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Newsvendor Selling to Loss-Averse Consumers with Stochastic Reference Points

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We study a newsvendor who sells a perishable asset over repeated periods to consumers with a given consumption valuation for the product. The market size in each period is random, following a stationary distribution. Consumers are loss averse with *stochastic* reference points that represent their beliefs about possible *price* and *product availability*. Given the distribution of reference points, they choose purchase plans to maximize their expected total utility, including gain-loss utility, before visiting the store, and follow the plans in the store. In anticipation of consumers' purchase plans, in each period, before demand uncertainty resolves, the firm chooses an initial order quantity. After the uncertainty resolves, the firm chooses a contingent price depending on the demand realization, with the option of clearing inventory by charging a sale price, and otherwise, posting a full price. Over repeated periods, the interaction of the firm's operational decisions about ordering and contingent pricing and the consumers' purchase actions results in a distribution of reference points, and, in equilibrium, this distribution is consistent with consumers' beliefs. Under this framework of *endogenized* reference points, we fully characterize the firm's optimal inventory and contingent pricing policies. We identify conditions under which the firm's expected price and profit are increasing in the consumer loss aversion level. We also show that the firm can prefer demand variability over no-demand uncertainty. We obtain a set of insights into how consumers' loss aversion affects the firm's optimal operational policies that are in stark contrast to those obtained in classic newsvendor models. As examples, the optimal full price increases in the initial order quantity; and the optimal full price decreases, while the optimal sales frequency increases, in the procurement cost.

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1. Introduction

Motivation and Research Questions. The U.S. grocery industry generates more than $600 billion in yearly sales with approximately 55% of sales derived from perishables (*Progressive Grocer* 2015, McNeill 2011). Consumers frequent supermarkets to visit the perishables department. Daily perishable goods such as bakery items, fruit cups, and sushi are delivered to the store (or, in some cases, produced in the store), processed in-house each day, and sold on that same day. If these products are not expected to be sold out by the end of the day, sales floor managers put discount tags on them. Marked-down items of daily perishable products differ little in quality because the loss of freshness during the day may not be significant, though they cannot be sold the next day. Customers who typically shop later in the day would encounter the same product sometimes at the full price and other times at a sale price. Even for non-daily perishable products, e.g., dairy products, many customers may pick up marked-down items that are close to their "best by" dates without noticing a difference in quality, or at least these marked-down prices may influence their price perception of the products over time. Moreover, out-of-stocks are common for supermarkets. The 2012 Supermarket Experience Survey, recently released by the Retail Feedback Group, states that on their last grocery trip, 12% of shoppers were unable to find at least one item they had planned to purchase that day (Numainville 2012).

As in the supermarket example, firms in many industries, e.g., fashionable apparel, face the problem of selling a fixed initial inventory of perishable assets over repeated sales horizons. (The horizon can be months or as short as a day.) For those firms, pricing is the only control to match supply with demand once they set up their initial inventory levels at the beginning of the sales horizon: If demand is low, firms run sales to boost demand; if demand is high, consumers may experience stockouts. According to an estimate by the management consulting firm A.T. Kearney, a typical apparel retailer sells between 40% and 45% of its inventory at a promotional price (D'Innocenzio 2012). In the microdata collected monthly by the U.S. Bureau of Labor Statistics on goods and services, including three major groups (processed food, unprocessed food, and apparel), the average sale price is about 25% to 30% off the regular price (Klenow and Kryvtsov 2008). Therefore, it is reasonable that repeat consumers develop a perception of the distribution of full and sale prices of perishable products and their availability.

Indeed, experimental studies indicate that consumers often evaluate economic outcomes, e.g., the price that must be paid, relative to a distribution of reference levels (see, e.g., Blinder 1998). Such an effect is particularly significant over repeated purchase interactions, as consumers tend to draw on a stream of past experiences as benchmarks: They form ideas about what the typical prices are, and they judge the value of a product based on the difference between these typical prices and the posted price. Moreover, consumers evaluate changes from possible reference levels differently depending on whether the changes are gains or losses. There is significant empirical evidence that consumers are *loss averse*, i.e., they weigh losses more heavily than equally sized gains (see, e.g., Kahneman et al. 1990). In particular, the evidence of reference price effects for frequently purchased products is strong (see, e.g., Mazumdar et al. 2005).

It is plausible that personal reference points tend to be rationalized within consumers, rather than given exogenously. This view has gained ground in the behavioral economics literature and was formalized by Koszegi and Rabin (2006) with many follow-up works. In particular, the application of the proposed framework leads to two countervailing effects of running sales, when consumers are loss averse with *endogenized* stochastic reference points: the negative *comparison effect* and the positive *attachment effect*. The comparison effect means that higher sales frequency increases the weight of sale prices in the loss-averse consumers' reference distribution, making consumers used to the sale price and less likely to purchase at the full price. The attachment effect, on the contrary, means that higher sales frequency increases consumers' psychological attachment to the habit of purchasing. Hence, to avoid the pain of not obtaining the product when there are no sales, consumers are willing to pay the full price.

This paper is motivated by the prevalent markdown practice over repeated sales horizons in the grocery and apparel industries, and by the literature on endogenized reference points. The paper investigates how, in the presence of endogenized reference points, the newsvendor should adjust its operational decisions, such as order quantity and contingent promotion decisions.

The Model. We consider a profit-maximizing newsvendor who sells a perishable product on a repeated basis and seeks to maximize its expected long-run average profit. Consumers have a given consumption valuation, and their market size is random for each sales season, following a stationary distribution. Moreover, consumers are loss averse with reference points that represent their *probabilistic beliefs* about the product's price and availability outcomes learned over time. Given these reference points, they develop *purchase plans* to maximize their total expected utility including gain-loss utility because of their loss-averse behavior. In anticipation of consumers' purchase plans, in each sales season, before demand uncertainty resolves, the firm chooses an initial order quantity; after the uncertainty resolves, the firm sets full or sale prices contingent on demand realization. The firm aims to sell the product at the full price; however, because the product is perishable, the firm may choose to run sales to avoid unsold, but already sunk, inventory, and also potentially to manipulate consumers' purchase plans. Over repeated seasons, the firm's operational decisions about ordering and contingent pricing result in a distribution of reference points. In equilibrium, this distribution should be consistent with consumers' probabilistic beliefs. In this sense, the consumers' reference points in our model are *endogenously* induced and influenced by their own purchase plans rather than provided as exogenous values.

Contributions. This paper makes three main contributions: First, we are the first to explicitly take into account consumers' loss aversion with *stochastic* reference points in relation to firms' operational decisions, such as the order quantity and contingent pricing policy. Second, we demonstrate that contingent pricing policies allow the firm not only to efficiently match supply with demand but also to profitably manipulate consumers' stochastic reference points by varying the frequency of sales. In line with this finding, we show that the firm can prefer demand variability over no-demand uncertainty, under the optimal contingent pricing strategy. This observation is in stark contrast to the results in a classic newsvendor setting. The reason behind this observation is that sales driven by an appropriate level of demand uncertainty can entice loss-averse consumers to form a purchase plan of buying up to a *higher* full price. Third, we show how consumers' loss aversion affects the firm's optimal operations decisions with insights significantly different from those in classic newsvendor settings (see, e.g., Petruzzi and Dada 1999). Based on our results, we caution that it is critical to take into account consumers' loss-averse behavior when designing markdown pricing algorithms; markdown algorithms that ignore consumer loss aversion can lead to substantially suboptimal solutions.

2. Related Literature

There are two bodies of literature that are closely related to our research. The first is the behavioral economics literature on consumers' loss aversion. The theory of loss aversion was first proposed by Kahneman and Tversky (1979). Recent development, such as Heidhues and Koszegi (2005, 2008, 2014) and Koszegi and Rabin (2006), considers stochastic reference points and the impact of consumer loss-averse behavior on firms' pricing strategies. Our research is distinct from these works in several aspects. These papers assume that market demand is deterministic, whereas we consider demand uncertainty. These works ignore inventory decisions and therefore assume that the product is always available. In contrast, our model endogenizes consumer reference points to depend on the *product's availability*, which is a key feature that leads to many novel insights (see Propositions 3, 4, and 9). Furthermore, compared to Heidhues and Koszegi (2014), stochastic reference points in our model are driven by the contingent pricing policy that is also used to combat demand uncertainty. Those operational decisions should not be tailored solely to manipulate consumer behavior but also to improve the firm's operational performance.

The second closely related stream is the pricing and revenue management literature on consumer reference-price effects (see Arslan and Kachani 2011 and §3.1.2 of Özer and Zheng 2012 for surveys). Recently, Popescu and Wu (2007) study a discrete-time infinite-horizon monopolistic pricing problem under a general nonlinear reference-dependent demand model. They show that the optimal price trajectory is either increasing or decreasing, and that using the optimal fixed price is close to optimal. Nasiry and Popescu (2011) study a version with the reference point as a weighted average of the lowest and most recent prices. Zhao and Stecke (2010) study a newsvendor who can advance-sell to loss-averse consumers, and the authors solve for the firm's optimal advance-selling strategy. Chen et al. (2014) study a periodic-review stochastic inventory model with reference price effects and show that the optimal inventory policy is a reference-price-dependent base-stock policy. All of these papers consider a *single* reference point for decision makers at the time of decision making. Whereas the assumption of a single reference point may be a reasonably good approximation of reality, as information becomes more abundant and accessible (e.g., because of websites such as Decide.com that provide historic price information), consumers' reference dependence in their decision making becomes more complicated. In contrast to these papers, we thus consider *probabilistic beliefs* as consumers' reference points in determining their purchase decisions. Consistent with Roels and Su (2014), who assume that the reference points can be engineered, we echo that the distribution of reference prices can be manipulated by operational policies. In our context, the reference points for repeated purchasers are driven by demand uncertainty and firms' inventory and contingent pricing policies.

A consumer behavior that is related to reference-price effects is regret and disappointment, incurred because of a mismatch between a reference point and a realization. Nasiry and Popescu (2012) characterize the effect of anticipated regret on consumer decisions and on firms' profits and policies in an advance selling context where buyers have uncertain valuations. Liu and Shum (2013) study a firm's optimal pricing and rationing decisions over two periods in anticipation of possible consumer disappointment caused by stockouts, and they show that the firm may benefit from such disappointment (see comparison of insights in §4.2). Özer and Zheng (2015) study a seller's optimal pricing and inventory strategies when anticipated regret and misperception of product availability affect consumers' purchase decisions. Whereas these papers assume a single reference point that leads to regret and disappointment, we focus on stochastic reference points. Our message is consistent with Özer and Zheng (2015) in the sense that both papers advocate for *nonstatic* pricing when consumers are loss averse (in their words, have regretful emotions). In Özer and Zheng (2015) there is no demand uncertainty, and the authors show the superiority of markdown pricing over everyday-low-price for a two-period formulation. In our setting of repeated sales horizons, there is demand uncertainty, and we show the advantage of contingent pricing over a deterministic price.

Two published papers consider stochastic reference points in the operations setting. Ho et al. (2010) consider managers' stochastic reference dependence and loss aversion when studying the ordering behavior in a multilocation inventory system. In a competitive newsvendor setting, Avcı et al. (2014) study managers' loss aversion and status-seeking behavior in making newsvendor ordering decisions, with stochastic reference points of the possible competitors' profit outcomes. Both papers assume that managers, as decision makers, are loss averse with stochastic reference points, whereas we focus on the loss-averse behavior of consumers and, more distinctively, how the firm should react to such behavior.

Several papers consider strategically forward-looking, but loss-neutral and nonemotional, consumers' behavior and its impact on firms' optimal inventory and pricing decisions. Liu and van Ryzin (2008) find that the firm can optimally set the rationing level in the markdown period to induce high-value consumers to buy in the early period. Gallego et al. (2008) study the inventory level that should be assigned to sales when strategic consumers adjust their expectations of sales from the firm's past actions over repeated seasons. These papers focus on capacity rationing decisions, whereas we consider optimal inventory and contingent pricing decisions. More importantly, while the profit gain for the firm in those contexts comes from better market segmentation, the profit gain in our model comes from inducing consumers to buy at a higher full price by manipulating their reference points.

We end this section with a brief comparison between our loss aversion model and those commonly applied in the operations management literature. There are two main differences. First, consistent with the recent works in behavioral economics, e.g., Koszegi and Rabin (2006), we assume that loss-averse consumers assess gains and losses in *two* dimensions, the *product* and *money*, separately. Second, we consider that by repeatedly visiting the store, consumers take into account the product availability and price outcomes and their frequencies. Hence, the consumers' reference level is not necessarily a single point. Instead, it is more likely a distribution of multiple points with respective probabilities. Two recent working papers that also apply the framework of Koszegi and Rabin (2006) to an operations context are Yang et al. (2014) on delay in a queueing system and Courty and Nasiry (2015) on quality-dependent consumer valuations.

3. The Model

We consider a single risk-neutral profit-maximizing firm selling a single perishable product over a short horizon on a repeated basis, e.g., a grocery store that sells fruit cups, which faces a sales horizon measured in days. The firm orders q units of the product at cost c per unit and sells to consumers who request a single unit of the product.

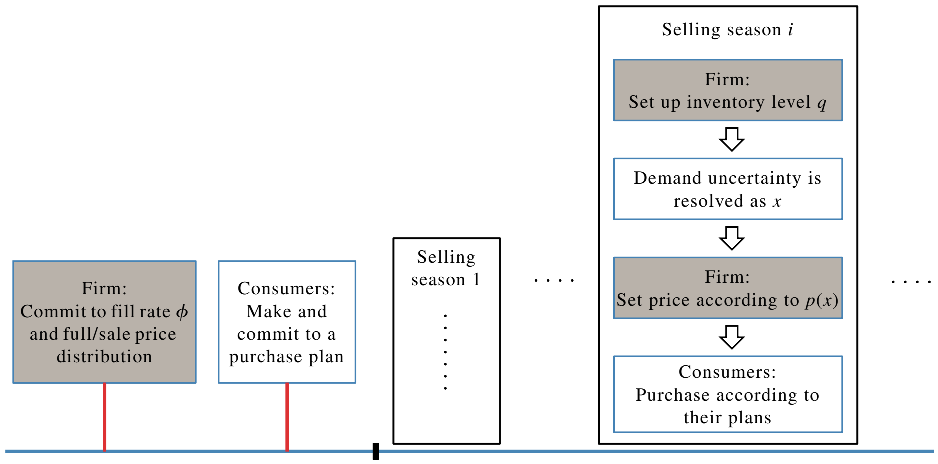
Consumer. There is a random number, D, of consumers, with a cumulative distribution function (cdf) $F(\cdot)$ and an expected value $\mathbb{E}(D) < \infty$. The consumers have a known consumption valuation $v(>c)$ of the product. Consumers are *loss averse* in a sense that will be discussed in further detail.

Sale Price. We assume that there is a sale price $s(<c)$ at which the firm can clear all on-hand inventory by selling to bargain hunters with valuation of the product at s. We fix the sale price as exogenously given and allow the full price to be optimized. The same qualitative insights can be obtained if one endogenizes both. The assumption $s < c$ is consistent with the empirical evidence of data from the U.S. Bureau of Labor Statistics: The sale price typically represents a sizable markdown from the marginal cost (see Shi 2012). This assumption is also commonly used in the operations literature; see, e.g., Cachon and Swinney (2009). Consistent with the U.S. Bureau of Labor Statistics, which defines a sale as a price cut available to *all* buyers, we assume that the firm *cannot* price-discriminate among consumers and bargain hunters when running sales. The one-price-for-all assumption may be stylized, but it simplifies analysis; moreover, it provides a lower bound on the firm's profitability when price discrimination is allowed. Because the valuation of bargain hunters is below the procurement cost, the firm primarily targets consumers for profitability. When the sale price is charged, we assume that the firm can prioritize selling to consumers. The store can screen regular consumers by requiring loyalty cards to enjoy discount, before making the same discount available to bargain hunters.

Inventory Availability. An important novel feature of our model is the consideration of product availability in the framework of stochastic reference points. Since the firm orders a limited quantity, a consumer may find the product out of stock when demand exceeds supply. As mentioned, this unavailability was ignored in the loss aversion models considered in the behavioral economics literature (e.g., Koszegi and Rabin 2006). However, it is an important feature in newsvendor problems. We assume that if there are more consumers than available products, then the products are rationed among consumers with equal probability (see, e.g., Su and Zhang 2008 for the same rationing rule). We define the *fill rate* as the long-run probability that the product is successfully procured when consumers are willing to buy it. The fill rate, together with the price distribution, influences loss-averse consumers' purchase decisions.

Contingent Pricing Scheme. For tractability, we restrict our attention to pricing schemes with easily implementable forms. Specifically, we focus on *two-price* contingent pricing schemes, in which only two prices—full and sale price—are charged, conditional on the realization of consumer demand, and in which the sale price is exogenously given as s. This pricing

Figure 1 (Color online) Sequence of Events



strategy implies that the firm occasionally runs sales to clear the market. Although it may seem restrictive to consider a two-price scheme, randomizing among a limited number of discrete prices can be more practical than randomizing in a continuous price interval, as suggested by Heidhues and Koszegi (2014). Moreover, the two-price contingent scheme provides a lower bound on the benefit of contingent pricing over static pricing with a single price. Let $\bar{p}(>s)$ denote the *full price*. In addition, let Ω be the set of demand realizations for which the price is set to s. Then, the contingent pricing scheme is

$$p(x) = \begin{cases} s & \text{if } x \in \Omega, \\ \bar{p} & \text{if } x \in \Omega^c \equiv \mathbb{R}^+ \backslash \Omega. \end{cases} \tag{1}$$

Sequence of Events. We describe the timing of the model, which is also illustrated in Figure 1. (i) Before the selling seasons, the firm decides and commits to the stock quantity q (the stocking decision can be delayed to the beginning of *each* selling season before demand uncertainty is resolved, which does not change the results of the model), the fill rate ϕ, and the contingent pricing policy $p(x)$ as in (1). The firm announces the fill rate ϕ, and the price distribution

$$g(p) = \begin{cases} \int_{x\in\Omega} dF(x) & \text{if } p = s, \\ \int_{x\in\Omega^c} dF(x) & \text{if } p = \bar{p}, \end{cases} \tag{2}$$

induced by the contingent pricing policy. (ii) Consumers commit to a purchase plan. During the selling seasons, the following events occur in sequence: (iii) At the beginning of each season, the firm purchases q units of inventory. (iv) The random demand from consumers realizes as x. (v) The firm sets the price $p = p(x)$ according to its contingent pricing policy. (vi) Consumers observe the price p and make their purchase decisions according to the plans they have mentally committed to. If the product is available and the price charged is one at which the consumers have planned to buy, they will purchase it. If there are more consumers willing to purchase the product than inventory available, units are randomly rationed.

Two comments with respect to the realism of this sequence of events are in order: First, in reality, even *without* the firm's announcement, consumers can infer the fill rate ϕ and the induced price distribution $g(p)$ over repeated interactions. Even if a firm does not announce its induced price distribution, it can develop a stable "reputation" for having committed to a price distribution. Second, we assume that the firm can observe the demand realization at the beginning of each sales horizon before setting the price. The same stylized assumption is made by many recent papers, e.g., Cachon and Feldman (2015). In addition, we note that quite often an accurate forecast of the total can be obtained after observing a relatively small fraction of the total demand (see, e.g., Fisher and Raman 1996). Thus, the assumption that the firm observes the actual demand at the beginning of a period may be less restrictive than it seems.

3.1. Consumer's Problem

A consumer's expected utility is the sum of her expected *consumption utility* and her expected *gain-and-loss utility*. The expected consumption utility results from the expected consumption outcome, which depends on the consumer's purchase decision and the availability of the product. The expected gain-and-loss utility captures the consumer's loss aversion when she compares a realized consumption outcome to other possible outcomes in her reference distribution.

Let the binary variable $b \in \{0, 1\}$ denote the consumer's purchase outcome, where $b = 1$ indicates that the consumer successfully procures the product and $b=0$ indicates otherwise. Note that the outcome $b=0$ occurs either because a consumer chooses not to purchase or because the product is unavailable even if the consumer chooses to purchase. A consumer's utility function has two components: product and money. Denote the consumption outcome by $\mathbf{k}=(k^v, k^p)$, where $k^v=vb$ is the valuation drawn from purchase outcome b, and $k^p=-pb$ is the monetary loss from purchase outcome b. Hence, the combined consumption utility is $C(\mathbf{k})=k^v+k^p=(v-p)b$. In addition, a loss-averse consumer compares her *actual* consumption outcome $\mathbf{k}=(k^v, k^p)$ to a *possible* consumption outcome $\mathbf{r}=(r^v, r^p)$ in her reference point distribution, where r^v is the reference consumption valuation and r^p is the reference out-of-pocket cost. Given the firm's contingent pricing policy (1), there exist three reference points; i.e., $\mathbf{r}\in\{(v,-s),(v,-\bar{p}),(0,0)\}$, if a consumer plans to buy at both the full and sale price. Comparing her actual consumption outcome to a reference point, the consumer obtains a *gain-and-loss utility* along both dimensions of product and money: $W(\mathbf{k}\mid\mathbf{r})=\eta(k^v-r^v)^+ + \eta\lambda(k^v-r^v)^- + \eta(k^p-r^p)^+ + \eta\lambda(k^p-r^p)^-$, where $\eta>0$, $\lambda\geq 1$, $a^+=\max\{a,0\}$ and $a^-=\min\{a,0\}$ for any real number a. Note that $\lambda\geq 1$ implies that the consumer feels losses more strongly than she does equally sized gains. Therefore, the consumer's total utility of a consumption outcome $\mathbf{k}$ conditional on a reference point $\mathbf{r}$ is $u(\mathbf{k}\mid\mathbf{r})=C(\mathbf{k})+W(\mathbf{k}\mid\mathbf{r})$. As a consumer's reference is her *probabilistic beliefs* about the possible outcomes, we use $\boldsymbol{\Gamma}(\cdot)$ to denote the probability distribution over $\mathbf{r}$. We call $\Gamma(\cdot)$ the consumer's *reference distribution* in order to distinguish it from a deterministic reference point. Therefore, the expected utility of a consumption outcome $\mathbf{k}$ conditional on the consumer's reference distribution is

$$U(\mathbf{k}\mid\boldsymbol{\Gamma})=\sum_{\mathbf{r}} u(\mathbf{k}\mid\mathbf{r})\boldsymbol{\Gamma}(\mathbf{r}). \tag{3}$$

Heidhues and Koszegi (2014) show that the consumer's purchase plan follows a *cutoff* structure: The consumer chooses to buy at any price lower than or equal to the cutoff price and not to buy at any higher price. Then, to induce consumers to always make a purchase, the full price $\bar{p}$ must be the cutoff price (we will use them interchangeably) and satisfy

$$U((v,-\bar{p})\mid\boldsymbol{\Gamma})=U((0,0)\mid\boldsymbol{\Gamma}), \tag{4}$$

where the reference distribution $\boldsymbol{\Gamma}$ is endogenously induced by the purchase plan with the cutoff price $\bar{p}$ (see Heidhues and Koszegi 2014, Definition 1).

Equation (4) holds even if product unavailability is taken into account. At any cutoff price $\bar{p}$, the utilities of purchasing and not purchasing, conditional on the same reference distribution $\boldsymbol{\Gamma}$, should be equal. Explicitly, if the fill rate of the product is ϕ when the consumer intends to make a purchase, equating the two utilities gives $\phi\cdot U((v,-\bar{p})\mid\boldsymbol{\Gamma}')+(1-\phi)\cdot U((0,0)\mid\boldsymbol{\Gamma}')=U((0,0)\mid\boldsymbol{\Gamma}')$, where $\boldsymbol{\Gamma}'$ is the reference distribution when unavailability is taken into account. The reduced equation is in the same form as (4).

Since each purchase plan is uniquely determined by the cutoff price $\bar{p}$, with a little abuse of notation, we use $\bar{p}$ to denote the consumer's purchase plan. The purchase plan serves as a *personal equilibrium* (see Heidhues and Koszegi 2014). For consumers, alternative purchase plans, such as not to purchase regardless of price and to purchase only at the sale price, can also serve as personal equilibria. However, they lead to trivial results. Hence, we restrict our focus to the personal equilibrium in which consumers can be induced to buy at both full and sale prices.

Now, we explain how a consumer forms her reference distribution $\boldsymbol{\Gamma}$, based on her plan as well as the information about the fill rate ϕ and price distribution $g(p)$. The consumer expects to find the product available at the sale price s, and the full price $\bar{p}$, with probabilities $\phi g(s)$ and $\phi g(\bar{p})$, respectively. In both cases, the consumer expects to buy. So, the outcome vector is $(v,-s)$ when $p=s$ and $(v,-\bar{p})$ when $p=\bar{p}$. In addition, the consumer expects the product to be unavailable with probability $1-\phi$, in which case her expected outcome vector is $(0,0)$. Thus, the consumer's reference distribution, $\boldsymbol{\Gamma}(\cdot)$, is

$$\boldsymbol{\Gamma}(\mathbf{r}; g(\cdot),\phi,\bar{p})=\begin{cases}\phi g(s) & \text{if } \mathbf{r}=(v,-s),\\ \phi g(\bar{p}) & \text{if } \mathbf{r}=(v,-\bar{p}),\\ 1-\phi & \text{if } \mathbf{r}=(0,0).\end{cases} \tag{5}$$

We can characterize the value of the credible cutoff price $\bar{p}$ by using (3)–(5) as follows.

Lemma 1. *The cutoff price $\bar{p}$ can be written as*

$$\bar{p}=v+\frac{\phi s\int_{\Omega}dF(x)-[1-\phi(1+\int_{\Omega^c}dF(x))]v}{1+\eta[\lambda-\phi(\lambda-1)\int_{\Omega^c}dF(x)]}(\lambda-1)\eta. \tag{6}$$

3.2. Firm's Problem

The firm has the option to deliberately ration the order quantity among consumers (see, e.g., Liu and van Ryzin 2008), though we will show shortly that the firm prefers to serve consumers with its on-hand inventory as much as possible. Let $\xi(p,q,x)$ denote the availability of product to consumers when the full price is p, the order quantity is q, and the demand is realized as x. Given the consumers' purchase plan $\bar{p}$, the firm will optimally set the full price at $\bar{p}$ and will need to further decide the order quantity q, the set of consumer demand realizations Ω that triggers the sales, and product availability $\xi(\bar{p},q,x)$ at each demand realization x, to maximize its expected profit. By our definition, the fill rate can be written as

$$\phi(\bar{p}, q) = \int_0^\infty \xi(\bar{p}, q, x)\, dF(x). \tag{7}$$

Accordingly, the firm's expected profit can be written as

$$\Pi(\bar{p}, q, \Omega, \xi(\bar{p}, q, x)) = sq \int_{x\in\Omega} dF(x) + \bar{p} \int_{x\in\Omega^c} \xi(\bar{p}, q, x) x\, dF(x) - cq. \tag{8}$$

To simplify the analysis and the exposition, we assume that consumers' decision making is influenced by the fill rate only, namely, the *aggregated* product availability, but not by the product availability at each price. We relax this assumption in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/msom.2015.0532). The firm's problem is to solve $\max_{\{\bar{p}, q, \Omega, \xi(\bar{p}, q, x)\}} \Pi(\bar{p}, q, \Omega, \xi(\bar{p}, q, x))$.

4. Market Equilibrium

In the market equilibrium, neither the firm nor consumers have incentives to deviate from their decisions. Specifically, given the firm's decisions on contingent pricing, inventory, and fill rate, the consumers' purchase plan is a personal equilibrium; in turn, given the consumers' purchase plan, the firm's decisions maximize its expected profit.

4.1. Optimal Pricing Policy

This section characterizes the firm's optimal contingent pricing scheme for *any* given order quantity level q. (We study the optimal choice of q in §4.2.) To streamline the analysis, we assume the following:

Assumption (V). $v/s \geq 1 + \eta\lambda/(1+\eta)$.

Assumption (V) requires that consumers' valuation of the product is sufficiently higher than the sale price s in proportion to their degree of loss aversion. For instance, if $\eta = 1$ and $\lambda = 2$ as suggested by experimentally observed values of the loss-aversion levels (see Ho and Zhang 2008), then we require $v \geq \frac{3}{2}s$. Under this assumption, we can avoid the trivial solution that the optimal full price collapses to the sale price. In the rest of the paper, we also ignore the trivial cases that the optimal contingent pricing scheme degenerates to a single price and that consumers' equilibrium purchase plan is to purchase only at the sale price. Hence, the following results are only applicable within the range where the optimal sales frequency is *nonzero*, as is usually observed in practice.

Proposition 1 (Optimal Contingent Pricing Policy). *Given any initial order quantity q, the optimal contingent pricing policy must have the following structure, which induces consumers to purchase at both prices.*

(i) *The pricing scheme is of a threshold form:*

$$p(x) = \begin{cases} s & \text{if } x \leq \tau^*, \\ \bar{p}^* & \text{if } x > \tau^*, \end{cases}$$

where

$$\bar{p}^* = v + \frac{\phi^* sF(\tau^*) - [1 - \phi^*(2 - F(\tau^*))]v}{1 + \eta[\lambda - \phi^*(\lambda - 1)(1 - F(\tau^*))]} (\lambda - 1)\eta > s, \tag{9}$$

$F(\tau^*)$ is the likelihood of running a sale at the sale price s, namely, the optimal sales frequency, and

$$\phi^* = \int_0^\infty \xi(\bar{p}^*, q, x)\, dF(x) = \int_0^\infty \frac{\min(x, q)}{x}\, dF(x) \tag{10}$$

is the optimal fill rate given the order quantity q.

(ii) *A nonzero optimal sales threshold $\tau^* \leq q$ is the solution to*

$$sq - \bar{p}^*\tau + \frac{\partial \bar{p}^*}{\partial F} \int_\tau^\infty \min\{x, q\}\, dF(x) = 0, \tag{11}$$

where $\partial \bar{p}^*/\partial F$ is the derivative of the optimal full price with respect to sales likelihood $F(\tau^*)$.

Proposition 1 provides several insights into the optimal contingent pricing policy. First, the optimal two-price scheme is in the form of a *threshold* structure: The firm will not run sales unless demand is sufficiently low. Second, it is optimal for the firm to sell as much to consumers as its inventory permits. By our assumption of priority rationing rules, this means that the product is always available to consumers when sales take place. The psychological gain from always having the product at the sale price reinforces consumers' positive attachment effect; that is, as consumers are more attached to the idea of obtaining the product, the potential loss of not having the product would be more painful, hence increasing consumers' willingness to pay for the product to prevent this loss. This effect still exists but is weaker when the firm cannot successfully prioritize consumers over bargain hunters. Hence, in the absence of such prioritization, our results provide a bound on the firm's optimal profit. Third, when consumers are loss averse with endogenized stochastic reference points and demand is uncertain, the firm may price the product at a level higher than consumers' consumption valuation v. By contrast, when consumers are not loss averse, the firm can only profitably price the product at lower than or equal to the consumer valuation. By (9), the optimal full price $\bar{p}^*$ is greater than v if and only if the numerator of the second term on the right-hand side of (9) is positive. The next proposition summarizes this result.

Proposition 2 (Attachment Effect and Fill Rate). *Under the optimal contingent pricing policy, consumers pay a full price higher than their consumption valuation, if and only if the sales frequency $F(\tau^*)$ and* *the fill rate ϕ^* simultaneously satisfy the following two conditions*:

$$\phi^* > \tfrac{1}{2}, \tag{12}$$

$$F(\tau^*) \le \frac{2\phi^* - 1}{\phi^*} \frac{v}{v-s} \equiv \hat{F}. \tag{13}$$

The implications of Proposition 2 are twofold. First, Condition (12) indicates that the optimal fill rate needs to be relatively high in order to make buying at the full price an attractive purchase plan in the first place. Second, Condition (13) indicates that the optimal sales frequency needs to be sufficiently low to prevent consumers from declining to buy at the full price. The intuition is as follows. When considering the trade-off between the positive attachment effect and the negative comparison effect, we observe that when the sales frequency increases, consumers' expectation of obtaining the product increases because of an increased sales chance. This attachment effect increases the consumers' feeling of loss if they do not buy when the full price is charged. However, when the sales frequency becomes overly high, then, compared to the higher possibility of buying the product at the sale price, consumers view buying the product at the full price to be a loss; and, the more frequent the sales, the more loss consumers feel if they buy the product at the full price. To dampen this negative comparison effect, the sales frequency should not be too high. As an immediate corollary of Proposition 2, we have the following result.

Corollary 1 (When to Discount Less Frequently Than "Optimal"). *Under Conditions* (12) *and* (13), *the optimal sales frequency is lower than the "optimal" sales frequency in the absence of consumers' loss aversion*, *i.e.*, *the sales frequency under the contingent pricing strategy that maximizes profitability for a particular period given its realized demand*.

Corollary 1 is *in contrast* to Cachon and Feldman (2015), where the firm benefits from discounting *more* frequently than the frequency under the "optimal" contingent discount policy, given the realized demand. The driving forces behind the two results are totally different. Unlike the "more frequently than optimal" discount strategy that entices non-loss-averse consumers to visit the store, the discount strategy in our model serves to induce loss-averse consumers to buy at a higher full price. In our setting with stochastic reference points, frequent deep discounting can be detrimental because it enhances the negative comparison effect, and the firm would want to avoid it.

4.2. Optimal Order Quantity

We discuss the firm's optimal choice of the order quantity, given that the firm implements the optimal contingent pricing scheme specified in Proposition 1. By Proposition 1, the firm's decision on the order quantity q determines the optimal fill rate ϕ^*; the optimal fill rate ϕ^* and the firm's decision on sales threshold τ jointly determine the optimal full price $\bar{p}^*$ and the sales frequency $F(\tau)$. Therefore, the firm's expected profit Π can be seen as a function of the firm's two decision variables, q and τ. Given a fixed order quantity q, a nonzero optimal sales threshold τ^* should satisfy the first order condition (11). Given a fixed sales threshold τ, the first order condition of the expected profit function $\Pi(\cdot)$ with respect to q is

$$\frac{\partial \Pi(q,\tau)}{\partial q} = sF(\tau) + \bar{p}^*(1-F(q)) + \frac{\partial \bar{p}^*}{\partial \phi^*}\frac{\partial \phi^*}{\partial q}\int_\tau^\infty \min\{x,q\}\,dF(x) - c = 0. \tag{14}$$

Note that $\partial\Pi(0,\tau)/\partial q = sF(\tau) + \bar{p}^* - c > 0$. In addition, since $\lim_{q\to\infty} 1 - F(q) = 0$ and $\lim_{q\to\infty} \partial\phi^*/\partial q = 0$, we have $\lim_{q\to\infty} \partial\Pi(q,\tau)/\partial q = sF(\tau) - c < 0$. Therefore, based on intermediate value theorem, there exists an optimal order quantity $q^* > 0$ such that $\partial\Pi(q^*,\tau)/\partial q = 0$. Given the consumer loss-aversion level λ and procurement cost c, the optimal order quantity q^* and nonzero sales threshold τ^* must simultaneously satisfy the two first order conditions (11) and (14).

5. Comparative Statics

In this section we study various comparative statics of the market equilibrium.

5.1. Order Quantity

We first investigate how the optimal full price $\bar{p}^*$, the optimal sales threshold τ^*, and the expected price change with respect to the initial order quantity level q by temporarily ignoring the ordering cost. (Unless otherwise specified, the monotonicity is in its weaker sense.) We note that the following result is the main reason that the effects of operational decisions on the firm's profitability in a market with loss-averse consumers differ from those in a market without loss-averse consumers.

Proposition 3 (Higher Availability, Larger Attachment Effect). *The optimal full price $\bar{p}^*$ is increasing and the optimal sales threshold τ^* is decreasing in the order quantity q. Therefore, the expected price $\hat{p}^* = sF(\tau^*) + \bar{p}^*(1 - F(\tau^*))$ is increasing in the order quantity q.*

The increasing monotonicity of the optimal full price in the order quantity is in contrast to the common intuition of the price-quantity relationship in a consumer market without loss-averse behavior: As the firm increases its initial order, the risk of overstocking increases. Hence, one would intuitively expect the firm to cut the price to reduce unsold inventory. The seeming anomaly occurs because, in a loss-averse consumer market, the firm can take advantage of the positive attachment effect by creating moderate sales, and the attachment effect is reinforced by higher product availability. In a classic newsvendor setting with non-loss-averse consumers, the fill rate does not play a role in influencing consumers' willingness to pay, and its impact on the firm's profitability is always along the single dimension of inventory risks. In a loss-averse consumer market, however, an increase in the fill rate can increase consumers' expectation of obtaining the product, and hence enhances the attachment effect, because they are more attached to the idea of buying the product. Consequently, consumers are willing to pay a higher full price to avoid such a loss. Proposition 3 implies the subtlety in determining the *optimal* order quantity when selling to a loss-averse consumer market.

5.2. Procurement Cost

Driven by the effects identified in Proposition 3, we demonstrate how the firm's optimal ordering and pricing decisions change with respect to the procurement cost. One might typically expect that the firm would sell the product at a higher price if it procured the units more expensively. This is true for a market with non-loss-averse consumers. However, if the firm sells to consumers who are loss averse, it should sell at a cheaper price when the procurement cost increases. The next proposition states this result.

Proposition 4 (Higher Cost, More Promotions and Lower Full Price). *We have the following*:

(i) *the optimal order quantity q^* is decreasing in the procurement cost c;*

(ii) *the optimal sales threshold τ^* is increasing in the procurement cost c;*

(iii) *both the optimal full price $\bar{p}^*$ and the expected price $\hat{p}^*$ are decreasing in the procurement cost c.*

Part (i) of Proposition 4 says that when the procurement cost increases, the firm optimally orders less. This result is consistent with that in a market without loss-averse consumers. Surprisingly, parts (ii) and (iii) of Proposition 4 show that a higher procurement cost would lead to a higher optimal sales frequency (because of an increased sales threshold) and a lower optimal full price, both of which are in consumers' favor. In particular, the finding that the optimal full price decreases in the procurement cost is in stark contrast to predictions by the marketing literature on cost pass-throughs in a non-loss-averse consumer market: The optimal full price is either equal to consumers' valuation if there is a single segment of consumers with a homogeneous valuation, or increasing in the procurement cost if there are multiple segments or a continuum of heterogeneous consumer valuations. The reason for this counterintuitive finding is that an increase in the procurement cost causes the firm to reduce the optimal order quantity (see part (i)). By Proposition 3, less availability reduces consumers' willingness to pay at the full price, and then the firm compensates for this reduction by reducing the optimal full price. Another natural response to reduced inventory availability is to slightly increase the sales frequency to boost the attachment effect while sustaining the full price.

5.3. Loss-Aversion Level

We consider the impact of consumers' loss-aversion level on the firm's strategies and profitability.

5.3.1. Exogenous Order Quantity. To gain intuition, we start with a fixed initial order quantity. Note that given a fixed order quantity q, the optimal fill rate ϕ^* is not affected by consumers' loss-aversion level λ explicitly; see Proposition 1(i). However, from Proposition 1(ii), we can see that the optimal sales threshold τ^* depends on λ explicitly, and so does the optimal sales frequency $F(\tau^*)$; we use the notation $\tau^*(\lambda)$ and $F(\tau^*(\lambda))$ to emphasize this dependency.

Proposition 5 (Comparative Statics on Loss-Aversion Level). *Given any fixed order quantity q, if $F(\tau^*(\lambda)) \le \hat{F}$, where $\hat{F}$ is defined in (13), the optimal sales frequency $F(\tau^*(\lambda))$ is decreasing in λ, and the optimal full price $\bar{p}^*$ is increasing in λ.*

Proposition 5 says that when the optimal sales frequency $F(\tau^*(\lambda))$ is below the threshold $\hat{F}$, it is decreasing in λ. As discussed above, the attachment effect can increase consumers' willingness to pay, so it has a *positive* impact on the firm's profitability. The comparison effect, in contrast, restrains the firm from charging a higher full price, so it has a *negative* impact on the firm's profitability. As consumers' loss aversion increases, both effects become more significant. Proposition 5 shows that when the optimal sales frequency is not too high, as consumers' loss aversion increases, the firm should lower the optimal sales frequency $F(\tau^*)$ to dampen consumers' feeling of loss caused by the comparison effect. At the same time, the firm can set the optimal full price higher by leveraging the attachment effect.

Recall that $F(\tau^*) \le \hat{F}$ is a necessary condition of the optimal full price being higher than consumers' consumption valuation (see Proposition 2). Hence, Proposition 5 also suggests that if the optimal full price is greater than v for some level of loss aversion, it will further increase if consumers become more loss averse. In view of (13), the condition that $F(\tau^*) \leq \hat{F}$ is more likely to hold, when the initial order level q is higher or when the valuation ratio v/s is smaller.

The next proposition discusses how the firm's optimal profit changes with respect to consumers' loss-aversion level. Given a fixed initial inventory level q, the firm's expected profit under the optimal contingent pricing policy is

$$\Pi^*(q) = \Pi(\bar{p}^*, q, \tau^*, \xi(\bar{p}^*, q, x)) = sqF(\tau^*) + \bar{p}^* \int_{\tau^*}^{\infty} \min\{x, q\}\, dF(x) - cq. \quad (15)$$

We can see that profitability depends on the loss-aversion level, essentially through the optimal sales threshold. Let $\tau^*(1)$ be the value of the optimal sales threshold $\tau^*(\lambda)$ at $\lambda = 1$ and $\tau^*(\infty) = \lim_{\lambda\to\infty}\tau^*(\lambda)$, and then we have the following results.

Proposition 6 (When Loss Aversion Benefits the Firm). *Given any fixed order quantity q,*

(i) *if $F(\tau^*(1)) \leq \hat{F}$, the firm's expected profit Π^* is strictly increasing in λ;*

(ii) *if $F(\tau^*(\infty)) \geq \hat{F}$, the firm's expected profit Π^* is strictly decreasing in λ;*

(iii) *otherwise, the firm's expected profit Π^* has a U-shape in λ, with a unique minimum $\lambda_{\min}$ such that $F(\tau^*(\lambda_{\min})) = \hat{F}$.*

Proposition 6 states that the firm's expected profit is either strictly monotone (increasing or decreasing), or it has a unique minimum in the loss-aversion parameter. The firm's profitability is influenced by the two competing effects—the positive attachment effect and the negative comparison effect. Both effects become more significant when consumer loss aversion increases. Part (i) states that if the sales frequency in the loss-neutral market (i.e., $\lambda = 1$) is less than the threshold $\hat{F}$, then the positive attachment effect always dominates, and the firm's expected profit will increase as consumers become more loss averse. In this case, the firm strictly benefits from loss aversion. Again, this is more likely the case when the initial order level q is higher or when the valuation ratio v/s is smaller. Part (ii) specifies an extreme in which the negative comparison effect always dominates. In this case, the firm's expected profit is hurt by consumers' loss-averse behavior. By the definition of $\hat{F}$, this extreme is more likely the case when the initial order level q is lower or when the valuation ratio v/s is larger. Part (iii) shows an intermediate case, in which the negative comparison effect first dominates for low levels of loss aversion, and then at some point the positive attachment effect takes over for higher levels of loss aversion. In this case, the firm may benefit from consumers' loss aversion if the level of loss aversion is sufficiently high.

5.3.2. Optimal Order Quantity.

We have the following results on comparative statics of the optimal order quantity with respect to the loss aversion level.

Proposition 7 (More Loss Aversion, Larger Initial Order). *One of the following two scenarios must hold:*

(i) *The firm's optimal order quantity q^* is increasing in λ.*

(ii) *The firm's optimal order quantity q^* has a U-shape in λ.*

Proposition 7 indicates that there is a threshold on the loss-aversion level λ, beyond which the more loss-averse consumers are, the higher the firm should set its initial order quantity. This result is consistent with Proposition 3, since a higher initial order quantity would lead to a higher fill rate and hence enhance the positive attachment effect. This insight is different from the message in Liu and Shum (2013) that limiting supply in a loss-averse (in their words, disappointment-averse) market can benefit the firm. In Liu and Shum (2013), loss-averse behavior of high-value consumers with a deterministic reference point can reinforce intertemporal market segmentation. However, we demonstrate that even with a single segment of consumers of a homogeneous valuation but in the presence of stochastic reference points, the firm is better off expanding supply to the market as much as possible. The ample supply coupled with contingent sales can induce loss-averse consumers with stochastic reference points to buy at the (higher) full price.

Define $\hat{F}(q^*) \equiv ((2\phi(q^*) - 1)/\phi(q^*))(v/(v-s))$, then the following proposition shows that the comparative statics results of Propositions 5 and 6 for exogenous order quantity also carry over when the initial order quantity is optimized.

Proposition 8 (Comparative Statics at Optimal Order Quantity). (i) *In the neighborhood where $F(\tau^*) \geq \hat{F}(q^*)$, the optimal sales frequency $F(\tau^*)$ is decreasing in λ and the optimal order quantity q^* is increasing in λ.*

(ii) *In the neighborhood where $F(\tau^*) \leq (or, \geq) \hat{F}(q^*)$, the firm's optimal expected profit $\Pi(\tau^*, q^*)$ is increasing (resp., decreasing) in λ.*

5.4. Demand Variability

Lastly, we consider the comparative statics with respect to the demand distribution. In particular, we show that given a fixed order quantity, the firm can benefit from demand variability when consumers are loss averse, as long as the variability is not significant. For illustration, we consider a simple two-point demand distribution as follows:

$$X = \begin{cases} d_l & \text{with probability } \theta, \\ d_h & \text{with probability } (1-\theta), \end{cases} \quad (16)$$

where $d_l < d_h$ and the mean demand is $\bar{d} = \theta d_l + (1-\theta)\,d_h$. To investigate the impact of demand variability on the firm's profitability, we fix the mean demand $\bar{d}$ and the probability θ but vary the distance between the low demand and the mean demand; i.e., $\sigma = \bar{d} - d_l$. The high demand, therefore, can be expressed as $d_h = (\bar{d} + \theta/(1-\theta))\sigma$, and the demand variance is $\mathrm{var}(X) = (\theta/(1-\theta))\sigma^2$. The demand variability is increasing in the parameter σ.

PROPOSITION 9 (WHEN DEMAND VARIABILITY BENEFITS THE FIRM). *Consider the demand distribution* (16). *For any fixed order quantity* q, *there exists a threshold on* σ *below which the firm's expected profit* $\Pi^*(\sigma)$ *is increasing in* σ. *More specifically,*

(i) *if* $q < \bar{d}$, $\Pi^*(\sigma)$ *is strictly increasing in* $\sigma \in [0,\ \bar{d}-q]$ *and is strictly decreasing in* $\sigma \in (\bar{d}-q,\ \bar{d}]$;

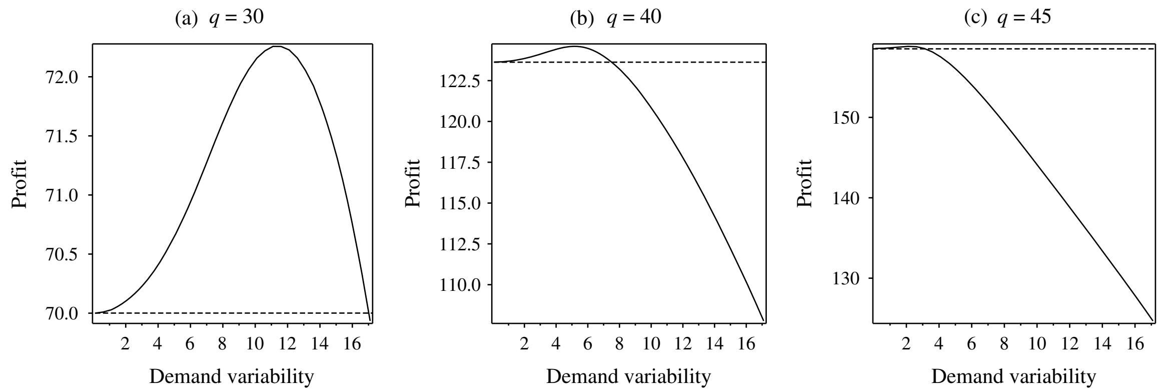
(ii) *if* $q \geq \bar{d}$, $\Pi^*(\sigma)$ *is increasing in* $\sigma \in [0,\ ((1-\theta)/\theta)\cdot(q-\bar{d})]$ *and is strictly decreasing in* $\sigma \in ((1-\theta)/\theta)\cdot(q-\bar{d}),\ \bar{d}]$.

Proposition 9 suggests that when consumers are loss averse, demand variability can be in the firm's favor. We use part (i) of Proposition 9 to illustrate this finding, which shows that for a fixed order quantity that is lower than the mean demand (noting that the optimal order quantity is lower than the mean demand when the overstocking cost is sufficiently high), sufficiently low levels of demand variability, e.g., $\sigma \in [0, \bar{d}-q]$, strictly benefit the firm. This is because when the demand variability σ increases from 0 to $\bar{d}-q$, the product availability conditional on demand being d_l, i.e., $q/(\bar{d}-\sigma)$, is strictly increasing in σ, whereas the product availability conditional on demand being d_h, i.e., $q/(\bar{d}+(\theta/(1-\theta))\sigma)$, is strictly decreasing in σ. In this case, the average fill rate is strictly increasing in demand variability. A higher fill rate strengthens the attachment effect, so loss-averse consumers are willing to accept a higher full price and the firm benefits from a higher profit margin when selling to loss-averse consumers, which is more than enough to offset the detriment of increased demand variability. When the demand variability σ further increases from $\bar{d}-q$ to $\bar{d}$, the order quantity is more than sufficient to satisfy all consumers' needs when demand is d_l, so the product availability conditional on demand being d_l is always 1, but the product availability conditional on demand being d_h continues to fall. As a result, the average fill rate strictly decreases. In this case, the attachment effect is continuously being weakened. To compensate for this decreasing attachment effect, the firm has to lower the full price to induce loss-averse consumers to buy, which, together with increased demand variability, leads to a reduction in the firm's expected profit.

These findings are in stark contrast to those in a newsvendor setting, where loss-averse consumer behavior is absent and demand variability only introduces a mismatch between demand and supply without affecting consumers' willingness to pay. Such a mismatch always cuts into the firm's profitability, so the newsvendor firm prefers deterministic demand to uncertain demand. In contrast, when consumers are loss averse, a not-too-high degree of demand variability strengthens the attachment effect created by occasional sales and induces a greater willingness to pay. When the enhanced positive side of the attachment effect more than compensates for the increased negative side of demand-and-supply mismatch, the firm is better off with demand variability. Proposition 9 also shows that when demand variability is sufficiently high, the attachment effect will be weakened because of too-frequent sales, so the firm's profitability is hurt by both a weakened attachment effect and the increased demand-and-supply mismatch. These findings stress the important role of consumer loss-averse behavior in studying firms' operational decisions.

For a more general demand distribution, we have similar observations. In Figure 2, three numerical examples show how the firm's expected profit changes in response to various levels of demand variability for a given initial order quantity that is lower than the mean demand, when the firm uses the optimal contingent pricing specified in Proposition 1. The parameters chosen for these numerical examples are $s=1.5$, $c=2$, $v=4$, $\eta=1$, $\lambda=2$, and a normally distributed random market size D with mean $\mathbb{E}(D)=50$, and we allow the standard deviation σ of the random demand D to vary. Consistent with Proposition 9, all three numerical examples have unimodular profit functions. In Figure 2(a), the firm's expected profit with demand variability $\sigma \in (0, 17)$, represented by the solid line, is higher than the firm's profit with no demand variability $\sigma = 0$, represented by the dotted line. Similar findings emerge in Figures 2(b) and 2(c), where the firm earns a higher profit when $\sigma \in (0, 7.7)$ and $\sigma \in (0, 3.1)$, respectively. Therefore, in these cases, the firm prefers some level of demand variability to no demand variability. Figure 2 also suggests that when the demand variability is sufficiently high, the negative impact of the demand variability will eventually dominate the positive benefit of the attachment effect, and the firm's expected profit will decrease. In conclusion, these numerical examples suggest that small or medium degrees of demand variability may benefit the firm, but high demand variability is always detrimental. We note that there exist examples in which any level of demand variability hurts the firm.

Figure 2 **Impact of Demand Variability on Profit:** $D \sim \mathrm{N}(50, \sigma^2)$



6. Conclusion

Consumers' loss-averse behavior with endogenized stochastic reference levels has largely been overlooked in the operations management literature. In addition, in the behavioral economics literature, the loss-averse consumers are assumed to have a deterministic market size, whereas the firms are assumed to have no supply constraints. We attempt to fill the gap between these two bodies of literature by considering the trade-offs in the inventory and contingent pricing policies of a newsvendor-type firm over repeated sales horizons. The firm orders a limited amount of inventory and sells it to loss-averse consumers with a random market size whose stochastic price reference levels are influenced by the firm's contingent markdown strategies and product availability. We fully characterize the optimal inventory and pricing policies, which include a contingent pricing strategy of a threshold form. We demonstrate that the firm can benefit from consumers' loss-averse behavior. The model reveals somewhat counterintuitive insights into how consumers' loss aversion affects the firm's operational decisions. Specifically, we show that when consumers are loss averse, (i) the firm may prefer demand variability over no-demand uncertainty; (ii) the optimal full price increases in the order quantity; (iii) the optimal full price decreases in the procurement cost; and (iv) the optimal sales frequency increases in the procurement cost. Again, we caution that these results hold up to the critical point where the optimal sales frequency degenerates to zero.

In the online appendix, we make a set of extensions. First, we consider the extension where the firm randomly rations inventory during sales among regular consumers and bargain hunters. Second, we consider a market of two consumer segments with heterogeneity in either loss aversion or consumption valuation. For these extensions, we show that the insights obtained from the base model can continue to hold under mild assumptions. Lastly, we consider a duopoly of newsvendors who compete in selling to loss-averse consumers. In contrast to Heidhues and Koszegi (2014) where there is no demand uncertainty and a deterministic price becomes an equilibrium strategy, we show that when there is demand uncertainty, setting a deterministic price would *not* be an equilibrium strategy. This is because in a market with random demand, contingent pricing benefits a firm not only by manipulating consumers' reference distribution but also by better matching demand with supply.

There are several limitations to our model. First, to simplify analysis, we assume that under contingent pricing, the firm can quickly learn the demand for the entire period. However, in reality, demand uncertainty may unfold over time, and the firm may not learn it all at once. Second, we assume that the sales horizons unfold repeatedly with a stationary demand distribution over time. In reality, market conditions can shift as time goes by. Third, we consider the equilibrium behavior over repeated interactions between loss-averse consumers and a farsighted newsvendor. Hence, we caution that our comparative statics results may not be applicable to predicting transient and myopic market interactions.

Despite these limitations, our stylized model captures the core tensions of how consumers' loss aversion with endogenized stochastic reference points influences the firm's optimal inventory and pricing decisions. The obtained insights may have several implications. For example, to automate pricing decision making, many retailers have begun using markdown pricing optimization software to determine the depth and frequency of their clearance events. As limited initial inventory hinders the firm from profitably selling at the full price to loss-averse consumers, our results suggest that the firm may want to build up initial stocks aggressively, more so than when selling to consumers who are not loss averse. Moreover, as running sales may profitably manipulate loss-averse consumers' reference points, the firm can be satisfied with limited replenishment ability, because beneficial occasional sales can be a natural outcome of the contingent pricing strategy driven by market uncertainty. As a result, when contingent pricing policies are employed in a market with loss-averse consumers, quick response initiatives may have limited additional benefit.

As a closing remark, our model is closely related to a repeated two-period newsvendor-type model, where in the first (regular-selling) period, a full price is posted and in the second (salvage) period, a sale price may be posted, depending on the sales volumes realized in the first period. If the demand in the regular-selling period is strong, there is little inventory left for the salvage period, so a full price may still be sustained; otherwise, a sale price is posted. In other words, the demand uncertainty in the first period drives the price for the second period. Consumers who only shop in the second period experience a distribution of full and sale prices. The loss-averse behavior of those consumers can be accounted for by the firm to make more profitable operational decisions. (The loss-averse behavior of those consumers who shop in the regular-selling period does not make a difference, if only a single price is posted for that period; the endogenized reference-point framework leads to a different outcome from that obtained by ignoring the reference-point effect only when the reference points are stochastic.) This setting, which is similar to our formulation, may be appropriate for modeling a newsvendor-type firm, e.g., a fashion retailer, who repeatedly makes initial ordering and end-of-horizon contingent markdown decisions (see, e.g., Cachon and Kok 2007). Our insights can also shed light on this alternative setting.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/msom.2015.0532.

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