Intertemporal Segmentation via Flexible-Duration Group Buying

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Abstract. Problem definition: We study a special form of group buying: the group buying succeeds only if the number of sign-ups reaches a preset threshold, with no duration constraint. Customers with heterogeneous valuations arrive sequentially and decide between signing up for the group buying or purchasing a regular product. To decide whether to join the group buying, customers need to estimate their expected waiting time, which varies depending on the cumulative sign-ups by the time of their arrival. The firm decides on the prices for the group-buying product and regular product, with the product quality levels and group-buying size exogenously determined. Academic/practical relevance: This type of group buying is often adopted for a special edition of the product and offered alongside a constantly available regular product. Methodology: We study the product line design with the group-buying sign-up behavior of customers characterized by the rational expectations equilibrium in a random pledging process. Results: We show that group buying with flexible duration can result in intertemporal customer segmentation, as different segments might be admitted at different times in the dynamic sign-up process. Such intertemporal segmentation is a natural discrimination scheme and has nontrivial implications. First, the efficiency loss due to waiting for enough sign-ups may decrease when a larger batch size is required for economic production. Second, as valuation heterogeneity in the market increases, the firm may not always benefit from offering group buying along with the regular product. Third, group buying can achieve a win-win-win situation for both high-end and low-end customers as well as the firm. Managerial implications: In addition to demonstrating the profitability of flexible-duration group buying, we show that the firm can strengthen its profitability by contingently setting prices or concealing sign-up information in group buying. We also confirm the robustness of our main insights by considering customers’ heterogeneous patience levels and horizontally differentiated products, among other factors.

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1. Introduction
1.1. Motivation
Beauty companies used to adhere to a strict “one in, one out” policy, in which slow-moving products were discontinued to make room for new ones (Boncompagni 2012). Due to the limited shelf space available, firms could only offer their bread-and-butter products. However, thanks to social network platforms and online forums, beauty companies can now offer special-edition products online, such as discontinued hot products, through a special program. For example, in February 2012, Bobbi Brown and MAC Cosmetics, two brands owned by the Estée Lauder Companies, ran two Facebook campaigns called “Bobbi Brings Back: Lip Color” and “MAC by Request,” respectively, asking their fans to sign up for discontinued lipsticks (Boncompagni 2012, Gattis 2012). Indeed, firms can only make profits with reissues when the number of sign-ups reaches a specific threshold because cosmetics (e.g., lipsticks) are always produced in batches to benefit from economies of scale (Brumberg 1986). Without these online campaigns, firms would lose the opportunity to satisfy customers who desperately browse the Internet for their favorite discontinued cosmetics and may have to settle for bread-and-butter products instead.

The cosmetic industry is not alone. First, many tourist platforms (e.g., Ctrip, the counterpart of Expedia in China) offer customized and private tour routes that differ from standard ones. However, these special routes will only materialize if the number of sign-ups exceeds a certain threshold, whereas there is no threshold for
standard routes. Second, as an emerging practice, on-demand transportation (e.g., Bus.com, Uber Central, Call-n-Ride) will be readily available if there are enough online requests. For example, Bus.com offers group buying of bus services with a customizable experience or for a cheaper price than current routes. Third, many platforms in China provide flexible-duration group-buying opportunities for food, apparel, electronics, and hotel reservations. For example, group buying on Pinduoduo.com has no preset deadline (Wong 2018), unlike the once popular group-buying website Groupon in its heyday of “a deal a day” (Girotra et al. 2013), or the crowdfunding website Kickstarter. In the flexible-duration cases, group buying is only unlocked when the number of sign-ups reaches a threshold. Thus, customers who sign up are uncertain about how long they are going to wait, whereas they have immediate access to the regular products or services from the same vendor through other online or offline channels.

Group buying with a deadline is successful if and only if the number of sign-ups exceeds a predetermined threshold before a specific date, that is, the duration is fixed (see, e.g., Hu et al. 2013, Liu and Tunca 2019). Given the relatively short and fixed duration of the campaign, for example, one day in the case of Groupon and LivingSocial’s “a deal a day,” waiting times tend to be the same for all pledgers. In contrast, group buying with flexible duration has a predetermined goal, but its duration is uncertain. Customers face different waiting costs depending on when they sign up, which cannot be ignored because of the relatively long and random waiting times. This difference between group buying with and without a deadline is similar to that between auctions with and without a deadline (Roth and Ockenfels 2002).

Although group buying with flexible duration is common, it is still unclear whether such a strategy effectively enhances the firm’s profitability, because it can cannibalize the sales of a regular product. Even if flexible-duration group buying is profitable, it is not clear how it should be designed and how it affects customers. In the base model, we consider group buying for a premium product. By and large, offering flexible-duration group buying for a premium special-edition product along with the regular product can be double-edged. On the positive side, it exploits customer preferences and allows for market expansion or better market segmentation. On the negative side, group buying with flexible duration inevitably means that customers, who might otherwise purchase the regular product, have to wait for the threshold to be met, resulting in efficiency loss. If the special-edition product is in low demand, gathering enough sign-ups can take a long time. To shorten the wait, the firm may promote the group-buying product to a larger market (beyond the diehards expected to by the special edition), but making the premium product available to regular customers can result in a lower profit margin and cannibalization of the regular market. Thus, it is crucial to understand the optimal design of group buying with flexible duration, which is a nontrivial task due to the interactions just mentioned.

### 1.2. Results

In view of these difficulties, we build a stylized model to understand the mechanism behind flexible-duration group buying. Two segments of customers with heterogeneous valuations arrive sequentially following a Poisson process. Upon arrival, customers can see the cumulative number of sign-ups up to their arrival time. Each customer decides between buying the regular product immediately and signing up for group buying of the special-edition product. Depending on when customers join, signing up for group buying has different waiting costs proportional to waiting times. For all customers, the time they have to wait for enough sign-ups is uncertain and factors into their decision making. In anticipation of the rational sign-up behavior of subsequent arrivals, customers make a rational expectation of their own waiting time. Anticipating customers’ sign-up behavior, the firm determines whether to offer group buying and how to optimally design it in terms of pricing and market segmentation. We show that answers to these questions critically depend on the interaction of supply and demand, in particular the batch size needed for economic production on the supply side and market valuation heterogeneity on the demand side.

In flexible-duration group buying, customers may wait a long time for the threshold to be met, particularly when the batch size for economic production is large. Hence, the loss of efficiency due to waiting cannot be assumed away and may need to be compensated for by the firm in its pricing. To mitigate this efficiency loss, the firm may invite different segments of customers to sign up at different times during the dynamic pledging process, which we refer to as intertemporal customer segmentation. As an important feature of flexible-duration group buying when customer waiting cost is considered, this intertemporal segmentation has the following implications.

First, the loss of efficiency due to waiting for enough sign-ups may decrease in the group-buying threshold that is assumed to be the batch size for economic production. This is perhaps surprising as one may expect the opposite. When the batch size is small, group buying optimally targets only the high-end segment of the market. However, as the batch size increases, the firm may be forced to also target the low-end segment by reducing the price, with only high-value customers signing up at the early stage.
and both segments admitted at the later stage of the group-buying process. Indeed, the pledging process can be accelerated by the participation of both segments. Therefore, both waiting times and efficiency loss can be reduced when the batch size becomes larger.

Second, as the valuation heterogeneity increases, the firm may not always want to offer group buying in addition to the regular product, though this is counterintuitive. As the valuation heterogeneity increases, common intuition would suggest that the firm has more incentive to resort to group buying, with the regular product and a premium product for group buying targeted at different segments. However, we show that the profitability of group buying is moderated by the batch size for economic production. Specifically, when the batch size is in an intermediate range, as valuation heterogeneity increases, the firm’s optimal strategy goes from no group buying, to group buying, to no group buying and finally back to group buying. This nonmonotonic behavior is driven by the possible intertemporal segmentation enabled by group buying. When both the batch size and valuation heterogeneity are in an intermediate range, group buying allows the firm to intertemporally segment customers, but as valuation heterogeneity further increases, the firm may find it optimal to serve the high-end customers with the regular product (not serving low-end customers) and eliminate the offering of group buying all together. This is because group buying requires customer waiting, which needs to be compensated for by a lower price, and inviting both segments to join at the late pledging process can make it necessary to lower the price even more. But when the valuation heterogeneity is sufficiently high, group buying can become profitable again, as now the group-buying campaign targets only the high-end customers.

Third, every stakeholder can benefit from the group-buying offer. In particular, group buying can increase the firm’s profitability when the inventory holding cost is high enough. Besides, its introduction always leads to an increase in the total sales volume. Furthermore, imagine that the group-buying product is always offered as the regular product, that is, both products are regularly offered and thus available without any wait. Despite this, customers will still prefer the flexible-duration group buying alongside the regular product, because the high-end segment is always strictly better off by paying less and obtains a larger surplus, and the low-end segment is always weakly better off, thanks to the intertemporal customer segmentation achieved. Specifically, group buying enables low-end customers to jump on the bandwagon to buy the premium product (i.e., intertemporal segmentation), which would not be affordable otherwise. Moreover, the participation of the low-end segment benefits high-end customers by shortening their waiting times.

We then extend our base model in several directions. First, we allow the firm to charge state-dependent prices for the group-buying product (referred to as contingent pricing) to different pledgers, with two endogenized price points. Perhaps counterintuitively, it can be optimal for the firm to offer a lower price to people signing up late. This is because when the batch size is below a certain threshold, group buying can succeed even without an early-bird discount; with a guaranteed success, the end-of-cycle discount becomes more effective than the early-bird discount. Indeed, as customers who sign up early expect to wait, the acceleration of the later pledging process can be anticipated by all of the early arrivals, shortening waiting times and increasing their willingness to pay. Hence, the firm can charge those customers who sign up early a higher price. In contrast, an early-bird discount accelerates the early pledging process but has no effect on the later arrivals, as each customer’s waiting time depends only on the cumulative number of sign-ups by the time of the customer’s arrival. This result contrasts starkly with the literature on traditional group buying with a deadline, which suggests that the early-bird discount is more profitable.

Second, we investigate unobservable group buying. Interestingly, the firm prefers to conceal sign-up information if the premium product is of sufficiently high quality. The reason is that the firm must persuade the first customer to sign up by setting an incentive-compatible price, as this customer bears the highest waiting cost. When the sign-up information is available, the first sign-up customer knows that he or she will wait the longest; thus the firm must compensate the customer by offering a sufficiently low price, which applies to everyone else and results in unextracted surpluses for all subsequent sign-ups. In contrast, with sign-up information concealed, customers use the average expected waiting time to make their sign-up decisions. Hence, the firm can benefit by offering less compensation to customers than in observable group buying, which further improves the profitability of unobservable group buying, particularly when the quality level of the group-buying product is sufficiently high.

We also study several other extensions. For example, we incorporate customers’ heterogeneous patience levels, investigate the group-buying of an inferior product, and explore the context of horizontally differentiated products. In the optimal flexible-duration group buying, we find that it is always those customers in the highest-priority class for the group-buying product who sign up first at the early stage. “Highest priority” can refer to either the highest-valuation segment or a
lower-valuation but most patient segment (in the context of vertically differentiated products) or the segment of customers who like the group-buying product most (in the context of horizontally differentiated products). Meanwhile, the firm may have an incentive to invite customers in the lower-priority classes for the group-buying product to sign up at a later stage.

1.3. Contributions
We make the following contributions. First, to the best of our knowledge, this work is the first to formally investigate group buying with flexible duration by studying its optimal design. Our model captures one of the key features of flexible-duration group buying, namely, customers’ waiting cost. Hence, our model offers a new operational perspective that complements the existing literature. Second, this study highlights the significance of taking into account operational considerations in the implementation of innovative marketing programs. In particular, the firm should consider customers’ sequential arrivals and their dynamic sign-up behavior in designing its group-buying offer. Third, this research recognizes the profitability of offering contingent product lines online via flexible-duration group buying. In general, if the inventory holding cost is high enough or customers are patient enough, offering the contingent product line through flexible-duration group buying is more profitable than offering a fixed product line. Finally, our work contributes by proposing a more natural discrimination scheme for firms, that is, intertemporal customer segmentation. Unlike most of the literature, in which the firm proactively discriminates between customers by offering time-varying prices, and customers may wait for lower prices, we show that in flexible-duration group buying, the firm does not need to proactively price-discriminate between different segments, since the price for the group-buying product remains unchanged. Instead, the firm can passively discriminate between customers based on their different expected waiting times (and costs) resulting from their sequential arrivals. Hence, intertemporal customer segmentation can be more natural and acceptable than those price discrimination policies.

2. Literature Review
From a variety of perspectives, many papers study the profitability of group buying, in which the campaign will be activated if at least a prespecified number of customers sign up. Anand and Aron (2003) are among the earliest to study group buying as a mechanism of offering a contingent quantity discount. They show that group buying offers a way to better respond to demand uncertainty because the reduced price will only be offered if the demand is sufficiently high. Chen and Zhang (2015) study interpersonal bundling, in which the group discount is effective only if the number of sign-ups belongs to a certain interval, with a minimum or maximum group size. Taking into account strategic customers, Marinesi et al. (2018) show that the firm can take advantage of the group-buying activation threshold to reveal the state of the market and reduce the mismatch between supply and demand. From a supply chain perspective, Chen and Roma (2011) study the group-buying discount received by two competing retailers, based on their aggregated purchasing quantity, in a two-level distribution channel. Focusing on social interaction, Jing and Xie (2011) show that group buying with a discounted price and a minimum group size can motivate informed customers to work as “sales agents” to acquire less informed customers.

In these papers, the customer sign-up process is often simplified. However, some research, including ours, zooms in on the group-buying process to explore the pledging dynamics. For example, incorporating strategic customer behavior, Surasvadi et al. (2017) propose a contingent markdown mechanism that resembles group buying, in which the discount price will be offered to not only the sign-ups but also all subsequent buyers if the number of sign-ups reaches a prespecified threshold or at the end of the selling season, whichever comes first. Liu and Tunca (2019) characterize the stochastic dynamic equilibrium behavior of consumers’ pledging in a group-buying scheme with two threshold levels, using a recursive differential and difference equation system. They empirically estimate consumer arrival rates and utility distributions utilizing data from group-buying events hosted by an online retailer, and provide empirical evidence of consumer network effects in group buying. They show that for low and high consumer arrival rates, the retailer sets a small price discount, whereas for an intermediate consumer arrival rate, the retailer sets a deeper price discount.

Three papers study the group-buying design from an information disclosure perspective. Hu et al. (2013) use a two-period model and show that revealing the cumulative number of sign-ups increases the group-buying success rate. Hu et al. (2020) consider a positive network externality among customers in a general form, which includes group buying as a special case, and show that it may not be optimal for the firm to disclose the number of sign-ups. Subramanian and Rao (2016) study consumer observational learning from deal sales displayed on Groupon and Living-Social and demonstrate that displaying daily sales volumes can benefit the high-type merchant by credibly signaling its type, thus enabling it to acquire new customers at a higher margin. In our context, we show...
in an extension that when customer waiting cost is considered, information transparency can be detrimental to the firm because it must compensate customers for the high waiting cost of early sign-ups.

All in all, group buying without a deadline is not a special case of group buying with a finite duration, though it may be considered as group buying with an infinite duration. With the duration changed from finite to infinite, the model changes fundamentally in the following respects. First, the success rate of a group-buying campaign with a finite duration is less than 1, that is, there is a positive probability that group buying may not reach the target within the fixed duration. By contrast, in our theoretical model, an optimally designed group-buying campaign is assured of success, even though the waiting times are random and can be long. Second, in the group-buying literature, we are not aware of any other paper explicitly taking customers’ waiting costs into account, likely due to these papers’ focus on relatively short fixed durations. However, waiting costs become a critical factor in customers’ decision making when the time to meet the group-buying threshold becomes long. As a feature, we incorporate customers’ waiting cost in our model and investigate its potential impacts on customers’ sign-up behavior and on the firm’s optimal group-buying design. We uncover a novel type of discrimination scheme enabled by customers’ waiting, that is, intertemporal customer segmentation. Third, even though intertemporal customer segmentation can exist in fixed-duration group buying (although it has not been explicitly studied in the existing literature on group buying; see Online Supplement A for a detailed comparison), the driving force behind customer sign-up behavior differs significantly from that of our flexible-duration group buying. To be specific, in fixed-duration group buying, the high-valuation customers can sign up at an earlier stage than the low-valuation customers due to different levels of risk borne by the two segments of customers, as the probability of a successful group-buying deal would be lower at the earlier stage of the campaign. By contrast, in flexible-duration group buying, the risk borne by the two segments of customers stays the same, as the probability of a successful group-buying deal is always 1. Thus, the driving force of the intertemporal customer segmentation in this case would be the heterogeneous waiting costs faced by customers who sign up at different times. Moreover, in fixed-duration group buying, if we extend the deadline to infinity without considering the customers’ waiting costs (as in the existing literature on group buying), there would be no difference in the risk of signing up, because the success rate is always 1. Under this circumstance, intertemporal customer segmentation would not exist, which further confirms that

our flexible-duration group buying is not a special case of fixed-duration group buying that does not consider waiting costs.

Our research is closely related to the literature on product line design that involves simultaneously offering products of different qualities. The earlier work includes Moorthy (1984), who develops a framework of market segmentation based on customer self-selection. The follow-ups theoretically model the variety and quality of products considering duopoly competition (Moorthy 1988, Desai 2001), time of product introduction (Moorthy and Png 1992), and communicating cost (Villas-Boas 2004), among others. In this stream, Netessine and Taylor (2007) take into account the supply side, which is very close to our work. Both papers analyze the optimal customer segmentation using two products and studying the interaction of supply and demand sides. But there are some distinct differences. Unlike their economic order quantity (EOQ) model in which two products are simultaneously produced in advance and wait for customers to purchase them (i.e., made to stock), in our study one product is produced beforehand, whereas the other will not be produced unless the group-buying deal succeeds. In other words, we treat group buying as a way to offer a contingent product line and customers have to wait for the group-buying product. From the system efficiency perspective, the decision whether to make to stock or make to group-buying order hinges on which cost is lower: the inventory holding cost versus the customers’ waiting cost. Hence, our study can be viewed as a complement to their research. In general, if the inventory holding cost is high enough or customers are patient enough, offering the product line through group buying is more profitable.

Our work also contributes to the growing body of literature on intertemporal price discrimination (Conlisk et al. 1984, Su 2007, Besbes and Lobel 2015, Garrett 2016, Öry 2017, Chen et al. 2019), in which the firm discriminates between different customer segments by using intertemporal pricing. It is worth noting that the intertemporal customer segmentation through flexible-duration group buying differs from intertemporal price discrimination. Most critically, in the papers mentioned previously, the firm proactively discriminates between different segments of customers by offering time-varying prices. By contrast, in our study, the price for the group-buying product remains unchanged. Instead, the firm passively discriminates between customers based on their different expected waiting times (and costs). Furthermore, intertemporal customer segmentation via group buying can achieve a win-win-win situation for all stakeholders, which is absent in intertemporal price discrimination. Last but not least, in those papers, the firm suffers if more...
customers are willing to wait, whereas in our setting, the firm can even have an incentive to induce more customers to wait.

In addition to customer cooperation via group buying, researchers also study collaboration between sellers through group buying. Examples include firms’ group buying from their upstream supplier to obtain a discount (Chen and Roma 2011), and suppliers’ group selling to their downstream firm to counteract the default risk (Huang et al. 2016) or customers’ strategic behavior (He et al. 2016).

Finally, our research is also related to some of the most popular business models driven by crowds (see Chen et al. 2019 for a review), such as crowdfunding (Hu et al. 2015, Chemla and Tinn 2019) and crowdfunding (Marinesi and Girotra 2013), all of which explicitly or implicitly adopt a predetermined deadline. We offer a framework for the optimal design of such business processes from a flexible-duration perspective. Also allowing for flexible duration, Araman and Caldentey (2016) let the firm decide when to stop a crowdfunding process, depending on the number of votes and the updated belief about the customer arrival rate. In contrast, the threshold of the group-buying campaign in our setting is exogenously determined by the production side and the campaign duration is random—an outcome of random customer arrivals with random valuations.

3. The Base Model

In this section, we introduce the setup for the base model with a two-point valuation distribution. We then focus on a prototypical setting where customers’ valuations are continuous and only the group-buying product is offered, to present preliminary results on customers’ sign-up behavior.

3.1. Model Setup

In the base model, we consider a monopolistic firm selling one regular product to customers who arrive according to a Poisson process with arrival rate \( \lambda \). Customers are heterogeneous in their valuations of product quality attributes and belong to one of the two segments: high-end segment with valuation \( H \), or low-end segment with valuation \( L (H > L > 0) \). The fractions of high- and low-end customers in the population are \( \gamma (0 < \gamma < 1) \) and \( 1 - \gamma \), respectively. Each customer intends to purchase at most one unit of one of the products, and the customer type is private information.

To better discriminate between high- and low-end customers, the firm can choose to offer group buying for a specific premium product. We assume that the regular product and the premium product are vertically differentiated in quality. (We consider horizontally differentiated products in Section 5.5.) The quality levels of the group-buying product and the regular product are \( \theta (\theta > 1) \) and \( 1 \), respectively. (We consider \( \theta < 1 \) in Section 5.4 and consider the case in which whether the premium product is a fit for customers is uncertain in Online Supplement J.) Although we interpret \( \theta \) as the quality level of the group-buying product, it can also represent a combination of product characteristics. With the group-buying offer, the firm posts its group-buying campaign at the beginning of a sales horizon, which is characterized by the price \( p \) for the premium product and the minimum number \( N \) of customers required, alongside the price \( r \) for the regular product. When the number of sign-ups for the premium product reaches the threshold \( N \), the firm immediately produces the premium product in a batch of size \( N \) and then delivers it to those who have signed up during the current campaign. Another group-buying cycle can start again. We assume that the threshold \( N \) is exogenously determined, which may stand for the batch size required for economic production. (We numerically analyze the endogenized batch size \( N \) in Online Supplement J.) To make the model parsimonious, we do not explicitly consider different production costs of the regular and the premium product. Instead, we assume different batch sizes in production, which is reasonable because of the following. Generally, the regular and premium products have different setup costs: \( K_1 \) and \( K_2 \), respectively. As the regular product is the bread-and-butter product of the firm, its production can be in a continuous flow, and thus \( K_1 \) is relatively low. In contrast, since the premium product is to some extent customized, its specialized development cost \( K_2 \) can be quite high. That is, \( K_1 \ll K_2 \).

Without loss of generality, we assume \( K_1 = 0 \) and \( K_2 = K \). Moreover, the setup cost \( K \) is equivalent to imposing a batch size \( N \) for economic production, implying that at least \( N \) units of the product are required to cover the setup cost. The firm determines the prices \( r \) and \( p \), which will remain the same throughout the selling season, to maximize the long-run average profit. (We consider the case in which the firm sets prices for the group-buying product contingently in Section 5.1.) Our model accommodates the situation in which group buying either occurs once or repeats multiple times.

Customers incur a waiting cost \( c > 0 \) per unit time. (We consider heterogeneous waiting costs in Section 5.3.) Everything else being equal, the longer customers wait, the less they want to join the group-buying campaign. Upon arrival, each customer decides whether to purchase the regular product immediately, or sign up and wait for the premium product (if the firm offers group buying), or buy nothing and exit the market. (We consider stand-alone group buying without any regular product in Section 3.2 and Online Supplement H.)
For simplicity, we assume a zero surplus for the outside option. A customer will sign up for the premium product if and only if doing so offers a benefit that is at least as high as purchasing the regular product. Moreover, customers cannot renege after their pledge. When joining the campaign, customers can see the prices \( p \) and \( r \) as well as the additional number of sign-ups needed to reach the target \( N \). In practice, the firm commonly discloses to customers the number of sign-ups in real time. (We will discuss an alternative information disclosure strategy in Section 5.2.) For ease of exposition, we denote the state of pledge-to-go by \( n (1 \leq n \leq N) \), which represents the additional number of sign-ups required to reach the threshold \( N \). In deciding whether to sign up, customers consider their own valuation and belief about the expected waiting time, denoted by \( w(n) \), which depends on the real-time state \( n \) of pledge-to-go. Last, all exogenous parameters, including \( H, L, \lambda, \gamma, c, \) and \( \theta \), are common knowledge.

### 3.2. A General Valuation Model

In this subsection, we consider a prototypical model in which only the group-buying product is offered in order to gain insight into customers’ sign-up behavior. We assume customers’ valuations follow a general distribution \( F(\cdot) \), which can be either continuous or discrete with a finite or unbounded range. For simplicity, we normalize \( \theta = 1 \), since we do not consider the regular product. All other assumptions remain the same as in Section 3.1.

**Theorem 1 (Sign-up Behavior).** For a given \( N \), there exist thresholds for customers’ valuations, \( \bar{v}_n, 1 \leq n \leq N \), such that upon arrival at any pledge-to-go state \( n \), customers whose valuations are no less than \( \bar{v}_n \) will be induced to sign up for group buying, and those whose valuations are below \( \bar{v}_n \) will not. Moreover, (i) \( \bar{v}_n \) is increasing in \( n \); and (ii) \( \Delta \bar{v}_n \equiv \bar{v}_{n+1} - \bar{v}_n \) is increasing in \( n \) if \( F(\cdot) \) is continuous. (The monotonicity in this paper is in its weaker sense unless otherwise stated.)

As Theorem 1 demonstrates, there exist \( N \) thresholds for customers’ valuations that govern customers’ sign-up behavior in equilibrium. More importantly, Theorem 1 also indicates that as the pledge-to-go state \( n \) decreases (i.e., as the time elapses), the admission threshold \( \bar{v}_n \) becomes lower. That is, more customers would be invited to sign up at the later stage of the group buying (as the expected waiting times are shortened), which we refer to as intertemporal customer segmentation, since the group-buying campaign admits different types of customers during the dynamic pledging process. Besides, Theorem 1(ii) shows that as the time goes, the admission threshold \( \bar{v}_n \) decreases at a slower pace, because it is better for the firm to admit fewer lower-valuation customers as the pressure to meet the threshold of a successful group-buying deal is reduced.

In the rest of the paper, for simplicity, we focus on the setting with a two-point valuation distribution, because we need to optimize the product line design. Our study can be extended to incorporate multiple segments of customers and the main insights would carry through.

### 4. Model Analysis

The base model is a Stackelberg game. In the first stage, the firm determines whether to offer group buying and decides on the price for the regular product and that for the group-buying product, if any. In the second stage, customers purchase the regular product, or sign up for group buying if it is offered, or exit the market, based on their own valuation and the real-time sign-up information upon their arrival. We analyze the game backward. In the second stage, we assume customers make rational expectations of future arrivals and solve customers’ equilibrium sign-up behavior, and then go back to the first stage to analyze the firm’s optimal decisions, in anticipation of customers’ rational behavior in the second stage. We adopt the concept of rational expectations equilibrium (REE), in which agents make choices based on their rational anticipations and available information, and the expectations are consistent with the realizations in equilibrium (see, e.g., Muth 1961 and Lucas 1972).

#### 4.1. Equilibrium Market Segmentation with Group Buying

In this subsection, we assume the group-buying premium product is always offered alongside the regular product, and then endogenize the group-buying offer decision in Section 4.3. We use the superscript \( G \) to emphasize the presence of the group-buying offer. In a group-buying campaign, the firm can target either high- or low-end customers to sign up for the group-buying product, or both, by manipulating prices \( p^G \) and \( r^G \). In our hypothesis, all customers who sign up will eventually get the group-buying product, although their expected waiting times differ. Theorem 1 characterizes customers’ dynamic sign-up behavior in the general setting, with Lemma A.1 in Online Appendix A customized for the setting with two segments of customers. Hence, there are five possible customer segmentations in terms of which segment signs up for the group buying in a given state \( n (1 \leq n \leq N) \): only high-end customers sign up (referred to as scenario \( \{H\} \)); high-end customers sign up first, then both high- and low-end customers sign up (referred to as scenario \( \{H; H + L\} \)); both high- and low-end customers sign up first, then both high- and low-end customers sign up (referred to as scenario \( \{H + L\} \)); only low-end customers sign up (referred to as scenario \( \{L\} \)); and finally, low-end customers do not sign up (referred to as \( \{0\} \)).
customers sign up first, then both high- and low-end customers sign up (referred to as scenario \{L; H + L\}). However, when \(\theta > 1\), only the first three scenarios can occur in equilibrium, which is summarized in the following proposition. By contrast, when \(\theta < 1\), only the last three are possible in equilibrium (see Proposition E.1 in Online Appendix E).

**Proposition 1 (Customer Segmentation with Group Buying).** Suppose \(\theta > 1\). With group buying, customer segmentation in equilibrium must be one of the following three scenarios: \{H\}, \{H; H + L\}, or \{H + L\}.

The three scenarios in Proposition 1 share a common feature, namely, the first sign-up customer is a high-end one. Denote by \(\bar{n}^G\) the so-called tipping state. That is, the customer segment(s) signing up for the group-buying product will change after the pledge-to-go state \(n\) drops to \(\bar{n}^G\). More specifically, after the number of \(N - \bar{n}^G\) sign-ups has been gathered, low-end customers will join the group-buying campaign. Take the scenario \{H; H + L\} as an example. Those customers who join early at any state \(n\), where \(\bar{n}^G < n < N\), must be high-end customers, whereas those who join later at any state \(n < \bar{n}^G\) can be either type. Thus, we can simply use the tipping state \(\bar{n}^G\) to characterize the three scenarios. In particular, \(\bar{n}^G = 0\) (never tip), \(1 < \bar{n}^G < N\) (intertemporal segmentation), and \(\bar{n}^G = N\) (always tip) correspond to scenarios \{H\}, \{H; H + L\}, and \{H + L\}, respectively.

As the firm cannot observe each customer’s type, it makes pricing decisions anticipating that each customer self-selects an incentive-compatible choice. To this end, the prices \(p^G\) and \(c^G\) should satisfy the individual rationality (IR) and incentive compatibility (IC) constraints for all pledge-to-go states. By analyzing the IR and IC constraints, we can view both \(p^G\) and \(c^G\), as well as the firm’s long-run average profit \(\bar{n}^G\), as functions of tipping state \(\bar{n}^G\). With group buying, customer segmentation in REE is characterized by the following proposition (see Figure 1 for an illustration).

**Proposition 2 (REE with Group Buying).** With group buying, for any given \(\theta > 1\) and \(N\), there exist two thresholds for the batch size \(\bar{N}_1 \equiv (\theta - 1)(1 - \gamma)L\lambda/c + 1 - \gamma\) and \(\bar{N}_2 \equiv (\theta - 1)(H - L)\gamma\lambda/c + 1 - \gamma\), such that the following holds:

(i) When \(H/L \leq 1/\gamma\), the firm sets prices such that
   (i.1) if \(N \leq \bar{N}_1\), \{H + L\} is an REE;
   (i.2) if \(N > \bar{N}_1\), \{H; H + L\} is an REE.

(ii) When \(H/L > 1/\gamma\), the firm sets prices such that
    (ii.1) if \(N \leq \bar{N}_2\), \{H\} is an REE;
    (ii.2) if \(N > \bar{N}_2\), \{H; H + L\} is an REE.

Proposition 2 reveals that the equilibrium is determined not only by the supply-side factor, namely, the batch size \(N\) for economic production, but also by the demand-side factor, namely, \(H/L\), a measure that captures the (dis)similarity of customer segments. A small \(H/L\) indicates that customer segments are close to each other, whereas a large \(H/L\) implies they are far apart. When the batch size \(N\) is sufficiently small and the valuation heterogeneity \(H/L\) is sufficiently low, the optimal strategy for the firm is to set prices so that all customers purchase the premium product through group buying (see subcase (i.1) in Proposition 2 and the region \{H + L\} in Figure 1). Indeed, since the premium product is more profitable (as \(\theta > 1\)), waiting times are short due to the small \(N\), and the difference of valuations between the two segments is not significant, the firm prefers to use only the premium product to capture the whole market.

When the valuation heterogeneity \(H/L\) is sufficiently high and the batch size \(N\) is not too large, the firm has an incentive to segment the market perfectly by completely excluding low-end customers from the group buying of the premium product, a strategy referred to as perfect customer segmentation (see subcase (ii.1) in Proposition 2 and the region \{H\} in Figure 1). In this case, the firm uses the premium product to target the high-end segment and the regular product to target the low-end segment. By combining subcases (i.1) and (ii.1), we find that when \(N\) is small enough, the market partition is mainly influenced by \(H/L\), which is similar to the situation in Netessine and Taylor (2007, figure 3), where the firm offers a product line of premanufactured products to customers with different valuations, when the production technology cost is intermediate.

When the batch size \(N\) is sufficiently large, our result is in stark contrast with the literature. Increasing the batch size \(N\) could be expected to further reinforce the firm’s price discrimination, by allowing only high-end customers to participate in group buying.
The intuition is that with a larger batch size $N$, waiting times would increase, thus only high-end customers would be willing to wait. Interestingly, when the pricing decisions are endogenized, we observe a distinct phenomenon: high-end customers sign up for group buying first; then, when there are enough sign-ups to reach the tipping point, both high- and low-end customers join the group-buying campaign (see subcases (i.2) and (ii.2) in Proposition 2 and the region $\{H; H + L \}$ in Figure 1). That is, the group buying admits different customer segments during the dynamic pledging process, which is referred to as intertemporal customer segmentation (see Figure 2 for an illustration). In other words, intertemporal customer segmentation outperforms perfect customer segmentation when the batch size $N$ is sufficiently large, which is somewhat counterintuitive. We summarize this observation as follows.

**Corollary 1** *(Comparison of Two Types of Segmentation)*

_With group buying, there exists a threshold for the batch size $N$, above which intertemporal customer segmentation outperforms perfect customer segmentation, and below which the opposite is true._

The underlying reason is as follows. As the batch size $N$ increases, waiting times will also increase. To maintain the attractiveness of group buying, the firm needs to shorten waiting times. The solution is to lower the group-buying price $p^G$ and allow some low-end customers to jump on the bandwagon and join the group buying, which only occurs at the later stage of the pledging process. In this way, the firm mitigates efficiency loss due to waiting by sacrificing some benefits of price discrimination. Moreover, since the later stage in which both segments sign up for the group-buying product can be a relatively short period (especially when the quality level of the group-buying product, the valuation heterogeneity, the proportion of high-end customers, or the arrival rate is high, or the unit-time waiting cost is low), the impact of no sales for the regular product is somewhat limited. Even if we take into account no sales of the regular product for the late sign-up stage by considering the firm’s holding cost of the regular product, the insights on intertemporal segmentation largely hold, except that the firm has less incentive to admit the low-end customers at the later stage of the pledging process.

Overall, intertemporal segmentation can benefit both the firm and the high-end segment. Indeed, high-end customers wait for less time to obtain the group-buying product. Besides, the participation of low-end customers admitted only during the late pledging process ensures that the group-buying price $p^G$ is neither too low nor too high, leaving some profitable margin to the firm and also some surpluses to the high-end customers. Interestingly, we find that efficiency loss may decrease with $N$, due to the firm’s endogenized pricing decisions. When the batch size is small, group buying optimally targets only the high-end segment, so the firm can benefit from setting a higher price for the premium product. However, as the batch size increases, it is profitable for the firm to encourage the low-end segment to sign up as well by lowering the group-buying price, so that the late pledging process is accelerated, waiting times are shortened, and thus efficiency loss is reduced.

**4.2. No-Group-Buying Benchmarks**

To validate the effectiveness and evaluate the potential profitability of group buying, we compare it with three widely used benchmark strategies. In the first and second benchmarks, the firm sells only the regular product. The first strategy is called the volume strategy, in which the firm always charges the low price $L$ to attract all customers. The second strategy is called the margin strategy, in which the firm charges the high price $H$ to target only the high-end market. The third strategy is called the product-line strategy, in which the firm simultaneously offers the regular and premium products. We use the superscripts $V$, $M$, and $P$ to denote the equilibrium outcomes when the firm adopts the volume, margin, and product-line strategies, respectively.

We assume that the firm incurs some inventory holding and extra management costs when offering the premium product at the beginning of the sales horizon. We denote by $h$ the inventory holding cost per unit time per unit of the premium product. The inventory holding cost is obviously different for the two products. For the ease of comparison, we normalize the inventory cost for the regular product to zero. This is reasonable since the regular product is the bread-and-butter product and usually less costly to hold than the premium product.

In the volume strategy, the firm sets the price $r^V = L$ and the corresponding long-run average profit is $\pi^V = L\lambda$, resulting in zero surpluses for low-end customers and positive surpluses for high-end customers. Thus, the long-run average customer surplus is $S^V = (H - L)\gamma\lambda$. 

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**Figure 2.** *(Color online) Illustration of a Pledging Process with Intertemporal Segmentation*
In the margin strategy, the firm sets the price \( r^M = H \) and the corresponding long-run average profit is \( \pi^M = Hy/\lambda \), leading to zero surpluses for both segments, thus \( S^M = 0 \). In the product-line strategy, the firm sets the prices \( r^P = L \) and \( p^P = (\theta - 1)H + L \), and the corresponding long-run average profit is \( \pi^P = (\theta - 1)H\gamma + L\lambda - (N + 1)h/2 \), resulting in zero surpluses for low-end customers and positive surpluses for high-end customers. Thus, the long-run average customer surplus is \( S^P = (H - L)\gamma \lambda \). Notice that \( p^P - r^P > p^G - r^G \), implying that with group buying, the firm prices discriminate between two segments less aggressively. Among the three strategies, the optimal one is described in Proposition A.1 in Online Appendix A. Put plainly, when the inventory holding cost is sufficiently low, it is optimal for the firm to offer both products using the product-line strategy, with the premium and regular products targeting high- and low-end customers, respectively; otherwise, only the regular product will be offered. When only the regular product is available, it is not surprising that the optimal price and the resulting market segmentation are entirely determined by valuation heterogeneity. The firm must weigh a high profit margin with a small market share against a large market share but a low profit margin. With a low level of valuation heterogeneity, it is more profitable for the firm to target both segments, not fully extracting surpluses from high-end customers. With a high level of valuation heterogeneity, the firm benefits more by targeting only the high-end segment.

### 4.3. Group Buying or Not

Now we compare the performance of group buying, from the perspectives of both the firm and the customers, against the three benchmarks. First, we use the firm’s long-run average profit as the criterion to investigate the condition under which the firm has an incentive to offer group buying.

**Theorem 2** (Profit Comparison). Suppose \( \theta > 1 \). There exists a threshold \( \bar{h} \) for the inventory holding cost and a threshold \( \bar{m} \) for the valuation heterogeneity such that the following hold:

(i) If \( h \leq \bar{h} \), it is optimal for the firm to adopt the product-line strategy.

(ii) If \( h > \bar{h} \),

(ii.1) if \( H/L \leq 1/\gamma \), as \( N \) increases, the firm’s optimal group-buying strategy varies from \( \{G(H + L), R(\emptyset)\} \rightarrow \{G(H; H + L), R(L; \emptyset)\} \rightarrow \{NG, R(H + L)\} \);

(ii.2) if \( 1/\gamma < H/L \leq \bar{m} \), as \( N \) increases, the firm’s optimal group-buying strategy varies from \( \{G(H), R(L)\} \rightarrow \{G(H; H + L), R(L; \emptyset)\} \rightarrow \{NG, R(H)\} \); and

(ii.3) if \( H/L > \bar{m} \), as \( N \) increases, the firm’s optimal group-buying strategy varies from \( \{G(H), R(L)\} \rightarrow \{NG, R(H)\} \).

Theorem 2 characterizes the firm’s optimal group-buying strategy and targeted customer segments. When the inventory holding cost \( h \) is low, the firm always prefers to sell both products simultaneously using the product-line strategy; that is, the firm will not choose group buying as a way to sell the premium product. By contrast, when the inventory holding cost \( h \) is high, the firm will never offer both products make-to-stock, but instead, may resort to group buying. Since this intuition always holds, for the remainder of the paper we focus on the cases where the inventory holding cost \( h \) is high by assuming the following.

**Assumption 1.** The inventory holding cost is higher than \( \bar{h} \), that is, \( h > \bar{h} \).

As mentioned earlier, in the cosmetic industry, shelf space is always limited and offering extra products is costly, which motivates firms to offer a contingent product online via group buying. If the inventory holding cost is high enough, or, as an equivalent, customers are patient enough (since \( \bar{h} \) increases in \( c \)), offering the premium product through group buying is potentially more profitable.

Figure 3 illustrates the regions in the parameter space where it is optimal to offer group buying, and the resulting market segmentation. In that figure, \( G \) and \( R \) denote the group-buying product and the regular product, respectively (whereas \( NG \) indicates that group buying is not offered); \( H, L, H + L, H; H + L, L; \emptyset \), and \( \emptyset \) denote the customer segments that purchase a specific product (whereas \( \emptyset \) means that no segment purchases the corresponding product). For instance, \( G(H; H + L), R(L; \emptyset) \) specifies a region in which the firm benefits more from offering group buying. More specifically, in the early sign-up process of the group buying, only the high-end segment joins the campaign, while the low-end segment purchases the regular product. In the late phase, both segments sign up for the group-buying campaign, while no one purchases the regular product.

As the difference between valuations of the two segments increases, the firm may not always want to offer group buying alongside the regular product. This is counterintuitive, as group buying provides the firm with an additional tool for price discrimination. As the valuation heterogeneity \( H/L \) increases, it would seem likely that the firm had more incentive to resort to group buying as an additional price discrimination scheme, with the regular product and group buying targeting different segments. However, we show that the profitability of group buying is moderated by the batch size for economic production \( N \) on the supply side. Besides, it should be noted that the quality level of the premium product \( \theta \) also influences the firm’s decision. For \( \theta > 2 \) and \( 1 < \theta \leq 2 \),
the optimal group-buying strategies differ when $H/L$ is high enough, because a small value of $\theta$ limits the profitability of group buying (see Figure 3).

Offering group buying means a trade-off between the benefit of price discrimination and the efficiency loss due to customer waiting. Therefore, the firm has an incentive to offer group buying when the benefit of price discrimination is sufficiently strong compared with the efficiency loss. The profitability of price discrimination increases with the valuation heterogeneity, whereas the efficiency loss increases with the batch size. How to strike a balance can be summarized in the following cases.

First, when the batch size $N$ is sufficiently small, the efficiency loss is small as well, and thus can be easily dominated by the benefit. Therefore, the firm has an incentive to offer group buying regardless of the level of valuation heterogeneity. With a low level of valuation heterogeneity, the firm invites both segments to sign up (see the region $G(H + L), R(\emptyset)$ in Figure 3). However, with a sufficiently high level of valuation heterogeneity, the firm invites only the high-end segment to join the group-buying campaign, leaving the low-end segment to buy the regular product (see the region $G(H), R(L)$ in Figure 3). This case is similar to the classical result on product line design with vertical differentiation (see, e.g., Moorthy 1984).

Second, when the batch size $N$ is sufficiently large, the efficiency loss due to customer waiting is significant and can no longer be compensated for by inviting the low-end segment to participate in group buying. In this case, the benefit of price discrimination is dominated by the efficiency loss. Consequently, it is no longer profitable for the firm to offer group buying and the firm sells only the regular product (see the regions $NG, R(H + L)$ and $NG, R(H)$ in Figure 3).

Last, when the batch size $N$ is in an intermediate range, as valuation heterogeneity increases from 1, the firm’s optimal strategy can go from no group buying, to group buying, to no group buying, and finally back to group buying (see the following corollary and the shaded region in Figure 3).

**Corollary 2 (Nonmonotonic Group-Buying Strategy).** When $N$ is in an intermediate range, the firm’s group-buying strategy is nonmonotonic in $H/L$. Specifically, as $H/L$ increases, the firm’s optimal group-buying strategy can vary from $NG \rightarrow G \rightarrow NG \rightarrow G$.

Corollary 2 suggests that when the batch size is moderate, there can be two disconnected ranges of valuation heterogeneity, in which the firm has an incentive to offer or not to offer group buying. This nonmonotonic behavior of offering group buying with respect to valuation heterogeneity is driven by the possible intertemporal segmentation enabled by group buying. In a homogeneous market (i.e., $H/L = 1$), there is no need for price discrimination. Thus, the firm prefers the volume strategy. As $H/L$ increases, the firm can segment the market by offering group buying. In this case, when $N$ is not too low, the efficiency loss cannot be ignored. Hence, the firm reduces efficiency loss by inviting the low-end segment to participate in group buying during the late pledging process. As a result, when both $N$ and $H/L$ are in an intermediate range, group buying allows the firm to intertemporally segment the market (see the change from $NG, R(H + L)$ to $G(H, H + L), R(L, \emptyset)$ in Figure 3). As $H/L$ further increases, the profit potential of price discrimination also increases. However, to offer group buying in this case, the firm needs to sacrifice the potentially high profit margin from the high-end segment to attract the low-end segment. Thus, group
buying becomes less attractive, and the firm switches to the margin strategy (see the change from \(G(H;H+L), R(L;\emptyset)\) to \(NG, R(H)\) in Figure 3). Finally, as \(H/L\) further increases, group buying can once again be more attractive with a different market composition (recall Corollary 1). Indeed, when \(H/L\) is large enough, by perfectly segmenting the market, the benefit of price discrimination via group buying overcomes the loss of efficiency due to customer waiting (see the change from \(NG, R(H)\) to \(G(H), R(L)\) in Figure 3).

From the supply side, as \(N\) increases, the group-buying strategy changes from \(G\) to \(NG\), regardless of the level of valuation heterogeneity \(H/L\). However, the corresponding customer segmentation depends on valuation heterogeneity. The optimal group-buying strategy has the following market segmentation. There exists a threshold for \(H/L\) such that if \(H/L\) is below the threshold, both high- and low-end segments participate in the group-buying campaign; otherwise, only the high-end segment joins the group-buying campaign. Recall that to offer group buying, the firm needs to balance the benefit of price discrimination and efficiency loss. As mentioned, when valuation heterogeneity \(H/L\) is sufficiently low, the firm has an incentive to reduce expected waiting times by inviting both segments to sign up. Specifically, when \(N\) is small enough, both segments are invited to sign up at the beginning of the group-buying campaign, whereas with a larger \(N\), the group-buying product targets the high-end segment first, then both segments. In contrast, when the valuation heterogeneity \(H/L\) is sufficiently high, only the high-end segment is invited to sign up, as the benefit of price discrimination dominates the efficiency loss due to customer waiting. Thus, the firm has less incentive to invite the low-end segment to join and reduce waiting times, and hence compensate for efficiency loss by sacrificing the high profit margin.

**Corollary 3 (Total Sales Volume Comparison).** The total sales volume with group buying is as high as the best of the other three benchmarks. Specifically, \(q^G = q^V > q^M\).

All the previous analyses use the profit criterion. However, in practice, the firm may also consider other factors, such as the market share. Corollary 3 shows that the total sales volume with group buying is as high as the best of the other three benchmarks. From the firm’s viewpoint, another merit of offering group buying alongside the regular product is that the firm can achieve a larger total market share. In our example from the cosmetic industry, where the shelf space is limited, offering an additional product line requires extra shelf space and may incur additional costs. However, the emerging online group-buying program provides an opportunity to offer additional products without requiring extra physical shelf space.

In addition, by offering group buying online, the firm does not incur inventory holding costs, as the group-buying product is manufactured only when enough buyers have been assembled. In sum, offering group buying of the premium product alongside the regular product is similar to offering a line of make-to-stock products, but without the disadvantages mentioned earlier.

### 4.4. Customer Welfare

From the customers’ perspective, we measure the long-run average customer surplus. With the optimal group buying, the long-run average customer surplus \(S^G\) is given by

\[
S^G = \sum_{n=1}^{N} S^G_H(n) \cdot \gamma \lambda \cdot P(n) + \sum_{n=1}^{N} S^G_L(n) \cdot (1 - \gamma) \lambda \cdot P(n)
\]

where the individual surpluses of high- and low-end customers at a state \(n\) are

\[
S^G_H(n) = \theta H - p^G - c \cdot w^G(n),
\]

\[
S^G_L(n) = \begin{cases} 
\theta L - p^G - c \cdot w^G(n) & \text{if } 1 \leq n \leq \bar{n}^G, \\
0 & \text{if } \bar{n}^G < n \leq N.
\end{cases}
\]

Moreover, \(P(n)\) represents the probability of state \(n(1 \leq n \leq N)\) in the optimal group-buying campaign, which can also be viewed as the proportion of time when state \(n\) occurs. Due to the memoryless property of the Poisson process, the expected waiting time \(w^G(n)\) of a customer who arrives at state \(n\) is independent of the customer’s arrival time. Furthermore, we denote by \(S^V_H, S^V_L, S^M_H, S^M_L, S^P_H, S^P_L\) and \(S^M\) the high- and low-end customer surpluses under the volume, margin, and product-line strategies, respectively.

**Theorem 3 (Customer Surplus Comparison).** The comparison of customer surpluses under different strategies is characterized as follows:

(i) **Customers as a whole always strictly benefit from group buying of the premium product alongside the regular one, that is, \(S^G > S^P = S^V > S^M\).**

(ii) **Both the high- and low-end segments always (weakly) benefit from group buying of the premium product alongside the regular one, that is, \(S^G_H > S^P_H = S^V_H > S^M_H\), \(S^G_L \geq S^P_L = S^V_L = S^M_L\).**

Theorem 3 illustrates that customers always benefit from the group-buying offer. Additionally, both high- and low-end customers always (weakly) benefit from
the group-buying offer. The underlying rationale is as follows. For high-end customers, even the first to sign up has a surplus gain from group buying (beyond individual surpluses obtained in the other three benchmarks), because the firm encourages high-end customers to sign up for the group-buying product rather than to purchase the regular product immediately. It is noteworthy that the individual surpluses for all of the subsequent high-end arrivals are even higher than that of the first sign-up, because they wait for less time. Consequently, the customer surplus for the high-end segment in group buying (which is strictly higher than $H - L$) is always strictly higher than the surplus in the other three benchmarks (i.e., $H - L$ or 0). By contrast, in the other three benchmarks, the individual surpluses for the low-end customers are always zero, as they either purchase the regular product at price $L$ or are priced out of the market. However, in group buying, some (or even all) low-end customers can obtain positive individual surpluses by jumping on the bandwagon to buy the (otherwise unaffordable) premium product. Similarly, the later a low-end customer arrives, the more individual surplus the customer obtains. Hence, the customer surplus for the low-end segment in group buying is always (weakly) higher than in the other three benchmarks. Moreover, the participation of the low-end segment can benefit high-end customers by reducing their waiting times, which manifests an intertemporal cooperation between the two segments. To summarize, every stakeholder can simultaneously benefit from the group-buying offer, leading to a win-win-win situation.

**Corollary 4.** As the arrival rate $\lambda$ increases, the following results hold:

(i) The region in which intertemporal customer segmentation is optimal shrinks. In the region where the intertemporal segmentation is optimal, as the arrival rate $\lambda$ increases,

(ii) low-end customers join the group-buying campaign at a later time;

(iii) the firm charges a higher price for the premium product, and enjoys a higher profit; and

(iv) the individual surpluses for both high- and low-end customers decrease.

Corollary 4 summarizes the impact of the customer arrival rate on intertemporal segmentation. With a higher arrival rate, the intertemporal segmentation is less likely to be adopted by the firm as an intermediate solution. Moreover, in the optimal intertemporal segmentation, low-end customers tend to join the group buying later. With a higher arrival rate, it takes a shorter amount of time to assemble enough high-end customers, which effectively alleviates the waiting costs, as the exogenously determined threshold size $N$ remains unchanged. Hence, the firm has less incentive to admit low-end customers into group buying and can charge a higher price, leading to a higher profit. For customers, while their waiting times are shortened, both segments suffer from an individual surplus loss, because the high-end customers and those low-end customers who join the group buying have to pay more, and furthermore, a larger number of low-end customers are excluded from enjoying positive surpluses in the group buying.

5. Extensions

We use a simplified base model to derive our main findings. It is necessary to verify that these findings are robust when certain assumptions are relaxed. In this section, we investigate a set of extensions. The details are relegated to the online supplements.

5.1. Contingent Pricing

In the base model, we focus on uniform pricing, in which the same price is charged for the group-buying product, regardless of when customers join the campaign. Nevertheless, as pledgers face different waiting times for the success of the group-buying campaign, it is natural to see whether contingent pricing, that is, offering the group-buying product at different prices to different pledgers based on their sign-up time, would be more profitable. Now we investigate the situation where the firm sets prices based on the pledge-to-go state $n$, with the superscript $C$ denoting the equilibrium outcome. For simplicity, we first assume that the firm can only adopt two endogenized price points $p^C_1$ and $p^C_2$, and only change prices once at a so-called tipping state, denoted by $\bar{n}^C$, where $0 \leq \bar{n}^C \leq N$ (see Online Appendix B and Online Supplement C for more details).

One main observation is that customer segmentation in equilibrium is largely consistent with the base model, which confirms the robustness of our main findings (see Proposition 2 and Proposition B.1 in Online Appendix B). Nevertheless, there are some differences. Figure 4 illustrates the change of the optimal customer segmentation when the firm switches from uniform pricing to contingent pricing. Notably, under contingent pricing, the intertemporal segmentation $\{H; H + L\}$ is more likely to sustain in equilibrium (see shaded regions (II) and (III) in Figure 4). Specifically, under uniform pricing, the firm cannot discriminate between high- and low-end customers in region (II), due to the low level of valuation heterogeneity. However, under contingent pricing, intertemporal price discrimination is viable. Conversely, in region (III), under uniform pricing, it is optimal for the firm to implement perfect segmentation due to the high level of valuation heterogeneity,
Proposition 3 (Early-Bird vs. End-of-Cycle Discount). Under contingent pricing, for any $H/L$, there exists a threshold for the batch size $N$ below which the firm offers the end-of-cycle discount, that is, $p_1^C > p_C^E > p_2^C$; otherwise, it offers the early-bird discount, that is, $p_1^E < p_C^E < p_2^C$.

As mentioned, in the context of group buying with a deadline (see, e.g., Hu et al. 2013, Liu and Tunca 2019), the firm may have more incentive to offer the early-bird discount to boost early sign-ups, which generates a cascade effect among the later arrivals, because one more early sign-up will increase the success rate of a campaign and boost the confidence of every subsequent arrival. Perhaps surprisingly, in the case of group buying with flexible duration, Proposition 3 says that the optimal price path can be decreasing. Contrary to conventional wisdom, it is optimal for the firm to offer a discount to customers who sign up late (i.e., $p_1^C > p_2^C$) when the batch size $N$ is not too high (see regions (I), (II), and (III) in Figure 4). In addition, for these cases, the optimal price for early (resp., late) sign-ups $p_1^E$ (resp., $p_2^E$) is higher (resp., lower) than the optimal price $p_C^E$ under uniform pricing, that is, $p_1^E > p_C^E > p_2^E$. The reason is that the firm can encourage customers to sign up either by lowering $p_1^C$ to prompt early sign-ups or by lowering $p_2^C$ to prompt late sign-ups. When the batch size $N$ is sufficiently large, the firm must offer an early-bird discount to jump-start the sign-up process (see the region above the dashed line in Figure 4); otherwise, no one would want to be the first to sign up due to the significantly high waiting cost, and the group-buying campaign would fail. In this case, to join the group-buying campaign, low-end customers arriving late pay more than high-end customers arriving early. Nevertheless, when the batch size $N$ is not large, group buying can succeed even without the early-bird discount, as the waiting cost borne by the first customer is not significantly high. With a guaranteed success, the end-of-cycle discount is more effective than the early-bird discount. The acceleration of the late pledging process can be anticipated by all of the early arrivals in REE, reducing waiting times and increasing their willingness to pay. Thus, the firm can charge a higher price for early sign-ups. In contrast, when the firm offers the early-bird discount, the acceleration of the early pledging process has no effect on the late arrivals, as waiting times only depend on the current state of pledge-to-go.

Proposition 4 (No Incentive to Behave Strategically). Suppose the firm adopts the optimal price path in Proposition 3 as if customers are myopic. Even when customers are forward-looking, in a pure-strategy equilibrium they would have no incentive to wait and act as if they were myopic.

In our study, we assume that customers need to decide whether to purchase the regular product now, sign up for the group-buying product now, or leave immediately without purchasing anything. In other words, we assume that customers are not forward-looking. However, if they were forward-looking, one might wonder whether they had an incentive to wait. Suppose the firm adopts the optimal price path in Proposition 3 as if customers are myopic. Proposition 4 says that even if customers were forward-looking, they would behave myopically in a pure-strategy equilibrium. The argument goes as follows. Although the regular product price does not change over time, there may exist an incentive to wait when the group-buying product has a decreasing price path, that is, $p_1^C > p_2^C$. Whether to buy now or wait until later is a valid question only to high- and low-end customers who arrive early at a state $n > n^C$ because they observe the higher price $p_1^E$. For any high-end customer arriving at state $n > n^C$, if the customer delayed the purchase, so would all high-end customers who arrive later, because their utilities of delaying the purchase would be even higher. Consequently, group buying would not succeed and everyone would earn zero surplus. In anticipation of this situation, it is a dominant strategy for high-end customers arriving early to sign up immediately without waiting. Any low-end customer arriving at state $n > n^C$ who delayed the purchase would earn a negative utility, because the lower price $p_2^E$ the customer hoped to wait for exactly extracts all the surplus of a low-end customer.
arriving later at state \( n = \bar{n}^C \). Hence, any low-end customer arriving early at state \( n > \bar{n}^C \) has no incentive to wait either. Moreover, in a mixed-strategy equilibrium, high-end customers may have a positive probability of waiting. But by the same logic mentioned earlier, there will be a non-zero probability of their immediate sign-up, which implies that our main insight into intertemporal customer segmentation would still hold to some extent.

In Online Appendix B, Proposition B.2 highlights the economic value of contingent pricing for the firm when offering group buying with flexible duration (see also Theorem B.1 in Online Appendix B and Figure C.1 in Online Supplement C for the optimal group-buying strategy under contingent pricing). Recall that under uniform pricing, the firm intertemporally discriminates between high- and low-end customers, either directly via different products (i.e., perfect segmentation) or indirectly via different sign-up times (i.e., intertemporal segmentation). Nevertheless, contingent pricing further allows the firm to discriminate by charging different prices over time even to customers in the same segment (i.e., intertemporal price discrimination). Contingent pricing cannot only enable intertemporal price discrimination, but can also unleash the full potential of intertemporal segmentation, which are two sides of the same coin. On the one hand, the firm can extract more surplus by charging different prices over time dependent on the state of the group-buying campaign. This corresponds to a simple utility transfer from a customer to the firm, without any new surplus generated for the total welfare. On the other hand, by encouraging more customers to sign up at the later stage, the entire sign-up process is shortened (see region (III) \( \{H\} \rightarrow \{H; H + L\} \) in Figure 4). As a result, contingent pricing reduces the total waiting time and generates additional welfare by reducing the efficiency loss. Taken together, contingent pricing enhances the profitability of group buying with flexible duration and provides the firm with a stronger incentive to offer this kind of group buying.

In the previous analyses, we consider a special case of contingent pricing with only two price points for the group-buying product. More generally, the firm can charge an entirely different price \( p^C_n \) at any state \( n \). Two possible price paths are illustrated in Figure 5. In both paths, the price monotonically increases from state \( N \) to \( \bar{n}^C + 1 \), for which only the high-end segment signs up, and from state \( \bar{n}^C \) to 1, for which both segments sign up. It is worth noting that Figure 5, (a) and (b), correspond to the end-of-cycle discount and the early-bird discount in Proposition 3, respectively (see the dashed lines in Figure 5). Moreover, when the firm can freely charge contingent prices, the entire customer surplus can be extracted by the firm (see the shaded regions in Figure 5, corresponding to the customer surplus left on the table when only two price points are allowed).

### 5.2. Unobservable Sign-up Information

In addition to the pricing policy, the firm may manipulate the information structure to better design group buying. In Online Appendix C and Online Supplement D, we explore the effect of concealing sign-up information in flexible-duration group buying, referred to as unobservable group buying, for which we use the superscript \( U \) to denote the equilibrium outcome. By contrast, the group buying where the firm reveals the pledge-to-go state \( n \) to customers is called observable group buying.

Figure 6 illustrates the change of the optimal customer segmentation from observable to unobservable group buying. One main observation is that the customer segmentation here differs significantly from that in the base model (see Proposition 2 and Proposition C.1 in Online Appendix C). Notice that intertemporal segmentation depends on an observable pledge-to-go state. Hence, in unobservable group buying, intertemporal segmentation does not exist (see shaded regions (I) and (II) in Figure 6). Although we have verified the profitability of intertemporal segmentation with observable sign-up information, the firm may arguably benefit even more from its absence. Indeed, the firm can actually charge a higher price for the group-buying product, that is, \( p^U > p^C \), because there is no need to compensate for a high waiting cost to encourage the first sign-up. (Note that we assume that once customers sign up, they cannot renege; otherwise, this claim does not hold.) We confirm this observation in Proposition C.2 in Online Appendix C (see also Theorem C.1 in Online Appendix C and Figure D.1 in Online Supplement D for the optimal unobservable group-buying strategy).

Notice that in group buying with a deadline, observable group buying may be favored by firms (see, e.g., Hu et al. 2013). Now in group buying with flexible duration, unobservable group buying is more profitable when \( \theta \) is sufficiently large (see Proposition C.2). As mentioned earlier, without knowing the pledge-to-go state \( n \), no matter when a customer joins the campaign, the customer’s expected waiting time is the same even if the actual waiting time differs. The firm can take advantage of this information uniformity by charging a higher price for the group-buying product compared with that in observable group buying, because it only needs to compensate for customers’ waiting cost at an average level, which can increase the firm’s total revenue, especially when the group-buying product is attractive enough (with a high value of \( \theta \)).
5.3. Heterogeneous Waiting Costs
Throughout the paper, we assume customers have the same waiting cost per unit time. However, customers can be heterogeneous along two dimensions, that is, valuation and patience (e.g., Su 2007). In Online Appendix D and Online Supplement E, we assume that high- and low-end customers have waiting costs per unit time $c_H$ and $c_L$, respectively. We show that our qualitative findings are preserved (see Lemma E.1 in Online Supplement E and Propositions D.1 and D.2 in Online Appendix D). Specifically, first, when $c_H \leq c_L$, which means the high-end customers are more patient than the low-end customers, we find that the equilibrium outcomes remain qualitatively unchanged. Moreover, in this case, intertemporal customer segmentation $\{H; H + L\}$ becomes less attractive to the firm compared with perfect segmentation $\{H\}$ than that in the base model (see Corollary D.1 in Online Appendix D). Second, when $c_L < c_H \leq \bar{c}_H$, indicating that the high-end customers have higher waiting costs than the low-end customers, but not significantly higher, we observe that the equilibrium outcomes still remain qualitatively unchanged. More important, in this case, intertemporal customer segmentation $\{H; H + L\}$ becomes even more attractive to the firm compared with perfect segmentation $\{H\}$ than that in the base model (see Corollary D.1). Third, when $c_H > \bar{c}_H$, which implies that the high-end customers are very impatient compared with the low-end customers, the firm still has an incentive to adopt intertemporal customer segmentation, but in a different way. That is, it is now optimal for the firm to invite low-end customers to sign up first and adopt the intertemporal segmentation $\{L; L + H\}$ because low-end customers are much more patient.

5.4. Inferior Group-Buying Product
In the model, we assume that the premium group-buying product is of high quality. In Online Appendix E and Online Supplement F, we consider a group-buying product that is inferior to the regular product, that is, $\theta < 1$. In the base model, the premium group-buying product targets the high-end segment, whereas the regular product targets the low-end segment. However, with the inferior group-buying product, the segment targets have changed with the group-buying product targeting the low-end segment and the regular product targeting the high-end segment (see Propositions E.1 and E.2 in Online Appendix E). Moreover, as in the base model, intertemporal customer segmentation still exists. As the segments targeted by the product line are reversed, the roles of different segments in managerial insights will be reversed as well. For example, when $\theta < 1$ (resp., $\theta > 1$), those high-end (resp., low-end) customers who join late in the group buying are the free-riders of the inferior (resp., premium) group-buying product, while the low-end (resp., high-end) customers who join early benefit from shorter waiting times. In addition, group buying of an inferior product becomes less profitable than that of a premium product. Compared with the volume and margin strategies, offering group buying of an inferior product benefits the firm only when the probability of the high-end segment $\gamma$ is sufficiently high.
high and the batch size $N$ is in an intermediate range (see Theorem E.1 in Online Appendix E).

### 5.5. Horizontally Differentiated Products

In the base model, we focus on vertically differentiated products. Nonetheless, our model can be applied to horizontally differentiated products. In Online Appendix F and Online Supplement G, we consider two segments of customers, with one preferring the group-buying product (denoted by type-G) and the other preferring the regular product (denoted by type-R). We assume that the two products are located at the two ends of a Hotelling line, that is, positions 0 and the line, along with offering the regular product. We explore the mechanism behind offering group buying of a special-edition product.

6. Conclusion

In this paper, we build a stylized model to understand the mechanism behind offering group buying of a special-edition product without a predetermined deadline, along with offering the regular product. We explore the interaction between customers’ sign-up dynamics and the firm’s optimal product design, including pricing and market segmentation. The current literature on traditional group buying with a predetermined deadline does not take into account customers’ waiting cost, and only focuses on the group-buying product itself. In contrast, we consider customers’ waiting cost as a unique operational feature in group buying. We investigate how to better design the product line by treating the product offered through group buying as a contingent product. We find that flexible-duration group buying can create an intertemporal cooperation among customers through their self-interested behavior. This kind of intertemporal customer segmentation occurs when high- and low-end customers are invited to participate in the group-buying campaign at different times during the pledging process.

Our study has the following practical implications. First, we show why flexible-duration group buying can be a profitable add-on alongside the regular product. Generally, if the inventory holding cost is high enough or customers are patient enough, offering a contingent product line through flexible-duration group buying is more profitable than offering a product line in the normal way, because the firm can trade the high holding cost with the low customer waiting cost. Second, we identify flexible-duration group buying as a more natural and fairer discrimination scheme. This is because the firm can passively discriminate between different segments by offering a constant price, which can be more acceptable and deemed fairer than intertemporal price discrimination. Third, through customers’ intertemporal cooperation, the firm can obtain a higher profit without hurting customers, that is, achieve a win-win-win situation. Finally, to better design flexible-duration group buying, the firm can adopt contingent pricing or conceal the sign-up information.

Thanks to the internet and social network platforms, the traditional retail industry has been revolutionized. Firms can now offer contingent product lines online via flexible-duration group buying. Whether to adopt this contingent product scheme and how to design it optimally depend heavily on both the supply- and demand-side characteristics of a firm. For example, beauty companies usually face limited shelf space (implying a high inventory holding cost), and hence, it is better for them to offer special-edition products through online group buying rather than a product line of make-to-stock products. Moreover, several group-buying platforms in the retail industry (e.g., Pinduoduo) have also resorted to flexible-duration group buying, since their inventory holding and offline management costs tend to be high. In addition, as the demand for fairness has risen in recent years, firms may want to avoid protractive price discrimination in order to nurture sustainable customer relationships. To this end, flexible-duration group buying can be a good option, because not only does it enable intertemporal customer segmentation through a one-price-for-all scheme, but also the price difference between the two products is reduced. Last, depending on the demand-side characteristics, firms could further optimize the design of flexible-duration group buying in terms of product offering, pricing, and information disclosure. For instance, tourist platforms could adopt contingent pricing to some extent as people are more likely to tolerate differentiated pricing in the travel industry.

Future research directions may include the following. First, one might consider alternative information structures adopted by the firm, such as
contingent information disclosure depending on the real-time pledge-to-go state. Second, group-buying campaigns for multiple products might also be worth studying, along with the question of how many campaigns the firm wants to launch simultaneously. Third, it would be interesting to examine the behavioral aspects of customers’ pledging decisions, such as risk aversion and bounded rationality. Finally, a model in which both the price and batch size are endogenously determined, or one with an alternative customer arrival pattern such as high-end customers arriving first followed by low-end, could be a fruitful future direction.

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