MANAGEMENT SCIENCE
Vol. 62, No. 3, March 2016, pp. 867–879
ISSN 0025-1909 (print) | ISSN 1526-5501 (online)

informs®
http://dx.doi.org/10.1287/mnsc.2015.2160

Liking and Following and the Newsvendor: Operations and Marketing Policies Under Social Influence

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We consider a monopolistic firm selling two substitutable products to a stream of sequential arrivals whose purchase decisions can be influenced by earlier purchases. Before demand realizes, the firm faces a newsvendor problem for the two products with economies of scale in production for each. When consumers are responsive to others' decisions, social influence amplifies demand uncertainty, leading to a lower profit for the firm. We propose three solutions for the firm to better cope with or even benefit from social influence: influencer recruitment and a reduced product assortment either before demand realization (ex ante) or under production postponement (ex post). First, the firm can offer promotional incentives to recruit consumers as influencers. We reveal an operational benefit of influencer marketing that a very small fraction of such influencers is sufficient to diminish sales' unpredictability. Second, as the potential substitutability between products increases due to social influence, the firm may leverage the increased substitutability and enjoy lower cost in production by reducing product assortment before demand realization. Last, under production postponement, the firm can take advantage of the way that social influence results in demand herding and reduce product varieties by reacting to preorder information.

Keywords: social influence; newsvendor; positive network externality; influencer marketing; production postponement; product assortment
History: Received July 11, 2013; accepted January 9, 2015, by Martin Lariviere, operations management. Published online in *Articles in Advance* August 18, 2015.

1. Introduction

Consumer choice is influenced by the actions taken by others. It can be shaped by word of mouth, e.g., a face-to-face recommendation by a friend, or by observational learning, e.g., by seeing what a stranger is wearing on the street. With advances in online shopping, the importance of social influence has never been greater. Retailers place Facebook "Like" or Twitter "Share" buttons on their product pages so consumers can share their purchases with friends on Facebook or "followers" on Twitter. A count of the likes or shares is updated in real time next to the buttons and is readily observable by anyone who visits the website. Moreover, it is common for firms to collect sales information in real time and share it with consumers. For example, Amazon updates the sales ranking information of tens of thousands popular products hourly and presents it on products' websites as part of the products' attributes.

Researchers have provided experimental and empirical evidence that social influence results in the phenomenon where "the rich get richer." Cai et al. (2009) report on a field experiment in a restaurant, observing an up to 20% increase in demand for the five most popular dishes when consumers are given ranking information. Such a "preferential attachment" effect or "Matthew effect" (Merton 1968) can be even stronger in the online consumer market. Using the sales ranking on Apple's App Store, Carare (2012) shows that the willingness to pay of consumers is significantly higher for a top-ranked product than for the same product when it is unranked. Moreover, the collective choice of masses is also reported to be highly random due to social influence. An experimental study shows that with the exposure to others' choices, an increasing level of unpredictability of market success is observed (Salganik et al. 2006). When individuals tend to conform to others within a society or a social group, product adoption becomes *path dependent*; the demand does not simply aggregate preexisting individual preferences. For example, apparel fashion trends can occur when random word of mouth takes over.

Even in the era of online shopping, the physical world still moves at a slow pace. Lead time for procurement and production is relatively long, so that most ordering decisions for short-life-cycle products are made ahead of the sales season and can hardly be changed in the season. For example, lead times are often more than three months in the apparel and footwear industries, whereas the fashion seasons themselves are as short as two to three months. Given the prevalence of social influence on consumption, we build a stylized model to answer the following research questions: What are the effects of social influence on the policies of a newsvendor-type firm? If its impact is negative, how can the firm counteract it? If its impact can be positive, how can the firm take advantage of it?

We consider a monopolistic firm selling two horizontally differentiated products in a market with a predictably stable size. Before the sales horizon, the firm faces a newsvendor ordering decision for the products. We assume economies of scale in production for each. During the sales season, consumers sequentially arrive and make purchase decisions that may be influenced by earlier adoptions. We show that consumers' responsiveness to others' purchase decisions is a key factor in shaping the distribution of aggregate sales. When consumers are relatively responsive to others' decisions, social influence amplifies demand uncertainty facing the firm. The demand uncertainty manifests itself as both *unpredictability* and *inequality*: the firm is unsure which product will be popular, and when one product becomes popular, it may be extremely popular. We show that, if the newsvendor does not actively adapt its policies, the impact of social influence is nonpositive.

We consider three responses for the firm. First, in a strategy of influencer recruitment, the firm may randomly target a small fraction of consumers, designated influencers, and offer economic incentives, like price discounts or coupons, tailored to them. Through social influence, the firm can manipulate the collective decisions of the market by injecting influencers into the sequential sales process. We show that a small fraction of influencers is sufficient to diminish demand uncertainty and provide an operational benefit. Second, because of social influence, the potential substitutability between products increases, as consumers are more inclined toward a popular product. We show that the firm may leverage the increased substitutability between products and enjoy lower cost in production by offering fewer product varieties before demand realization. This strategy is shown to be profitable when consumers are sufficiently sensitive to others' decisions. Third, on the supply side, if the firm is able to delay production, we show that, similar to the strategy of ex ante reduced product assortment, the firm may be better off producing fewer product varieties after observing preorder information. This strategy takes advantage of the way that social influence results in demand herding and does not require consumers to be highly sensitive to others' decisions.

1.1. Literature Review

Our work is related to the literature on social influence and observational learning (see a recent work by Young 2009 on social learning and the references therein). In papers such as Banerjee (1992) and Bikhchandani et al. (1992), individuals sequentially make decisions, conditioning both on one's endowed private signal about the state of the world and on all predecessors' decisions. Despite some similarities, our work differs in the following two aspects. First, we assume that consumers have complete knowledge on the quality of the products; that is, the products belong to search goods, instead of experience goods (see Cabral 2000). Second, and more importantly, our focus is on the impact of social influence on the newsvendor and the firm's counteracting policies, whereas the literature on social learning is primarily focused on the social influence process itself.

In the social learning literature, there are Bayesian and non-Bayesian approaches to modeling how individuals are influenced by others. Similar to the non-Bayesian approach, we specify decision rules regarding how later arrivals are influenced by earlier adoptions. This approach allows us to simplify the learning process under social influence and focus on the firm's policies. It also enables us to capture some real-life phenomena that are difficult to model using a Bayesian approach.

The demand model in our paper shares some similarities with the works on forecasting and inventory management of fashion goods where previous sales are assumed to stimulate future demand. Ravindran (1972) investigates the optimal stocking level and the optimal duration of the selling season in a single-period inventory model. Hartung (1973) considers a multiperiod version with a fixed duration of the selling season. In studying the impact of inventory cost on retail assortment breadth, van Ryzin and Mahajan (1999) consider a trend-following model. Petruzzi and Monahan (2003) study the optimal time when a firm should shift to a secondary market from the primary selling season. Unlike these models where demand is assumed to follow an exogenous distribution, we *endogenize* demand uncertainty by modeling the consumers' adoption process as a path-dependent process, relate the demand uncertainty to the consumers' sensitivity to others' decisions, and design strategies to shape the aggregate demand by influencing consumers' decisions.

Our work belongs to an emerging body of research on "social" operations management, where social interaction among individual decision makers or segments of individuals is explicitly modeled. Tereyağoğlu and Veeraraghavan (2012) address pricing and production decisions when consumers value the uniqueness of products to display their exclusivity and social status. Our work considers the opposite type of social behavior, where consumers value a product more if it is more popular in the population. Similar consumer behavior is studied in a queueing setting by Veeraraghavan and Debo (2009) and their subsequent works on herding in queues, where consumers infer service quality from the length of queues using a Bayesian approach. Whereas they focus on the signaling effect of queue lengths with rational herding as a result, we study the impact of social-influence-induced herding on sales uncertainty and propose marketing and operational policies to react to it. Papanastasiou et al. (2014), Papanastasiou and Savva (2014), and Yu et al. (2016) investigate the implications of sharing and learning opinions about product quality on a single-product firm's pricing decision. Lobel et al. (2015) study how to optimally attract new customers through social interactions using a referral program. We consider a two-product newsvendor problem and focus on the benefits of various strategies in mitigating the negative impact of social influence. Last, see Hu et al. (2013, 2015a, b), Wu et al. (2015), and Hu and Wang (2015) for a stream of works on information and revenue management in various contexts with social interactions, such as online group buying, crowdfunding, sales of network goods, and service systems.

2. The Model and Analyses

2.1. The Setup

Consider a monopolistic firm selling two *horizontally* differentiated products to a market of heterogeneous consumers. The two products, labeled A and B, are located at the ends of a unit Hotelling line $[0, 1]$. Consumer preferences are distributed along the line. For instance, the two products may vary in their color or style, for which consumers have different preferences. We consider a market with a fixed market size M. Because we focus on how social influence affects consumers' choices between the firm's two products, the demand for each product does have uncertainty. This setup fits well to a market that has a relatively predictable consumer base with consumers differing in tastes. For example, in the well-known Sport Obermeyer case, the firm commits to produce 20,000 units of fashionable ski parkas in total for the season, which may imply a relatively predictable total market size. The firm must decide how many units to produce for each design/product. This is driven by consumer tastes and fashion trends generated under social influence. Instead of working with the absolute number of units to produce, we normalize the total market size to 1 and consider the market share of each product.

The firm faces a newsvendor problem for the two products, with economies of scale in production. Without loss of generality, the products are assumed to have zero salvage value. In the event of stockout, unmet demand is lost without additional stockout penalty. The firm needs to determine the production quantities of the two products before observing demand. The scale economy in production is in the form of a fixed cost of producing each of the two products, denoted as K, and a constant marginal supply cost, c, per unit of market share. The cost structure is assumed to be identical for both products. (The case of no economies of scale, i.e., $K = 0$, is a special case.)

We consider a multiperiod, dynamic process where consumers sequentially arrive one per time period. Since the market size is finite, the sales horizon has a finite length. Upon arrival, each consumer is able to observe the decisions made ahead of him. The firm knows the probability distribution of the consumer-intrinsic preferences. This can be achieved through comprehensive market research before launching products. However, the firm still faces the uncertainty of the demand realization because of the path-dependent nature of the sales process under social influence.

2.2. Demand Side

2.2.1. Individual's Decision Making. We analyze how consumers' purchase decisions are shaped by individual preference and social influence. We assume that consumers are uniformly distributed over the preference line segment with an individual's preference/type denoted by $\delta \in [0, 1]$. The utility of choosing a product is given by the sum of a base utility, a disutility measuring the distance between an individual's ideal location δ and the location of the product, and a *social utility*, less the price. Let r_i, $i = A, B$, denote the base utility of product i, which is assumed to be homogeneous across all consumers. A constant marginal disutility θ is incurred for a unit distance between a consumer's ideal preference and the position of the product. Consumers know the aggregate purchase decisions of their predecessors. Let $\mathbf{x} = (x_A, x_B)$, where x_i is the fraction of one's predecessors who have purchased product i, $i = A, B$. The social utility for product i is specified as a power function x_i^α, with $\alpha \geq 1$. Assume the price, p, is the same for the symmetric products. The utility of choosing product i for a type-δ consumer can be written as

$$u_A(\mathbf{x}) = r_A - \theta\delta + x_A^\alpha - p,$$
$$u_B(\mathbf{x}) = r_B - \theta(1-\delta) + x_B^\alpha - p. \quad (1)$$

The social utility term for product i, x_i^α, is increasing in the fraction of consumers who have purchased product i, x_i. This form expresses the value of *conformity* to popular preferences in a society. For simplicity, we assume $r_A = r_B \equiv r$ in the following analysis, with obtained insights carried over to the case where r_A and r_B are not equal. Furthermore, we normalize θ to 1.

Denote by $N_{i,t}$ the number of consumers who have purchased product i at the end of any period t. Let $\mathbf{N}_t = (N_{A,t}, N_{B,t})$. Assume the initial condition $\mathbf{N}_2 = (1, 1)$; that is, at the end of the second period, we assume one consumer purchased product A, and another purchased product B. This assumption gives a symmetric initial start for both products before any social influence takes place. It avoids trivial solutions where all consumers prefer the same product. Let $\mathbf{X}_t = (X_{A,t}, X_{B,t})$, where $X_{i,t} = N_{i,t}/t$ denotes the fraction of consumers who have purchased product i at the end of period t. Given $\mathbf{X}_t = \mathbf{x}$, the consumer arriving at period $t+1$ will purchase product i if and only if $u_i(\mathbf{x}) \geq u_{-i}(\mathbf{x})$ and $u_i(\mathbf{x}) \geq 0$. Since δ is uniformly distributed, the probability that a consumer arriving at period $t+1$ will purchase product i is given by

$$f_i(\mathbf{x}) = P(u_i(\mathbf{x}) \geq u_{-i}(\mathbf{x}), u_i(\mathbf{x}) \geq 0) = \Pi_{[0,1]}\left[\min\left\{r - p + x_i^\alpha, \frac{x_i^\alpha - x_{-i}^\alpha + 1}{2}\right\}\right], \quad i = A, B, \tag{2}$$

where $\Pi_{[0,1]}[a]$ is the projection of a onto the line segment $[0, 1]$; that is, $\Pi_{[0,1]}[a] = 1$ if $a > 1$, $\Pi_{[0,1]}[a] = 0$ if $a < 0$, and $\Pi_{[0,1]}[a] = a$ otherwise. Let $\mathbf{f}(\mathbf{x}) = (f_A(\mathbf{x}), f_B(\mathbf{x}))$. In essence, the random sales process in our model is a generalized Pólya urn process (Hill et al. 1980). We call $\mathbf{f}(\mathbf{x})$ the *updating probability function* that generates this stochastic process.

The case with no social influence is defined as when consumers make decisions independently and solely contingent on their intrinsic preferences. Consumers' utilities are the same as specified in (1) except without the social utility term; that is, for a consumer of type δ, his utilities of purchasing product A and product B are given by $\tilde{u}_A = r - \delta - p$ and $\tilde{u}_B = r - (1-\delta) - p$, respectively. This case is equivalent to the limiting scenario of (1) as $\alpha \to \infty$, and it serves as a benchmark in our comparison below. When there exists no social influence, the probability that a random consumer arriving at period $t+1$ will purchase product i is given by

$$\tilde{f}_i = P(\tilde{u}_i \geq \tilde{u}_{-i}, \tilde{u}_i \geq 0) = \Pi_{[0,1/2]}[r - p], \quad i = A, B. \tag{3}$$

If $r - p \geq 1/2$, the market is *completely covered*, and each product covers one-half of the market. Otherwise, each product acts like a local monopoly and captures $r - p$ market share. We start our analysis under the assumption of complete market coverage when there exists no social influence.

Assumption 1 (Complete Coverage Without Social Influence). $r - p \geq 1/2$.

We first show that when the market is completely covered without social influence, it is also completely covered with social influence. This result is summarized in Lemma 1; that is, social influence will not have a negative impact on the total market size. Admittedly, we disregard the potential positive impact of social influence on the market size by assuming that the market is completely covered without social influence. This base model is more suitable in explaining how social influence will affect which specific product a consumer chooses, rather than whether a consumer will enter the market. In §4, we extend our model to incomplete market coverage. In this extension, we show that social influence can *expand* the market size for a product, and this effect can be substantial so that the firm is better off producing fewer product varieties.

Lemma 1 (Complete Market Coverage Under Social Influence). *Under Assumption 1, the market is completely covered with social influence.*

By Lemma 1, a consumer will purchase one of the two products upon arrival. Thus, given $\mathbf{X}_t = \mathbf{x}$, the probability that a consumer arriving at period $t+1$ will purchase product i is reduced to

$$f_i(x_i) = \frac{x_i^\alpha - (1 - x_i)^\alpha + 1}{2}, \quad i = A, B. \tag{4}$$

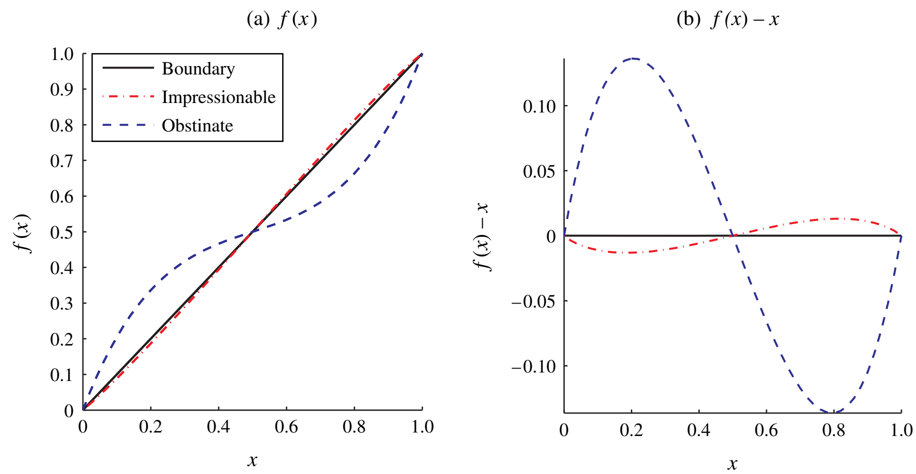
In our model, with Assumption 1 and a fixed market size, it suffices for us to focus discussion on the demand for one product, say, product A. The demand for product B can be derived immediately. Thus, we will omit the product subscript in the notation, i.e., $f(x)$ denotes the probability of a consumer choosing product A given that the fraction of his predecessors choosing product A is x.

2.2.2. Consumers' Sensitivity to Social Influence. The generated sales process can have dramatically different properties depending on the social utility exponent α.

Definition 1 (Consumers' Sensitivity to Social Influence). We define three scenarios based on the values of α: (1) the impressionable scenario, when $1 < \alpha < 2$; (2) the boundary scenario, when $\alpha = 1$ or 2; (3) the obstinate scenario, when $\alpha > 2$.

Figure 1 shows the values of $f(x)$ and $f(x) - x$ with various α. First, we notice that $f(x)$ is always equal to $1/2$ when evaluated at $x = 1/2$, regardless of the value of α. This is consistent with the intuition that when

Figure 1 (Color online) The Values of $f(x)$ and $f(x)-x$ as α Varies



there is an equal number of purchasers for both products, consumers base their decisions solely upon their intrinsic preferences. Second, Figure 1 shows that the curve $f(x)$ becomes flatter within the small neighborhood around $x=1/2$ as α increases. In this neighborhood, with $1<\alpha<2$, consumers respond actively to the difference between the market shares of the two products; specifically, consumers are more likely to be lured toward the more popular product. However, as α increases, consumers become less responsive to social influence unless one product gains a dominant popularity. In the boundary scenario, the likelihood of purchasing a product is proportional to the observed market share. The case of $1<\alpha<2$ implies a stronger social influence than the boundary scenario, whereas $\alpha>2$ implies a weaker social influence than the boundary scenario. Thus, we interpret the power of the social influence term, α, as the consumers' sensitivity with respect to others' decisions.

Most of the empirical literature on social influence assumes the boundary case, $f(x)=x$, where individuals are proportionally influenced by the average decisions of the previous adopters (see, e.g., Manski 1993). However, by varying α, we span the whole spectrum of consumer sensitivities to social influence. We will show shortly that the aggregate decisions of the entire population vary significantly across the three scenarios. Our definition of the different scenarios can be taken as a categorization of product types. For some products, consumers are more likely to stick to their intrinsic preferences, whereas for the others, social influence plays a much more crucial role. Our analysis sheds some light on how a firm should respond to social influence based on the type of products it offers. We conclude that the specification of social utility, x^α, leads to intuitive outcomes supported by experimental evidence (e.g., Salganik et al. 2006). It is also consistent with rational herding theory from the economic perspective (e.g., Banerjee 1992, Bikhchandani et al. 1992) and conformism theory from the psychological perspective (e.g., Cialdini and Goldstein 2004).

2.2.3. Limiting Distribution of Aggregate Demand. The firm is particularly interested in the impact of social influence on the aggregate demand of each of the products. Because it is very cumbersome, if not impossible, to derive the market share distribution of product A for a finite market, we resort to the limiting distribution of X_t and use it as an approximation of the market share in our constant market size setting. We provide both theoretical and numerical support for this approximation (see the online appendix, available as supplemental material at http://dx.doi.org/10.1287/mnsc.2015.2160). First, we are able to analytically derive the convergence rate for some cases. For example, we show that if $f(x)=x$ has a single root, denoted by x^*, and $f'(x^*)<1/2$, then the approximation error is $O(\sqrt{(\ln\ln t)/t})$. Second, we conduct extensive numerical experiments to see how accurate the approximation is under general conditions. We observe that the approximation has an accuracy of 1%–2% when the total market size is in the thousands.

We next show that the asymptotic distribution of X_t depends solely on the updating probability function $f(x)$. Lemma 2 summarizes the support for the limiting distribution of X_t. It is a direct consequence of Hill et al. (1980, Corollary 3.1 and Theorem 4.2).

Lemma 2 (Asymptotic Distribution). *X_t converges almost surely to a random variable X^* with support contained in the set $\{x\colon f(x)=x, f'(x)\le 1\}$.*

The points within the *smaller* set $\{x\colon f(x)=x, f'(x)<1\}$ are referred to as the *stable equilibria* of the demand process. The basic idea of the convergence proof is as follows. The conditional expectation of X_{t+1}, given the state $X_t=x_t$, is $E[X_{t+1}\mid X_t=x_t]=x_t+(1/(t+1))[f(x_t)-x_t]$. Thus the conditional mean of the increment $X_{t+1} - X_t$ shares the same sign as that of $f(x_t) - x_t$. After a finite number of periods, the process will converge to the points where the increment is zero, i.e., those roots of the equation $f(x) = x$. The condition $f'(x) < 1$ ensures that the equilibrium is robust to small perturbations, and thus it is called the *stability constraint*. Our model is a dynamic view of how a rational expectations equilibrium is reached (Easley and Kleinberg 2010, Section 17.4). The limiting distribution of the sales process characterized by (4) is summarized in the next lemma.

Lemma 3 (Limit of Aggregate Demand). *The distribution of $X^* \equiv \lim_{t\to\infty} X_t$ is*

(i) *Bernoulli with support on* $\{0, 1\}$ *and the success probability* $P(X^* = 1) = 1/2$ *under the impressionable scenario;*

(ii) *uniform in* $[0, 1]$ *under the boundary scenario;*

(iii) *the constant* $1/2$ *under the obstinate scenario.*

Lemma 3 provides some justification to our definition of the three scenarios. Under the impressionable scenario ($1 < \alpha < 2$), consumers are easily influenced by others, and thus eventually one of the two products crowds out the other with an equal chance. Under the obstinate scenario ($\alpha > 2$), the intrinsic preference dominates one's decision making, and the aggregate demand converges to the average preference across the entire population *as if* consumers are not aware of others' decisions. In this case, even though social influence *does* play a role in an individual's decision making, it has *no* impact at the aggregate level in the limit. The boundary scenario is a compromise of the other two scenarios that leads to a uniformly distributed limiting distribution.

Lemma 3 shows that the impact of social influence is twofold: (1) On the (potentially) positive side, under the impressionable scenario it leads to a dominant popularity of one of the two products, and thus the firm may enjoy lower supply costs if there exist economies of scale in production. (2) On the negative side, under the boundary and impressionable scenarios, social influence amplifies the demand uncertainty facing the firm, because the realized demand is path dependent, making it harder to predict sales and decide production decisions ex ante.

2.3. Impact of Social Influence

In this section, we investigate the impact of social influence on the newsvendor's profitability through a revenue–cost analysis. Recall that X^* denotes the random limit that the sales process converges to, i.e., the market share for product A in the limit. Denote by q_i, $i = A, B$, the firm's choice of production quantity for product i. Then, the newsvendor's expected profit can be written as

$$\pi(q_A, q_B) \equiv pE[\min\{q_A, X^*\} + \min\{q_B, 1 - X^*\}] - [c(q_A + q_B) + 2K]. \quad (5)$$

This is a formulation of the newsvendor problem for two products, where the first term expresses the total revenue from selling the two products, and the second term expresses the production cost.

By (3) and Assumption 1, we know that the market share of each product is equal to 1/2 in the limit when there exists no social influence. Given the distribution of X^* under social influence as summarized in Lemma 3, we can derive the firm's profit with social influence and compare it with the profit without social influence. The result is summarized as follows.

Proposition 1 (Impact of Social Influence).

(i) *Under the impressionable scenario and the boundary scenario, the firm makes a lower profit with social influence.*

(ii) *Under the obstinate scenario, the firm makes the same profit in both cases.*

Proposition 1 shows that social influence by itself never benefits the newsvendor. This implies that if a multiproduct newsvendor already has a desired market coverage, the firm would want to curb social influence, e.g., by not posting sales information on its website. Because of the path-dependent nature of the sales process, there is greater demand uncertainty introduced by social influence. The demand uncertainty manifests itself as both unpredictability and inequality. If the newsvendor does not actively adapt its policies, social influence simply works against the firm.

3. Strategies Under Social Influence

If social influence is inevitable, a newsvendor-type firm may suffer. How should the firm adapt its strategies? In addition, there could be a positive side of social influence: One of the products may become extremely popular, and thus the firm would be able to enjoy a lower production cost due to economies of scale. Is it possible for the firm to alleviate the demand uncertainty generated by social influence while enjoying economies of scale in production at the same time? In this section, we propose three strategies to counteract and take advantage of social influence.

3.1. Influencer Recruitment

The firm may be able to manipulate the collective decisions of the population by using social influence to its advantage. Suppose the firm randomly targets a small fraction of consumers, say, $\gamma \in (0, 1]$, and offers them an economic incentive, like a price discount or coupon, to entice them to purchase a particular product, say, product A. We would expect that other consumers would be influenced by the decisions of these targeted consumers. Thus, product A may become the more popular one, possibly allowing the firm to enjoy economies of scale in production ex ante.

We call the targeted consumers *influencers* and the remaining consumers *followers*. We focus on the policy of *randomly* recruiting influencers. However, in practice, advances in information technology may allow firms to learn consumers' preferences with great accuracy, and hence offer promotional incentives tailored to individual consumers. It is expected that with more *targeted* marketing campaigns, the influencers can be recruited more cost-efficiently. Thus, our analysis is conservative in evaluating the strategy of influencer recruitment.

On another note, consumers are assumed to be influenced by aggregate sales. In reality, certain consumers, like celebrities, have an unparalleled impact on the population, and the firm may be better off targeting them. Industry has adopted this influencer-marketing strategy with some success. However, the measurement of the success of such campaigns using sales data is still inadequate. Our analysis illustrates the often-neglected operational benefit from recruiting influencers.

A randomly selected influencer, after being offered the discounted price βp, where $\beta \in [0,1)$, would choose product A with probability

$$f_{I,A}(\mathbf{x}) = \Pi_{[0,1]}\left[\min\left\{r - \beta p + x_A^\alpha, \frac{x_A^\alpha - x_B^\alpha + 1 + (1-\beta)p}{2}\right\}\right].$$

When there are influencers, given Assumption 1 and $\beta \in [0,1)$, the market is completely covered. Thus, the choice probabilities of the influencers can be simplified as $f_{I,A}(x) = \Pi_{[0,1]}[(x^\alpha - (1-x)^\alpha + 1 + (1-\beta)p)/2]$. The choice probabilities of the followers remain given by (4).

Under the scheme of randomly recruiting influencers, an arrival is either a follower with probability $1-\gamma$, or is recruited as an influencer with probability γ. Hence, the probability that an arrival prefers product A over product B is the weighted sum of the probabilities that a follower or an influencer chooses product A. Consequently, the updating probability function, given the state $X_t = x$, can be written as

$$f(x) = (1-\gamma)\frac{x^\alpha - (1-x)^\alpha + 1}{2} + \gamma \Pi_{[0,1]}\left[\frac{x^\alpha - (1-x)^\alpha + 1 + (1-\beta)p}{2}\right]. \quad (6)$$

One question of particular interest is the fraction of influencers required to diminish demand uncertainty resulting from social influence. We show that in many cases it is sufficient to recruit only a *small* fraction of influencers.

Proposition 2 (Unique Limit Point with Influencers). *The set of convergence points of the sales process characterized by updating probability function* (6) *is a singleton if*

(i) $\gamma > 1 - 2^{\alpha-1}/\alpha$ *and* $\gamma(1-\beta)p \geq 2/\alpha - 1/2$ *under the impressionable scenario* (*moreover,* $1 - 2^{\alpha-1}/\alpha \leq 5.8\%$ *when* $1 < \alpha < 2$);

(ii) $\gamma > 0$ *under the boundary scenario* (*moreover, the singleton is* 1).

For the obstinate scenario, even without influencers, the sales process would converge to a deterministic outcome. For the boundary scenario, recall that the process converges to a uniform distribution when there are no influencers. Somewhat surprisingly, Proposition 2(ii) implies that any *tiny* fraction of influencers ($\gamma > 0$) is sufficient to eliminate the demand uncertainty in the limit. Furthermore, under the boundary scenario, the updating probability function $f(x)$ is reduced to $x + \gamma \min\{1-x, (1-\beta)p/2\}$, whose *unique* stable equilibrium is 1. The influencers bias the collective adoption process so that the entire population purchases the same product in the limit. Again, surprisingly, this result is *independent* of the fraction of influencers.

Under the impressionable scenario, each individual's decision is greatly influenced by the decisions from those who arrive before him and may have preferences in either direction. Our numerical experiments suggest that the impressionable scenario converges slower than the boundary scenario and tend to imply that the impressionable scenario converges slightly slower than the obstinate scenario (see Figure OA.1 in the online appendix). Thus, in the random sales process, a relatively larger fraction of influencers seems needed to eliminate demand uncertainty in the limit under the impressionable scenario than under the other two scenarios. Proposition 2 shows that even in this seemingly most unfavorable situation, a small fraction of influencers is sufficient: for $1 < \alpha < 2$, $\gamma > 1 - 2^{\alpha-1}/\alpha$. As $1 - 2^{\alpha-1}/\alpha \leq 5.8\%$, *at most* 5.8% of the consumers need to be influencers to completely eliminate demand uncertainty in the limit under the impressionable scenario.

The cost of recruiting influencers is quantified by the term $\gamma(1-\beta)p$, i.e., the product of the fraction of recruited influencers and the discount given to shape the decisions of these randomly targeted consumers. Proposition 2 indicates that the cost of recruiting influencers is negligible under the boundary scenario, because the uncertainty in demand vanishes in the limit for any γ as long as $\gamma(1-\beta)p > 0$. However, under the impressionable scenario, the firm needs to spend the minimal cost of $(2/\alpha - 1/2)$ to eliminate demand uncertainty. Thus, when consumers are relatively sensitive to others' decisions, the strategy of recruited influencers is profitable if and only if the benefit from reduced demand uncertainty outweighs the cost of recruiting influencers.

3.2. Product Assortment Decisions: Ex Ante vs. Ex Post

3.2.1. Reduced Product Assortment Before Demand Realization. With social influence, a consumer is more willing to tolerate the difference between the position of a product and his ideal preference if the product is popular in the population; that is, the potential substitutability between the products increases due to social influence. Consequently, the firm may leverage this increased substitutability among alternatives and enjoy a lower production cost by offering fewer product varieties. Here we consider the case when the firm reduces its product line from two products to one product and investigate when reduced product assortment may benefit the firm.

Without loss of generality, suppose that the firm chooses to manufacture only product A. Similar to the base model, a consumer upon arrival is able to observe the fraction of his predecessors who have purchased product A. The utility of a δ-type consumer purchasing product A is similar to u_A in (1). A consumer purchases the single product A as long as the utility is not less than 0. Given individuals' uniformly distributed preference, the probability that a random consumer arriving at period $t+1$ will purchase product A, when $X_t = x$, is $f(x) = \Pi_{[0,1]}[r - p + x^\alpha]$. Again, the consumer's purchase likelihood increases in the proportion of his predecessors who have purchased the product. In the context of producing only one product, Assumption 1 is no longer required. The market is not guaranteed to be covered completely with only one product even if $r - p \geq 1/2$ holds. Next, we derive the equilibrium that the process eventually converges to.

Lemma 4 (Limit of Aggregate Demand with One Product). *The aggregate demand X^* converges to the unique point of 1 if one of the following conditions holds*:

(i) (Value-driven regime) $r - p \geq 1$;

(ii) (Influence-driven regime) $r - p < 1$ *and* $\alpha \leq \bar{\alpha}$, *where $\bar{\alpha}$ is the unique solution of*

$$\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)} = r - p. \tag{7}$$

Otherwise, X^* follows a Bernoulli distribution with support on $\{x^*_\alpha, 1\}$, where x^*_α is the smaller root of $r - p + x^\alpha = x$.

Lemma 4 indicates that all consumers purchase product A in the limit, if either the intrinsic surplus of purchasing the product is sufficiently large, i.e., $r - p \geq 1$, or consumers are sufficiently sensitive to the decisions of others, i.e., $\alpha \leq \bar{\alpha}$. The first condition can be derived from the standard Hotelling model without social influence, whereas the second condition reveals the impact of social influence. Even if the intrinsic surplus is not large enough to cover the entire Hotelling line segment, all consumers may still purchase the product in the limit due to the social utility. When the intrinsic surplus is small, i.e., $r - p < 1$, and consumers are not sensitive to others' decisions, i.e., $\alpha > \bar{\alpha}$, there exist two limiting equilibria. The process may converge to 1, but it may also converge to an equilibrium where only x^*_α fraction of consumers purchase product A.

When producing only product A, the firm's expected profit is given by $\pi(q_A) = pE[\min\{q_A, X^*\}] - c \cdot q_A - K$. Under the conditions when the process is guaranteed to converge to 1 as shown in Lemma 4, the firm's optimal profit becomes $(p - c) - K$, with $q^*_A = 1$, which is higher than the highest profit that a firm can potentially obtain when producing both products, i.e., $(p - c) - 2K$. Under those conditions, the firm is able to cover the entire market with a single product and thus save the fixed cost that would be incurred when producing the second product.

Proposition 3 (Profit Comparison with Reduced Varieties). *The firm gains a higher profit by producing one product under either the value-driven regime or influence-driven regime.*

The benefit of producing only one product comes from two sources. On the demand side, the firm faces less uncertainty by producing one single product, and thus the firm can estimate demand more accurately ex ante and plan production accordingly. On the supply side, the firm is able to enjoy economies of scale in production by saving the fixed cost of producing the other product.

Our view of a reduced product assortment can potentially explain the shrinking list of video games that was observed in the past few years (Wingfield 2013). In 2012, only half as many new games were released in American stores as in 2008. Video games exert strong social externality, especially in an online environment, where players are mostly attracted to the titles with the biggest pool of players. With fewer product varieties, the firms can merge budgets to create blockbusters. Though our model ignores competition, the benefits of the strategy may still be valid for a competitive market. Our analysis suggests the need to carefully evaluate the impact of social influence on consumers' adoptions. The firm may want to utilize a strategy of reduced product assortment ex ante only when consumers are sufficiently sensitive to the decisions of others.

3.2.2. Endogenous Product Assortment Decisions Under Production Postponement. Production flexibility can be another solution to cope with the greater uncertainty induced by social influence. Flexibility in production technology enables firms to respond to preorder or early sales information in a relatively short period of time. Production postponement can also be achieved through delayed differentiation. This policy of production postponement has been well studied in the operations literature in the context of demand uncertainty. For example, early sales information typically improves the accuracy of demand forecasting. The ability to quickly respond to early sales information can result in less overstocking and higher product availability. In the context of capacity planning, early sales information also helps the firm make a more informed capacity planning decision, and thus can significantly improve the firm's profit (Boyacı and Özer 2010). However, the value of production postponement has not been studied previously in the presence of social influence. Preorders or early sales can be more than a good indicator of future market prospect; it also shapes purchasing decisions of later arrivals through social influence. As a result, demand uncertainty under social influence can be significantly reduced if the firm is able to postpone production until it observes sufficient preorder or early sales information.

We seek to rigorously evaluate the value of production postponement for a two-product newsvendor under social influence. Because we assume a fixed market size, the value of production postponement in our context is completely attributable to better reaction to social influence: It is easy to see that production postponement has zero benefits when there is no social influence. We consider the case where the firm can delay production until after observing T periods of preorder sales, for some given T. This would require a sufficiently flexible manufacturing system where the firm can produce in small batches or allow consumers to preorder products before they are actually produced. Such a preorder strategy is gaining traction in the fashion industry. For example, Moda Operandi, an online fashion merchant, allows consumers to preorder "looks" straight from the runway. (In the online appendix, we investigate how the *optimal* production time depends on consumers' sensitivity to others' decisions and show in an example that the optimal production time can be highly sensitive to consumers' sensitivity to others' decisions.)

We assume that Assumption 1 holds, i.e., the market is completely covered when there exists no social influence. Thus, the market is also completely covered under social influence. The profit function of the firm is the same as (5), except that the distribution of X^* no longer follows the results in Lemma 3, given the preorders from the first T consumers. We summarize the updated distribution of X^* with preorder information as follows. The obstinate scenario is trivial because the market shares of the two products can be accurately predicted in the limit as 1/2 even without any preorder information. The results under the boundary and impressionable scenarios directly follow from Zhu (2009, Theorem 2.1.4) and Auriol and Benaim (2000, Theorem 2), respectively.

Lemma 5 (Limit of Aggregate Demand with Preorders). *Given that $N_T = n < T$, where $T > 2$, the distribution of X^* is*

(i) *a Bernoulli distribution with support $\{0, 1\}$ under the impressionable scenario* (*in particular, there exists a positive number k such that*

$$P(X^* = 1)\begin{cases} \geq g(T, k) & \text{if } n/T > 1/2, \\ \leq 1 - g(T, k) & \text{otherwise}, \end{cases}$$

where $g(T, k) \equiv (1 - 2/(kT))e^{-k/T}$);

(ii) *a* Beta$(n, T - n)$ *distribution under the boundary scenario;*

(iii) *the constant* 1/2 *under the obstinate scenario.*

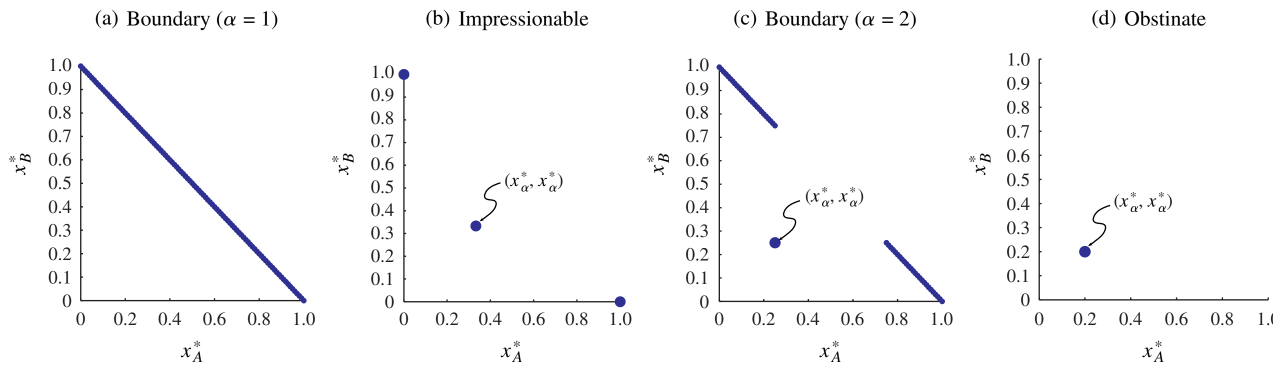
Lemma 5 implies that the level of demand uncertainty facing the firm decreases in the production delay under the impressionable and boundary scenarios. Consider first the impressionable scenario. From Lemma 3, we see that without preorder information, one of the two products will be extremely popular, and each of the two products has an equal chance of becoming the hit. With preorder information, the firm would naturally expect product A to become the popular one if the fraction of consumers purchasing product A at the end of period T (i.e., X_T) is greater than 1/2, and would expect product B to become the popular one otherwise. The trend may still be reversed by the later arrivals, although the likelihood of a reversal is no more than $1 - g(T, k)$, which is decreasing in T; that is, uncertainty still exists for future sales due to the path-dependent nature of the process, although the likelihood of a flip is decreasing with more preorder information. Similarly, under the boundary scenario the level of demand uncertainty decreases with the production delay, because the variance of X^* under T periods of delay is $n(T - n)/(T^2(T + 1)) \leq 1/(4(T + 1))$ (when $T > 2$), which is less than 1/12, the variance of the uniform distribution on $[0, 1]$.

After collecting preorder information, the firm may find it unprofitable to manufacture both products. Next, we investigate under what conditions one product may be dropped, when the firm has the option of not producing a product after observing T periods of preorders.

Proposition 4 (Product Assortment Under Production Postponement). *Given that $N_T = n < T$ where $T > 2$,*

(i) *under the impressionable scenario, if $n/T < 1/2$, the firm would choose not to produce product A if $p(1 - g(T, k)) \leq c + K$; if $n/T \geq 1/2$, the firm would not produce product B if $p(1 - g(T, k)) \leq c + K$;*

Figure 2 (Color online) An Illustration of Potential Equilibria When the Market is Not Guaranteed to be Covered Completely



(ii) *under the boundary scenario, the firm would choose not to produce product A if and only if* $((p\cdot n)/T)\cdot I_{I^{-1}_{1-c/p}(n,T-n)}(n+1,T-n)\le K$, *and the firm would not produce product B if and only if* $((p\cdot n)/T)[1-I_{I^{-1}_{c/p}(n,T-n)}(n+1,T-n)]\le K$, *where* $I_x(\alpha,\beta)$ *and* $I_x^{-1}(\alpha,\beta)$ *denote the cumulative distribution function and inverse cumulative distribution function of* $Beta(\alpha,\beta)$, *respectively.*

The demand prediction becomes more accurate with preorder information. If a product is not likely to become popular such that the contribution from selling the product is able to cover the fixed production cost K, then the firm is better off dropping the product from its product line. Compared with the strategy of reducing the product assortment ex ante (see §3.2.1), the viability of doing so under production postponement does not require consumers' deriving high surplus from the products (as in the value-driven regime) or being highly sensitive to others' decisions (as in the influence-driven regime). However, it does require a flexible manufacturing system such that products can be produced and delivered to markets quickly.

4. Incomplete Coverage

In this section, we relax Assumption 1 and investigate how the impact of social influence changes when $0<r-p<1/2$, i.e., the market is not completely covered in the absence of social influence. We ignore the case when $r-p\le 0$ because it is trivial—nobody purchases the products. The probability that a consumer arriving at period $t+1$ will purchase product i is still specified by (2) given that $\mathbf{X}_t=\mathbf{x}$. Denote by (X_A^*, X_B^*) the random limit the process converges to. The proposition below summarizes the limit when $0<r-p<1/2$.

Proposition 5 (Limit of Aggregate Demand Under Incomplete Coverage). *Let* x_α^* *denote the smallest root of* $r+x^\alpha-p=x$, *if it exists. Under the boundary and obstinate scenarios, the support of* (X_A^*, X_B^*) *is the same as that in Lemma 3 if and only if* $r-p\ge 1/2-(1/2)^\alpha$. *Under the impressionable scenario, the support of* (X_A^*, X_B^*) *is the same as that in Lemma 3 if and only if* $r-p\ge \alpha^{1/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)}$. *Otherwise,*

(i) *under the impressionable scenario,* (X_A^*, X_B^*) *converges to* $(0,1)$, *or* $(1,0)$, *or* (x_α^*, x_α^*);

(ii) *under the boundary scenario, when* $\alpha=2$, (X_A^*, X_B^*) *converges to either* $(x^*, 1-x^*)$, *for some* $x^*\in[0,(1-\sqrt{1-4(r-p)})/2]\cup[(1+\sqrt{1-4(r-p)})/2,1]$, *or* (x_α^*, x_α^*);

(iii) *under the obstinate scenario, the process converges to a unique equilibrium* (x_α^*, x_α^*).

Potential equilibria when the market is not covered completely are illustrated in Figure 2. Under the impressionable scenario, besides the two equilibria where either one product captures the entire market, another equilibrium emerges where each product covers x_α^* share of the market. Under the boundary scenario, we need to differentiate between when $\alpha=1$ and when $\alpha=2$. The value of α affects the absolute value of a consumer's utility and thus whether or not a consumer purchases. When $\alpha=1$, the market is completely covered, and the process can converge to any point between 0 and 1. When $\alpha=2$, the complete coverage is realized if and only if one product attracts the majority of the entire population, i.e., the market share of the popular product is greater than or equal to $(1+\sqrt{1-4(r-p)})/2$, for any $r-p\in(0,1/4)$. Under the obstinate scenario, instead of each product covering half of the entire market, each product only covers a smaller x_α^* share of the market.

The result indicates a *positive* impact of social influence—an increase in the market size. Interestingly, the entire market will be fully covered under social influence, even if it is only partially covered without social influence. This happens when $r-p\in[1/2-(1/2)^\alpha, 1/2)$ under the boundary and obstinate scenarios, and when $r-p\in[\alpha^{1/(1-\alpha)}-\alpha^{\alpha/(1-\alpha)}, 1/2)$ under the impressionable scenario. The combined *market expansion* from the two products, due to social influence, can be as large as $(1/2)^{\alpha-1}$, which is substantial when α is small.

5. Conclusion

Social influence is a double-edged sword. On the positive side, the total or individual product market size increases due to an increased social utility; on the negative side, the demand faced by the firm is more uncertain. If the firm already has a desired coverage in the absence of social influence, the negative side of social influence dominates, resulting in a lower profit for the firm. We propose three solutions for the firm: influencer recruitment and an endogenized product assortment either before demand realization or under production postponement.

Our proposed consumer choice model provides an *endogenous* way to capture how fashion trends proliferate. The multiperiod framework can be used to study many operational problems that previously considered "fashion" as exogenous demand uncertainty. For example, the model can be used to study dynamic pricing when there is social influence.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mnsc.2015.2160.

Acknowledgments

The authors thank Martin Lariviere, the associate editor, and two anonymous reviewers for their guidance and constructive comments. The authors also thank Saibal Ray for his valuable comments. Financial support from the Natural Sciences and Engineering Research Council of Canada [Grant 261986, RGPIN-2015-06757] is gratefully acknowledged.

Appendix. Proofs

Proof of Lemmas. See the online appendix. □

Proof of Proposition 1. When there exists no social influence, we know that the market share of each product converges to 1/2 in the limit. Thus, the firm obtains the highest profit ordering $q_A = q_B = 1/2$, with an optimal profit of $p - c - 2K$.

Next consider the case with social influence. Under the obstinate scenario, the market share of product A, X^*, also converges to 1/2 in the limit, as if consumers are not influenced by the decisions of others'. Thus, the firm makes the same profit as that in the case with no social influence.

Under the boundary scenario and the impressionable scenario, the aggregate demand X^* is a random variable as shown in Lemma 3, and we will show that the firm's profit given by (5) is always no greater than $p - c - 2K$. Consider first the case when $q_A + q_B > 1$. Then we have $pE[\min\{q_A, X^*\} + \min\{q_B, 1-X^*\}] - [c(q_A+q_B) + 2K] \le pE[X^* + (1-X^*)] - c - 2K = p - c - 2K$, where the inequality is due to the first-order stochastic dominance of $X^* + (1-X^*)$ over $\min\{q_A, X^*\} + \min\{q_B, 1-X^*\}$, and $q_A + q_B > 1$.

Next consider the case when $q_A + q_B \le 1$. We have $pE[\min\{q_A, X^*\} + \min\{q_B, 1 - X^*\}] - [c(q_A + q_B) + 2K] \le (p-c)(q_A+q_B) - 2K \le p - c - 2K$, where the first inequality is due to the first-order stochastic dominance of $q_A + q_B$ over $\min\{q_A, X^*\} + \min\{q_B, 1-X^*\}$, and the second inequality is due to $q_A + q_B \le 1$. Thus, we obtain the announced result. □

Proof of Proposition 2. Because $1/2(x^\alpha - (1-x)^\alpha + 1) \in [0,1]$, $\forall x \in [0,1]$ and $\beta \in [0,1]$, we can rewrite the updating probability function, as shown in (6), as

$$f(x) = \frac{x^\alpha - (1-x)^\alpha + 1}{2} + \gamma \min\left\{1 - \frac{x^\alpha - (1-x)^\alpha + 1}{2}, \frac{(1-\beta)p}{2}\right\}.$$

Let $f_1(x) = (x^\alpha - (1-x)^\alpha + 1)/2 + \gamma((1-\beta)p/2)$, and $f_2(x) = (1-\gamma)((x^\alpha - (1-x)^\alpha + 1)/2) + \gamma$. When $(1-\beta)p > 2$, $(1-\beta)p/2 > 1 - (x^\alpha - (1-x)^\alpha + 1)/2$, $\forall x \in [0,1]$, and thus $f(x)$ is reduced to $f_2(x)$. When $(1-\beta)p \in [0,2]$, there exists an $x' \in [0,1]$, such that $f(x) = f_1(x)$, $\forall x < x'$, and $f(x) = f_2(x)$, $\forall x \ge x'$, where x' is the unique solution of $x^\alpha - (1-x)^\alpha = 1 - (1-\beta)p$. Notice that $(1-\beta)p \ge 1$ implies $x' \le 1/2$.

As the mapping f: $[0,1] \to [0,1]$ is monotone nondecreasing, the existence of the fixed points, i.e., $f(x) = x$, is guaranteed by Tarski's (1955) fixed point theorem. Next, we show that the fixed point is unique and stable when either condition is satisfied.

Consider first when $\alpha = 1$ or 2. Equation (6) is reduced to $f(x) = x + \gamma \min\{1-x, ((1-\beta)p)/2\}$. The updating probability function $f(x)$ crosses the diagonal line only once at $x = 1$, when $1 - \beta > 0$, and $f'(1) = 1 - \gamma < 1$ for all $\gamma > 0$.

When $\alpha \in (1,2)$, it is easy to verify that both $f_1(x)$ and $f_2(x)$ are convex when $x \in [0,1/2]$ and concave when $x \in [1/2,1]$. The point $x = 1$ is guaranteed to be an equilibrium because $f(1) = 1$ and $f'(1) < 1$. To this end, we only need to show that there exists no fixed point in $[0,1)$ when condition (i) is satisfied. Consider first when $(1-\beta)p \in [1,2]$, i.e., $x' \le 1/2$. In this case, $f(x) = \min\{f_1(x), f_2(x)\}$ for all $x \in [0,1/2]$, and $f(x)$ is reduced to $f_2(x)$ for all $x \in [1/2,1]$. Let $g_i(x) = f_i(x) - x$, $i = 1, 2$. As a linear combination of a concave function and a linear function, $g_2(x)$ is concave as well when $x \in [1/2,1]$, and its minimum is reached at either 1/2 or 1. We have $g_2(1/2) = \gamma/2 > 0$ due to $\gamma > 0$, and $g_2(1) = 0$. Thus, we conclude that $f(x)$ does not have a fixed point within $[1/2,1)$.

Next we derive the conditions under which there exists no equilibrium in $[0,1/2]$. As $f_i(x)$ is convex within $[0,1/2]$, $g_i(x)$ is convex as well when $x \in [0,1/2]$. The minimum of $g_1(x)$ is reached at $g_1'(x) = 0 \Leftrightarrow x^{\alpha-1} + (1-x)^{\alpha-1} = 2/\alpha$. Denote by $\tilde{x}_1$ the solution of $x^{\alpha-1} + (1-x)^{\alpha-1} = 2/\alpha$. It is easy to verify that $x^{\alpha-1} + (1-x)^{\alpha-1} \in [1, 2^{2-\alpha}]$ for all $x \in [0,1/2]$. Because $2^{2-\alpha} > 2/\alpha$ for all $\alpha \in (1,2)$, $\tilde{x}_1$ is within the region $[0,1/2]$. Next we show that under condition (i), $g_1(\tilde{x}_1) > 0$. Because $\tilde{x}_1 \in [0,1/2]$, we have $\tilde{x}_1^\alpha + (1-\tilde{x}_1)^\alpha \le \tilde{x}_1^{\alpha-1} + (1-\tilde{x}_1)^{\alpha-1} = 2/\alpha$, which is equivalent to $\tilde{x}_1^\alpha - (1-\tilde{x}_1)^\alpha \ge 2\tilde{x}_1^\alpha - 2\alpha$. Thus, $g_1(\tilde{x}_1) \ge \tilde{x}_1^\alpha - 1/\alpha + 1/2 + \gamma((1-\beta)p/2) - \tilde{x}_1 \ge \tilde{x}_1^2 - 1/\alpha + 1/2 + \gamma((1-\beta)p/2) - \tilde{x}_1 \ge (\tilde{x}_1 - 1/2)^2 + 1/4 - 1/\alpha + \gamma((1-\beta)p/2)$. The second inequality is due to $\alpha \in (1,2)$. Thus under the condition that $\gamma(1-\beta)p \ge 2/\alpha - 1/2$, $g_1(\tilde{x}_1)$ is guaranteed to be greater than 0, and there exists no fixed point for $f_1(x)$ within $[0,1/2]$. Next we derive the conditions under which there exists no fixed point within $[0,1/2]$ for $f_2(x)$. The minimum of $g_2(x)$ is reached at $g_2'(x) = 0 \Leftrightarrow x^{\alpha-1} + (1-x)^{\alpha-1} = 2/((1-\gamma)\alpha)$. Let us denote the solution of $x^{\alpha-1} + (1-x)^{\alpha-1} = 2/((1-\gamma)\alpha)$ as $\tilde{x}_2$. So if $\tilde{x}_2$ is outside the interval $[0,1/2]$ (in this case, the minimum of

$g_2(x)$ is reached at either 0 or 1/2, which are both greater than 0), then $f_2(x)$ does not intersect with the diagonal line within $[0, 1/2]$. Because $x^{\alpha-1} + (1-x)^{\alpha-1} \in [1, 2^{2-\alpha}]$ for all $x \in [0, 1/2]$, one sufficient condition for $\tilde{x}_2 \notin [0, 1/2]$ is that $2/((1-\gamma)\alpha) > 2^{2-\alpha}$, which is equivalent to $\gamma > 1 - 2^{\alpha-1}/\alpha$. When $(1-\beta)p < 1$, $f(x) = f_1(x)$ for all $x \in [0, 1/2]$, and $f(x) = \min\{f_1(x), f_2(x)\}$ for all $x \in [1/2, 1]$. Thus, a unique stable equilibrium is guaranteed under the condition that $\gamma(1-\beta)p \geq 2/\alpha - 1/2$. Similarly, when $(1-\beta)p > 2$, $f(x)$ is reduced to $f_2(x)$ for all $x \in [0, 1]$, and the condition for a unique stable equilibrium is given by $\gamma > 1 - 2^{\alpha-1}/\alpha$. Summarizing the conditions under the two scenarios, we thus obtain the announced result. □

Proof of Proposition 4. Given that the market is completely covered, the profit from producing the two products is given by $\pi_A(q_A) = pE[\min\{q_A, X^*\}] - c \cdot q_A - K$, $\pi_B(q_B) = pE[\min\{q_B, 1 - X^*\}] - c \cdot q_B - K$. Under the impressionable scenario, the demand in the limit X^* follows a Bernoulli distribution with support $\{0, 1\}$, as shown in Lemma 5. Consider first when $n/T > 1/2$. Suppose the optimal production quantity of product A is given by q_A^*. Because $P(X^* = 1) \leq 1 - g(T, k)$, the profit from producing q_A^* market share of product A satisfies $\pi_A(q_A^*) \leq p[(1 - g(T, k)) \cdot q_A^* + g(T, k) \cdot 0] - c \cdot q_A^* - K = [p(1 - g(T, k)) - c] \cdot q_A^* - K$. Because $q_A^* \in [0, 1]$, the optimal profit from producing product A is no greater than 0 if $p(1 - g(T, k)) \leq c + K$, and thus the firm would not produce product A. Similarly, when $n/T \geq 1/2$ and $p(1 - g(T, k)) \leq c + K$, the firm would obtain a nonpositive profit from producing product B.

Under the boundary scenario, Lemma 5 shows that demand of product A, i.e., X^*, follows a $Beta(n, T-n)$ distribution. To streamline the proof, we first prove one intermediate result and summarize it in the lemma below. Assume that X^* follows a general continuous probability distribution $\Phi(\cdot)$ with density function $\phi(\cdot)$. With constant marginal cost c, the optimal production quantities of the two products are given by the following lemma. (See the online appendix for a proof.)

Lemma 6. *Given the constant marginal cost c, the optimal production quantities are given by $q_A^* = \Phi^{-1}(1 - c/p)$ and $q_B^* = 1 - \Phi^{-1}(c/p)$.*

Given that X^* follows a $Beta(n, T-n)$ distribution, we have $q_A^* = I_{1-c/p}^{-1}(n, T-n)$ and $q_B^* = 1 - I_{c/p}^{-1}(n, T-n)$. Plugging q_A^* into the profit function from producing product A, we have

$$\begin{aligned}
\pi_A(q_A^*) &= pE[\min\{q_A^*, X^*\}] - c \cdot q_A^* - K \\
&= p\left[\int_0^{q_A^*} x \cdot \frac{x^{n-1}(1-x)^{T-n-1}}{B(n, T-n)}\, dx \right. \\
&\qquad \left. + \int_{q_A^*}^1 q_A^* \cdot \frac{x^{n-1}(1-x)^{T-n-1}}{B(n, T-n)}\, dx\right] - c \cdot q_A^* - K \\
&= \frac{p}{B(n, T-n)} \int_0^{q_A^*} x^n (1-x)^{T-n-1}\, dx - K \\
&= \frac{p \cdot n}{T} I_{q_A^*}(n+1, T-n) - K \\
&= \frac{p \cdot n}{T} I_{I_{1-c/p}^{-1}(n, T-n)}(n+1, T-n) - K.
\end{aligned}$$

The firm will not produce product A if the optimal profit $\pi_A(q_A^*)$ is nonpositive. Similarly, we can derive the optimal profit from producing product B, which is given by $\pi_B(q_B^*) = ((p \cdot n)/T)[1 - I_{I_{c/p}^{-1}(n, T-n)}(n+1, T-n)] - K$. The firm will produce product B if and only if the optimal profit is positive, and thus we obtain the announced result. □

Proof of Proposition 5. Under the assumption of incomplete coverage without social influence, i.e., $0 < r - p < 1/2$, the random limits of the process can be divided into two categories: either the market is completely covered under social influence or there exists no overlap in the market covered by the two products, i.e., each product acts like a local monopoly. Under the first case, the market equilibrium (x_A^*, x_B^*) is given by

$$\frac{1 + x_A^\alpha - x_B^\alpha}{2} = x_A, \quad \frac{1 - x_A^\alpha + x_B^\alpha}{2} = x_B, \tag{8}$$

subject to the constraints $(1 + x_A^{*\alpha} - x_B^{*\alpha})/2 \leq \Pi_{[0,1]}[r + x_A^{*\alpha} - p]$ and $(1 - x_A^{*\alpha} + x_B^{*\alpha})/2 \leq \Pi_{[0,1]}[r + x_B^{*\alpha} - p]$. The solutions of equation set (8) are summarized in Lemma 3. Under the boundary scenario, the solution is $(x^*, 1 - x^*)$, $\forall x^* \in [0, 1]$. When $\alpha = 1$, plugging $(x^*, 1 - x^*)$ into the constraints, we have $(1 + x^* - (1 - x^*))/2 = x^* \leq \Pi_{[0,1]}[r - p + x^*]$ and $(1 - x^* + (1 - x^*))/2 = 1 - x^* \leq \Pi_{[0,1]}[r - p + 1 - x^*]$. Because $r - p > 0$, the constraints are satisfied for all $x^* \in [0, 1]$, and thus the support of (X_A^*, X_B^*) is given by $\{(x^*, 1 - x^*), \forall x^* \in [0, 1]\}$. When $\alpha = 2$, the solutions of equation set (8) need to satisfy the following constraints: $(1 + x^{*2} - (1 - x^*)^2)/2 = x^* \leq \Pi_{[0,1]}[r - p + x^{*2}]$ and $(1 - x^{*2} + (1 - x^*)^2)/2 = 1 - x^* \leq \Pi_{[0,1]}[r - p + (1 - x^*)^2]$. The two inequalities are satisfied for all $x^* \in [0, 1]$ when $r - p \geq 1/4$. Otherwise, the constraints are satisfied for $x^* \in [0, (1 - \sqrt{1 - 4(r - p)})/2] \cup [(1 + \sqrt{1 - 4(r - p)})/2, 1]$.

Under the impressionable scenario, Lemma 3 shows that the process converges to either $(0, 1)$ or $(1, 0)$. It is easy to verify that both $(0, 1)$ and $(1, 0)$ satisfy the constraints. Under the obstinate scenario, the process is shown to converge to $(1/2, 1/2)$. Plugging $(1/2, 1/2)$ into the first constraint, we have $(1 + x_A^{*\alpha} - x_B^{*\alpha})/2 = 1/2 \leq \Pi_{[0,1]}[r - p + (1/2)^\alpha] \Leftrightarrow r - p \geq 1/2 - (1/2)^\alpha$. Plugging $(1/2, 1/2)$ into the second constraint yields the same condition. Consequently, $(1/2, 1/2)$ constitutes an equilibrium under the obstinate scenario if and only if $r - p \geq 1/2 - (1/2)^\alpha$.

Next consider the random limits when there exists no overlap in the coverage of the two products. In this case, random limit (x_A^*, x_B^*) is given by $\Pi_{[0,1]}[r + x_A^\alpha - p] = x_A$, $\Pi_{[0,1]}[r + x_B^\alpha - p] = x_B$, subject to the constraints $\Pi_{[0,1]}[r + x_A^{*\alpha} - p] < (1 + x_A^{*\alpha} - x_B^{*\alpha})/2$ and $\Pi_{[0,1]}[r + x_B^{*\alpha} - p] < (1 - x_A^{*\alpha} + x_B^{*\alpha})/2$. Similar to the proof of Lemma 4, we can show that (x_α^*, x_α^*) constitutes an equilibrium when $\alpha > \bar{\alpha}$, where x_α^* is the smaller root of $r + x^\alpha - p = x$, and $\bar{\alpha}$ is given by $\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)} = r - p$; that is $r + x^\alpha - p = x$ has two solutions within $[0, 1]$ if and only if $\alpha > \bar{\alpha}$. Plugging (x_α^*, x_α^*) into the first constraint, we have $\Pi_{[0,1]}[r + x_\alpha^{*\alpha} - p] = x_\alpha^* < (1 + x_\alpha^{*\alpha} - x_\alpha^{*\alpha})/2 = 1/2$. Let $g(x) = r + x^\alpha - p - x$. $x_\alpha^* < 1/2$ if and only if either $g(1/2) < 1/2$ or $g(1/2) \geq 1/2$ and $\arg\min_x g(x) < 1/2$. Notice that $g(1/2) = r + (1/2)^\alpha - p < 1/2 \Leftrightarrow r - p < 1/2 - (1/2)^\alpha$. It is easy to verify that $g(x)$ is convex for all $\alpha \geq 1$, and $\arg\min_x g(x) = (1/\alpha)^{1/(\alpha-1)}$. Thus, we have that $\arg\min_x g(x) < 1/2$ is equivalent to $\alpha \in (1, 2)$. Because

$\alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)} \geq 1/2 - (1/2)^\alpha$, $\forall\, \alpha \geq 1$, (x_α^*, x_α^*) constitutes as an equilibrium if and only if either $r - p < 1/2 - (1/2)^\alpha$ and $\alpha \in \{1\} \cup [2, +\infty)$ or $r - p < \alpha^{1/(1-\alpha)} - \alpha^{\alpha/(1-\alpha)}$ and $\alpha \in (1, 2)$. Thus, we obtain the announced result. □

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