Online Appendix to “Intertemporal Segmentation via Flexible-Duration Group Buying”

A. Premium Group-Buying Product

Lemma A.1 (Customers’ Sign-up Behavior). For any \( \theta \neq 1 \) and \( N \), if high-end (low-end) customers sign up at state \( n (1 \leq n \leq N) \), then all high-end (low-end) customers sign up at any subsequent state \( n' (1 \leq n' \leq n) \).

Proof of Lemma A.1. See online supplement. \( \square \)

Proposition A.1 (No-Group-Buying Benchmarks). In the absence of group buying as an option, there exists a threshold for the inventory holding cost, \( \hat{h}_1 \), such that

(i) if \( h \leq \hat{h}_1 \), it is optimal for the firm to offer both products by using the product-line strategy, and high-end customers purchase the premium product while low-end customers purchase the regular product;

(ii) if \( h > \hat{h}_1 \), the firm offers only the regular product,

(ii-1) when \( H/L \leq 1/\gamma \), it is optimal for the firm to adopt the volume strategy, and both high- and low-end customers purchase the regular product;

(ii-2) when \( H/L > 1/\gamma \), it is optimal for the firm to adopt the margin strategy, and only high-end customers purchase the regular product.

Proof of Proposition A.1. See online supplement. \( \square \)

Proof of Theorem 1. Consider a customer with valuation \( v \) who arrives at pledge-to-go state \( n (1 \leq n \leq N) \). She would like to sign up if and only if \( v - p - c \cdot w(n) \geq 0 \). Denote the threshold for customers’ valuation at state \( n \) as \( \bar{v}_n \).

Note that \( v \) follows either a continuous or a discrete distribution. First, consider the case of a general continuous distribution. We assume \( v \in [0,v_m] \) with a finite upper bound \( v_m \) or \( v \in [0,\infty) \).

Thus, \( \bar{v}_n = p + c \cdot w(n) \), where \( w(n) = \frac{1}{\lambda} \sum_{k=1}^{n-1} \frac{1}{F(v_k)} \) is the expected waiting time at state \( n \) and \( \bar{F}(v) = 1 - F(v) \). Then, define the difference between two neighboring thresholds as

\[
\Delta v_n \equiv \bar{v}_{n+1} - \bar{v}_n = c \cdot w(n+1) - c \cdot w(n) = \frac{c}{\lambda} \sum_{k=1}^{n} \frac{1}{F(\bar{v}_k)} - \frac{c}{\lambda} \sum_{k=1}^{n-1} \frac{1}{F(\bar{v}_k)} = \frac{c}{\lambda F(\bar{v}_n)} > 0,
\]

where \( 1 \leq n < N \). Thus, \( \bar{v}_n \) is increasing in \( n \), and \( \Delta v_n \) is also increasing in \( n \). Moreover, we have

\[
\bar{v}_n = \bar{v}_1 + \frac{c}{\lambda} \left[ \frac{1}{F(\bar{v}_1)} + \frac{1}{F(\bar{v}_2)} + \cdots + \frac{1}{F(\bar{v}_{n-1})} \right] = \bar{v}_1 + \frac{c}{\lambda} \sum_{k=1}^{n-1} \frac{1}{F(\bar{v}_k)}.
\] (A.1)
The firm’s long-run average profit can be written as

\[
\pi(\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_N) = \sum_{n=1}^{N} \left[ p^*(\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_N) \cdot \lambda \cdot \bar{F}(\bar{v}_n) \cdot P(n) \right]
\]

\[
= \frac{\bar{v}_1 \lambda \sum_{n=1}^{N} \bar{F}(\bar{v}_n) [w(n + 1) - w(n)]}{w(N + 1)}
\]

\[
= \frac{cN \lambda \bar{v}_1 \bar{F}(\bar{v}_N)}{\lambda(\bar{v}_N - \bar{v}_1) \bar{F}(\bar{v}_N) + c},
\]

where the second equation follows from \(p^*(\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_N) = \bar{v}_n - \frac{c}{\lambda} \sum_{k=1}^{n-1} \frac{1}{F(\bar{v}_k)} \) for any \(n \ (1 \leq n \leq N)\) and the last equation follows from the expression of \(w(n)\) and (A.1). For ease of exposition, define \(\pi(\bar{v}_1, \bar{v}_N) \equiv \frac{cN \lambda \bar{v}_1 \bar{F}(\bar{v}_N)}{\lambda(\bar{v}_N - \bar{v}_1) \bar{F}(\bar{v}_N) + c}\). Therefore, the firm’s optimization problem can be reduced to

\[
\max_{\bar{v}_1, \bar{v}_N} \pi(\bar{v}_1, \bar{v}_N) \quad \text{s.t.} \quad \bar{v}_n = \bar{v}_{n-1} \cdot \frac{c}{\lambda \bar{F}(\bar{v}_{n-1})}, \quad \text{for any} \ (1 < n \leq N),
\]

which demonstrates that the problem for an arbitrary \(N \geq 2\) under a general continuous distribution can be technically challenging. Moreover, \(\bar{v}_N < v_m\) always holds, because otherwise, if \(\bar{v}_N = v_m\), \(\bar{F}(\bar{v}_N) = 0\), and then \(\pi(\bar{v}_1, \bar{v}_N) = 0\) always holds.

Second, consider the case of a general discrete distribution. We assume \(v \in \{v_1, v_2, \ldots, v_M\}\), where \(0 \leq v_1 < v_2 < \ldots < v_M\) and \(M\) is arbitrary. Thus, \(\bar{v}_n = v_j\), where \(v_{j-1} < p + c \cdot w(n) \leq v_j, \ 0 < j \leq M, \) and \(w(n) = \frac{1}{\lambda} \sum_{k=1}^{n-1} \frac{1}{F(v_k)}\). Then, for any \(n \ (1 \leq n < N)\), \(\bar{v}_{n+1} = v_{j'}\), where \(v_{j'-1} < p + c \cdot w(n+1) \leq v_{j'}\) and \(j' \leq M\). If \(j' = j\), then \(\bar{v}_{n+1} = \bar{v}_n\). If \(j' > j\), then there must exist some \(i \ (j+1 \leq i \leq M)\) such that \(v_j < p + c \cdot w(n+1) \leq v_i\), and hence, \(\bar{v}_{n+1} = v_i > v_j = \bar{v}_n\). Thus, \(\bar{v}_n\) is increasing in \(n\). Moreover, \(\bar{v}_N \leq v_m\). \(\square\)

**Proof of Proposition 1.** By Lemma A.1, if the high-end customers sign up first, there are three possible scenarios: \(\{H\}\), \(\{H; H + L\}\), and \(\{H + L\}\), which are defined as Case I for ease of analysis. Similarly, if the low-end customers sign up first, there are also three possible scenarios: \(\{L\}\), \(\{L; H + L\}\), and \(\{L + H\}\), which are defined as Case II. Here we rule out the scenario \(\{L + H\}\) in Case II to avoid repetition. We use the tipping state \(\bar{x}^G\) to stand for different scenarios.\(^1\) Specifically, in Case I, \(\bar{x}^G = 0\), \(1 \leq \bar{x}^G < N\), and \(\bar{x}^G = N\) represent scenarios \(\{H\}\), \(\{H; H + L\}\), and \(\{H + L\}\), respectively. In Case II, \(\bar{x}^G = 0\) and \(1 \leq \bar{x}^G < N\) represent scenarios \(\{L\}\) and \(\{L; H + L\}\), respectively.

For Case II, the IR and IC constraints are

\(^1\)\(n^G\) denotes the largest integer that is less than or equal to \(\bar{x}^G\), thus we use \(\bar{x}^G\) instead of \(n^G\) in the Online Appendices and Supplements for preciseness.
\[
\begin{cases}
IR_L : \theta L - p^G - c \cdot w^G(n) \geq 0 & 1 \leq n \leq N, \\
IC_L : \theta L - p^G - c \cdot w^G(n) \geq L - r^G & 1 \leq n \leq N, \\
IR_{H1} : H - r^G \geq 0 & \bar{x}^G < n \leq N, \\
IC_{H1} : H - r^G \geq \theta H - p^G - c \cdot w^G(n) & \bar{x}^G < n \leq N, \\
IR_{H2} : \theta H - p^G - c \cdot w^G(n) \geq 0 & 1 \leq n \leq \bar{x}^G, \\
IC_{H2} : \theta H - p^G - c \cdot w^G(n) \geq H - r^G & 1 \leq n \leq \bar{x}^G,
\end{cases}
\]

where \( 0 \leq \bar{x}^G < N \) and the expected waiting time \( w^G(n) \) is

\[
w^G(n) = \begin{cases} \\
\frac{n-1}{\bar{x}^G} + \frac{n-\bar{x}^G-1}{(1-\gamma)\lambda} & 1 \leq n \leq \bar{x}^G, \\
\bar{x}^G < n \leq N.
\end{cases}
\]

In equilibrium, \( IR_L \) is binding at state \( n = N \). Thus, \( p^G(\bar{x}^G) = \theta L - c \cdot w^G(N) \). Define two prices \( \bar{r}_1^G(\bar{x}^G) \equiv p^G(\bar{x}^G) - (\theta - 1) H + c \cdot w^G(\bar{x}^G) \), \( \bar{r}_2^G(\bar{x}^G) \equiv p^G(\bar{x}^G) - (\theta - 1) L + c \cdot w^G(N) \). \( \bar{r}_1^G(\bar{x}^G) < \bar{r}_2^G(\bar{x}^G) \) always holds because \( H > L \) and \( w^G(n) \) is monotonously increasing in \( n \). To satisfy \( IC_{H1} \) and \( IC_L \), the price \( r^G \) should meet the constraints \( r^G < \bar{r}_1^G(\bar{x}^G) \) and \( r^G \geq \bar{r}_2^G(\bar{x}^G) \). Since \( \bar{r}_1^G(\bar{x}^G) < \bar{r}_2^G(\bar{x}^G) \) when \( \theta > 1 \), it is impossible to satisfy these two constraints at the same time. Therefore, Case II cannot become the equilibrium.

Using similar logic, we can show that Case I can become the equilibrium, which proves the proposition. Refer to the proof of Proposition 2 for details of Case I. \( \square \)

**Proof of Proposition 2.** We continue the detailed analysis for Case I in this part (see the definition in the proof of Proposition 1). In the base model, the IR and IC constraints are

\[
\begin{cases}
IR_{H} : \theta H - p^G - c \cdot w^G(n) \geq 0 & 1 \leq n \leq N, \\
IC_{H} : \theta H - p^G - c \cdot w^G(n) \geq H - r^G & 1 \leq n \leq N, \\
IR_{L1} : L - r^G \geq 0 & \bar{x}^G < n \leq N, \\
IC_{L1} : L - r^G \geq \theta L - p^G - c \cdot w^G(n) & \bar{x}^G < n \leq N, \\
IR_{L2} : \theta L - p^G - c \cdot w^G(n) \geq 0 & 1 \leq n \leq \bar{x}^G, \\
IC_{L2} : \theta L - p^G - c \cdot w^G(n) \geq L - r^G & 1 \leq n \leq \bar{x}^G,
\end{cases}
\]

where \( 0 \leq \bar{x}^G \leq N \) and the expected waiting time \( w^G(n) \) is

\[
w^G(n) = \begin{cases} \\
\frac{n-1}{\bar{x}^G} + \frac{n-\bar{x}^G-1}{(1-\gamma)\lambda} & 1 \leq n \leq \bar{x}^G, \\
\bar{x}^G < n \leq N.
\end{cases}
\]

In equilibrium, \( IR_{L1} \) is binding. Thus, \( r^G(\bar{x}^G) = L \). Whether \( IC_H \) or \( IC_{L2} \) is binding in equilibrium depends on the relative size of \( \bar{x}^G \). Since \( w^G(n) \) is monotonously increasing in \( n \), no matter whether \( IC_H \) or \( IC_{L2} \) is binding, the binding must happen at the largest possible state \( n \). Define two prices \( \bar{p}_1^G(\bar{x}^G) \equiv \theta L - c \cdot w^G(\bar{x}^G) \), \( \bar{p}_2^G(\bar{x}^G) \equiv (\theta - 1) H + L - c \cdot w^G(N) \), and we know that \( \bar{p}_1^G(\bar{x}^G) < \bar{p}_2^G(\bar{x}^G) \) if and only if \( \bar{x}^G > \bar{x}_1^G \), where \( \bar{x}_1^G \equiv N - 1 + \gamma - (\theta - 1)(H - L)\gamma/c \). Then, we can write the price \( p^G \) as the function of tipping state \( \bar{x}^G \):

\[
p^G(\bar{x}^G) = \begin{cases} \\
\bar{p}_2^G(\bar{x}^G) & 1 \leq \bar{x}^G \leq \bar{x}_1^G, \\
\bar{p}_1^G(\bar{x}^G) & \bar{x}_1^G < \bar{x}^G \leq N.
\end{cases}
\]
We then derive the firm’s long-run average profit $\pi^G$, also as the function of tipping state $\bar{x}^G$:

$$\pi^G(\bar{x}^G) = \left[ p^G(\bar{x}^G) \cdot \gamma + r^G(\bar{x}^G) \cdot (1 - \gamma) \lambda \right] \cdot P(n > \bar{x}^G) + \left[ p^G(\bar{x}^G) \cdot \lambda \right] \cdot P(n \leq \bar{x}^G)$$

$$= \frac{p^G(\bar{x}^G) \cdot \gamma \lambda + L(1 - \gamma) \lambda \cdot \frac{N - \bar{x}^G}{N} + p^G(\bar{x}^G) \cdot \frac{\bar{x}^G}{\lambda}}{N - (1 - \gamma) \bar{x}^G}.$$

For $\bar{x}^G_1 < \bar{x}^G \leq N$, plugging $p^G(\bar{x}^G) = p^G_1(\bar{x}^G)$ into $\pi^G(\bar{x}^G)$, we have

$$\pi^G(\bar{x}^G) = \frac{\theta L \gamma \lambda N + L(1 - \gamma) \lambda (N - \bar{x}^G) - c \gamma N(\bar{x}^G - 1)}{N - (1 - \gamma) \bar{x}^G}.$$

Taking the first-order derivative of $\pi^G(\bar{x}^G)$ w.r.t. $\bar{x}^G$, we have

$$\frac{\partial \pi^G(\bar{x}^G)}{\partial \bar{x}^G} = \frac{\gamma N [L(1 - \gamma)(\theta - 1) - c(N - 1 + \gamma)]}{[N - (1 - \gamma) \bar{x}^G]^2}.$$

We can see that $\frac{\partial \pi^G(\bar{x}^G)}{\partial \bar{x}^G} > 0$ if and only if $N \leq \tilde{N}_1$. Therefore, for $\bar{x}^G_1 < \bar{x}^G \leq N$, when $N \leq \tilde{N}_1$, the firm sets $\bar{x}^G = N$; when $N > \tilde{N}_1$, the firm sets $\bar{x}^G = \bar{x}^G_1$. Note that $\bar{x}^G_1 > 0$ if and only if $N > \tilde{N}_2$.

For $1 \leq \bar{x}^G \leq \bar{x}^G_1$, plugging $p^G(\bar{x}^G) = p^G_2(\bar{x}^G)$ into $\pi^G(\bar{x}^G)$, we have

$$\pi^G(\bar{x}^G) = \frac{[(\theta - 1)H + L] \gamma \lambda N + L(1 - \gamma) \lambda (N - \bar{x}^G) - c N(N - 1) + c(1 - \gamma) N \bar{x}^G}{N - (1 - \gamma) \bar{x}^G}.$$

Taking the first-order derivative of $\pi^G(\bar{x}^G)$ w.r.t. $\bar{x}^G$, we have

$$\frac{\partial \pi^G(\bar{x}^G)}{\partial \bar{x}^G} = \frac{(1 - \gamma) N [c + (\theta - 1) H \gamma \lambda]}{[N - (1 - \gamma) \bar{x}^G]^2} > 0.$$

Since $\frac{\partial \pi^G(\bar{x}^G)}{\partial \bar{x}^G} > 0$ always holds, for $1 \leq \bar{x}^G \leq \bar{x}^G_1$, the firm always sets $\bar{x}^G = \bar{x}^G_1$.

Comparing the results above, we define two thresholds for the batch size $N$:

$$\tilde{N}_1 = \frac{(\theta - 1)(1 - \gamma)L \lambda}{c} + 1 - \gamma,$$

$$\tilde{N}_2 = \frac{(\theta - 1)(H - L) \gamma \lambda}{c} + 1 - \gamma,$$

which determines the optimal tipping state $\bar{x}^G$. Besides, $\tilde{N}_1$ increases in $L$. For a given $L$, $\tilde{N}_2$ increases in $H/L$. $\tilde{N}_2 > \tilde{N}_1$ if and only if $H/L > 1/\gamma$. Thus, the REE when offering group buying is

(i) when $H/L \leq 1/\gamma$,

1) if $N \leq \tilde{N}_1$, $\bar{x}^G = N$, $r^G = L$, $p^G = \theta L - c(N - 1)/\lambda$, and $\pi^G = \theta L \lambda - c(N - 1)$;

2) if $N > \tilde{N}_1$, $\bar{x}^G = \bar{x}^G_1$, $r^G = L$, $p^G = \theta L + \gamma (\theta - 1)(H - L) - c(N - 2 + \gamma)/\lambda$, and $\pi^G = \pi^G_1$;

(ii) when $H/L > 1/\gamma$,

1) if $N \leq \tilde{N}_2$, $\bar{x}^G = 0$, $r^G = L$, $p^G = (\theta - 1)H + L - c(N - 1)/(\gamma \lambda)$, and $\pi^G = (\theta - 1)H \gamma \lambda + L \lambda - c(N - 1)$;
(2) if $N > \tilde{N}_2$, $\bar{x}^G = \bar{x}_1^G$, $r^G = L$, $p^G = \theta L + \gamma (\theta - 1)(H - L) - c(N - 2 + \gamma)/\lambda$, and $\pi^G = \pi_1^G$; where

$$\pi_1^G \equiv \frac{(\theta - 1)(H - L)L(1 - \gamma)\gamma \lambda^2 + (\theta - 1)cHN \gamma^2 \lambda + cL\lambda [1 + \theta(1 - \gamma)\gamma N + \gamma^2(N + 1) - 2\gamma] - c^2 \gamma N(N - 2 + \gamma)}{c[1 + \gamma(N - 2 + \gamma)] + (\theta - 1)(H - L)(1 - \gamma)\gamma \lambda}.$$

**Proof of Theorem 2.** The range in which one strategy dominates the others follows directly by comparing the profits. Define the following thresholds for the inventory holding cost $h$:

$$\bar{h}_2 \equiv \frac{2c(N - 1) + 2(\theta - 1)(H \gamma - L)\lambda}{N + 1},$$

$$\bar{h}_3 \equiv \frac{2c(N - 1)}{N + 1},$$

$$\bar{h}_4 \equiv \frac{2\lambda A_1[(\theta - 1)H \gamma + L] - 2A_2}{(N + 1)A_1},$$

$$\bar{h} \equiv \min \{\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_4\},$$

where $A_1 \equiv c[1 + \gamma(N - 2 + \gamma)] + (\theta - 1)(H - L)(1 - \gamma)\gamma \lambda$ and $A_2 \equiv (\theta - 1)(H - L)L(1 - \gamma)\gamma \lambda^2 + (\theta - 1)cHN \gamma^2 \lambda + cL\lambda [1 + \theta(1 - \gamma)\gamma N + \gamma^2(N + 1) - 2\gamma] - c^2 \gamma N(N - 2 + \gamma)$.

Define the following four thresholds for the batch size $N$:

$$\bar{N}_3 \equiv \frac{(\theta - 1)H \gamma}{c} + 1,$$

$$\bar{N}_4 \equiv \frac{(\theta - 1)H[(1 - \gamma)L + \gamma H]\lambda}{c} + 2 - \gamma,$$

$$\bar{N}_5 \equiv \frac{[L + (\theta - 2)H \gamma]\lambda}{c} + 1,$$

$$\bar{N}_6 \equiv \frac{[(\gamma + (1 - \gamma)\theta)L + \gamma(\theta - 2)H]\lambda}{2c} + 1 - \frac{\gamma}{2} + \frac{\sqrt{A_3}}{2c\gamma},$$

where $A_3 \equiv [4(1 - \gamma)(L - H \gamma)\lambda(c(1 - \gamma) + (L - H)(\theta - 1)\gamma \lambda) + \gamma(c(2 - \gamma) + (L + (1 - \gamma)\theta + H \gamma(\theta - 2)\lambda^2)] \gamma$.\n
$\bar{N}_3$ increases in $L$. For a given $L$, $\bar{N}_4$ increases in $H/L$, $\bar{N}_5$ increases in $H/L$ when $\theta > 2$ while decreasing in $H/L$ when $1 < \theta \leq 2$, and $\bar{N}_6$ decreases in $H/L$. Note that $\bar{N}_4 > \bar{N}_3 > \bar{N}_1$ always holds. For simplicity of notation, we further define the following threshold for the batch size $N$:

$$\bar{N} \equiv \begin{cases} \bar{N}_4 & H/L \leq 1/\gamma, \\ \max\{\bar{N}_5, \bar{N}_6\} & H/L > 1/\gamma. \end{cases}$$

When $\theta > 2$, $\bar{N}_5 > 0$ always holds; while when $1 < \theta \leq 2$, $\bar{N}_5 > 0$ if and only if $H/L < \bar{m}$, where $\bar{m} > 1/\gamma$ is a threshold for the valuation heterogeneity $H/L$, defined as the unique solution to the equation $\tilde{N}_2 = \tilde{N}_6$. \qed

**Proof of Theorem 3.** The customer surpluses under the volume, margin, and product-line strategies are: $S^V = S^V_H = (H - L)\gamma \lambda$, $S^V_L = 0$; $S^M = S^M_H = S^M_L = 0$; $S^P = S^P_H = (H - L)\gamma \lambda$, $S^P_L = 0$. As for the group buying, the customer surpluses are
\[ S^G = \sum_{n=1}^{N} S^G_n \cdot \gamma \cdot P(n) + \sum_{n=1}^{\bar{x}_1^G} S^G_n \cdot (1-\gamma) \cdot P(n), \]

\[ S^G_H = \sum_{n=1}^{N} S^G_n \cdot \gamma \cdot P(n), \]

\[ S^G_L = \sum_{n=1}^{\bar{x}_1^G} S^G_n \cdot (1-\gamma) \cdot P(n), \]

where \( S^G_n = \theta H - p^G - c \cdot w^G(n) \) and \( S^G_L = \theta L - p^G - c \cdot w^G(n) \) are the individual surpluses for the high- and low-end customers at state \( n \), respectively. By \( IC_H, IR_{L1} \) and \( IR_{L2} \), we know \( S^G_H(n) \geq H - L \) and \( S^G_L(n) \geq 0 \) hold for any \( n \) \((1 \leq n \leq N)\). The former equation does not always hold for all states, while when \( H/L \geq 1/\gamma \) and \( N < \bar{N}_2 \), \( S^G_L(n) = 0 \) holds for all states. In addition, since \( \sum_{n=1}^{N} P(n) = 1 \), and \( P(n) > 0 \) for any \( n \) \((1 \leq n \leq N)\), we have \( S^G > (H - L)\gamma \lambda, S^G_H > (H - L)\gamma \lambda, \) and \( S^G_L > 0 \), which prove the theorem. \( \square \)

**B. Contingent Pricing**

**Proposition B.1 (REE under Contingent Pricing).** Under contingent pricing, for any given \( \theta > 1 \) and \( N \), there exist two thresholds for the batch size, \( \bar{N}^G_2 \equiv (\theta - 1)(L - H\gamma)\lambda/c \) and \( \bar{N}^G_3 \equiv (\theta - 1)(H\gamma - L)\gamma\lambda/c \), such that

1. when \( H/L \leq 1/\gamma \), the firm sets prices so that
   - (i-1) if \( N \leq \bar{N}^G_2 \), \( \{H + L\} \) is an REE;
   - (i-2) if \( N > \bar{N}^G_2 \), \( \{H; H + L\} \) is an REE;
2. when \( H/L > 1/\gamma \), the firm sets prices so that
   - (ii-1) if \( N \leq \bar{N}^G_3 \), \( \{H\} \) is an REE;
   - (ii-2) if \( N > \bar{N}^G_3 \), \( \{H; H + L\} \) is an REE.

**Proposition B.2 (Profitability of Contingent Pricing).** Compared with uniform pricing, contingent pricing always increases the firm’s profit and enhances the firm’s incentive to offer group buying.

**Theorem B.1 (Profit Comparison under Contingent Pricing).** Suppose \( \theta > 1 \).

1. If \( H/L \leq 1/\gamma \), as \( N \) increases, the firm’s optimal group-buying strategy changes from \( \{G(H + L), R(\emptyset)\} \to \{G(H; H + L), R(L; \emptyset)\} \to \{NG, R(H + L)\} \).
2. If \( H/L > 1/\gamma \), as \( N \) increases, the firm’s optimal group-buying strategy changes from \( \{G(H), R(L)\} \to \{G(H; H + L), R(L; \emptyset)\} \to \{NG, R(H)\} \).
C. Unobservable Group Buying

**Proposition C.1 (REE in Unobservable Group Buying).** In unobservable group buying, for any given $\theta > 1$ and $N$, the firm sets prices so that

(i) when $H/L \leq 1/\gamma$, $\{H + L\}$ is an REE;

(ii) when $H/L > 1/\gamma$, $\{H\}$ is an REE.

**Theorem C.1 (Profit Comparison in Unobservable Group Buying).** Suppose $\theta > 1$.

(i) If $H/L \leq 1/\gamma$, as $N$ increases, the firm’s optimal group-buying strategy changes from $\{G(H + L), R(\emptyset)\} \rightarrow \{NG, R(H + L)\}$.

(ii) If $H/L > 1/\gamma$, as $N$ increases, the firm’s optimal group-buying strategy changes from $\{G(H), R(L)\} \rightarrow \{NG, R(H)\}$.

**Proposition C.2 (Profitability of Unobservable Group Buying).** When $\theta$ is sufficiently large, compared with observable group buying, unobservable group buying increases the firm’s profit and its incentive to offer group buying.

D. Heterogeneous Waiting Costs

**Proposition D.1 (Customer Segmentation with Heterogeneous Waiting Costs).** Suppose $\theta > 1$. If customers have heterogeneous waiting costs, there exists a threshold for the waiting cost, $\bar{c}_H$, such that

(i) when $c_H \leq \bar{c}_H$, with group buying, customer segmentation in equilibrium must be one of the following three scenarios: $\{H\}$, $\{H; H + L\}$, or $\{H + L\}$;

(ii) when $c_H > \bar{c}_H$, with group buying, customer segmentation in equilibrium must be one of the following three scenarios: $\{L\}$, $\{L; L + H\}$, or $\{L + H\}$.

**Proposition D.2 (REE with Heterogeneous Waiting Costs).** If customers have heterogeneous waiting costs, for any given $\theta > 1$ and $N$, there exists a threshold for the waiting cost, $\bar{c}_H$, and three thresholds for the batch size, $\bar{N}_1^D \equiv (\theta - 1)(1 - \gamma)L\lambda/c_L + 1 - \gamma$, $\bar{N}_2^D \equiv (\theta - 1)(H - L)\gamma\lambda/c_H + 1 - \gamma c_L/c_H$, and $\bar{N}_3^D \equiv (\theta - 1)H\gamma\lambda/c_H + \gamma$, such that

(i) when $c_H \leq \bar{c}_H$, the firm sets prices so that

(i-i) when $H \leq [\gamma c_L + (1 - \gamma)c_H]L/([\gamma c_L] - (c_H - c_L))/[(\theta - 1)\lambda$],

(i-i-1) if $N \leq \bar{N}_1^D$, $\{H + L\}$ is an REE;

(i-i-2) if $N > \bar{N}_1^D$, $\{H; H + L\}$ is an REE;

(i-ii) when $H/L > [\gamma c_L + (1 - \gamma)c_H]L/([\gamma c_L] - (c_H - c_L))/[(\theta - 1)\lambda$],

(i-iii-1) if $N \leq \bar{N}_2^D$, $\{H\}$ is an REE;
(i-ii-2) if $N > \bar{N}_2^D$, \{H; H + L\} is an REE;

(ii) when $c_H > \bar{c}_H$, the firm sets prices so that

(iii) if $N \leq \bar{N}_3^D$, \{L + H\} is an REE;

(iv) if $N > \bar{N}_3^D$, \{L; L + H\} is an REE.

**Corollary D.1 (Effect of Heterogeneous Waiting Costs on Customer Segmentation).**

If customers have heterogeneous waiting costs, compared with the base model,

(i) when $c_H \leq c_L$, the attractiveness of \{H; H + L\} to the firm compared with \{H\} decreases;

(ii) when $c_L < c_H \leq \bar{c}_H$, the attractiveness of \{H; H + L\} to the firm compared with \{H\} increases.

**E. Inferior Group-Buying Product**

**Proposition E.1 (Customer Segmentation with Group Buying).** Suppose $\theta < 1$. With group buying, the customer segmentation must be one of the following three scenarios: \{L\}, \{L; L + H\}, or \{L + H\}.

**Proposition E.2 (REE with Inferior Group-Buying Product).** With inferior group buying, for any given $\theta < 1$ and $N$, there exists a threshold for the batch size $\bar{N}_1^1$, and the firm sets prices so that

(i) if $N \leq \bar{N}_1^1$, \{L\} is an REE;

(ii) if $N > \bar{N}_1^1$, \{L; L + H\} is an REE.

**Theorem E.1 (Profit Comparison with Inferior Group-Buying Product).**

Compared with the volume and margin strategies, it is profitable for the firm to offer group buying with an inferior product (i.e., $\theta < 1$) if and only if $N$ is in an intermediate range, which is not empty if $\gamma$ is sufficiently high.

**F. Horizontally Differentiated Products**

**Proposition F.1 (REE with Horizontally Differentiated Products).** In the context of horizontally differentiated products, there exists a threshold for the batch size $\bar{N}_1^H \equiv (H - L)\gamma \lambda / c + 1 - \gamma$, and the firm sets prices so that

(i) if $N \leq \bar{N}_1^H$, \{G\} is an REE;

(ii) if $N > \bar{N}_1^H$, \{G; G + R\} is an REE.

**Theorem F.1 (Profit Comparison with Horizontally Differentiated Products).**

In the context of horizontally differentiated products, there exists a threshold for the inventory holding cost $\bar{h}^H$, above which it is optimal for the firm to offer the product line via flexible-duration group buying rather than doing so noncontingently, and below which the opposite is true.