition 1 shows that the alternating prices (p_{2i-1}^n, p_{2i}^n) are $(\Omega - p_0, p_0)$ if $b = \bar{c}$ and $(\Omega - \frac{b}{\bar{c}}(\Omega - b), \Omega - b)$ if $b > \bar{c}$.

For the farmers with production cost $c \le p_{2i}^n$, let B_1 denote their welfare difference between in the presence and in the absence of strategic farmers, then we have

$$B_1 = \int_0^{p_{2i}^n} \left\{\bar{p} - c - (\frac{p_{2i-1}^n + p_{2i}^n}{2} - c)\right\}\frac{1}{\bar{c}}dc = \begin{cases} 0 & \text{if } b = \bar{c}, \\ \frac{1}{2}\frac{b}{\bar{c}^2}(\bar{c} - b)(1 - \frac{\Omega}{\bar{c}+b})(\Omega - b) < 0 & \text{if } b > \bar{c}, \end{cases}$$

where the last inequality holds because $b > \bar{c}$ and $b \le \Omega \le b + \bar{c}$. For the farmers with production cost $p_{2i}^n < c \le \bar{p}$, let B_2 denote the welfare difference between in the presence and in the absence of strategic farmers, then

$$B_2 = \int_{p_{2i}^n}^{\bar{p}} \left\{\bar{p} - c - \frac{1}{2}(p_{2i}^n - c)\right\}\frac{1}{\bar{c}}dc = \begin{cases} \int_{p_{2i}^n}^{\bar{p}} \frac{b-c}{2\bar{c}}dc > 0, & \text{if } b = \bar{c}, \\ \frac{3}{4}\frac{b^2}{\bar{c}}(1 - \frac{\Omega}{\bar{c}+b})^2 > 0 & \text{if } b > \bar{c}. \end{cases}$$

For the farmers with production cost $\bar{p} < c \le p_{2i-1}^n$, let B_3 denote the welfare difference between in the presence and in the absence of strategic farmers, then

$$B_3 = \int_{\bar{p}}^{p_{2i-1}^n} \left\{0 - \frac{1}{2}(p_{2i}^n - c)\right\}\frac{1}{\bar{c}}dc = \int_{\bar{p}}^{p_{2i-1}^n} \frac{1}{2}(c - p_{2i}^n)\frac{1}{\bar{c}}dc > 0,$$

where the last inequality holds because $c > \bar{p} > p_{2i}^n$. For the farmers with production cost $c > p_{2i-1}^n$, let B_4 denote the welfare difference between in the presence and in the absence of strategic farmers. It is easy to see that $B_4 = 0$.

Clearly, if $b = \bar{c}$, then $\sum_{i=1}^4 B_i > 0$ and thus the aggregate welfare of all farmers in the presence of strategic farmers is higher than that in the absence of strategic farmers. Suppose $b > \bar{c}$, we have

$$B_1 + B_2 = \frac{b}{4\bar{c}^2}\left(1 - \frac{\Omega}{\bar{c} + b}\right)\left(b\bar{c} + 2b^2 + (2\bar{c}^2 - 2b^2 - 3b\bar{c})\frac{\Omega}{\bar{c} + b}\right) > 0,$$

where the last inequality holds by $\frac{\Omega}{\bar{c}+b} < \frac{2b^2+b\bar{c}}{2b^2+3b\bar{c}-2\bar{c}^2}$ which is true because $\bar{p} - (\Omega - b) \ge \frac{2b\bar{c}(b-\bar{c})}{2b^2+3b\bar{c}-2\bar{c}^2}$. Given $B_3 > 0$ and $B_4 = 0$, it follows immediately that $\sum_{i=1}^4 B_i > 0$. This completes the proof of Part (iii). $\square$

F. Proof of Proposition 5

Each part of Proposition 5 corresponds to each of the following three Lemmas.

LEMMA F.1. *Suppose Assumptions 1-5 hold and* $p^o > \bar{p}$.

1. *Suppose* $\alpha > \alpha_0$.
   (a) *If* $\alpha > \alpha_1$, *then* $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c} + (1-\alpha)b}$.
   (b) *If* $\alpha \leq \alpha_1$, *then the price process does not converge.*
2. *Suppose* $\alpha \leq \alpha_0$.
   (a) *If* $\alpha > \alpha_2$, *then* $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c} + (1-\alpha)b}$.
   (b) *If* $\alpha \leq \alpha_1$, *then the price process does not converge.*

Proof of Lemma F.1. Part 1: Suppose $\alpha > \alpha_0$, which is equivalent to

$$\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 < p^o. \tag{F.1}$$

Part 1(a): We will show p_t converges if $\alpha > \alpha_1$ and does not converge if $\alpha = \alpha_1$. Note that $\alpha \geq \alpha_1$ is equivalent to $b(1-\alpha) \leq \bar{c}$.

We first show by induction that for any $t \geq 1$

$$p_t = \Big\{1 - \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^t\Big\}\frac{\Omega - \alpha\frac{b}{\bar{c}}p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} + \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^t p_0 < p^o, \tag{F.2}$$

from which whether p_t converges can be checked immediately. By (8), $p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 < p^o$, where the equality holds because $p_0 < \bar{p} < p^o \leq \bar{c}$ and the inequality holds because of (F.1). This establishes that (F.2) holds when $t = 1$.

Suppose (F.2) holds when $t = i-1$. Next we will show (F.2) holds when $t = i$. One has

$$\begin{aligned}
p_i &= \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{i-1} && [\text{by } p^o \leq \bar{c} \text{ and } p_{i-1} < p^o \leq \bar{c}]\\
&= \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}\Big\{\{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i-1}\}\frac{\Omega - \alpha\frac{b}{\bar{c}}p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} + \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i-1}p_0\Big\}\\
&= \Big\{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i\Big\}\frac{\Omega - \alpha\frac{b}{\bar{c}}p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} + \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i p_0\\
&< \Big\{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i\Big\}\frac{(1-\alpha)\frac{b}{\bar{c}}p_0 + p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} + \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i p_0 && [\text{by (F.1)}]\\
&= \frac{\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i + (1-\alpha)\frac{b}{\bar{c}}}{1+(1-\alpha)\frac{b}{\bar{c}}}p_0 + \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i}{1+(1-\alpha)\frac{b}{\bar{c}}}p^o\\
&< \frac{\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i + (1-\alpha)\frac{b}{\bar{c}}}{1+(1-\alpha)\frac{b}{\bar{c}}}p^o + \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^i}{1+(1-\alpha)\frac{b}{\bar{c}}}p^o && [\text{by } p_0 < p^o \text{ and } (1-\alpha)\frac{b}{\bar{c}} \leq 1]\\
&= p^o.
\end{aligned}$$

This completes the induction. Therefore, if $\alpha > \alpha_1$ which is $(1-\alpha)\frac{b}{\bar{c}} < 1$, then $\lim_{t\to\infty} p_t = \frac{\Omega - \alpha\frac{b}{\bar{c}}p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c}+(1-\alpha)b}$. If $\alpha = \alpha_1$, then the price alternates between $\Omega - \alpha\frac{b}{\bar{c}}p^o - p_0$ and p_0.

Part 1(b): We will show the price does not converge when $\alpha < \alpha_1$. Note that $b(1-\alpha) > \bar{c}$.

We first show there exists an $i \geq 0$ such that $p_i \geq p^o$. Suppose for a contradiction that $p_i < p^o$ for any $i \geq 0$, then as in Part 1(a), we obtain (F.2). Since $(1-\alpha)\frac{b}{\bar{c}} > 1$, there must exist an i such that $p_i \geq p^o$, contradicting our supposition.

Define $j = \min\{i : p_i \geq p^o, p_{i-k} < p^o, \forall k = 1, \ldots, i\}$. That is, period j is the first time that the price goes above p^o. Note that $p_j = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{j-1} \geq p^o$ and $p_{j-1} < \bar{p}$ (otherwise, $p_j \geq p^o$ cannot hold), so it follows the proof of Part 2(b) that p_t does not converge.

Part 2: Suppose $\alpha < \alpha_0$, which is equivalent to

$$\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 \geq p^o. \tag{F.3}$$

Part 2(a): We will show if $\alpha > \alpha_2$, that is, $b < \frac{\bar{c}}{\sqrt{1-\alpha}}$, then $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c}+(1-\alpha)b}$.

According to Part 1(a), it suffices to show there exists an $i \geq 0$ such that $p_{2i} < \bar{p}$ and $p_{2i+1} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{2i} < p^o$. Then it follows the proof of Part 1(a).

Observe that

$$\begin{aligned}
&\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) - \bar{c} \\
\leq&\Omega - \alpha\frac{b}{\bar{c}}\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) - \bar{c} && [\text{by } p^o > \bar{p} = \frac{\bar{c}}{\bar{c}+b}\Omega] \\
=&\frac{\bar{c}^2 - (1-\alpha)b^2}{\bar{c}}\Big(\frac{\Omega}{\bar{c}+b} - 1\Big) \\
\leq&0. && [\text{by } b < \frac{\bar{c}}{\sqrt{1-\alpha}} \text{ and } \Omega \leq \bar{c}+b]
\end{aligned} \tag{F.4}$$

In the following we derive the formula of p_{2i+1}. We have $p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 \geq p^o$, where the last inequality holds because of (F.3). Moreover, $p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 < \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) < \bar{c}$ where the last inequality holds because of (F.4). It follows by $p^o \leq p_1 \leq \bar{c}$ that $p_2 = \Omega - \frac{b}{\bar{c}}p_1 < \Omega - \frac{b}{\bar{c}}p^o < \Omega - \frac{b}{\bar{c}}\bar{p} = \bar{p} < p^o \leq \bar{c}$. Since $p_2 < p^o < \bar{c}$, $p_3 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_2 \leq \bar{c}$ where the last inequality holds because $p_2 \geq \Omega - b$ and (F.4). If $p_3 \leq p^o$, then we are done. Suppose $p_3 > p^o$, then $p_4 = \Omega - \frac{b}{\bar{c}}p_3 < \Omega - \frac{b}{\bar{c}}p^o < \bar{p} < p^o \leq \bar{c}$, by which it follows that $p_5 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_4$.

Continuing in this fashion, we obtain that if $p_{2i-1} > p^o$, then $p_{2i} < \bar{p}$ and

$$p_{2i+1} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{2i} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}\Big(\Omega - \frac{b}{\bar{c}}p_{2i-1}\Big)$$

$$= \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega - \alpha\frac{b}{\bar c}p^o + (1-\alpha)\frac{b^2}{\bar c^2}p_{2i-1}$$
$$= \frac{\big(1-(1-\alpha)\frac{b}{\bar c}\big)\Omega - \alpha\frac{b}{\bar c}p^o}{1-(1-\alpha)\frac{b^2}{\bar c^2}} + \Big((1-\alpha)\frac{b^2}{\bar c^2}\Big)^i\Big\{p_1 - \frac{\big(1-(1-\alpha)\frac{b}{\bar c}\big)\Omega - \alpha\frac{b}{\bar c}p^o}{1-(1-\alpha)\frac{b^2}{\bar c^2}}\Big\}.$$

Note that $(1-\alpha)\frac{b^2}{\bar c^2}<1$, so $\lim_{i\to\infty}p_{2i+1} = \frac{\big(1-(1-\alpha)\frac{b}{\bar c}\big)\Omega-\alpha\frac{b}{\bar c}p^o}{1-(1-\alpha)\frac{b^2}{\bar c^2}}$. Recall that we expect to show there exists an $i\geq 0$ such that $p_{2i}<\bar p$ and $p_{2i+1}<p^o$. To end the proof, it suffices to show $\lim_{i\to\infty}p_{2i+1}<p^o$. One has

$$\lim_{i\to\infty}p_{2i+1}-p^o = \frac{\big(1-(1-\alpha)\frac{b}{\bar c}\big)\Omega-\alpha\frac{b}{\bar c}p^o}{1-(1-\alpha)\frac{b^2}{\bar c^2}} - p^o = \frac{1}{1-(1-\alpha)\frac{b^2}{\bar c^2}}\Big\{\Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega - \Big(1+\alpha\frac{b}{\bar c}-(1-\alpha)\frac{b^2}{\bar c^2}\Big)p^o\Big\}$$
$$< \frac{1}{1-(1-\alpha)\frac{b^2}{\bar c^2}}\Big\{\Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega - \Big(1+\alpha\frac{b}{\bar c}-(1-\alpha)\frac{b^2}{\bar c^2}\Big)\frac{\bar c}{\bar c+b}\Omega\Big\} = 0.$$

where the last inequality holds because $p^o > \frac{\bar c}{\bar c+b}\Omega$ and $(1-\alpha)\frac{b^2}{\bar c^2}<1$. This completes the proof of Part 2(a).

Part 2(b): We will show if $\alpha<\alpha_1$, that is, $b\geq\frac{\bar c}{1-\alpha}$, then p_t does not converge.
Again, we have $p_1 = \Omega-\alpha\frac{b}{\bar c}p^o-(1-\alpha)\frac{b}{\bar c}p_0\geq p^o$. If $p_1>\bar c$, then $p_2=\Omega-b$, and thus $p_3 = \Omega-\alpha\frac{b}{\bar c}p^o-(1-\alpha)(\Omega-b)\geq\Omega-\alpha\frac{b}{\bar c}\bar c-(1-\alpha)(\Omega-b) = \alpha(\Omega-b)+(1-\alpha)b>\bar c$ where the first inequality holds because $p^o\leq\bar c$ and the second inequality holds because $\Omega>b$ and $b\geq\frac{\bar c}{1-\alpha}$. Thus, the price will alternate between $\Omega-\alpha\frac{b}{\bar c}p^o-(1-\alpha)(\Omega-b)$ and $\Omega-b$ and we are done. To end the proof, we assume $p_1\leq\bar c$ hereafter. Then $p_2=\Omega-\frac{b}{\bar c}p_1\leq\Omega-\frac{b}{\bar c}p^o<\Omega-\frac{b}{\bar c}\bar p=\bar p<p^o<\bar c$. It follows that $p_3=\Omega-\alpha\frac{b}{\bar c}p^o-(1-\alpha)\frac{b}{\bar c}p_2 = \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega-\alpha\frac{b}{\bar c}p^o+(1-\alpha)\frac{b^2}{\bar c^2}p_1$. One can check

$$\begin{aligned}
p_3-p_1 &= \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega-\alpha\frac{b}{\bar c}p^o+\Big((1-\alpha)\frac{b^2}{\bar c^2}-1\Big)p_1\\
&\geq \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Omega-\alpha\frac{b}{\bar c}p^o+\Big((1-\alpha)\frac{b^2}{\bar c^2}-1\Big)p^o \quad [\text{by } p_1\geq p^o \text{ and } b\geq\frac{\bar c}{1-\alpha}>\frac{\bar c}{\sqrt{1-\alpha}}]\\
&= \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Big(\Omega-(1+\frac{b}{\bar c})p^o\Big)\\
&\geq \Big(1-(1-\alpha)\frac{b}{\bar c}\Big)\Big(\Omega-(1+\frac{b}{\bar c})\frac{\bar c}{\bar c+b}\Omega\Big) \quad [\text{by } p^o\geq\frac{\bar c}{\bar c+b}\Omega \text{ and } 1-(1-\alpha)\frac{b}{\bar c}\leq 0]\\
&= 0.
\end{aligned}$$

Continuing in this fashion, we could show if $p_{2i-1}<\bar c$, then $p_{2i+1}>p_{2i-1}$. That is, if there does not exist an i such that $p_{2i-1}>\bar c$, then p_{2i-1} is increasing in i. This monotonicity and $p_1\geq p^o$ indicates that $p_{2i-1}>p^o$, and thus $p_{2i}=\Omega-\frac{b}{\bar c}p_{2i-1}<p^o$, decreasing in i. Hence, the price does not converge. Note that if there exists an i such that $p_{2i-1}>\bar c$, then similar to the above case that $p_1>\bar c$, the price will alternate between $\Omega-\alpha\frac{b}{\bar c}p^o-(1-\alpha)(\Omega-b)$ and $\Omega-b$. $\square$

LEMMA F.2. *Suppose Assumptions 1-5 hold and $p^o=\bar p$.*

1. *If $\alpha > \alpha_2$, then the market price converges to $\bar{p}$.*
2. *Otherwise, the price process does not converge.*

Proof of Lemma F.2. Part 1: We will show p_t converges if $\alpha > \alpha_2$ and does not converge if $\alpha = \alpha_2$. Note that $\alpha \geq \alpha_2$ is equivalent to $(1-\alpha)b^2 \leq \bar{c}^2$.

We first prepare some results. Observe that

$$\begin{aligned}
&\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) - \bar{c} \\
=&\Omega - \alpha\frac{b}{\bar{c}}\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) - \bar{c} && [\text{by } p^o = \frac{\bar{c}}{\bar{c}+b}\Omega] \\
=&\frac{\bar{c}^2 - (1-\alpha)b^2}{\bar{c}}\Big(\frac{\Omega}{\bar{c}+b} - 1\Big) \\
\leq&0. && [\text{by } (1-\alpha)b^2 < \bar{c}^2 \text{ and } \Omega \leq \bar{c}+b]
\end{aligned} \tag{F.5}$$

Then we will show by induction that

$$p_{2t+1} = \bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^t(p_1 - \bar{p}), \quad p_{2t+2} = \Omega - \frac{b}{\bar{c}}p_{2t+1}. \tag{F.6}$$

We first show (F.6) holds when $t = 1$. One has

$$p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 > \Omega - \alpha\frac{b}{\bar{c}}\bar{p} - (1-\alpha)\frac{b}{\bar{c}}\bar{p} = \bar{p},$$

where the last inequality holds because $p_0 < \bar{p}$. Moreover,
$p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 < \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) < \bar{c}$ where the last inequality holds because of (F.5). It follows by $p^o \leq p_1 \leq \bar{c}$ that $p_2 = \Omega - \frac{b}{\bar{c}}p_1 < \Omega - \frac{b}{\bar{c}}p^o = \bar{p} = p^o \leq \bar{c}$. Since
$p_2 < p^o \leq \bar{c}$,
$p_3 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_2 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - \frac{b}{\bar{c}}p_1) = \bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)(p_1 - \bar{p}) > \bar{p}$ where the last inequality holds because $p_1 > \bar{p}$. Moreover, $p_3 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_2 \leq \bar{c}$ where the last inequality holds because $p_2 \geq \Omega - b$ and (F.5). Observe that $p^o = \bar{p} < p_3 \leq \bar{c}$, so $p_4 = \Omega - \frac{b}{\bar{c}}p_3$. This establishes that (F.6) holds for $t = 1$.

Suppose (F.6) holds for $t = i - 1$. Next we show (F.6) holds for $t = i$. We have

$$\begin{aligned}
p_{2i+1} &= \Omega - \alpha\frac{b}{\bar{c}}\bar{p} - (1-\alpha)\frac{b}{\bar{c}}p_{2i} = \Omega - \alpha\frac{b}{\bar{c}}\bar{p} - (1-\alpha)\frac{b}{\bar{c}}\Big(\Omega - \frac{b}{\bar{c}}p_{2i-1}\Big) \\
&= \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)\Omega - \alpha\frac{b}{\bar{c}}\bar{p} + (1-\alpha)\frac{b^2}{\bar{c}^2}p_{2i-1} \\
&= \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)\Omega - \alpha\frac{b}{\bar{c}}\bar{p} + (1-\alpha)\frac{b^2}{\bar{c}^2}\Big\{\bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^{i-1}(p_1 - \bar{p})\Big\} \\
&= \bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^i(p_1 - \bar{p}) > \bar{p} = p^o,
\end{aligned}$$

where the last inequality holds because $p_1 > \bar{p}$. Note also that $p_{2i+1} = \Omega - \alpha\frac{b}{\bar{c}}\bar{p} - (1-\alpha)\frac{b}{\bar{c}}p_{2i} < \bar{c}$ because $p_{2i} > \Omega - b$ and (F.5), so $p_{2i+2} = \Omega - \frac{b}{\bar{c}}p_{2i+1}$. This completes the induction.

Therefore, if $\alpha > \alpha_2$, then $(1-\alpha)\frac{b^2}{\bar{c}^2} < 1$, hence $\lim_{t\to\infty} p_{2i+1} = \lim_{t\to\infty} p_{2i+2} = \bar{p}$. If $\alpha = \alpha_2$, then $(1-\alpha)\frac{b^2}{\bar{c}^2} = 1$, hence p_t alternates between p_1 and $\Omega - \frac{b}{\bar{c}}p_1$.

Part 2: We will show if $\alpha < \alpha_2$, that is, $(1-\alpha)\frac{b^2}{\bar{c}^2} > 1$, then the price does not converge.

We take a brief detour that

$$\begin{aligned}
&\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b)\\
=&\frac{\bar{c}}{\bar{c}+b}\Omega\Big(1-(1-\alpha)\frac{b^2}{\bar{c}^2}\Big) + (1-\alpha)\frac{b^2}{\bar{c}}\\
\geq&\bar{c}\Big(1-(1-\alpha)\frac{b^2}{\bar{c}^2}\Big) + (1-\alpha)\frac{b^2}{\bar{c}} \qquad \Big[\text{by } \Omega \leq \bar{c}+b \text{ and } 1-(1-\alpha)\frac{b^2}{\bar{c}^2} < 0\Big]\\
=&\bar{c}.
\end{aligned} \tag{F.7}$$

Then we show there exists an $i \geq 0$ such that $p_{2i+1} \geq \bar{c}$. Suppose there does not exist such an i, that is, $p_t \leq \bar{c}$ for any t. Note also that

$$p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 > \Omega - \alpha\frac{b}{\bar{c}}\bar{p} - (1-\alpha)\frac{b}{\bar{c}}\bar{p} = \bar{p} = p^o,$$

then as in Part 2(a) of Lemma F.1, we have

$$p_{2i+1} = \frac{\big(1-(1-\alpha)\frac{b}{\bar{c}}\big)\Omega - \alpha\frac{b}{\bar{c}}p^o}{1-(1-\alpha)\frac{b^2}{\bar{c}^2}} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^i\Big\{p_1 - \frac{\big(1-(1-\alpha)\frac{b}{\bar{c}}\big)\Omega - \alpha\frac{b}{\bar{c}}p^o}{1-(1-\alpha)\frac{b^2}{\bar{c}^2}}\Big\} = \bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^i(p_1 - \bar{p})$$

where the last equality holds because $p^o = \bar{p}$. Note that $p_1 > \bar{p}$, so p_{2i+1} is increasing in i because $(1-\alpha)\frac{b^2}{\bar{c}^2} > 1$, by which we know there must exist an i such that $p_{2i+1} \geq \bar{c}$.

Let $j = \min\{i : p_{2i+1} \geq \bar{c}\}$. Since $p_{2j+1} \geq \bar{c} \geq p^o$, $p_{2j+2} = \Omega - bF(p_{2j+1}) = \Omega - b$, and $p_{2j+3} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{2j+2} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b) \geq \bar{c}$, where the last inequality holds because of (F.7). It can be checked that the price alternates between $\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b)$ and $\Omega - b$. □

Lemma F.3. *Suppose Assumptions 1-5 hold and $p^o < \bar{p}$. Then the price process does not converge.*

Proof of Lemma F.3. Whenever the low price in each cycle falls into the range of $[p^o, \bar{p}]$ which is a necessary process for convergence, the contract is not effective anymore and thus the price process will stop converging because the price does not converge when $b \geq \bar{c}$ in the model with naïve farmers only and no contract. Therefore, the price does not converge if $p^o < \bar{p}$. □

Proof of Proposition 5 The result follows immediately by combining Lemmas F.1, F.2, and F.3. □

G. Proof of Proposition 6, Corollaries 2 and 3

Proof of Proposition 6. We first show under some conditions $p^{o*} = \bar{p}$ is the unique optimal solution to SE's problem (10). We consider three scenarios.

Scenario 1: Suppose $p^{o*} > \bar{p}$. Proposition 5 implies that $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^{o*}-\bar{p})}{\bar{c}+(1-\alpha)b} < \bar{p}$. Then there exists a t' such that for any $t \geq t'$, $\pi_t(p^{o*}) = (p_t - p^{o*})q_t^o < 0$, which violates the profit constraint in problem (10). Hence, $p^{o*} > \bar{p}$ cannot be optimal.

Scenario 2: Suppose $p^{o*} = \bar{p}$, then $\lim_{t\to\infty} p_t = \bar{p}$. In the long run, both types of farmers receive a surplus as follows. (i) If $c \leq \bar{p}$, then $\lim_{i\to\infty} \bar{w}_i^n(c; p^{o*} = \bar{p}) = \lim_{i\to\infty} \bar{w}_i^s(c; p^{o*} = \bar{p}) = \bar{p} - c$. (ii) If $c > \bar{p}$, then $\lim_{i\to\infty} \bar{w}_i^n(c; p^{o*} = \bar{p}) = \lim_{i\to\infty} \bar{w}_i^s(c; p^{o*} = \bar{p}) = 0$.

Scenario 3: Suppose $p^{o*} < \bar{p}$. Proposition 5 shows that the price process does not converge. Moreover, when the lower price in any cycle is lower than p^{o*}, the contract will take effect and the price will go above p^{o*}. When the lower price goes above p^{o*}, the contract will not take effect and the price will diverge below p^{o*}. So the lower price keeps going above and below p^{o*}. To complete the proof, we will show the aggregate welfare of all farmers in every possible case is lower than that in Scenario 2.

Note that the prices in each cycle i are (p_{2i-1}, p_{2i}) where $p_{2i} < \bar{p} < p_{2i-1}$. We have two cases.

Case 1: Suppose $p_{2i-1} \leq \bar{c}$ for any $i \geq 1$. In this case, $p_{2i} = \Omega - \frac{b}{\bar{c}} p_{2i-1}$, so
$\frac{p_{2i-1}+p_{2i}}{2} = \frac{\Omega+(1-\frac{b}{\bar{c}})p_{2i-1}}{2} < \frac{\Omega+(1-\frac{b}{\bar{c}})\frac{\bar{c}}{\bar{c}+b}\Omega}{2} = \bar{p}$.

The type-N farmers' welfare is as follows. (i) If $c \leq p_{2i-2}$, then $\bar{w}_i^n(c; p^{o*} < \bar{p}) = \frac{p_{2i-1}+p_{2i}}{2} - c < \bar{p} - c$. (ii) If $p_{2i-2} < c \leq p_{2i-1}$, then $\bar{w}_i^n(c; p^{o*} < \bar{p}) = \frac{1}{2}(p_{2i} - c)$. (iii) If $c > p_{2i-1}$, then $\bar{w}_i^n(c; p^{o*} < \bar{p}) = 0$.

As for type-S farmers, there are two possible subcases. Subcase 1: $p_{2i-2} < p^{o*}$. That is, the type-S farmers accept the contract in period $2i-1$ and their welfare is as follows. (i) If $c \leq p^{o*}$, then $\bar{w}_i^s(c; p^{o*} < \bar{p}) = \frac{p^{o*}+p_{2i}}{2} - c < \bar{p} - c$. (ii) If $p^{o*} < c \leq p_{2i-1}$, then $\bar{w}_i^s(c; p^{o*} < \bar{p}) = \frac{1}{2}(p_{2i} - c)$. (iii) If $c > p_{2i-1}$, then $\bar{w}_i^s(c; p^{o*} < \bar{p}) = 0$. Subcase 2: $p_{2i-2} \geq p^{o*}$. That is, no type-S farmers accept the contract in period $2i-1$ and their welfare is the same as type-N farmers.

Comparing with farmers' welfare in Scenario 2, we obtain

$$\bar{w}_i^n(c; p^{o*} < \bar{p}) \leq \lim_{i\to\infty} \bar{w}_i^n(c; p^{o*} = \bar{p}) \quad \text{for any } 0 \leq c \leq \bar{c}$$
$$\bar{w}_i^s(c; p^{o*} < \bar{p}) \leq \lim_{i\to\infty} \bar{w}_i^s(c; p^{o*} = \bar{p}) \quad \text{for any } 0 \leq c \leq \bar{c}.$$

Therefore,

$$\int_0^{\bar{c}} \lim_{i\to\infty} \left(\bar{w}_i^s(c; p^{o*} < \bar{p}) + \bar{w}_i^n(c; p^{o*} < \bar{p})\right)\frac{1}{\bar{c}}dc \leq \int_0^{\bar{c}} \lim_{i\to\infty} \left(\bar{w}_i^s(c; p^{o*} = \bar{p}) + \bar{w}_i^n(c; p^{o*} = \bar{p})\right)\frac{1}{\bar{c}}dc.$$

Case 2: Suppose there exists a $j \geq 1$ such that $p_{2j-1} > \bar{c}$. In this case, $p_{2j} = \Omega - b$, so $p_{2j+1} = \Omega - \frac{b}{\bar{c}}(\Omega - b) > \bar{c}$, and thus $p_{2j+2} = \Omega - b$. Continuing in this fashion, we obtain that $p_{2i} = \Omega - b = \underline{c}$ and $p_{2i+1} = \Omega - \frac{b}{\bar{c}}(\Omega - b)$ for any $i \geq j$. That is, no type-S farmers accept the contract and the price dynamics is the same as the model with all farmers being naïve and no SE. Hence, the aggregate welfare of all farmers is also the same as that with all farmers being naïve and no SE.

Note that the aggregate welfare of all farmers in Scenario 2 is the same as that when price is stabilized at $\bar{p}$ with a sufficient number of strategic farmers. Therefore, by Part (iii) of Proposition 3, we have if $b = \bar{c}$ or $b > \bar{c}$ and $\bar{p} - (\Omega - b) \geq \frac{2b\bar{c}(b-\bar{c})}{2b^2+3b\bar{c}-2\bar{c}^2}$, then

$$\int_0^{\bar{c}} \lim_{i\to\infty} \Big(\bar{w}_i^s(c; p^{o*} < \bar{p}) + \bar{w}_i^n(c; p^{o*} < \bar{p})\Big)\frac{1}{\bar{c}}dc \leq \int_0^{\bar{c}} \lim_{i\to\infty} \Big(\bar{w}_i^s(c; p^{o*} = \bar{p}) + \bar{w}_i^n(c; p^{o*} = \bar{p})\Big)\frac{1}{\bar{c}}dc.$$

Combining the above two cases, we see $p^{o*} < \bar{p}$ cannot be optimal. This establishes that $p^{o*} = \bar{p}$ is the unique optimal solution.

Part (i): We will show the SE receives a positive profit in the short run and the profit diminishes to 0 in the long run.

According to the proof of Lemma F.2, the market price when $p^o = \bar{p}$ is as follows,

$$p_{2i-1} = \bar{p} + \Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^{i-1}(p_1 - \bar{p}) > \bar{p} \quad \text{and} \quad p_{2i} = \Omega - \frac{b}{\bar{c}}p_{2i-1} < \bar{p}.$$

In each cycle, since $p_{2i-2} < p^o = \bar{p} < p_{2i-1}$, type-S farmers accept the contract in period $2i-1$ and do not accept in period $2i$. Hence, $\pi_{2i-1}(p^o; p_{2i-1}) = (p_{2i-1} - \bar{p})q^o_{2i-1} > 0$ and $\pi_{2i}(p^o; p_{2i}) = 0$. Using the fact that $\lim_{i\to\infty} p_{2i-1} = \bar{p}$, we conclude that $\lim_{i\to\infty} \pi_{2i-1}(p^o; p_{2i-1}) = 0$.

Part (ii): We make the welfare comparison in both short- and long-run.

We use p_t and p_t^n to denote the price in the presence of SE's contract and in the model with naïve farmers only, respectively. If $\alpha = 0$, then by Assumption (4), we have $g(\alpha) \geq 1$. Proposition 1 shows that the alternating price is $(2\bar{p} - p_0, p_0)$ if $b = \bar{c}$ and $(\Omega - \frac{b}{\bar{c}}(\Omega - b), \Omega - b)$ if $b > \bar{c}$. Hence, the naïve farmers' welfare $w_i^n(c)$ is as follows. (i) If $c \leq p^n_{2i-2}$, then

$$w_i^n(c) = \frac{p^n_{2i-1} + p^n_{2i}}{2} - c = \begin{cases} \bar{p} - c & \text{if } b = \bar{c}, \\ \frac{1}{2}\Big(\Omega - \frac{b}{\bar{c}}(\Omega - b) + \Omega - b\Big) & \text{if } b > \bar{c}. \end{cases}$$

(ii) If $p^n_{2i-2} < c \leq p^n_{2i-1}$, then $w_i^n(c) = \frac{1}{2}(p^n_{2i} - c)$. (iii) If $c > p^n_{2i-1}$, then $w_i^n(c) = 0$. Next we will derive the welfare of each type of farmers and make a comparison with $w_i^n(c)$. Note that for each i,

$$p^n_{2i-2} \leq p_{2i-2} < \bar{p} < p_{2i-1} \leq p^n_{2i-1} \quad \text{and} \quad p^n_{2i} \leq p_{2i}.$$

Part (ii)(a): We will make the welfare comparison in the short run.

The type-S farmers' welfare is as follows. (i) If $c \leq p^o$, then $\bar{w}_i^s(c) = \frac{p^o + p_{2i}}{2} - c < \bar{p} - c$. (ii) If $p^o < c \leq p_{2i-1}$, then $\bar{w}_i^s(c) = \frac{1}{2}(p_{2i} - c) < 0$. (iii) If $c > p_{2i-1}$, then $\bar{w}_i^s(c) = 0$. The type-N farmers' welfare is as follows. (i) If $c \leq p_{2i-2}$, then

$$\bar{w}_i^n(c) = \frac{p_{2i-1} + p_{2i}}{2} - c = \frac{\Omega - (\frac{b}{\bar{c}} - 1)p_{2i-1}}{2} - c \begin{cases} = \bar{p} - c & \text{if } b = \bar{c}, \\ < \frac{\Omega - (\frac{b}{\bar{c}} - 1)\frac{\bar{c}}{\bar{c}+b}\Omega}{2} - c = \bar{p} - c & \text{if } b > \bar{c} \end{cases}$$

(ii) If $p_{2i-2} < c \leq p_{2i-1}$, then $\bar{w}_i^n(c) = \frac{1}{2}(p_{2i} - c) < 0$. (iii) If $c > p_{2i-1}$, then $\bar{w}_i^n(c) = 0$.

Comparing with $w_i^n(c)$ above, we have that for any $c > p_{2i-2}^n = \underline{c}$,

$$\bar{w}_i^s(c) \geq w_i^n(c) \quad \text{and} \quad \bar{w}_i^n(c) \geq w_i^n(c).$$

For any $c \leq p_{2i-2}^n = \underline{c}$,

$$\bar{w}_i^s(c) < w_i^n(c) \quad \text{and} \quad \bar{w}_i^n(c) \leq w_i^n(c)$$

where the above equality holds only when $b = \bar{c}$.

Part (ii)(b): We will make the welfare comparison in the long run.

In the long run, the price converges to $\bar{p}$. Hence, both types of farmers' welfare is just the same as that in the model with a sufficient number of strategic farmers. Therefore, the result follows immediately by Part (ii) of Proposition 3.

Part (ii)(c): Observe that the total welfare of all farmers when $p^o = \Omega - b$ is the same as that in the model with all farmers being naïve and no SE. So Part (ii)(c) follows immediately by the optimality of $p^{o*} = \bar{p}$ to problem (10). □

Proof of Corollary 2 Lemma F.2 implies that if $p^o = \bar{p}$, then $\lim_{t\to\infty} p_t = \bar{p}$. Corollary 2 follows immediately. □

Proof of Corollary 3. Recall that in the model with naïve farmers only, if $b = \bar{c}$, then the price alternates between $\Omega - p_0$ and p_0. If $b > \bar{c}$, then there exists an i' such that in each cycle $i > i'$ the price alternates between $\Omega - \frac{b}{\bar{c}}(\Omega - b)$ and $\Omega - b$. Let (p_{2i-1}^n, p_{2i}^n) denote the alternating prices in the model with naïve farmers only, and (p_{2i-1}, p_{2i}) denote the price in the presence of SE's contract. Note that

$$p_{2i-2}^n \leq p_{2i-2} < \bar{p} < p_{2i-1} \leq p_{2i-1}^n \quad \text{and} \quad p_{2i}^n \leq p_{2i}.$$

(i) Since $c_1 \leq p_{2i-2}^n$, the proof of Part (ii) of Proposition 6 gives us that

$$\bar{w}_i^s(c_1) = \frac{p^o + p_{2i}}{2} - c_1,$$

$$\bar{w}_i^n(c_1) = \frac{p_{2i-1} + p_{2i}}{2} - c_1 = \frac{p_{2i-1} + \Omega - \frac{b}{\bar{c}} p_{2i-1}}{2} - c_1 = \frac{\Omega - (\frac{b}{\bar{c}} - 1)p_{2i-1}}{2} - c_1,$$
$$w_i^n(c_1) = \frac{p_{2i-1}^n + p_{2i}^n}{2} - c_1 = \frac{p_{2i-1}^n + \Omega - \frac{b}{\bar{c}} p_{2i-1}^n}{2} - c_1 = \frac{\Omega - (\frac{b}{\bar{c}} - 1)p_{2i-1}^n}{2} - c_1.$$

Since $p^b = \bar{p} < p_{2i-1}$, we have $\bar{w}_i^s(c_1) < \bar{w}_i^n(c_1)$. Note that $p_{2i-1} > p_{2i-1}^n$, so $\bar{w}_i^n(c_1) < w_i^n(c_1)$. Therefore,

$$\bar{w}_i^s(c_1) < \bar{w}_i^n(c_1) < w_i^n(c_1). \tag{G.1}$$

(ii) If $p_{2i-2}^n < c_2 \le \bar{p}$, then

$$\bar{w}_i^s(c_2) = \frac{p^o + p_{2i}}{2} - c_2,$$
$$\bar{w}_i^n(c_2) = \begin{cases} \frac{p_{2i-1}+p_{2i}}{2} - c_2 & \text{if } p_{2i-2}^n < c_2 \le p_{2i-2}, \\ \frac{1}{2}(p_{2i}^n - c_2) & \text{if } p_{2i-2} < c_2 \le \bar{p}, \end{cases}$$
$$w_i^n(c_2) = \frac{1}{2}(p_{2i}^n - c_2).$$

If $p_{2i-2}^n < c_2 \le p_{2i-2}$, then $\bar{w}_i^s(c_1) - \bar{w}_i^s(c_2) = c_2 - c_1$, $\bar{w}_i^n(c_1) - \bar{w}_i^n(c_2) = c_2 - c_1$, and $w_i^n(c_1) - w_i^n(c_2) > c_2 - c_1$, and hence $\bar{w}_i^s(c_1) - \bar{w}_i^s(c_2) \le \bar{w}_i^n(c_1) - \bar{w}_i^n(c_2) \le w_i^n(c_1) - w_i^n(c_2)$.
If $p_{2i-2} < c_2 \le \bar{p}$, then one can check $\bar{w}_i^s(c_2) \ge \bar{w}_i^n(c_2) \ge w_i^n(c_2)$. Therefore, by (G.1), we have $\bar{w}_i^s(c_1) - \bar{w}_i^s(c_2) \le \bar{w}_i^n(c_1) - \bar{w}_i^n(c_2) \le w_i^n(c_1) - w_i^n(c_2)$.
(iii) If $\bar{p} < c_2 \le p_{2i-1}^n$, then

$$\bar{w}_i^s(c_2) = \bar{w}_i^n(c_2) = \begin{cases} \frac{1}{2}(p_{2i} - c_2) < 0 & \text{if } \bar{p} < c_2 \le p_{2i-1}, \\ 0 & \text{if } p_{2i-1} < c_2 \le p_{2i-1}^n, \end{cases}$$
$$w_i^n(c_2) = \frac{1}{2}(p_{2i}^n - c_2) < 0.$$

One can check $\bar{w}_i^s(c_2) = \bar{w}_i^n(c_2) > w_i^n(c_2)$. Therefore, by (G.1), we have $\bar{w}_i^s(c_1) - \bar{w}_i^s(c_2) \le \bar{w}_i^n(c_1) - \bar{w}_i^n(c_2) \le w_i^n(c_1) - w_i^n(c_2)$.
(iv) If $c_2 > p_{2i-1}^n$, then $\bar{w}_i^s(c_2) = \bar{w}_i^n(c_2) = w_i^n(c_2) = 0$. Therefore, by (G.1), we have $\bar{w}_i^s(c_1) - \bar{w}_i^s(c_2) \le \bar{w}_i^n(c_1) - \bar{w}_i^n(c_2) \le w_i^n(c_1) - w_i^n(c_2)$. □

H. Proof of Proposition 7

Proof of Proposition 7 As shown in Proposition 6, Parts (i) and (iii) hold immediately. To complete the proof, it suffices to show the firm obtains a higher profit and $\bar{p}$ is an optimal solution to problem (12) if $b = \bar{c}$.

Part 1: We will show the firm obtains a higher profit, that is, incurs a lower cost, in both short and long run.

Suppose there is no contract, i.e., $p^o=0$, then the price alternates between two prices p^n_{2i-1} and p^n_{2i} (we omit the short divergence periods when $b>\bar{c}$). Note that if $b=\bar{c}$, then $(p^n_{2i-1},p^n_{2i})=(\Omega-p_0,p_0)$. If $b>\bar{c}$, then $(p^n_{2i-1},p^n_{2i})=(\Omega-\frac{b}{\bar{c}}(\Omega-b),\Omega-b)$. In this case,

$$
\begin{aligned}
&f_{2i-1}(p^o;p^o=0)+f_{2i}(p^o;p^o=0)\\
=&p^n_{2i-1}q_{2i-1}+p^m(d-q_{2i-1})+p^n_{2i}d=p^n_{2i-1}\frac{\Omega-p^n_{2i-1}}{b}+p^m\big(d-\frac{\Omega-p^n_{2i-1}}{b}\big)+p^n_{2i}d\\
>&(p^n_{2i-1}+p^n_{2i})d=\begin{cases}\Omega d & \text{if } b=\bar{c},\\ \big(\Omega-(\frac{b}{\bar{c}}-1)(\Omega-b)\big)d & \text{if } b>\bar{c},\end{cases}
\end{aligned}
$$

where the first equality holds because the market supply in period $2i-1$ can not satisfy the demand by our assumption that $d>\lim_{i\to\infty}\frac{\Omega-p^n_{2i-1}}{b}$, and the inequality holds because $p^m>p^n_{2i-1}$. Observe that the costs in the short run and in the long run are the same when the price is alternating forever. Next we will derive the cost when the firm offers a contract with $p^o=\bar{p}$ and then make a comparison with $f_{2i-1}(p^o;p^o=0)+f_{2i}(p^o;p^o=0)$ in both short and long run.

Part 1(a): We will make the comparison in the short run.

Suppose the firm offers a contract with $p^o=\bar{p}$, then according to the proof of Lemma F.2, the market price is as follows,

$$
p_{2i-1}=\bar{p}+\Big((1-\alpha)\frac{b^2}{\bar{c}^2}\Big)^{i-1}(p_1-\bar{p}) \quad\text{and}\quad p_{2i}=\Omega-\frac{b}{\bar{c}}p_{2i-1}.
$$

Note that p_{2i-1} is decreasing in i and $\lim_{i\to\infty}p_{2i-1}=\bar{p}$, so there exists an i' such that $p_{2i-1}<\Omega-bd$ for any $i\geq i'$. When $i\geq i'$, we have

$$
\begin{aligned}
&f_{2i-1}(p^o;p^o=\bar{p})+f_{2i}(p^o;p^o=\bar{p})\\
=&q^o_{2i-1}p^o+(d-q^o_{2i-1})p_{2i-1}+dp_{2i}<dp_{2i-1}+dp_{2i}=d(p_{2i-1}+p_{2i})\\
=&d\big(p_{2i-1}+\Omega-\frac{b}{\bar{c}}p_{2i-1}\big)=d\big(\Omega-(\frac{b}{\bar{c}}-1)p_{2i-1}\big)\begin{cases}=\Omega d & \text{if } b=\bar{c},\\ <d\Big(\Omega-(\frac{b}{\bar{c}}-1)(\Omega-d)\Big) & \text{if } b>\bar{c}.\end{cases}\\
<&f_{2i-1}(p^o;p^o=0)+f_{2i}(p^o;p^o=0).
\end{aligned}
$$

Therefore, the firm incurs a lower cost than that without the contract in the short run.

Part 1(b): We will make the long run comparison.

Given $\lim_{i\to\infty}p_{2i-1}=\lim_{i\to\infty}p_{2i}=\bar{p}$, it follows that

$$
\lim_{i\to\infty}f_{2i-1}(p^o;p^o=\bar{p})+f_{2i}(p^o;p^o=\bar{p})=2\bar{p}d=\begin{cases}\Omega d & \text{if } b=\bar{c},\\ \frac{2\bar{c}}{\bar{c}+b}\Omega d & \text{if } b>\bar{c}.\end{cases}
$$

Observe that $\frac{2\bar{c}}{\bar{c}+b}\Omega-\big\{\Omega-(\frac{b}{\bar{c}}-1)(\Omega-b)\big\}=\frac{b}{\bar{c}(\bar{c}+b)}(b-\bar{c})(\Omega-b-\bar{c})<0$, so $\lim_{i\to\infty}f_{2i-1}(p^o;p^o=\bar{p})+f_{2i}(p^o;p^o=\bar{p})<f_{2i-1}(p^o;p^o=0)+f_{2i}(p^o;p^o=0)$. Therefore, the firm incurs a lower cost than without the contract in the long run.

Part 2: We will show $p^o = \bar{p}$ is an optimal solution to problem (12) if $b = \bar{c}$. We consider two cases. Case 1: $p^o \geq \bar{p} = \frac{\Omega}{2}$. According to Proposition 5, $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c}+(1-\alpha)b} = \frac{\Omega - \alpha p^o}{2-\alpha} < \bar{p}$. Assumption $d < \frac{\Omega - \bar{p}}{b}$ means that the market supply at price $\bar{p}$ exceeds the demand d, therefore the focal market supply at price $\frac{\Omega-\alpha p^o}{2-\alpha}$ which is lower than $\bar{p}$ also exceeds the demand d. Thus,

$$\lim_{i\to\infty} f_{2i-1}(p^o; p^o \geq \bar{p}) + f_{2i}(p^o; p^o \geq \bar{p}) = 2\Big\{\alpha \frac{p^o}{\bar{c}} p^o + (d - \alpha \frac{p^o}{\bar{c}}) \frac{\Omega - \alpha p^o}{2-\alpha}\Big\}.$$

Taking derivatives to p^o yields that the unconstrained optimal solution $p^{o*} = \frac{\Omega + bd}{4}$, which is smaller than $\bar{p} = \frac{\Omega}{2}$ because $bd < \Omega$ by our assumption. Therefore, the convexity indicates that $\lim_{i\to\infty} f_{2i-1}(p^o; p^o = \bar{p}) + f_{2i}(p^o; p^o = \bar{p}) < \lim_{i\to\infty} f_{2i-1}(p^o; p^o > \bar{p}) + f_{2i}(p^o; p^o > \bar{p})$.

Case 2: $p^o < \bar{p} = \frac{\Omega}{2}$. If $p^o \leq p_0$, then the contract is not effective and the price alternates between $\Omega - p_0$ and p_0. If $p^o > p_0$, then once the low price in each cycle goes above p^o, the contract will not be effective anymore, and thus the price alternates between two constant prices. To summarize, in Case 2 the price will alternate. Suppose the price alternates between p_{2i-1} and p_{2i}. Note that $p_{2i} = \Omega - p_{2i-1}$. We have

$$\begin{aligned}
&\lim_{i\to\infty} f_{2i-1}(p^o; p^o < \bar{p}) + f_{2i}(p^o; p^o < \bar{p}) \\
&= \begin{cases} p_{2i-1}d + p_{2i}d = \Omega d & \text{if } p_{2i-1} < \Omega - bd, \\ p_{2i-1}q_{2i-1} + p^m(d - q_{2i-1}) + p_{2i}d > p_{2i-1}d + p_{2i}d = \Omega d & \text{otherwise.} \end{cases}
\end{aligned}$$

Note that $\lim_{i\to\infty} f_{2i-1}(p^o; p^o = \bar{p}) + f_{2i}(p^o; p^o = \bar{p}) = 2d\bar{p} = \Omega d$, so $\lim_{i\to\infty} f_{2i-1}(p^o; p^o = \bar{p}) + f_{2i}(p^o; p^o = \bar{p}) \leq \lim_{i\to\infty} f_{2i-1}(p^o; p^o < \bar{p}) + f_{2i}(p^o; p^o < \bar{p})$.

Combining the above two cases, we conclude that $p^{o*} = \bar{p}$ is an optimal solution to firm's problem (12) if $b = \bar{c}$. $\square$

I. Proof of Propositions 8 and 9

Before the proof of Proposition 8, we first have the following preparations:

$$\begin{aligned}
q_t^{A,n} &= (1-\alpha)\mathsf{P}(u_t^{A,n} \geq u_t^{B,n}) = (1-\alpha)\mathsf{P}(p_{t-1}^A - c \geq p_{t-1}^B - (\bar{c} - c)) \\
&= (1-\alpha)\mathsf{P}(c \leq \frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2}) = (1-\alpha)F(\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2}), \\
q_t^{B,n} &= (1-\alpha)[1 - F(\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2})], \\
q_t^{A,s} &= \alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2}), \\
q_t^{B,s} &= \alpha[1 - F(\frac{p_t^A - p_t^B + \bar{c}}{2})].
\end{aligned}$$

Thus,

$$p_t^A = \Omega - b(q_t^{A,n} + q_t^{A,s}) = \Omega - b(1-\alpha)F(\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2}) - b\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2}), \tag{I.1}$$

$$p_t^B = \Omega - b(q_t^{A,n} + q_t^{A,s}) = \Omega - b + b(1-\alpha)F(\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2}) + b\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2}), \quad \text{(I.2)}$$

$$p_t^A - p_t^B = b\Big\{1 - 2(1-\alpha)F(\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2}) - 2\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2})\Big\}. \quad \text{(I.3)}$$

LEMMA I.1. *Suppose Assumptions 1-3 hold and $g(\alpha) \le 1$. Then $|p_t^A - p_t^B| \le \bar{c}$ for any $t \ge 1$. Moreover, $p_t^A - p_t^B = -g(\alpha)(p_{t-1}^A - p_{t-1}^B)$ for any $t \ge 2$.*

Proof of Lemma I.1. Note that $g(\alpha) \le 1$ is equivalent to $b(1-2\alpha) \le \bar{c}$. To show $|p_t^A - p_t^B| \le \bar{c}$, we have three cases.

Case 1: Suppose $p_{t-1}^A - p_{t-1}^B > \bar{c}$. Then (I.3) reduces to $p_t^A - p_t^B = b\{1 - 2(1-\alpha) - 2\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2})\}$. If $p_t^A - p_t^B > \bar{c}$, then $p_t^A - p_t^B = b\{1 - 2(1-\alpha) - 2\alpha\} = -b < \bar{c}$, which arrives a contradiction. If $p_t^A - p_t^B < -\bar{c}$, then $p_t^A - p_t^B = b\{1 - 2(1-\alpha)\} = -b(1-2\alpha) \ge -\bar{c}$, which arrives a contradiction. Hence, it must be the case that $|p_t^A - p_t^B| \le \bar{c}$.

Case 2: Suppose $p_{t-1}^A - p_{t-1}^B < -\bar{c}$. Then (I.3) reduces to $p_t^A - p_t^B = b\{1 - 2\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2})\}$. If $p_t^A - p_t^B > \bar{c}$, then $p_t^A - p_t^B = b(1-2\alpha) \le \bar{c}$, which arrives a contradiction. If $p_t^A - p_t^B < -\bar{c}$, then $p_t^A - p_t^B = b > -\bar{c}$, which arrives a contradiction. Hence, it must be the case that $|p_t^A - p_t^B| \le \bar{c}$.

Case 3: Suppose $|p_{t-1}^A - p_{t-1}^B| \le \bar{c}$. Then (I.3) reduces to

$$p_t^A - p_t^B = b\Big\{1 - 2(1-\alpha)\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2\bar{c}} - 2\alpha F(\frac{p_t^A - p_t^B + \bar{c}}{2})\Big\}. \quad \text{(I.4)}$$

If $p_t^A - p_t^B > \bar{c}$, then $p_t^A - p_t^B = b\{1 - 2(1-\alpha)\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2\bar{c}} - 2\alpha\} \le b(1-2\alpha) \le \bar{c}$, which arrives a contradiction. If $p_t^A - p_t^B < -\bar{c}$, then
$p_t^A - p_t^B = b\{1 - 2(1-\alpha)\frac{p_{t-1}^A - p_{t-1}^B + \bar{c}}{2\bar{c}}\} \ge b\{1 - 2(1-\alpha)\} = -b(1-2\alpha) \ge -\bar{c}$, which arrives a contradiction. Hence, it must be the case that $|p_t^A - p_t^B| \le \bar{c}$.

This establishes that $|p_t^A - p_t^B| \le \bar{c}$ for any $t \ge 1$. Consequently, (I.4) holds for any $t \ge 2$ and reduces to $p_t^A - p_t^B = -\frac{(1-\alpha)b}{\bar{c}+\alpha b}(p_{t-1}^A - p_{t-1}^B)$. □

LEMMA I.2. *Suppose Assumptions 1-3 hold and $g(\alpha) > 1$. Then there exists an $i \ge 0$ such that $p_{i+2j}^A - p_{i+2j}^B < -\bar{c}$ and $p_{i+2j+1}^A - p_{i+2j+1}^B > \bar{c}$ for any $j \ge 0$.*

Proof of Lemma I.2. We first show there exists an $i \ge 0$ such that $|p_i^A - p_i^B| > \bar{c}$. Suppose for a contradiction that there does not exist such an i, that is, $|p_t^A - p_t^B| \le \bar{c}$ for any $t \ge 1$. Thus, (I.3) can be reduced to $p_t^A - p_t^B = -g(\alpha)(p_{t-1}^A - p_{t-1}^B)$. Hence,

$$|p_t^A - p_t^B| = g(\alpha)|p_{t-1}^A - p_{t-1}^B|. \quad \text{(I.5)}$$

Given $g(\alpha) > 1$, by (I.5), there must exist an $i \ge 0$ such that $|p_i^A - p_i^B| > \bar{c}$.

To end the proof, it suffices to show (i) If $p^A_{t-1} - p^B_{t-1} > \bar{c}$, then $p^A_t - p^B_t < -\bar{c}$. (ii) If $p^A_{t-1} - p^B_{t-1} < -\bar{c}$, then $p^A_t - p^B_t > \bar{c}$.
We first show Case (i). If $p^A_{t-1} - p^B_{t-1} > \bar{c}$, then (I.3) reduces to $p^A_t - p^B_t = b\{1 - 2(1-\alpha) - 2\alpha F(\frac{p^A_t - p^B_t + \bar{c}}{2})\}$. If $p^A_t - p^B_t > \bar{c}$, then $p^A_t - p^B_t = b\{1 - 2(1-\alpha) - 2\alpha\} = -b < \bar{c}$, which arrives a contradiction. If $|p^A_t - p^B_t| \leq \bar{c}$, then $p^A_t - p^B_t = -g(\alpha)(p^A_{t-1} - p^B_{t-1}) < -g(\alpha)\bar{c} < -\bar{c}$, which arrives a contradiction. Hence, it must be the case that $p^A_t - p^B_t < -\bar{c}$. The proof of Case (ii) follows a similar way. $\square$

Proof of Proposition 8. Part (i): If $g(\alpha) > 1$, then it follows Lemma I.2 that $p^A_t - p^B_t$ does not converge. If $g(\alpha) = 1$, by Lemma I.1, we have $p^A_t - p^B_t = -(p^A_{t-1} - p^B_{t-1})$, and hence, $p^A_t - p^B_t$ does not converge. Therefore, p^A_t and p^B_t do not converge.
Part (ii): If $g(\alpha) < 1$, then it follows Lemma I.1 that $p^A_t - p^B_t = -g(\alpha)(p^A_{t-1} - p^B_{t-1}) = [-g(\alpha)]^{t-1}(p^A_1 - p^B_1)$. By (I.1) and (I.2), we have

$$p^A_t = \Omega - \frac{b}{2} - \frac{1}{2}[-g(\alpha)]^{t-1}(p^A_1 - p^B_1)$$
$$p^B_t = \Omega - \frac{b}{2} + \frac{1}{2}[-g(\alpha)]^{t-1}(p^A_1 - p^B_1)$$

and therefore,

$$\lim_{t\to\infty} p^A_t = \lim_{t\to\infty} p^B_t = \Omega - \frac{b}{2}. \quad \square$$

Before we prove Proposition 9, we first derive some expressions and demonstrate one lemma. The random yield quantity of two crops in period t are

$$Q^{A,s}_t = \gamma_t \hat{q}^{A,s}_t = \gamma_t \alpha \mathsf{P}(u^{A,s}_t \geq u^{B,s}_t) = \gamma_t \alpha F(\frac{\mathsf{E}[\gamma_t P^A_t] - \mathsf{E}[\gamma_t P^B_t] + \bar{c}}{2}),$$
$$Q^{B,s}_t = \gamma_t \hat{q}^{B,s}_t = \gamma_t \alpha \mathsf{P}(u^{A,s}_t < u^{B,s}_t) = \gamma_t \alpha [1 - F(\frac{\mathsf{E}[\gamma_t P^A_t] - \mathsf{E}[\gamma_t P^B_t] + \bar{c}}{2})],$$
$$Q^{A,n}_t = \gamma_t \hat{q}^{A,n}_t = \gamma_t (1-\alpha) \mathsf{P}(u^{A,n}_t \geq u^{B,n}_t) = \gamma_t (1-\alpha) F(\frac{P^A_{t-1} - P^B_{t-1} + \bar{c}}{2}),$$
$$Q^{B,n}_t = \gamma_t \hat{q}^{B,n}_t = \gamma_t (1-\alpha) \mathsf{P}(u^{A,n}_t < u^{B,n}_t) = \gamma_t (1-\alpha) [1 - F(\frac{P^A_{t-1} - P^B_{t-1} + \bar{c}}{2})].$$

The market clearing prices of Crops A and B are

$$P^A_t = \Omega - b(Q^{A,s}_t + Q^{A,n}_t) = \Omega - b\alpha\gamma_t F(\frac{\mathsf{E}[\gamma_t P^A_t] - \mathsf{E}[\gamma_t P^B_t] + \bar{c}}{2}) - b(1-\alpha)\gamma_t F(\frac{P^A_{t-1} - P^B_{t-1} + \bar{c}}{2}), \tag{I.6}$$
$$P^B_t = \Omega - b(Q^{B,s}_t + Q^{B,n}_t) = \Omega - b\alpha\gamma_t[1 - F(\frac{\mathsf{E}[\gamma_t P^A_t] - \mathsf{E}[\gamma_t P^B_t] + \bar{c}}{2})] - b(1-\alpha)\gamma_t[1 - F(\frac{P^A_{t-1} - P^B_{t-1} + \bar{c}}{2})]. \tag{I.7}$$

Thus,

$$P^A_t - P^B_t = b\alpha\gamma_t[1 - 2F(\frac{\mathsf{E}[\gamma_t P^A_t] - \mathsf{E}[\gamma_t P^B_t] + \bar{c}}{2})] + b(1-\alpha)\gamma_t[1 - 2F(\frac{P^A_{t-1} - P^B_{t-1} + \bar{c}}{2})], \tag{I.8}$$

$$P_t^A\gamma_t - P_t^B\gamma_t = \{\alpha[1-2F(\frac{\mathsf{E}[\gamma_t P_t^A]-\mathsf{E}[\gamma_t P_t^B]+\bar{c}}{2})] + (1-\alpha)[1-2F(\frac{P_{t-1}^A - P_{t-1}^B+\bar{c}}{2})]\}b\gamma_t^2,$$

$$\mathsf{E}[P_t^A\gamma_t] - \mathsf{E}[P_t^B\gamma_t] = \Big\{\alpha[1-2F(\frac{\mathsf{E}[\gamma_t P_t^A]-\mathsf{E}[\gamma_t P_t^B]+\bar{c}}{2})] + (1-\alpha)[1-2F(\frac{P_{t-1}^A - P_{t-1}^B+\bar{c}}{2})]\Big\}b\mathsf{E}[\gamma_t^2]. \tag{I.9}$$

Lemma I.3. *Suppose Assumptions 1-3 hold. If* $b(1-2\alpha)\mathsf{E}[\gamma^2] \le \bar{c}$*, then*

$$P_t^A - P_t^B = \begin{cases} -\frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}(\gamma_t^2)}(P_{t-1}^A - P_{t-1}^B) & \text{if } |P_{t-1}^A - P_{t-1}^B| \le \bar{c}, \\ -\frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}(\gamma_t^2)}\bar{c} & \text{if } P_{t-1}^A - P_{t-1}^B > \bar{c}, \\ \frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}(\gamma_t^2)}\bar{c} & \text{if } P_{t-1}^A - P_{t-1}^B < -\bar{c}. \end{cases} \tag{I.10}$$

Proof of Lemma I.3. To complete the proof, it is essential to derive the expression of $\mathsf{E}[\gamma_t P_t^A] - \mathsf{E}[\gamma_t P_t^B]$. For the ease of exposition, let $Y = \mathsf{E}[\gamma_t P_t^A] - \mathsf{E}[\gamma_t P_t^B]$. We have three cases.

<u>Case 1:</u> Suppose $|P_{t-1}^A - P_{t-1}^B| \le \bar{c}$.

By this supposition, (I.9) reduces to

$$Y = \big\{-(1-\alpha)\frac{P_{t-1}^A - P_{t-1}^B}{\bar{c}} + \alpha[1-2F(\frac{Y+\bar{c}}{2})]\big\}b\mathsf{E}[\gamma_t^2]. \tag{I.11}$$

If $Y > \bar{c}$, then by (I.11),

$$Y = \big\{-(1-\alpha)\frac{P_{t-1}^A - P_{t-1}^B}{\bar{c}} - \alpha\big\}b\mathsf{E}[\gamma_t^2] \le (1-\alpha-\alpha)b\mathsf{E}[\gamma_t^2] = (1-2\alpha)b\mathsf{E}[\gamma_t^2] \le \bar{c},$$

contradicting the assumption that $Y > \bar{c}$. The first inequality above holds because $P_{t-1}^A - P_{t-1}^B \ge -\bar{c}$.

If $Y < -\bar{c}$, then by (I.11),

$$Y = [-(1-\alpha)\frac{P_{t-1}^A - P_{t-1}^B}{\bar{c}} + \alpha]b\mathsf{E}[\gamma_t^2] \ge [-(1-\alpha)+\alpha]b\mathsf{E}[\gamma_t^2] = -(1-2\alpha)b\mathsf{E}[\gamma_t^2] \ge -\bar{c},$$

contradicting the assumption that $Y < -\bar{c}$. The first inequality above holds because $P_{t-1}^A - P_{t-1}^B \le \bar{c}$.

Hence, it must be the case that $|Y| \le \bar{c}$. Then by (I.11),

$$Y = \big\{-(1-\alpha)\frac{P_{t-1}^A - P_{t-1}^B}{\bar{c}} - \alpha\frac{Y}{\bar{c}}\big\}b\mathsf{E}[\gamma_t^2]$$
$$Y = -\frac{(1-\alpha)b\mathsf{E}[\gamma_t^2]}{\bar{c}+\alpha b\mathsf{E}[\gamma_t^2]}(P_{t-1}^A - P_{t-1}^B).$$

Plugging back into (I.8) yields

$$P_t^A - P_t^B = -\frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}[\gamma_t^2]}(P_{t-1}^A - P_{t-1}^B).$$

<u>Case 2:</u> Suppose $P_{t-1}^A - P_{t-1}^B > \bar{c}$.

By the supposition, (I.9) reduces to

$$Y = \big\{ -(1-\alpha) + \alpha[1 - 2F(\frac{Y+\bar{c}}{2})]\big\} b\mathsf{E}[\gamma_t^2]. \tag{I.12}$$

If $Y > \bar{c}$, then $Y = [-(1-\alpha) - \alpha] b\mathsf{E}[\gamma_t^2] = -b\mathsf{E}[\gamma_t^2] < \bar{c}$, contradicting the assumption that $Y > \bar{c}$. If $Y < -\bar{c}$, then $Y = [-(1-\alpha) + \alpha] b\mathsf{E}[\gamma_t^2] = -b(1-2\alpha)\mathsf{E}[\gamma_t^2] \geq -\bar{c}$, contradicting the assumption that $Y < -\bar{c}$.

Hence, it must be the case that $|Y| \leq \bar{c}$. Then by (I.12),

$$Y = \big\{ -(1-\alpha) - \alpha\frac{Y}{\bar{c}}\big\} b\mathsf{E}[\gamma_t^2],$$
$$Y = -\frac{(1-\alpha)b\mathsf{E}[\gamma_t^2]}{\bar{c} + b\mathsf{E}[\gamma_t^2]}\bar{c}.$$

Plugging back into (I.8) yields

$$P_t^A - P_t^B = -\frac{(1-\alpha)b\gamma_t}{\bar{c} + \alpha b\mathsf{E}[\gamma_t^2]}\bar{c}.$$

<u>Case 3:</u> Suppose $P_{t-1}^A - P_{t-1}^B < -\bar{c}$. Symmetric to Case 2. □

Proof of Proposition 9. Observing (I.6) and (I.7), in order to see whether P_t^A and P_t^B converge, it is essential to study whether $P_t^A - P_t^B$ converges. Hence, in each part we will first analyze the convergence of $P_t^A - P_t^B$, and then come back to the convergence of prices.

<u>Part 1:</u> We will show if $I(\alpha,\gamma) < 0$, then the market prices converge in probability towards $\bar{P}$. We first prove the convergence of $P_t^A - P_t^B$ if $I(\alpha,\gamma) < 0$. By (I.10), we have

$$|P_t^A - P_t^B| \leq \frac{(1-\alpha)b\gamma_t}{\bar{c} + \alpha b\mathsf{E}[\gamma_t^2]} \cdot \frac{(1-\alpha)b\gamma_{t-1}}{\bar{c} + \alpha b\mathsf{E}[\gamma_{t-1}^2]} \cdots \frac{(1-\alpha)b\gamma_1}{\bar{c} + \alpha b\mathsf{E}[\gamma_1^2]} |P_0^A - P_0^B|,$$
$$\ln|P_t^A - P_t^B| \leq \sum_{i=1}^{t} \ln\frac{(1-\alpha)b\gamma_i}{\bar{c} + \alpha b\mathsf{E}[\gamma_i^2]} + \ln|P_0^A - P_0^B|.$$

Doing the same operation to both sides of the above inequality, we obtain

$$\frac{\ln|P_t^A - P_t^B| - \ln|P_0^A - P_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}{\sqrt{t \cdot var[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}} \leq \frac{\sum_{i=1}^{t}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]} - \mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]]}{\sqrt{t \cdot var[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}}.$$

Note that the right hand side of the above inequality converges to the standard normal distribution $N(0,1)$ as $t \to \infty$, according to Central Limit Theorem. Thus,

$$\lim_{t\to\infty} \mathsf{P}(|P_t^A - P_t^B| \geq \epsilon) = \lim_{t\to\infty} \mathsf{P}(\ln|P_t^A - P_t^B| \geq \ln\epsilon)$$
$$= \lim_{t\to\infty} \mathsf{P}(\frac{\ln|P_t^A - P_t^B| - \ln|P_0^A - P_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}{\sqrt{t \cdot var[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}} \geq \frac{\ln\epsilon - \ln|P_0^A - P_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}{\sqrt{t \cdot var[\ln\frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}})$$

$$\leq \lim_{t\to\infty} \mathsf{P}(\frac{\sum_{i=1}^{t}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b \mathsf{E}[\gamma_i^2]} - \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b \mathsf{E}[\gamma_i^2]}]]}{\sqrt{t\cdot var[\ln \frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}} \geq \frac{\ln\epsilon - \ln|P_0^A - P_0^B| - t\mathsf{E}[\ln \frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}{\sqrt{t\cdot var[\ln \frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}]}})$$
$$= Pr(N(0,1)\geq\infty) = 0,$$

where the second last equality holds because the left hand side above is a standard normal distribution and in the right hand side $\mathsf{E}[\ln \frac{(1-\alpha)b\gamma}{\bar{c}+\alpha b\mathsf{E}[\gamma^2]}] = I(\alpha,\gamma) < 0$.

Given $P_t^A - P_t^B$ converges to 0 in probability, it follows immediately by (I.8) that $\lim_{t\to\infty} Pr(|\mathsf{E}[P_t^A\gamma_t] - \mathsf{E}[P_t^B\gamma_t]| < \epsilon) = 1$. Plugging back into (I.6) and (I.7) yields $\lim_{t\to\infty} Pr(|P_t^A - \bar{P}| > \epsilon) = \lim_{t\to\infty} Pr(|P_t^B - \bar{P}| > \epsilon) = 0$.

Part 2: We will show the convergence if $I(\alpha,\gamma)=0$ and non-convergence if $I(\alpha,\gamma)>0$.

Part 2(a): We first prove the convergence of $P_t^A - P_t^B$ if $I(\alpha,\gamma)=0$ and non-convergence if $I(\alpha,\gamma)>0$.

Before analyzing $P_t^A - P_t^B$, we first prepare some useful results about $\widetilde{P}_t^A - \widetilde{P}_t^B$ which is defined as follows,

$$\widetilde{P}_t^A - \widetilde{P}_t^B = -\frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}[\gamma_t^2]}(\widetilde{P}_{t-1}^A - \widetilde{P}_{t-1}^B),$$
$$|\widetilde{P}_0^A - \widetilde{P}_0^B| \leq \bar{c}.$$

We have

$$|\widetilde{P}_t^A - \widetilde{P}_t^B| = \frac{(1-\alpha)b\gamma_t}{\bar{c}+\alpha b\mathsf{E}[\gamma_t^2]}\cdot\frac{(1-\alpha)b\gamma_{t-1}}{\bar{c}+\alpha b\mathsf{E}[\gamma_{t-1}^2]}\cdots\frac{(1-\alpha)b\gamma_1}{\bar{c}+\alpha b\mathsf{E}[\gamma_1^2]}|\widetilde{P}_0^A - \widetilde{P}_0^B|.$$

Thus,

$$\ln|\widetilde{P}_t^A - \widetilde{P}_t^B| = \sum_{i=1}^{t}\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]} + \ln|\widetilde{P}_0^A - \widetilde{P}_0^B|.$$

Through some algebra, we get

$$\frac{\ln|\widetilde{P}_t^A - \widetilde{P}_t^B| - \ln|\widetilde{P}_0^A - \widetilde{P}_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} = \frac{\sum_{i=1}^{t}[\ln\gamma_i - \mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} \longrightarrow N(0,1).$$

Then we have

$$\lim_{t\to\infty}\mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| < \epsilon) = \lim_{t\to\infty}\mathsf{P}(\ln|\widetilde{P}_t^A - \widetilde{P}_t^B| < \ln\epsilon)$$
$$= \lim_{t\to\infty}\mathsf{P}(\frac{\ln|\widetilde{P}_t^A - \widetilde{P}_t^B| - \ln|\widetilde{P}_0^A - \widetilde{P}_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} < \frac{\ln\epsilon - \ln|\widetilde{P}_0^A - \widetilde{P}_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}})$$
$$= \lim_{t\to\infty}\mathsf{P}(\frac{\sum_{i=1}^{t}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]} - \mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} < \frac{\ln\epsilon - \ln|\widetilde{P}_0^A - \widetilde{P}_0^B| - t\mathsf{E}[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}{\sqrt{t\cdot var[\ln\frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}})$$

$$= \lim_{t\to\infty} \mathsf{P}\Big(\frac{\sum_{i=1}^{t}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]} - \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]]}{\sqrt{t\cdot var[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} < \frac{\ln\epsilon - \ln|\widetilde{P}_0^A - \widetilde{P}_0^B|}{\sqrt{t\cdot var[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}} - \sqrt{\frac{t}{var[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]}}\mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}]\Big).$$

Hence,

$$\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| < \epsilon) = \begin{cases} Pr(N(0,1) < -\infty) = 0 \text{ if } \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}] = I(\alpha,\gamma) > 0, \\ Pr(N(0,1) < 0) = 1/2 \text{ if } \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}] = I(\alpha,\gamma) = 0. \end{cases}$$

By the same process, we could show

$$\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| > 1) = \begin{cases} 1 \quad \text{if } \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}] = I(\alpha,\gamma) > 0, \\ 1/2 \text{ if } \mathsf{E}[\ln \frac{(1-\alpha)b\gamma_i}{\bar{c}+\alpha b\mathsf{E}[\gamma_i^2]}] = I(\alpha,\gamma) = 0. \end{cases}$$

Observing that $P_t^A - P_t^B$, within the range of $[-1,1]$, follows a similar process as $\widetilde{P}_t^A - \widetilde{P}_t^B$. Once $P_t^A - P_t^B$ jumps out of $[-1,1]$, it nevertheless will be pulled back into $[-1,1]$.

If $I(\alpha,\gamma) > 0$, then $\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| < \epsilon) = 0$ and $\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| > 1) = 1$, implying that even if $P_t^A - P_t^B$ can be pulled back, the process will inevitably keep growing out of the range $[-1,1]$. Thus, $P_t^A - P_t^B$ does not converge.

If $I(\alpha,\gamma) = 0$, then $\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| < \epsilon) = 1/2$ and $\lim_{t\to\infty} \mathsf{P}(|\widetilde{P}_t^A - \widetilde{P}_t^B| > 1) = 1/2$, implying that the process $P_t^A - P_t^B$ within the range of $[-1,1]$ will end up with two possible results: one is out of the range of $[-1,1]$ and the other is 0, each with half probability. Since the process will be pulled back whenever it gets out of $[-1,1]$ and each time there is 1/2 probability that it will converge to 0 whenever it is in $[-1,1]$, the process will eventually converge to 0. That is, $\lim_{t\to\infty} \mathsf{P}(|P_t^A - P_t^B| < \epsilon) = 1$ for any $\epsilon > 0$.

Part 2(b): We analyze whether P_t^A and P_t^B converge.

We have if $I(\alpha,\gamma) = 0$, then the sequence $P_t^A - P_t^B$ will converge to 0. It now follows by (I.8) that $\lim_{t\to\infty} Pr(|\mathsf{E}[P_t^A\gamma_t] - \mathsf{E}[P_t^B\gamma_t]| < \epsilon) = 1$. Plugging back into (I.6) and (I.7) yields

$$\lim_{t\to\infty} Pr(|P_t^A - \bar{P}| > \epsilon) = \lim_{t\to\infty} Pr(|P_t^B - \bar{P}| > \epsilon) = 0.$$

If $I(\alpha,\gamma) > 0$, then $P_t^A - P_t^B$ does not converge. Therefore, by (I.6) and (I.7) the prices (P_t^A, P_t^B) do not converge. $\square$

J. Proof of Results in Appendix B

Before the proof of Proposition B.1, we first introduce a lemma.

Lemma J.1. *Suppose Assumptions 1-3 hold. In the model with backward looking strategic farmers, if $b \le \bar{c}$, then the recursive equation of market price $p_t (t \ge 1)$ is*

$$p_t = \Omega - \frac{b}{\bar{c}}\Big((1-\frac{\alpha}{2})p_{t-1} + \frac{\alpha}{2}p_{t-2}\Big).$$

Proof of Lemma J.1. If $0 \le p_{t-1} \le \bar{c}$ and $0 \le \frac{p_{t-1}+p_{t-2}}{2} \le \bar{c}$ for any $t \ge 0$, then (B.2) reduces to

$$p_t = \Omega - \frac{b}{\bar{c}}\Big((1-\frac{\alpha}{2})p_{t-1} + \frac{\alpha}{2}p_{t-2}\Big).$$

To complete the proof, we will prove by induction that $0 \le p_{t-1} \le \bar{c}$ and $0 \le \frac{p_{t-1}+p_{t-2}}{2} \le \bar{c}$ for any $t \ge 0$.

By Assumption 2 and the convention that $p_i = 0$ for any $i < 0$, we have $0 \le p_0 \le \bar{c}$ and $0 \le \frac{p_0+p_{-1}}{2} \le \bar{c}$.

For any given $t \ge 1$, suppose $0 \le p_{t-1} \le \bar{c}$ and $0 \le \frac{p_{t-1}+p_{t-2}}{2} \le \bar{c}$, then we have

$$\begin{aligned} p_t &= \Omega - \frac{b}{\bar{c}}\Big((1-\frac{\alpha}{2})p_{t-1} + \frac{\alpha}{2}p_{t-2}\Big) < \Omega - \frac{b}{\bar{c}}\Big((1-\frac{\alpha}{2})(\Omega - b) + \frac{\alpha}{2}(\Omega - b)\Big) \\ &= \Omega - \frac{b}{\bar{c}}(\Omega - b) = (1-\frac{b}{\bar{c}})(\Omega - b - \bar{c}) + \bar{c} \le \bar{c}. \end{aligned}$$

Therefore, $0 \le \frac{p_t+p_{t-1}}{2} \le \bar{c}$. We have now established that $0 \le p_{t-1} \le \bar{c}$ and $0 \le \frac{p_{t-1}+p_{t-2}}{2} \le \bar{c}$ for any $t \ge 0$. $\square$

Proof of Proposition B.1. We first show the existence of the limit. By Lemma J.1,

$$p_t - \bar{p} = -\frac{b}{\bar{c}}\Big((1-\frac{\alpha}{2})(p_{t-1} - \bar{p}) + \frac{\alpha}{2}(p_{t-2} - \bar{p})\Big). \tag{J.1}$$

Since $b \le \bar{c}$, we have $|p_t - \bar{p}| \le (1-\frac{\alpha}{2})|p_{t-1} - \bar{p}| + \frac{\alpha}{2}|p_{t-2} - \bar{p}|$, and so $|p_t - \bar{p}| - |p_{t-1} - \bar{p}| \le -\frac{\alpha}{2}(|p_{t-1} - \bar{p}| - |p_{t-2} - \bar{p}|)$. Therefore, $\lim_{t\to\infty} |p_t - \bar{p}|$ exists. Remember that we expect to show $\lim_{t\to\infty} p_t - \bar{p}$ exists. Suppose $\lim_{t\to\infty} |p_t - \bar{p}| = A > 0$ (Note that $A \ne 0$. Otherwise, we are done). Thus, for any given $\epsilon > 0$, there exists a T such that for any $t > T$,

$$|p_t - \bar{p}| \in (A - \epsilon, A + \epsilon). \tag{J.2}$$

Suppose for a contradiction that $\lim_{t\to\infty} p_t - \bar{p}$ does not exist. It follows from (J.2) that there must exist a t_1 with $t_1 - 1 > T$ and $t_1 - 2 > T$ such that $p_{t_1-1} - \bar{p} \in (-A-\epsilon, -A+\epsilon)$ and $p_{t_1-2} - \bar{p} \in (A-\epsilon, A+\epsilon)$. Therefore, by (J.1), $p_{t_1} - \bar{p} = -\frac{b}{\bar{c}}(1-\frac{\alpha}{2})(p_{t_1-1} - \bar{p}) - \frac{b}{\bar{c}}\frac{\alpha}{2}(p_{t_1-2} - \bar{p}) \in (\frac{b}{\bar{c}}(1-\alpha)A - \epsilon', \frac{b}{\bar{c}}(1-\alpha)A + \epsilon')$. Clearly, $|p_{t_1} - \bar{p}| \notin (A-\epsilon, A+\epsilon)$, contradicting the fact (J.2). Consequently, $\lim_{t\to\infty} p_t - \bar{p}$ does exist. Second, we compute the limit. Let $\lim_{t\to\infty} p_t - \bar{p} = B$. Then by (J.1), we obtain $B = 0$. $\square$

K. Proof of Results in Appendix C

Proof of Proposition C.1. Incorporating farmer exit, the market dynamics becomes,

$$p_t = \Omega - \alpha b F(p_t) - (1-\alpha) b F(\min\{p_{t-1}, c_{t-1}\}) \tag{K.1}$$

where c_{t-1} denotes the threshold such that at the end of period $t-1$, the farmers with production cost $c > c_{t-1}$ exit the market and the rest stay in the market (for simplicity, we consider the farmers who do not plant because of high production cost above the relatively higher price in each cycle as that they exit the market).

Part 1: We show there exists an i' such that $c_i \leq \bar{p}$ for any $i \geq i'$. Suppose for a contradiction that there does not exist such an i', that is, $c_t > \bar{p}$ for any $t \geq 1$. We consider two cases.

Case 1: For any fixed t, if $p_{t-1} < \bar{p}$, then by (K.1), $p_t = \Omega - \alpha b F(p_t) - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}$ where the last equality holds because $p_{t-1} < \bar{p} < c_{t-1}$. Next we show $p_t > \bar{p}$. Suppose for a contradiction that $p_t \leq \bar{p}$, then $p_t = \Omega - \alpha\frac{b}{\bar{c}}p_t - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}$, from which we obtain $p_t = \bar{p} - g(\alpha)(p_{t-1} - \bar{p}) > \bar{p}$ where the last inequality holds because $p_{t-1} < \bar{p}$, arriving a contradiction. So it must be the case that $p_t > \bar{p}$.

Case 2: For any fixed t, if $p_{t-1} > \bar{p}$, then by (K.1),

$$p_t < \Omega - \alpha b F(p_t) - (1-\alpha)\frac{b}{\bar{c}}\bar{p} \tag{K.2}$$

where the last inequality holds because $\min\{p_{t-1}, c_{t-1}\} > \bar{p}$. Next we show $p_t \leq \bar{c}$. Suppose for a contradiction that $p_t > \bar{c}$, then $p_t < \Omega - \alpha b - (1-\alpha)\frac{b}{\bar{c}}\bar{p} = \frac{\bar{c}+\alpha b}{\bar{c}+b}\Omega - \alpha b$. Since $\frac{\bar{c}+\alpha b}{\bar{c}+b}\Omega - \alpha b - \bar{c} = (\bar{c}+\alpha b)(\frac{\Omega}{\bar{c}+b} - 1) < 0$ because $\Omega < \bar{c} + b$ by Assumption 1, we have $p_t \leq \bar{c}$, which arrives a contradiction. So it must be the case that $p_t \leq \bar{c}$. Hence, (K.2) reduces to $p_t < \Omega - \alpha\frac{b}{\bar{c}}p_t - (1-\alpha)\frac{b}{\bar{c}}\bar{p}$, from which we obtain $p_t < \bar{p}$.

The above two cases show that if $c_t > \bar{p}$ for any $t \geq 1$, then the price will fluctuate above and below $\bar{p}$ constantly, indicating that the planting farmers with $c > \bar{p}$ will incur a loss in each cycle and thus exit the market eventually. As a result, the left farmers in the market must be the ones with $c \leq \bar{p}$.

Part 2: We show the price converges.

Case 1: For any fixed $i \geq i'$, if $p_i < \bar{p}$, then by (K.1), $p_{i+1} \geq \Omega - \alpha b F(p_{i+1}) - (1-\alpha)\frac{b}{\bar{c}}p_i$, where the last inequality holds because $\min\{p_i, c_i\} \leq p_i$. Next we show $p_{i+1} > \bar{p}$. Suppose for a contradiction that $p_{i+1} \leq \bar{p}$, then $p_{i+1} \geq \Omega - \alpha\frac{b}{\bar{c}}p_{i+1} - (1-\alpha)\frac{b}{\bar{c}}p_i$, from which we obtain $p_{i+1} > \bar{p} - g(\alpha)(p_i - \bar{p}) > \bar{p}$, arriving a contradiction. So it must be the case that $p_{i+1} > \bar{p}$.

Case 2: For any fixed $i \geq i'$, if $p_i > \bar{p}$, then by (K.1), $p_{i+1} = \Omega - \alpha b F(p_{i+1}) - (1-\alpha)\frac{b}{\bar{c}}c_i$. Next we show $p_{i+1} > \bar{p}$. Suppose for a contradiction that $p_{i+1} < \bar{p}$, then $p_{i+1} = \Omega - \alpha\frac{b}{\bar{c}}p_{i+1} - (1-\alpha)\frac{b}{\bar{c}}c_i$, from which we obtain $p_{i+1} = \bar{p} - g(\alpha)(c_i - \bar{p}) > \bar{p}$, arriving a contradiction. Hence it must be the case that $p_{i+1} > \bar{p}$.

Taking the above cases into consideration, we have $p_{i+1} > \bar{p}$ for any $i \geq i'$. Note that $c_i \leq \bar{p}$, so each period all naïve farmers with $c \leq c_i$ will cultivate the crop and sell it at price p_{i+1} above their production cost earning a positive surplus, and thus no more naïve farmers exit the market. Hence $c_i = c_{i+1} = \cdots = c_\infty$.

Therefore, for any $t \geq i'+1$, $p_t = \Omega - \alpha b F(p_t) - (1-\alpha)\frac{b}{\bar{c}} c_t$ where the last equality holds because $c_t \leq \bar{p} < p_{t-1}$. Since $c_t = c_{t+1} = \cdots = c_\infty$, we have $p_t = p_{t+1} = \cdots = p_\infty$. That is, the price converges to p_t, higher than $\bar{p}$.

L. Proof of Results in Appendix D.1

Before the proof of Proposition D.1, we first introduce three lemmas.

Lemma L.1. *Suppose Assumptions 1-6 hold. For any feasible solution to optimization problem (D.1), the contract is not effective in period $2i$ for any $i \geq 1$.*

Proof of Lemma L.1. Part 1: We show if $p_{2i-2} < \bar{p}$, then $p_{2i-1} > \bar{p}$ for any fixed $i \geq 1$.

If $p^o_{2i-1} \leq p_{2i-2}$, then $p_{2i-1} = \Omega - bF(p_{2i-2}) > \Omega - bF(\bar{p}) = \bar{p}$ and we are done. Hence, we need only consider the case when $p^o_{2i-1} > p_{2i-2}$. Note that $\bar{p} = \frac{\bar{c}}{\bar{c}+b}\Omega < \bar{c}$, since $p_{2i-2} < \bar{p}$, so $p_{2i-2} < \bar{c}$. Consequently,

$$p_{2i-1} = \Omega - b\alpha F(p^o_{2i-1}) - b(1-\alpha)\frac{p_{2i-2}}{\bar{c}}. \tag{L.1}$$

We take a brief detour to show $p^o_{2i-1} \leq \bar{c}$. Suppose for a contradiction that $p^o_{2i-1} > \bar{c}$, then

$$\begin{aligned} p_{2i-1} &= \Omega - b\alpha - b(1-\alpha)\frac{p_{2i-2}}{\bar{c}} < \Omega - b\alpha - b(1-\alpha)\frac{\Omega - b}{\bar{c}} = \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)\Omega - \alpha b + (1-\alpha)\frac{b^2}{\bar{c}} \\ &\leq \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)(b+\bar{c}) - \alpha b + (1-\alpha)\frac{b^2}{\bar{c}} = \bar{c} < p^o_{2i-1}, \end{aligned}$$

where the second last inequality holds because $\Omega \leq b + \bar{c}$ and $1 - (1-\alpha)\frac{b}{\bar{c}} \geq 0$. Thus, $\pi_{2i-1}(p^o_{2i-1}) = (p_{2i-1} - p^o_{2i-1})q^o_{2i-1} < 0$, violating the profit constraint in Problem (D.1). Therefore, it must be the case that $p^o_{2i-1} \leq \bar{c}$ as claimed.

Plugging $p^o_{2i-1} \leq \bar{c}$ back into (L.1) yields $p_{2i-1} = \Omega - \alpha\frac{b}{\bar{c}}p^o_{2i-1} - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2}$. The profit constraint $\pi_{2i-1}(p^o_{2i-1}) \geq 0$ requires that $p_{2i-1} \geq p^o_{2i-1}$, by which we derive $p^o_{2i-1} \leq \frac{\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2}}{1+\alpha\frac{b}{\bar{c}}}$. We have

$$\begin{aligned} p_{2i-1} &= \Omega - \alpha\frac{b}{\bar{c}}p^o_{2i-1} - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2} \geq \Omega - \alpha\frac{b}{\bar{c}}\frac{\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2}}{1+\alpha\frac{b}{\bar{c}}} - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2} \\ &= \frac{1}{1+\alpha\frac{b}{\bar{c}}}\Big(\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i-2}\Big) > \frac{1}{1+\alpha\frac{b}{\bar{c}}}\Big(\Omega - (1-\alpha)\frac{b}{\bar{c}}\bar{p}\Big) = \bar{p}. \end{aligned}$$

Part 2: We show that if $p_{t-1} > \bar{p}$, then $p^o_t \leq p_{t-1}$ and $p_t < \bar{p}$.

Suppose for a contradiction that $p_t^o > p_{t-1}$, then

$$p_t = \Omega - b\alpha F(p_t^o) - b(1-\alpha)F(p_{t-1}) < \Omega - bF(\bar{p}) = \bar{p},$$

where the inequality holds because $p_t^o > p_{t-1} > \bar{p}$. Thus, $\pi_t(p_t^o) = (p_t - p_t^o)q_t^o < 0$, violating the profit constraint in problem (D.1). Therefore, it must be the case that $p_t^o \le p_{t-1}$, which indicates that the contract will not be effective in period t. Moreover, $p_t = \Omega - bF(p_{t-1}) < \Omega - bF(\bar{p}) = \bar{p}$. Recall that $p_0 < \bar{p}$. Combining Parts 1 and 2, we see that $p_{2i} < \bar{p} < p_{2i-1}$ and $p_{2i}^o \le p_{2i-1}$ for any $i \ge 1$. Therefore, the contract is not effective in period $2i$ for any $i \ge 1$. $\square$

Lemma L.2. *Suppose Assumptions 1-6 hold. For any fixed t, suppose $p_{t-1} < \bar{p}$. Consider the following one-period problem.*

$$\begin{aligned} \min_{p_t^b} \quad & (p_t - \bar{p})^2 && \text{(L.2)} \\ s.t. \quad & p_t = \Omega - b\alpha F(p_t^o) - b(1-\alpha)F(p_{t-1}), \\ & p_{t-1} < p_t^o \le p_t. && \text{(L.3)} \end{aligned}$$

Then $p_t^{o*} = \bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - p_{t-1}) \le \bar{c}$.

Proof of lemma L.2. We first derive the formula of p_t. It follows by $p_{t-1} < \bar{p}$ and $\bar{p} = \bar{c}\frac{\Omega}{\bar{c}+b} \le \bar{c}$ that $p_{t-1} < \bar{c}$. Next we show $p_t^o \le \bar{c}$. Suppose for a contradiction that $p_t^o > \bar{c}$. On one hand, $\pi_t \ge 0$ requires that $p_t \ge p_t^o$, hence it follows that $p_t > \bar{c}$. On the other hand,

$$\begin{aligned} p_t &= \Omega - b\alpha - b(1-\alpha)\frac{p_{t-1}}{\bar{c}} && [\text{by } p_t^o > \bar{c} \text{ and } p_{t-1} \le \bar{c}] \\ &\le \Omega - b\alpha - b(1-\alpha)\frac{\Omega - b}{\bar{c}} && [\text{by } p_{t-1} \ge \Omega - b] \\ &= \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)\Omega - \alpha b + (1-\alpha)\frac{b^2}{\bar{c}} \\ &\le \Big(1 - (1-\alpha)\frac{b}{\bar{c}}\Big)(b + \bar{c}) - \alpha b + (1-\alpha)\frac{b^2}{\bar{c}} && [\text{by } \Omega \le b + \bar{c} \text{ and } 1 - (1-\alpha)\frac{b}{\bar{c}} \ge 0] \\ &= \bar{c}, \end{aligned}$$

which arrives a contradiction. Hence, it must be the case that $p_t^o \le \bar{c}$. Since $p_{t-1} \le \bar{c}$ and $p_t^o \le \bar{c}$, we have

$$p_t = \Omega - \alpha\frac{b}{\bar{c}}p_t^o - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}. \tag{L.4}$$

Plugging (L.4) back into (L.3) gives $p_{t-1} < p_t^o \le \frac{\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}}{1+\alpha\frac{b}{\bar{c}}}$. Hence, problem (L.2) can be written as

$$\min_{p_t^o} \quad \Big(\frac{b}{\bar{c}+b}\Omega - \alpha\frac{b}{\bar{c}}p_t^o - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}\Big)^2 \tag{L.5}$$

$$\text{s.t.} \quad p_{t-1} < p_t^o \leq \frac{\Omega - (1-\alpha)\frac{b}{\bar{c}} p_{t-1}}{1+\alpha\frac{b}{\bar{c}}}.$$

Taking derivatives to the objective function in (L.5) yields the unconstrained optimal solution $\frac{1}{\alpha}\left(\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)p_{t-1}\right)$. Observe that

$$\frac{1}{\alpha}\left(\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)p_{t-1}\right) - \frac{\Omega - (1-\alpha)\frac{b}{\bar{c}} p_{t-1}}{1+\alpha\frac{b}{\bar{c}}} = \frac{(1-\alpha)\bar{c}}{\alpha(\bar{c}+\alpha b)}\left(\frac{\bar{c}}{\bar{c}+b}\Omega - p_{t-1}\right) > 0,$$

where the last inequality holds because $p_{t-1} < \bar{p} = \frac{\bar{c}}{\bar{c}+b}\Omega$. Therefore, the convexity indicates that $p_t^{o*} = \frac{\Omega-(1-\alpha)\frac{b}{\bar{c}}p_{t-1}}{1+\alpha\frac{b}{\bar{c}}} = \bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - p_{t-1})$. One can verify that $p^{o*} \leq \bar{c}$. $\square$

Lemma L.3. *Suppose Assumptions 1-6 hold.*

1. *Suppose $\Omega - b \leq \widetilde{p}_{2i} < p_{2i} < \bar{p}$. If $\widetilde{p}^o_{2i+1} = \bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - \widetilde{p}_{2i})$ and $p^o_{2i+1} = \bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - p_{2i})$, then $\widetilde{p}_{2i+2} < p_{2i+2} < \bar{p}$. That is, implementing an optimal contract based on a lower low price in cycle i will admit a lower low price in cycle $i+1$.*
2. *Suppose $\Omega - b \leq \widetilde{p}_{2i} = p_{2i} < \bar{p}$. If $\widetilde{p}^o_{2i+1} = 0$ and $p^o_{2i+1} = \bar{p} + \frac{(1-\alpha)b}{\bar{c}+\alpha b}(\bar{p} - p_{2i})$, then $\widetilde{p}_{2i+2} < p_{2i+2} < \bar{p}$. That is, offering no contract will admit a lower low price than implementing the optimal contract in odd period $2i+1$.*

Proof of Lemma L.3. Part 1: Note that $\bar{c} \geq p^o_{2i+1} > p_{2i}$, so

$$\begin{aligned} p_{2i+1} &= \Omega - \alpha\frac{b}{\bar{c}}p^o_{2i+1} - (1-\alpha)\frac{b}{\bar{c}}p_{2i} = \Omega - \alpha\frac{b}{\bar{c}}\frac{\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i}}{1+\alpha\frac{b}{\bar{c}}} - (1-\alpha)\frac{b}{\bar{c}}p_{2i} \\ &= \frac{1}{1+\alpha\frac{b}{\bar{c}}}\left(\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i}\right) > \frac{1}{1+\alpha\frac{b}{\bar{c}}}\left(\Omega - (1-\alpha)\frac{b}{\bar{c}}\bar{p}\right) = \bar{p}. \end{aligned}$$

where the last inequality holds because $p_{2i} < \bar{p}$. Lemma L.1 implies that no contract is effective in period $2i+2$. Hence, $p_{2i+2} = \Omega - bF(p_{2i+1})$. Note also that

$$\begin{aligned} p_{2i+1} - \bar{c} &= \frac{1}{1+\alpha\frac{b}{\bar{c}}}\left(\Omega - (1-\alpha)\frac{b}{\bar{c}}p_{2i}\right) - \bar{c} \leq \frac{1}{1+\alpha\frac{b}{\bar{c}}}\left(\Omega - (1-\alpha)\frac{b}{\bar{c}}(\Omega - b)\right) - \bar{c} \\ &= \left(1 - (1-\alpha)\frac{b}{\bar{c}}\right)(\Omega - b - \bar{c}) < 0, \end{aligned}$$

so $p_{2i+2} = \Omega - \frac{b}{\bar{c}}p_{2i+1}$. By the same process, we obtain $\widetilde{p}_{2i+2} = \Omega - \frac{b}{\bar{c}}\widetilde{p}_{2i+1}$. Observe that $\widetilde{p}_{2i+1} > p_{2i+1}$ by $\widetilde{p}_{2i} < p_{2i}$, so $\widetilde{p}_{2i+2} \leq p_{2i+2}$.

Part 2: By $\widetilde{p}^o_{2i+1} = 0$, it follows that $\widetilde{p}_{2i+1} = \Omega - \frac{b}{c}\widetilde{p}_{2i} > \Omega - \frac{b}{\bar{c}}\bar{p} = \bar{p}$. According to Lemma L.1, no contract is effective in period $2i+2$. Thus, $\widetilde{p}_{2i+2} = \Omega - bF(\widetilde{p}_{2i+1}) \leq \Omega - \frac{b}{\bar{c}}\widetilde{p}_{2i+1}$.

On the other hand, given $p_{2i} < p^o_{2i+1}$, we have $p_{2i+1} = \Omega - \alpha\frac{b}{\bar{c}}p^o_{2i+1} - (1-\alpha)\frac{b}{\bar{c}}p_{2i}$. As in Part 1, we have $p_{2i+2} = \Omega - \frac{b}{\bar{c}}p_{2i+1}$.

Observe that $\widetilde{p}_{2i+1} > p_{2i+1}$ by $\widetilde{p}_{2i} = p_{2i} < p^o_{2i+1}$, so $\widetilde{p}_{2i+2} < p_{2i+2}$. $\square$

Proof of Proposition D.1. Lemma L.1 implies that the infinite horizon problem can be viewed as an infinite number of independent problem, each of which consists of one cycle (two periods), and Lemma L.2 gives the optimal solution of each independent problem. Moreover, Lemma L.3 shows that with the optimal operation, the prices will be closer to the market limiting price in the current cycle if they are closer to the market limiting price in the last cycle. Combing the three lemmas, we arrive the conclusion. □

Before we prove Proposition D.2, we first introduce a lemma.

LEMMA L.4. *Suppose Assumptions 1-6 hold. For any fixed t, suppose $p_{t-1} < \bar{p}$. Consider the following one-period problem.*

$$\min_{p_t^o} \quad (p_t - \bar{p})^2 \tag{L.6}$$
$$s.t. \quad p_t = \Omega - b\alpha F(p_t^o) - b(1-\alpha)F(p_{t-1}),$$
$$p_{t-1} < p_t^o \leq \bar{c}. \tag{L.7}$$

If $\alpha < \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$, then $p_t^{o*} = \bar{c}$.

Proof of lemma L.4. We first derive the formula of p_t. To achieve this, we expect to show $p_t^o \leq \bar{c}$ and $p_{t-1} \leq \bar{c}$. Since $p_{t-1} < \bar{p}$ and $\bar{p} = \bar{c}\frac{\Omega}{\bar{c}+b} \leq \bar{c}$, it follows immediately that $p_{t-1} < \bar{c}$. Note that $p_t^o > \bar{c}$ has the same dynamic effect as that $p_t^o = \bar{c}$, so without loss of generality we assume $p_t^o \leq \bar{c}$. Therefore, we have

$$p_t = \Omega - \alpha\frac{b}{\bar{c}}p_t^o - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}. \tag{L.8}$$

Hence, problem (L.6) can be written as

$$\min_{p_t^o} \quad \Big(\frac{b}{\bar{c}+b}\Omega - \alpha\frac{b}{\bar{c}}p_t^o - (1-\alpha)\frac{b}{\bar{c}}p_{t-1}\Big)^2 \tag{L.9}$$
$$\text{s.t.} \quad p_{t-1} < p_t^o \leq \bar{c}.$$

Taking derivatives to the objective function in (L.9) yields the unconstrained optimal solution $\frac{1}{\alpha}\big(\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)p_{t-1}\big)$. Observe that

$$\frac{1}{\alpha}\Big(\frac{\bar{c}}{\bar{c}+b}\Omega - (1-\alpha)p_{t-1}\Big) - \bar{c} = \frac{1}{\alpha}\Big(\bar{p} - (1-\alpha)p_{t-1} - \alpha\bar{c}\Big) > 0,$$

where the last inequality holds because $\alpha < \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$. Therefore, the convexity indicates that $p_t^{o*} = \bar{c}$. □

Proof of Proposition D.2. Part (a): We show the case if $\alpha \geq \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$.
Note that $p_1^o = \frac{\bar{p}-p_0}{\alpha} + p_0 \leq \bar{c}$ by $\alpha \geq \frac{\bar{p}-p_0}{\bar{c}-p_0}$, so

$$\begin{aligned} p_1 &= \Omega - \alpha b F(p_1^o) - (1-\alpha) b F(p_0) = \Omega - \alpha \frac{b}{\bar{c}} p_1^o - (1-\alpha)\frac{b}{\bar{c}} p_0 \\ &= \Omega - \alpha \frac{b}{\bar{c}}\Big(\frac{\bar{p}-p_0}{\alpha} + p_0\Big) - (1-\alpha)\frac{b}{\bar{c}} p_0 = \bar{p}. \end{aligned}$$

Part (b): We show the case if $\alpha < \frac{\bar{p}-p_{t-1}}{\bar{c}-p_{t-1}}$.
As in Lemma L.1, we could show for any solution to optimization problem (D.1) (without the profit constraint), the contract is not effective in period $2i$ for any $i \geq 1$. That is, the infinite horizon problem can be viewed as an infinite number of independent problem, each of which consists of one cycle. Moreover, Lemma L.4 gives the optimal solution of such independent problem. Combining these two lemmas completes the proof of Part (b). $\square$

M. Proof of Results in Appendix D.2

Before we prove Proposition D.3, we first introduce a lemma.

LEMMA M.1. *Suppose Suppose Assumptions 1-5 hold and $\alpha < \hat{\alpha}$.*
(a) If $\alpha > \alpha_1$, then $\lim_{t\to\infty} p_t = \bar{p} - \frac{\alpha b(p^o - \bar{p})}{\bar{c}+(1-\alpha)b}$.
(b) If $\alpha \leq \alpha_1$, then the price process does not converge.

Proof of Lemma M.1. Note that $\alpha < \hat{\alpha}$ is equivalent to

$$\Omega - \alpha \frac{b}{\bar{c}} p^o - b(1-\alpha) < \bar{c}. \tag{M.1}$$

Part 1: We will show the price converges if $\alpha > \alpha_1$ and does not converge if $\alpha = \alpha_1$. Note that $\alpha \geq \alpha_1$ is the same to

$$b(1-\alpha) \leq \bar{c}.$$

We have two cases.
Case 1: Suppose $\Omega - \alpha \frac{b}{\bar{c}} p^o - (1-\alpha)\frac{b}{\bar{c}} p_0 \leq \bar{c}$.
We will show by induction that for any $t \geq 1$,

$$p_t = \frac{1 - \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^t}{1 + (1-\alpha)\frac{b}{\bar{c}}}\Big(\Omega - \alpha\frac{b}{\bar{c}} p^o\Big) + \Big(-(1-\alpha)\frac{b}{\bar{c}}\Big)^t p_0 \leq \bar{c}, \tag{M.2}$$

from which Part 1 follows immediately.
Note that $p^o \leq \bar{c}$ and $p_0 \leq \bar{c}$, so $p_1 = \Omega - \alpha \frac{b}{\bar{c}} p^o - (1-\alpha)\frac{b}{\bar{c}} p_0 \leq \bar{c}$, where the last inequality holds because of the supposition in Case 1. Hence, (M.2) holds when $t = 1$. Suppose (M.2) holds when $t = i-1$, and we will show it holds when $t = i$. One can check

$$p_i = \Omega - \alpha \frac{b}{\bar{c}} p^o - (1-\alpha)\frac{b}{\bar{c}} p_{i-1} \qquad \text{[by } p^o \leq \bar{c} \text{ and } p_{i-1} \leq \bar{c}]$$

$$
\begin{aligned}
&= \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}\Big\{\frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i-1}}{1+(1-\alpha)\frac{b}{\bar{c}}}\Big(\Omega - \alpha\frac{b}{\bar{c}}p^o\Big) + \Big(-(1-\alpha)\frac{b}{\bar{c}}\Big)^{i-1} p_0\Big\} \\
&= \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i}}{1+(1-\alpha)\frac{b}{\bar{c}}}\Big(\Omega - \alpha\frac{b}{\bar{c}}p^o\Big) + \Big(-(1-\alpha)\frac{b}{\bar{c}}\Big)^{i} p_0 \\
&\le \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i}}{1+(1-\alpha)\frac{b}{\bar{c}}}\Big(1-\alpha)\frac{b}{\bar{c}}p_0 + \bar{c}\Big) + \Big(-(1-\alpha)\frac{b}{\bar{c}}\Big)^{i} p_0 \qquad \Big[\text{by } \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 \le \bar{c}\Big] \\
&= \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i}}{1+(1-\alpha)\frac{b}{\bar{c}}}\bar{c} + \frac{(1-\alpha)\frac{b}{\bar{c}} + \big(-(1-\alpha)\frac{b}{\bar{c}}\big)^{i}}{1+(1-\alpha)\frac{b}{\bar{c}}}p_0 \\
&\le \bar{c}. \qquad [\text{by } p_0 \le \bar{c}]
\end{aligned}
$$

This establishes that (M.2) holds for any $t \ge 1$.

Therefore, if $\alpha > \alpha_1$, that is, $b(1-\alpha) < \bar{c}$, then $\lim_{t\to\infty} p_t = \frac{\Omega-\alpha\frac{b}{\bar{c}}p^o}{1+(1-\alpha)\frac{b}{\bar{c}}} = \bar{p} - \frac{\alpha b(p^o-\bar{p})}{\bar{c}+(1-\alpha)b}$. If $\alpha = \alpha_1$, the price alternates between p_0 and $\Omega - \alpha\frac{b}{\bar{c}}p^o - p_0$.

<u>Case 2:</u> Suppose $\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 > \bar{c}$.

We will show there exists an $i \ge 0$ such that $p_i \le \bar{c}$ and $\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_i \le \bar{c}$, then the analysis follows the same as Case 1.

We have $p_1 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_0 > \bar{c}$, where the last inequality holds because of the supposition in Case 2. Then $p_2 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)b < \bar{c}$, where the last inequality holds because of (M.1). Thus,

$$
\begin{aligned}
p_3 &= \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_2 = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}\Big(\Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)b\Big) \\
&= \Big(1-(1-\alpha)\frac{b}{\bar{c}}\Big)\Big(\Omega - \alpha\frac{b}{\bar{c}}p^o\Big) + (1-\alpha)\frac{b}{\bar{c}}b(1-\alpha) \\
&\le \Big(1-(1-\alpha)\frac{b}{\bar{c}}\Big)\Big(b(1-\alpha) + \bar{c}\Big) + (1-\alpha)\frac{b}{\bar{c}}b(1-\alpha) \\
&= \bar{c},
\end{aligned}
$$

where the last inequality holds because (M.1) and $b(1-\alpha) \le \bar{c}$. This completes the proof of Case 2.

<u>Part 2:</u> We will show if $\alpha < \alpha_1$, that is, $b(1-\alpha) > \bar{c}$, then the price process does not converge.

We first show there exists an $i \ge 0$ such that $p_i > \bar{c}$. Suppose for a contradiction that there does not exist such an i; i.e, $p_t \le \bar{c}$ for any t. Then by the same process as Case 1 of Part 1, we obtain $p_t = \frac{1-\big(-(1-\alpha)\frac{b}{\bar{c}}\big)^t}{1+(1-\alpha)\frac{b}{\bar{c}}}(\Omega - \alpha\frac{b}{\bar{c}}p^o) + \Big(-(1-\alpha)\frac{b}{\bar{c}}\Big)^t p_0$. Since $(1-\alpha)\frac{b}{\bar{c}} > 1$, there must exist an $i \ge 0$ such that $p_i > \bar{c}$, which arrives a contradiction.

Now we show the price does not converge. It follows by $p_i > \bar{c}$ that $p_{i+1} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)b < \bar{c}$, where the last inequality holds because of (M.1). Then

$$
p_{i+2} = \Omega - \alpha\frac{b}{\bar{c}}p^o - (1-\alpha)\frac{b}{\bar{c}}p_{i+1} = \Big(1-(1-\alpha)\frac{b}{\bar{c}}\Big)\Big(\Omega - \alpha\frac{b}{\bar{c}}p^o\Big) + (1-\alpha)\frac{b}{\bar{c}}b(1-\alpha)
$$

$$> \left(1-(1-\alpha)\frac{b}{\bar{c}}\right)\left((1-\alpha)b+\bar{c}\right)+(1-\alpha)\frac{b}{\bar{c}}b(1-\alpha)=\bar{c},$$

where the last inequality holds by $1-(1-\alpha)\frac{b}{\bar{c}}<0$ and (M.1). Continuing in this fashion, we find that for any $j \geq 0$,

$$p_{i+2j+1}=\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)b<\bar{c}, \quad \text{and} \quad p_{i+2j+2}=\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)\frac{b}{\bar{c}}p_{i+2j+1}>\bar{c}.$$

That is, the price alternates between two constant prices and does not converge. $\square$

Proof of Proposition D.3. Part (i): Note that $\alpha \geq \hat{\alpha}$ is equivalent to

$$\Omega-\alpha\frac{b}{\bar{c}}p^{b}-b(1-\alpha)\geq\bar{c}.$$

We have

$$p_1=\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)\frac{b}{\bar{c}}p_0\geq\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)b\geq\bar{c},$$

where the first inequality holds because $p_0 \leq \bar{c}$. Consequently, $p_2=\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)b\geq\bar{c}$. One can check that for any $t \geq 1$, $p_t=\Omega-\alpha\frac{b}{\bar{c}}p^{o}-(1-\alpha)b\geq\bar{c}$.

Parts (ii) and (iii): It follows immediately from Lemma M.1. $\square$