A. Proofs

This online appendix provides all proofs of propositions, and lemmas in our paper. Since it is a trivial case for our analysis that $s(M) = E_s(M)$ in probability 1, in this appendix we always assume $\text{Var}(s(M)) > 0$.

Proof of Lemma 1. Let $H(\alpha) = \bar{G}(p - s(\alpha m))$ for any $m \geq 0$. We have

$$H'(\alpha) = g(p - s(\alpha m))s'(\alpha m)m.$$ (A.1)

By Assumption 1, it is easy to see that $H(\alpha)$ is nondecreasing and (weakly) convex in $\alpha$ if $p > v^0 + s(m)$ and $m \leq d^0$.

Since $p > v^0 + s(m)$, it follows that $H(1) = \bar{G}(p - s(m)) < \bar{G}(v^0) \leq 1$. Also we have $H(0) \geq 0$. Therefore, $H(\alpha)$ can only have one fixed point on $[0, 1)$ in this case.

Proof of Lemma 2. We prove the properties in part (i) and (ii) only for $\alpha_F(\cdot)$, since the properties of $d_F(\cdot)$ are direct sequences of the properties of $\alpha_F(\cdot)$.

By Equation (1), we have

$$\frac{\partial \alpha_F(m)}{\partial m} = \frac{g(p - s(\alpha_F(m)m))s'(\alpha_F(m)m)\alpha_F(m)}{1 - g(p - s(\alpha_F(m)m))s'(\alpha_F(m)m)m}.$$ (A.2)

Note that $g(p - s(\alpha_F(m)m))s'(\alpha_F(m)m)m = H'(\alpha_F(m))$ where $H'(\cdot)$ is defined by Equation (A.1). If $p > v^0 + s(m)$ and $m \leq d^0$, by the convexity of $H(\cdot)$, we have

$$H'(\alpha_F(m)) \leq \frac{H(1) - H(\alpha_F(m))}{1 - \alpha_F(m)} \leq \frac{1 - \alpha_F(m)}{1 - \alpha_F(m)} = 1;$$

hence, the right-hand side of (A.2) is well defined and nonnegative. Then from (A.2) and Assumption 1, we can see that $\alpha_F(m)$ is (weakly) increasing and (weakly) convex in $m$ as long as $p > v^0 + s(m)$ and $m \leq d^0$; if we also have $G(p) > 0$ and $s'(\cdot) > 0$, then $\frac{\alpha_F(m)}{m} > 0$ and strictly increasing, that is, “increasing” and “convex” properties here are in a strict sense.

For part (ii), we have $H(1) = \bar{G}(p - s(m)) = 1$ if $p \leq v + s(m)$. Thus the largest fixed point of $H(\alpha) = \alpha$, i.e., $\alpha_F(m)$, is 1. Therefore, the proof is completed.

Proof of Lemma 3. The proof is analogous to the proof of Lemma 1.

Before we prove Proposition 1, we give the following lemma:
LEMMA A.1. Let \( X \) be a random variable and \( f(\cdot) \) and \( g(\cdot) \) be increasing functions. Suppose \( f(X), g(X) \) and \( f(X)g(X) \) all have finite expectations. Then \( \mathbb{E}[f(X)g(X)] \geq f(X) \cdot \mathbb{E}g(X) \).

Proof of Lemma A.1. Define \( x_0 = \inf_x \{ g(x) \geq \mathbb{E}g(X) \} \). We have

\[
\mathbb{E}[f(X)(g(X) - \mathbb{E}g(X))] = \mathbb{E}[f(X)(g(X) - \mathbb{E}g(X))]; X > x_0] + \mathbb{E}[f(X)(g(X) - \mathbb{E}g(X)); X < x_0] \\
+ \mathbb{E}[f(X)(g(X) - \mathbb{E}g(X)); X = x_0] \\
\geq \mathbb{E}[f(x_0)(g(X) - \mathbb{E}g(X)); X > x_0] + \mathbb{E}[f(x_0)(g(X) - \mathbb{E}g(X)); X < x_0] \\
+ \mathbb{E}[f(x_0)(g(X) - \mathbb{E}g(X)); X = x_0] \\
= f(x_0) \cdot \mathbb{E}[g(X) - \mathbb{E}g(X)] = 0.
\]

Thus the lemma is proved. \( \square \)

Proof of Proposition 1. Part (ii) holds because, if \( p \leq v + \mathbb{E}s(M), \alpha_F(m) \leq \alpha_N = 1 \) for any \( m \geq 0 \), which yields \( R_N \geq R_F \). Next we will prove part (i).

From lemma A.1, we see \( \mathbb{E}d_F(M) \geq \mathbb{E}\alpha_F(M)\mathbb{E}M \). Consequently, to prove \( \mathbb{E}d_F(M) \geq d_N \), it suffices to prove \( \mathbb{E}\alpha_F(M) \geq \alpha_N \).

Given \( p \geq v^0 + s(\overline{m}) \), we have \( G(\cdot) \) is concave on \( [v^0, \infty) \). From the definition of \( \alpha_F(M) \), i.e., Equation (1), we have

\[
\mathbb{E}\alpha_F(M) = \mathbb{E}G\left(p - s(\alpha_F(M))\mathbb{E}M\right) \geq \mathbb{E}G\left(p - \mathbb{E}s(\alpha_F(M))\mathbb{E}M\right).
\]

Note that when \( p > v^0 + s(\overline{m}) \), by Lemma 3, \( \alpha_N \) is the unique solution to (2). Thus it suffices to prove

\[
\mathbb{E}\alpha_F(M) \geq \mathbb{E}G\left(p - \mathbb{E}s(M\mathbb{E}\alpha_F(M))\right). \tag{A.3}
\]

In fact, if (A.3) holds, then \( \alpha_N \) must be between 0 and \( \mathbb{E}\alpha_F(M) \). To show (A.3), it suffices to show that

\[
\mathbb{E}s(\alpha_F(M)) \geq \mathbb{E}s(M\mathbb{E}\alpha_F(M)). \tag{A.4}
\]

Now we prove (A.4). For any \( M \), because of the convexity of \( s(\cdot) \), we have

\[
s\left(M\alpha_F(M)\right) \geq s\left(M\mathbb{E}\alpha_F(M)\right) + [\alpha_F(M) - \mathbb{E}\alpha_F(M)] \cdot M \cdot s'(M\alpha_F(M)). \tag{A.5}
\]

By Lemma A.1 and taking expectation of both sides of (A.5) with respect to \( M_1 \), we have (A.4). This completes the proof. \( \square \)
Proof of Lemma 4. We will only prove the properties of $\alpha_F(\cdot)$ in part (i) and (ii), because the properties of $d_F(\cdot)$ follow directly.

From Equation (5) we see that $\alpha_F(p_1, p_2, m) = \tilde{\alpha}_F(p_1, p_2, m)$ if $m \geq m_0$. By comparing Equations (1) and (4), we have that $\tilde{\alpha}_F(p_1, p_2, m)$, a function of $m$, has the same properties as $\alpha_F(m)$, defined as the largest fixed point of (4). Therefore, this lemma follows from Lemma 2.

Proof of Lemma 5. (i) We have known that
\[
H_F(x) - H_N(x) = \delta \left[ -G^{-1}(1 - x) + p_2 - \mathbb{E}\left( \max\{x, \alpha_N(p_2)\} M \right) \right] +
- \delta \mathbb{E} \left[ -G^{-1}(1 - x) + p_2 - s \left( \max\{x, \tilde{\alpha}_F(p_2, M)\} M \right) \right].
\]

For $x \leq \alpha_N(p_2)$, $G^{-1}(1 - x) - p_2 + \mathbb{E}\left( \max\{x, \alpha_N(p_2)\} M \right) \geq G^{-1}(1 - \alpha_N(p_2)) - p_2 + \mathbb{E}(\alpha_N(p_2) M) = 0$. Thus in this case we have $H_F(x) - H_N(x) \leq 0$, which yields $\alpha_{1,F}(p_1, p_2) \leq \alpha_{1,N}(p_1, p_2)$.

We now consider the case when $x > \alpha_N(p_2)$. In this case, we have
\[
\mathbb{E} \left[ -G^{-1}(1 - x) + p_2 - s \left( \max\{x, \tilde{\alpha}_F(p_2, M)\} M \right) \right] +
= \mathbb{E} \left[ -G^{-1}(1 - x) + p_2 - s \left( \max\{x, \tilde{\alpha}_F(p_2, M)\} M \right) \right] \cdot 1_{\{\tilde{\alpha}_F(p_2, M) \leq x\}},
\]
and
\[
\left[ -G^{-1}(1 - x) + p_2 - \mathbb{E}\left( \max\{x, \alpha_N(p_2)\} M \right) \right] +
= \mathbb{E} \left[ -G^{-1}(1 - x) + p_2 - s(x M) \right].
\]

Since
\[
\left[ -G^{-1}(1 - x) + p_2 - s \left( \max\{x, \tilde{\alpha}_F(p_2, M)\} M \right) \right] \cdot 1_{\{\tilde{\alpha}_F(p_2, M) \leq x\}}
\geq -G^{-1}(1 - x) + p_2 - s(x M), \ \forall M,
\]
it follows that $H_F(x) - H_N(x) \leq 0$. Hence we still have $\alpha_{1,F}(p_1, p_2) \leq \alpha_{1,N}(p_1, p_2)$.

(ii) If $p_1 \geq p_2$, we have $\alpha_{1,N}(p_1, p_2) \leq \alpha_N(p_2)$. Thus we have
\[
0 \leq H_N(\alpha_{1,N}) = G^{-1}(1 - \alpha_{1,N}) - p_1 - \delta [G^{-1}(1 - \alpha_{1,N}) - p_2],
\]
which yields $G^{-1}(1 - \alpha_{1,N}) \geq p_1 \geq p_2$. Here, we write $\alpha_{1,N}(p_1, p_2)$ as $\alpha_{1,N}$ for short. Therefore, we conclude that $H_F(\alpha_{1,N}) - H_N(\alpha_{1,N}) = 0$, which yields $\alpha_{1,F}(p_1, p_2) = \alpha_{1,N}(p_1, p_2)$.

Proof of Proposition 2. (i) As alluded to in the proof of Proposition 1, $\mathbb{E}\tilde{d}_F(p_2, M) \geq d_N(p_2)$ if $p_2 \geq v^0 + s(\overline{m})$ and $\overline{m} \leq d^0$. Since $d_F(p_1, p_2, M) \geq \tilde{d}_F(p_2, M)$, we have that $\mathbb{E}d_F(p_1, p_2, M) \geq d_N(p_2)$.
Consider \( p_1 \geq p_2 \). In this case, we have \( d_N(p_1, p_2) = d_N(p_2) \leq Ed_F(p_1, p_2, M) \). By Lemma 5, we also have \( d_1, F(p_1, p_2) = d_1, N(p_1, p_2) \). Therefore, \( R_F(p_1, p_2) \geq R_N(p_1, p_2) \).

Then consider \( p_1 < p_2 \). We also consider \( \delta = 0 \). In this case, we have \( H_F(x) = H_N(x) \) for all \( x \in [0, 1] \), which yields \( d_1, F(p_1, p_2) = d_1, N(p_1, p_2) \). Since we have assumed \( \text{Var}(s(M)) > 0 \) at the beginning of this appendix, either of the following two results holds: (a) \( Ed_2, F(p_1, p_2, M) > d_2, N(p_1, p_2) \), (b) \( d_2, N(p_1, p_2) = d_2, F(p_1, p_2, \overline{m}) = 0 \). Thus we must have either that \( R_F(p_1, p_2) > R_N(p_1, p_2) \) for \( \delta = 0 \) or that \( R_F(p_1, p_2) = R_N(p_1, p_2) \) for all \( \delta \). Therefore, there must be a threshold \( \delta_c > 0 \) such that \( R_F(p_1, p_2) \geq R_N(p_1, p_2) \) when \( \delta \leq \delta_c \).

(ii) If \( p_2 \leq \min\{ p_1, 2 + \varepsilon s(M) \} \), then we have \( d_1, F(p_1, p_2) = d_1, N(p_1, p_2) \) by Lemma 5 and \( Ed_F(p_1, p_2, M) \leq d_N(p_1, p_2) \) by Lemma 4(ii). Therefore, \( R_F(p_1, p_2) \leq R_N(p_1, p_2) \). \( \square \)

**Proof of Lemma 6.** First we prove \( \alpha^*_2, N > 0 \). Suppose \( \alpha^*_2, N = 0 \). Thus \( \alpha^*_1, N = \alpha^*_N \) must be the largest fixed point of the following equation:

\[
\alpha = G(p^*_1, N - \delta E s(\alpha M)),
\]

and hence \( R_N(p^*_1, N, p^*_2, N) = p^*_1, N\alpha^*_N \). If we take \( p_2, N = p^*_1, N \) and take \( p_1, N \) such that \( p_1, N > p^*_2, N \) and \( G(p^*_1, N) > 0 \), then \( R_N(p_1, N, p_2, N) > R_N(p^*_1, N, p^*_2, N) \). That is a contradiction. Hence we have \( \alpha^*_2, N > 0 \).

Consider \( \alpha^*_2, N(p_1, N, p_2, N) > 0 \). Thus \( \alpha_N \) is the largest fixed point of the following equation:

\[
\alpha = G(p_2, N - E s(\alpha M)),
\]

which implies that \( \alpha_N \) depends only on \( p_2, N \). Hence, \( R_N(p_1, N, p_2, N) = p_2, N\alpha_N(p_2, N) + (p_1, N - p_2, N)\alpha_1, N(p_1, N, p_2, N) \). Since \( R_N(p_2, N, p_2, N) > R_N(p_1, N, p_2, N) \) if \( p_1, N < p_2, N \), it follows that \( p^*_1, N \geq p^*_2, N \). \( \square \)

**Proof of Proposition 3.** (i) From Proposition 2(i) and Lemma 6, we see that, to prove \( R^*_F \geq R_F(p^*_1, N, p^*_2, N) \geq R_N \), it suffices to prove \( p^*_2, N \geq v^0 + s(\overline{m}) \).

We first show \( p^*_1, N \geq r \). Suppose \( p^*_1, N < r \), which yields \( p^*_2, N < r \). We have \( R_N(r_1, r_1) \geq r_1 \cdot G(r_1)EM \geq rEM > R_N(p^*_1, N, p^*_2, N) \). That is a contradiction. Hence, \( p^*_1, N \geq r \).

Now we prove \( p^*_2, N \geq v^0 + s(\overline{m}) \). Suppose \( p^*_2, N < v^0 + s(\overline{m}) \). Then we have that \( \alpha_1, N(p^*_1, N, r_2) \geq \alpha_1, N(p^*_1, N, p^*_2, N) \), which yields \( R_1, N(p^*_1, N, r_2) \geq R^*_1, N \), and that \( R_2, N(p^*_1, N, r_2) \geq r_2 \cdot P\{ r_2 \leq V \leq \frac{r_2 - \overline{m}}{1 - \delta} \} \geq (v^0 + s(\overline{m}))EM > R^*_2, N \). That is a contradiction. Therefore, we must have \( p^*_2, N \geq v^0 + s(\overline{m}) \), which completes the proof.
(ii) Now we show that, for any $p_{1,F}$ and $p_{2,F}$, there always exist $p_{1,N}$ and $p_{2,N}$ such that $R_N(p_{1,N},p_{2,N}) \geq R_F(p_{1,F},p_{2,F})$. We first consider the case in which $p_{2,F} \leq \bar{v} + \Es(M)$. Take $p_{2,N} = p_{2,F}$ and $p_{1,N} = p_{1,F}$. Thus $d_N(p_{2,F}) = EM \geq Ed_F(p_{1,F},p_{2,F},M)$. By Lemma 5(i) we have $R_N(p_{1,N},p_{2,N}) \geq R_F(p_{1,F},p_{2,F})$.

Then we consider $p_{2,F} = r(\bar{v} + \Es(M))$ where $r > 1$. Consider also $p_{1,F} > \bar{v} + \Es(M)$. Since $\P\{V \leq \beta EM\} = 1$, it follows that 0 is the unique fixed point of (7). Hence, $\alpha_{1,F}(p_{1,F},p_{2,F}) \leq \alpha_{1,N}(p_{1,F},p_{2,F}) = 0$. The $F$ setting boils down to the $N$ setting, earning 0 profit. Next, we consider $p_{1,F} \leq \bar{v} + \Es(M)$.

If $s(M) < \min\{p_{2,F}^*, v^0, s(d^n)\}$, then we can see from Lemma 1 that $\tilde{\alpha}_F(p_{2,F}^*, M)$ is the unique fixed point of (4), and furthermore we have $\alpha_F(p_{2,F}^*, M) = 0$ because $\P\{V \leq \beta EM\} = 1$. Thus we have

$$R_F(p_{1,F},p_{2,F}) = p_{1,F} \alpha_{1,F} EM + r(\bar{v} + \Es(M))E[\max\{0, \tilde{\alpha}_F(p_{2,F}, M) - \alpha_{1,F}\}M]$$

$$\leq p_{1,F} \alpha_{1,F} EM + r(\bar{v} + \Es(M))(1 - \alpha_{1,F})E[M \cdot 1_{\{s(M) \geq \min\{p_{2,F}^*, v^0, s(d^n)\}\}}]$$

$$\leq (\bar{v} + \Es(M)) \alpha_{1,F} EM + (\bar{v} + \Es(M))(1 - \alpha_{1,F}) EM$$

$$= (\bar{v} + \Es(M)) EM$$

$$= R_N(\bar{v} + \Es(M), \bar{v} + \Es(M)),$$

where the last inequality uses the fact that $E[M \cdot 1_{\{s(M) \geq \min\{p_{2,F}^*, v^0, s(d^n)\}\}}] \leq \frac{EM}{r}$. Therefore, we conclude that $R_N^* \geq R_F^*$.

Before proving Corollary 1, we first introduce some symbols and some lemmas (see below). Let $v_{1,N}(p_1, p_2)$ and $v_{2,N}(p_1, p_2)$ be the lowest valuations among consumers buying respectively in periods 1 and 2 in the $N$ setting. If they are interior points of the support of $V$, then we must have

$$v_{1,N}(p_1, p_2) = \frac{p_1 - \delta p_2}{1 - \delta} = \frac{p_1 - p_2}{1 - \delta} + p_2,$$

$$v_{2,N}(p_1, p_2) = p_2 - \Es(\alpha_N(p_2)M)$$

Under state-independent pricing, the firm’s problem in the $N$ setting is equivalent to

$$\max_{p_{1,p_2}} R_N(p_1, p_2) = p_1 G(v_{1,N}) EM + p_2 \left(G(v_{1,N}) - G(v_{2,N})\right) EM.$$  \hfill (A.6)

Let $v_{1,N}^*$ and $v_{2,N}^*$ be values of $v_{1,N}(\cdot)$ and $v_{2,N}(\cdot)$ at $p_{1,N}^*$, $p_{2,N}^*$. 

**Proof of Corollary 1.** By the FOCs of (A.6), we have

$$G(v_{1,N}^*) = (p_{1,N}^* - p_{2,N}^*) g(v_{1,N}^*) \frac{1}{1 - \delta}.$$
Since $g(\cdot)$ is nonincreasing on $(0, +\infty)$, it follows that $(v_{1,N}^* - v_{2,N}^*)g(v_{1,N}^*) \leq G(v_{1,N}^*) - G(v_{2,N}^*)$. Since $v_{1,N}^* - v_{2,N}^* \geq \frac{p_{1,N}^* - p_{2,N}^*}{1-\delta}$, we have

$$G(v_{1,N}^*) \leq G(v_{1,N}^*) - G(v_{2,N}^*),$$

which yields $\alpha_{1,N}^* \leq \alpha_{2,N}^*$ and $\alpha_{1,N}^* \leq 1/2$.

To prove this corollary, it suffices to prove $p_{2,N}^* \geq v^0 + s(\bar{m})$. Suppose $p_{2,N}^* < v^0 + s(\bar{m})$. Then we must have $\frac{R_N^*}{EM} < p_{1,N}^*G(v_{1,N}^*) + (v^0 + s(\bar{m}))/G(v_{1,N}^*)$. Take $p_{2,N} = r(v^0 + s(\bar{m}))$ where $r > 1$ and take $p_{1,N}$ such that $p_{1,N} - p_{1,N}^* = \delta(r(v^0 + s(\bar{m})))$, which implies $v_{1,N}(p_{1,N}, p_{2,N}) = v_{1,N}^*$ if $v_{2,N} \leq v_{1,N}^*$. Next we prove $R_N(p_{1,N}, p_{2,N}) > R_N^*$ for some $r > 1$.

If $v_{2,N} \leq v_{1,N}^*$, then we have

$$\frac{R_N(p_{1,N}, p_{2,N}) - R_N^*}{EM} > r(v^0 + s(\bar{m}))[G(v_{1,N}^*) - G(v_{2,N})] + p_{1,N}^*G(v_{1,N}^*) - p_{2,N}^*G(v_{1,N}^*) - p_{1,N}G(v_{1,N}^*)$$

$$= r(v^0 + s(\bar{m}))[G(v_{1,N}^*) - G(v_{2,N})] - p_{2,N}^*G(v_{1,N}^*) + (p_{1,N}^* - p_{1,N})G(v_{1,N})$$

$$\geq r(v^0 + s(\bar{m}))[(r - 1)G(v_{1,N}^*) - rG(v_{2,N})]$$

$$\geq (v^0 + s(\bar{m}))[\frac{r - 1}{2} - rG(v_{2,N})].$$

where the last inequality uses the fact that $\alpha_{1,N}^* \leq 1/2$. Since $v_{2,N} \leq p_{2,N} = r(v^0 + s(\bar{m}))$, we have

$$R_N(p_{1,N}, p_{2,N}) > R_N^* \Leftrightarrow G(v_{2,N}) \leq \min\left\{\frac{(r - 1)}{2r}, G(v_{1,N}^*)\right\}$$

$$\Leftrightarrow G(r(v^0 + s(\bar{m}))) \leq \frac{(r - 1)}{2r}$$

$$\Leftrightarrow P\{V \geq r(v^0 + s(\bar{m}))\} \geq \frac{r + 1}{2r}.$$ 

Therefore, if $P\{V \geq r(v^0 + s(\bar{m}))\} \geq \frac{r + 1}{2r}$ for some $r > 1$, then $R_F^* \geq R_N^*$. \hfill \square

**Proof of Proposition 4.** Let $p_{1,N}^*$ and $p_{2,N}^*$ be the optimal prices in the $N$ setting. And let $\alpha_{1,N}^*$ and $\alpha_{2,N}^*$ be the corresponding adoption fractions. Let $\alpha_N^* = \alpha_{1,N}^* + \alpha_{2,N}^*$. We have $p_{1,N}^* \geq p_{2,N}^*$. By the definition of $\alpha_{2,N}(\cdot)$, we have

$$p_{2,N}^* = G^{-1}(1 - \alpha_N^*) + \beta\alpha_N^* EM.$$ 

where $G^{-1}(x) = \inf_{y \geq 0} \{y : G(y) \geq x\}$. We will prove that there exist $p_{1,F}$ and $\{p_{2,F}(M)\}$ such that $R_F(p_{1,F}, p_{2,F}) \geq R_N^*$. Note that this proposition will hold if we prove this result.

Take $p_{1,F} = p_{1,N}^*$ and $p_{2,F}(M) = G^{-1}(1 - \alpha_N^*) + s(\alpha_N^* M)$. Thus at least $\alpha_N^*$ fraction of customers will buy the good in the $F$ setting for any $M > 0$, since $p_{2,F}(M)$ and $\alpha_N^*$ satisfy the REE condition.
(9), implying $\tilde{\alpha}_F(p_{2,F}(M),M) \geq \alpha^*_N$ and furthermore $\alpha_F(p_{1,F},p_{2,F},M) \geq \alpha^*_N$ for any $M > 0$ by Equation (10). Hence, $d_F(p_{1,F},p_{2,F},M) \geq d^*_N$ for any $M > 0$.

For any $M$, we have

$$G^{-1}(1 - \alpha^*_N) - p_{2,F}(M) + s\left(\max\{\alpha^*_N, \tilde{\alpha}(p_{2,F}(M),M)\}\right)M$$

$$= G^{-1}(1 - \alpha^*_N) - p_{2,F}(M) + s(\alpha^*_N M)$$

$$\geq G^{-1}(1 - \alpha^*_N) - p_{2,F}(M) + s(\alpha^*_N M)$$

$$= 0.$$

Since $E_{p_{2,F}}(M) = p^*_N$, we can see from Equation (11) that $\alpha_{1,F}(p_{1,F},p_{2,F}) = \alpha^*_1$, which yields $d_{1,F}(p_{1,F},p_{2,F}) \geq d^*_1$. We also have

$$R_F(p_{1,F},p_{2,F}) = (p_{1,F} - E_{p_{2,F}}(M))d_{1,F}(p_{1,F},p_{2,F}) + E[p_{2,F}(M)d_F(p_{1,F},p_{2,F},M)]$$

$$\geq (p_{1,F} - p^*_2)d^*_1 + p^*_2d^*_N$$

$$= R^*_N,$$

which completes the proof. \hfill \Box

**Proof of Lemma 7.** As in the proof of Proposition 4, we take $p_{2,F}(M) = G^{-1}(1 - \alpha^*_N) + s(\alpha^*_N M)$ where $\alpha^*_N \geq \alpha_1$. Thus we have

$$R^*_N(\alpha_1, M) = \max_{p_2} p_2 \alpha_2_F(\alpha_1, p_2, M) M \geq p_{2,F}(M)(\alpha^*_N - \alpha_1) M.$$

It follows that $E R^*_N(\alpha_1, M) \geq E[p_{2,F}(M)(\alpha^*_N - \alpha_1) M] \geq p^*_2(\alpha^*_N - \alpha_1) E M = R^*_N(\alpha_1)$, where the last inequality holds by Lemma A.1. \hfill \Box

The proof of Proposition 5 makes use of the following lemma.

**Lemma A.2.** Suppose $\sup \left\{ \frac{\bar{g}(x)}{\bar{g}(x)} : g(x) > 0 \right\}$ is finite. Then $R^*_N$ is continuous in $\delta \in [0,1)$; $\alpha^*_N$ and $R^*_N$ are left-continuous in $\delta \in (0,1)$; furthermore, $\alpha^*_N \to 0$ and $R^*_N \to 0$, as $\delta \to 1^-$. \hfill \Box

**Proof of Lemma A.2.** Let $y_2(\alpha_1, p_{2,N}) = p_{2,N} [\alpha_N(p_{2,N}) - \alpha_1]$. We note that the second-period problem in the $N$ setting is equivalent to

$$\max_{p_{2,N}} \left\{ \max_{p_{2,N}} y_2(\alpha_1, p_{2,N}), E s(M)(1 - \alpha_1) \right\}.$$
We can see that $y_2^*$ is continuous in $\alpha_{1,N}$. By the submodularity of $y_2$ with respect to $\alpha_1$ and $p_{2,N}$, we have $p_{2,N}^*(\alpha_{1,N})$ decreasing in $\alpha_1$. We can also have $p_{2,N}^*$ right-contiguous in $\alpha_{1,N}$. Let

$$\hat{y}_2^*(\alpha_{1,N}) = p_{2,N}^*(\alpha_{1,N})\alpha_{\gamma}(\alpha_{1,N}) = y_2^*(\alpha_{1,N}) + p_{2,N}^*(\alpha_{1,N})\alpha_{1,N}.$$ 

It can be seen that $\hat{y}_2^*$ is right-continuous in $\alpha_{1,N}$. By the Envelope theorem, we have

$$\frac{\partial \hat{y}_2^*}{\partial \alpha_{1,N}} = \frac{\partial y_2^*(\alpha_{1,N})}{\alpha_{1,N}} + p_{2,N}^*(\alpha_{1,N}) + \frac{\partial p_{2,N}^*(\alpha_{1,N})}{\partial \alpha_{1,N}}\alpha_{1,N} = \frac{\partial p_{2,N}^*(\alpha_{1,N})}{\partial \alpha_{1,N}}\alpha_{1,N} \leq 0.$$ 

Hence $\hat{y}_2^*$ is decreasing in $\alpha_{1,N}$.

Let $y_1(\delta, v_{1,N}) = \frac{R_N(\delta, v_{1,N})}{EM}$. Since $v_{1,N} - p_{2,N} = \frac{p_{1,N} - p_{2,N}}{1 - \delta}$ for $\delta < 1$, it follows that

$$y_1(\delta, v_{1,N}) = [p_{1,N} - p_{2,N}(v_{1,N})]G(v_{1,N}) + \hat{y}_2(v_{1,N}) = (1 - \delta)(v_{1,N} - p_{2,N}(v_{1,N}))\hat{G}(v_{1,N}) + \hat{y}_2(v_{1,N}).$$

We note that $\max_v vG(v)$ is bounded if $\sup \left\{ \frac{G(x)}{g(x)} : g(x) > 0 \right\}$ is finite. Thus, $(v_{1,N}^* - p_{2,N}(v_{1,N}^*))\hat{G}(v_{1,N})^* + \hat{y}_2^*(v_{1,N})$ must be bounded for all $\delta < 1$. It follows that $(1 - \delta)(v_{1,N}^* - p_{2,N}(v_{1,N}^*))\hat{G}(v_{1,N})^*$ is continuous in $\delta \in (0, 1)$. Since

$$\left| \max_v y_1(\delta_1, v) - \max_v y_1(\delta_2, v) \right| \leq \max_v \left| y_1(\delta_1, v) - y_1(\delta_2, v) \right|,$$

we can deduce that $y_1^*(\delta) = y_1(\delta, v_{1,N}^*)$ is continuous in $\delta \in [0, 1)$, i.e., $R_N^*(\delta)$ is continuous in $\delta \in [0, 1)$. Hence $\hat{y}_2^*(v_{1,N}^*)$ is continuous in $\delta \in [0, 1)$. Since $\hat{y}_2^*(v_{1,N}^*)$ is increasing and left-continuous in $v_{1,N}$, it follows that $v_{1,N}^*$ is left-continuous in $\delta \in (0, 1)$, implying $\alpha_{1,N}^*$ is left-continuous in $\delta \in (0, 1)$. We have shown $y_2^*(v_{1,N})$ is continuous in $v_{1,N}$. It follows that $y_2^*(v_{1,N}^*)$ is left-continuous in $\delta \in (0, 1)$, i.e., $R_{2,N}^*(\delta)$ is left-continuous in $\delta \in (0, 1)$, which yields that $R_{1,N}^*(\delta)$ is left-continuous in $\delta \in (0, 1)$.

Consider $\delta \to 1^-$. By the monotonicity of $\hat{y}_2^*(v_{1,N})$, we must have $v_{1,N}^* \to \infty$ (or the supremum of the support of $g(\cdot)$), implying $\alpha_{1,N}^* \to 0$. We note that $p_{2,N}^*$ is bounded for all $\delta$ because $\max_v v\hat{G}(v)$ is bounded. Hence $y_1(1, v_{1,N}^*) = \hat{y}_2^*(v_{1,N}^*) = p_{2,N}^*(\alpha_{1,N}^* + \alpha_{2,N}^*) \to p_{2,N}^*\alpha_{2,N}^*$, therefore it follows that $R_{1,N}^* \to 0$. \hfill \qed

**Proof of Proposition 5.** It can be seen from Lemma A.2 that, for any sufficiently small $\epsilon > 0$, there exists $\delta_c < 1$ such that $\alpha_{1,N}^*(\delta) < \epsilon$ and $R_{1,N}^*(\delta) < \epsilon$ if $1 > \delta > \delta_c$. For any $\delta > \delta_c$ ($\delta < 1$), take $p_{1,F}(\delta)$ such that $\alpha_{1,F}^*(\delta) = \alpha_{1,N}^*(\delta)$ ($> 0$). In this case, the total profits in the $F$ and $N$ setting mainly come from the second-period profits. More precisely, we have

$$R_F(\delta, p_{1,F}(\delta)) - R_N^*(\delta) \geq -\epsilon + ER_{2,F}(\alpha_{1,N}^*, M) - R_{2,N}^*(\alpha_{1,N}^*)$$
As alluded to in the proof of Lemma 7, $R_{2,F}(\alpha_1, M)$ is strictly convex in $M$, which yields that $E R_{2,F}(\alpha_1, M) - R_{2,N}(\alpha_1) > 0$ for all $\alpha_1$ as long as $\text{Var}(M) > 0$. Since $E R_{2,F}(\alpha_1, M) - R_{2,N}(\alpha_1) > 0$, it follows that there exists $\epsilon > 0$ such that $\epsilon < E R_{2,F}(\alpha_1, M) - R_{2,N}(\alpha_1)$ for all $0 < \alpha_1 < \epsilon$. Therefore, there must be a $\delta_c < 1$ such that $R_F(\delta, p_{1,F}(\delta)) - R_N^*(\delta) \geq 0$ if $1 > \delta > \delta_c$. \qed