Online Appendix to
"Surge Pricing and Two-Sided Temporal Responses in Ride-Hailing"

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Proof of Lemma 1. We consider two cases: $p_1 \geq \frac{p_2}{\alpha}$ and $p_1 \leq \frac{p_2}{\alpha}$.

Case 1: $p_1 \geq \frac{p_2}{\alpha}$. For the riders with a valuation $v \leq \frac{p_2}{\alpha}$, they do not try to take a ride in both periods. We next study the riders with a valuation $v \geq \frac{p_2}{\alpha}$. Note that these customers, if left in Period 2, always try to take a ride in Period 2. Let U_R denote his utility of trying to take a ride immediately and U_W denote the utility of waiting for Period 2. It is easy to see $U_R = \rho_1^r(v-p_1) + (1-\rho_1^r)\rho_2^r(\alpha v - p_2)$ and $U_W = \rho_2^r(\alpha v - p_2)$. It follows that $U_R \geq U_W$, if and only if $v \geq \frac{p_1 - \rho_2^r p_2}{1-\alpha\rho_2^r}$. By $p_1 \geq \frac{p_2}{\alpha}$, we have $\frac{p_1-\rho_2^r p_2}{1-\alpha\rho_2^r} \geq p_1$.

Case 2: $p_1 \leq \frac{p_2}{\alpha}$. We consider two separate scenarios.

- $v \geq \frac{p_2}{\alpha}$. Similar to Case 1, we have $U_R \geq U_W$, if and only if $v \geq \frac{p_1-\rho_2^r p_2}{1-\alpha\rho_2^r}$. Moreover, by $p_1 \leq \frac{p_2}{\alpha}$, it follows that $\frac{p_1-\rho_2^r p_2}{1-\alpha\rho_2^r} \leq p_1 \leq \frac{p_2}{\alpha}$. Therefore, customers with $v \geq \frac{p_2}{\alpha}$ try to take a ride immediately in period 1, and if they are not matched, they will continue trying to take a ride in Period 2.
- $v < \frac{p_2}{\alpha}$. These riders will never try to take a ride in Period 2. If $v \geq p_1$, they will try to take a ride in period 1; otherwise if $v < p_1 < \frac{p_2}{\alpha}$, the riders will not take a ride in both periods. Q.E.D.

Proof of Proposition 1. If no transaction takes place in Period 2, then the platform's profit can be expressed as $\pi = \min\{(1-p)r, 1\}p$. It is easy to check that the optimal solution is $p^* = 1/2$ with $\pi^* = r/4$ if $r \leq 2$; otherwise, $p^* = 1 - 1/r$ with $\pi^* = 1 - 1/r$. In addition, if $r \leq 2$ and $p^* = 1/2$, the highest valuation for the left riders is 1/2. Therefore, to guarantee no transaction in Period 2, we need the condition $c/\gamma \geq 1/2\alpha$. Similarly, when $r \geq 2$ and $p^* = 1-1/r$, we need the condition $c/\gamma \geq \alpha(1-1/r)$. Q.E.D.

Proof of Lemma 2. (i) $q(p_2)$ is decreasing in p_2 when $p_2 \geq \alpha\underline{v}$ and $p_2 \leq \alpha\underline{v}$. Moreover, $q(p_2)$ is continuous at $p_2 = \alpha\underline{v}$. Therefore, the desired result directly follows. (ii) We consider two separate cases. If $q(p_2) \leq y + d$, then obviously we have $\rho_2^r = 1$ for any p_2. If $q(p_2) \geq y + d$, then (2) is reduced to $\pi_2^* = \max_{p_2}(1-\gamma)(y+d)p_2$, subject to $q(p_2) \geq y + d$. It is easy to see that p_2^* must satisfy that $q(p_2^*) = y + d$, which also implies $\rho_2^r = 1$. Therefore, in both cases, we have $\rho_2^r = 1$. Q.E.D.

Proof of Lemma 3. As shown in Lemma 2, it suffices to only consider the scenario $q(p_2) \leq d + y$. By (1), we need to consider two cases: $\frac{p_2}{\alpha} \leq \underline{v}$ and $\underline{v} \leq \frac{p_2}{\alpha} \leq 1$. For each case, we compute the corresponding optimal profit, and then we compare the profits and find the optimal solutions.

Case 1: $\frac{p_2}{\alpha} \le \underline{v}$, i.e., $p_2 \le \alpha\underline{v}$. In this case, we have $q(p_2) = (1-\underline{v})(1-\rho_1^r)r + (\underline{v} - \frac{p_2}{\alpha})r$. By Lemma 2(ii), the problem of the platform in Period 2 in this case can be described as

$$\pi_2^{1*} = \max_{p_2} (1-\gamma)[(1-\underline{v})(1-\rho_1^r) + (\underline{v} - \frac{p_2}{\alpha})]rp_2 \tag{EC.1}$$

$$\text{s.t.} \qquad (1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r} \le \frac{p_2}{\alpha} \le \underline{v}. \tag{EC.2}$$

where π_2^{1*} is the optimal profit of Case 1 in Period 2.

We next focus on the scenario $(1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r} \le \underline{v}$, i.e., $(1-\underline{v})(1-\rho_1^r)r \le d+y$. Note that if $(1-\underline{v})(1-\rho_1^r)r \ge d+y$, which implies that the total number of available drivers is no larger than that of high-valuation riders who tried to buy in period 1, then the platform has no incentive to set a price lower than $\alpha\underline{v}$. Define $m_1(p_2) \equiv (1-\gamma)[(1-\underline{v})(1-\rho_1^r) + (\underline{v} - \frac{p_2}{\alpha})]p_2 r$. Let $m_1'(p_2) = 0$. We have $p_2 = \frac{\alpha}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}] \equiv p_2^1$. Therefore,

- if $\frac{p_2^1}{\alpha} \ge \underline{v}$, i.e., $(1-\underline{v})(1-\rho_1^r) \ge \underline{v}$, then $p_2^* = \alpha\underline{v}$. Because the optimal decision is taken at the boundary point $p_2^{1*} = \alpha\underline{v}$, the profit in this subcase must be no more than that in Case 2.
- If $(1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r} \le \frac{p_2^1}{\alpha} \le \underline{v}$, i.e., $(1-\underline{v})(1-\rho_1^r) \le \underline{v}$ and $\frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r \le d+y$, then $p_2^{1*} = p_2^1$ and $\pi_2^{1*} = \frac{1}{4}(1-\gamma)[(1-\underline{v})(1-\rho_1^r) + \underline{v}]^2\alpha r$.
- If $(1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r} \ge \frac{p_2^1}{\alpha}$, i.e., $\frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r \ge d+y$, then $p_2^{1*} = [(1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r}]\alpha$ and $\pi_2^{1*} = (1-\gamma)(d+y)[(1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r}]\alpha$.

Case 2: $\underline{v} \le \frac{p_2}{\alpha} \le 1$. In this case, we have $q(p_2) = (1-\frac{p_2}{\alpha})(1-\rho_1^r)r$. Then the problem of the platform in Period 2 in this case can be described as

$$\pi_2^{2*} = \max_{p_2} (1-\gamma)(1-\frac{p_2}{\alpha})(1-\rho_1^r)rp_2 \tag{EC.3}$$

$$\text{s.t.} \qquad \max\{1 - \frac{d+y}{(1-\rho_1^r)r}, \underline{v}\} \le \frac{p_2}{\alpha} \le 1 \tag{EC.4}$$

where π_2^{2*} is the optimal profit in Case 2. Define $m_2(p_2) \equiv (1-\gamma)(1-\frac{p_2}{\alpha})(1-\rho_1^r)rp_2$. Let $m_2'(p_2) = 0$. We have $p_2 = \frac{\alpha}{2}$. Therefore, the optimal solution to (EC.3), denoted as p_2^{2*}, can be expressed as $p_2^* = \max\{1 - \frac{d+y}{(1-\rho_1^r)r}, \underline{v}, \frac{1}{2}\}\alpha$. Specifically,

- when $\underline{v} \ge 1 - \frac{d+y}{r}$, i.e., $(1-\underline{v})(1-\rho_1^r)r \le d+y$: if $\underline{v} \le \frac{1}{2}$, we have $p_2^{2^*} = \frac{\alpha}{2}$ and $\pi_2^{2^*} = \frac{1}{4}(1-\gamma)(1-\rho_1^r)r\alpha$; otherwise, we have $p_2^{2^*} = \underline{v}\alpha$ and $\pi_2^{2^*} = (1-\underline{v})(1-\rho_1^r)\alpha\underline{v}r$.
- when $\underline{v} \le 1 - \frac{d+y}{r}$: if $\frac{1}{2} \ge 1 - \frac{d+y}{(1-\rho_1^r)r}$, i.e., $\frac{1}{2}(1-\rho_1^r)r \le d+y$, we have $p_2^{2^*} = \frac{\alpha}{2}$ and $\pi_2^{2^*} = \frac{1}{4}(1-\gamma)(1-\rho_1^r)\alpha r$; otherwise, we have $p_2^{2^*} = [1 - \frac{d+y}{(1-\rho_1^r)r}]\alpha$ and $\pi_2^{2^*} = (1-\gamma)(d+y)[1 - \frac{d+y}{(1-\rho_1^r)r}]\alpha$.

By comparing the two profits in Case 1 and Case 2, we can find the optimal solutions, i.e., $\pi_2^* = \max\{\pi_2^{1*}, \pi_2^{2*}\}$. The desired results directly follow. Q.E.D.

Proof of Proposition 2. As shown in Lemma 3, there are four possible solutions in Period 2. Since we focus on the scenario $\underline{v} \ge p_1 \ge \frac{p_2}{\alpha}$, only Solutions 3 and 4 can arise in equilibrium. To see this, note that in Solutions 1 and 2, $\frac{p_2^*}{\alpha} \geq \underline{v}$. Moreover, the platform could set a price p_1, such that $(1-\underline{v})r \geq 1$ or $(1-\underline{v})r \leq 1$. We first study the case $(1-\underline{v})r \geq 1$, and then we show that the optimal solution under this case must satisfy $(1-\underline{v})r = 1$, which implies it suffices to consider only the case $(1-\underline{v})r \leq 1$.

Suppose $(1-\underline{v})r \geq 1$. It follows that $\rho_1^r = \frac{1}{(1-\underline{v})r}$, and $(1-\underline{v})(1-\rho_1^r) + \underline{v} = 1 - \frac{1}{r}$. If Solution 3 is the optimal solution in Period 2, then $p_2^* = \frac{1}{2}(1-\frac{1}{r})\alpha$ and $\rho_2^d = \frac{1}{2}(1-\frac{1}{r})r/(d+y)$. Moreover, due to $\gamma p_2^* \rho_2^d = c$, we also have $\rho_2^d = \frac{c}{\gamma p_2^*}$. Then it follows $d + y = \frac{1}{4}(1-\frac{1}{r})^2 \alpha r \frac{\gamma}{c}$ and $\pi_2^* = \frac{1}{4}(1-\gamma)(1-\frac{1}{r})^2 \alpha r$. Then the platform's profit over two periods is given by $\pi = (1-\gamma)p_1 + \frac{1}{4}(1-\gamma)(1-\frac{1}{r})^2 \alpha r$. Moreover, since we have $\underline{v} = \frac{p_1 - p_2^*}{1-\alpha}$, it is easy to see the optimal p_1 takes the value such that $(1-\underline{v})r = 1$. If Solution 4 is the optimal solution in Period 2, by $\gamma p_2^* \rho_2^d = c$, we have $p_2^* = \frac{c}{\gamma}$. Then by $(1-\underline{v})(1-\rho_1^r) + \underline{v} = 1 - \frac{1}{r}$ and $\frac{p_2^*}{\alpha} = (1-\underline{v})(1-\rho_1^r) + \underline{v} - \frac{d+y}{r}$, we have $d + y = (1 - \frac{1}{r} - \frac{c}{\alpha\gamma})r$, and $\pi_2^* = (1-\gamma)(1-\frac{1}{r} - \frac{c}{\alpha\gamma})r\frac{c}{\gamma}$. It follows that $\pi = (1-\gamma)p_1 + (1-\gamma)(1-\frac{1}{r}-\frac{c}{\alpha\gamma})r\frac{c}{\gamma}$. Again, since $\underline{v} = \frac{p_1 - p_2^*}{1-\alpha}$, it is easy to see the optimal p_1 takes the value such that $(1-\underline{v})r = 1$. Therefore, it suffices to consider only the case $(1-\underline{v})r \leq 1$.

Suppose $(1-\underline{v})r \leq 1$ and Solution 3 is the optimal solution in Period 2. It follows that $\rho_1^r = 1$, $\rho_2^d = \frac{\underline{v}r}{2(d+y)}$, $p_2^* = \frac{1}{2}\underline{v}\alpha$, $\pi_2^* = \frac{1}{4}(1-\gamma)\alpha \underline{v}^2 r$ and $d = \frac{\alpha\gamma r}{4c}(\underline{v})^2 + (1-\underline{v})r - 1$. Then by $\underline{v} = \frac{p_1 - p_2^*}{1-\alpha}$, we have $p_1 = (1-\frac{\alpha}{2})\underline{v}$. Taking this and $\pi_2 = \frac{1}{4}(1-\gamma)\underline{v}^2 \alpha r$ into π, we have $\pi = (1-\gamma)(1-\frac{p_1}{1-\frac{\alpha}{2}})rp_1 + (1-\gamma)(\frac{p_1}{2-\alpha})^2 \alpha r$. Let $\frac{d\pi}{dp_1} = 0$, we have $p_1 = \frac{(2-\alpha)^2}{2(4-3\alpha)}$. In addition, recall that $(1-\underline{v})r \leq 1$, i.e., $p_1 \geq (1-\frac{1}{r})(1-\frac{\alpha}{2})$. Then $p_1^* = \max\{\frac{(2-\alpha)^2}{2(4-3\alpha)}, (1-\frac{1}{r})(1-\frac{\alpha}{2})\}$. Specifically, if $\frac{(2-\alpha)^2}{2(4-3\alpha)} \geq (1-\frac{1}{r})(1-\frac{\alpha}{2})$, i.e., $r \leq \frac{4-3\alpha}{2(1-\alpha)}$, we have $p_1^* = \frac{(2-\alpha)^2}{2(4-3\alpha)}$, $\underline{v} = \frac{2-\alpha}{4-3\alpha}$ and $p_2^* = \frac{(2-\alpha)}{2(4-3\alpha)}$. To guarantee $d + y \geq \frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r$ in Period 2, we need the condition $\frac{c}{\gamma} \leq \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$. Otherwise, if $r \geq \frac{4-3\alpha}{2(1-\alpha)}$, we have $p_1^* = (1-\frac{1}{r})(1-\frac{\alpha}{2})$, $\underline{v} = 1 - \frac{1}{r}$ and $p_2^* = \frac{\alpha}{2}(1-\frac{1}{r})$. To guarantee $d + y \geq \frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r$ in Period 2, we need the condition $\frac{c}{\gamma} \leq \frac{\alpha}{2}(1-\frac{1}{r})$.

Next suppose $(1-\underline{v})r \leq 1$ and Solution 4 is the optimal solution in Period 2. Since $\rho_2^d = 1$, we always have $p_2^* = \frac{c}{\gamma}$. In addition, because $p_2^* = (\underline{v} - \frac{d+y}{r})\alpha$ in Solution 4, we have $d + y = (\underline{v} - \frac{c}{\alpha\gamma})r$ and thus $\pi_2^* = (1-\gamma)(\underline{v} - \frac{c}{\alpha\gamma})r\frac{c}{\gamma}$. Taking this and $\underline{v} = \frac{p_1 - \frac{c}{\gamma}}{1-\alpha}$ into π, it follows $\pi^* = (1-\gamma)(1-\frac{p_1 - \frac{c}{\gamma}}{1-\alpha})rp_1 + (1-\gamma)(\frac{p_1 - \frac{c}{\gamma}}{1-\alpha} - \frac{c}{\alpha\gamma})r\frac{c}{\gamma}$. Let $\frac{d\pi}{dp_1} = 0$, we have $p_1 = \frac{1-\alpha}{2} + \frac{c}{\gamma}$. In addition, recall that $(1-\underline{v})r \leq 1$, i.e., $p_1 \geq (1-\frac{1}{r})(1-\alpha) + \frac{c}{\gamma}$ in this case. Then $p_1^* = \max\{\frac{1-\alpha}{2} + \frac{c}{\gamma}, (1-\frac{1}{r})(1-\alpha) + \frac{c}{\gamma}\}$. Specifically, if $\frac{1-\alpha}{2} + \frac{c}{\gamma} \geq (1-\frac{1}{r})(1-\alpha) + \frac{c}{\gamma}$, i.e. $r \leq 2$, then $p_1^* = \frac{1-\alpha}{2} + \frac{c}{\gamma}$, $\underline{v} = \frac{1}{2}$ and $p_2^* = \frac{c}{\gamma}$. To guarantee $d + y \leq \frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r$, we need the condition $\frac{c}{\gamma} \geq \frac{\alpha}{4}$. To guarantee $\underline{v} \geq p_1$, we need the condition $\frac{c}{\gamma} \leq \frac{\alpha}{2}$. If $r \geq 2$, then $p_1^* = (1-\frac{1}{r})(1-\alpha) + \frac{c}{\gamma}$, $\underline{v} = 1 - \frac{1}{r}$ and $p_2^* = \frac{c}{\gamma}$. To guarantee $d + y \leq \frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r$, we need the condition $\frac{c}{\gamma} \geq \frac{\alpha}{2}(1-\frac{1}{r})$. To guarantee $\underline{v} \geq p_1$, we need the condition $\frac{c}{\gamma} \leq \alpha(1-\frac{1}{r})$.

The drivers' surplus is given by $S_D = \gamma(p_1V_1 + p_2V_2) - dc$, where V_i is transaction volume in period i and d is the number of new drivers arrived in Period 2. Then the total social welfare is given by $W = S + S_D + \pi$, where $S = (\frac{1+\underline{v}}{2} - p_1)(1-\underline{v})r + \frac{\alpha\underline{v}-p_2}{2}(\underline{v} - \frac{p_2}{\alpha})r$ is the riders' surplus and $\pi = (1-\gamma)(p_1(1-\underline{v})r + p_2(\underline{v} - \frac{p_2}{\alpha})r)$ is the platform's profit. In addition, W can also be expressed as $W = \frac{1+\underline{v}}{2}(1-\underline{v})r + \frac{\alpha\underline{v}+p_2}{2}(\underline{v} - \frac{p_2}{\alpha})r - dc$.

Finally, we next study the overlaps regions. First, there is no overlap between SSP and GSP. The boundary is $\frac{c}{\gamma} = \frac{\alpha}{2}$ when $r \le 2$ (for SSP3 and GSP1) and $\frac{c}{\gamma} = \alpha(1-\frac{1}{r})$ (for SSP4 and GSP2). Second, there is no overlap between the SSP2 and SSP4. Third, there are indeed overlaps between SSP1 and SSP3, i.e., $\frac{\alpha}{4} \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$ and $r \le 2$. Fourth, there are overlaps between SSP1 and SSP4, i.e., $\frac{\alpha}{2}(1-\frac{1}{r}) \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$ and $2 \le r \le \frac{4-3\alpha}{2(1-\alpha)}$. Obviously, for the overlapped regions, the platform will set the price such that the maximized profit is obtained. We first provide the results on profit comparison for the overlap between SSP1 and SSP3 ($\frac{\alpha}{4} \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$ and $r \le 2$).

Lemma EC.1. *There always exists* $\beta_0 \equiv \frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} - \alpha[\frac{(2-\alpha)^2}{4(4-3\alpha)} - \frac{1-\alpha}{4}]}$ *such that SSP1 dominates SSP3 when* $\frac{\alpha}{4} \le \frac{c}{\gamma} \le \beta_0$*; otherwise when* $\beta_0 \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$*, SSP3 dominates SSP1.*

Proof of Lemma EC.1. The gap between the profits in two equilibria is given by $r[\frac{(2-\alpha)^2}{4(4-3\alpha)} - \frac{1-\alpha}{4} - \frac{c}{\gamma} + \frac{1}{\alpha}(\frac{c}{\gamma})^2]$. Let $f(\beta) \equiv \frac{1}{\alpha}\beta^2 - \beta + \frac{(2-\alpha)^2}{4(4-3\alpha)} - \frac{1-\alpha}{4}$. Note that $f(\beta)$ is convex and decreasing when $\frac{\alpha}{4} \le \beta \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$. Therefore, it suffices to show $f(\beta = \frac{\alpha}{4}) \ge 0$ and $f(\beta = \frac{\alpha(2-\alpha)}{2(4-3\alpha)}) \le 0$. Moreover, it is easy to check that $f(\beta = \frac{\alpha}{4}) = \frac{\alpha^2}{16(4-3\alpha)} \ge 0$ and$f(\beta = \frac{\alpha(2-\alpha)}{2(4-3\alpha)}) = -\frac{(1-\alpha)\alpha^2}{4(4-3\alpha)^2} \le 0$. The desired result follows from that β_0 is the solution to $f(\beta = 0)$ when $\frac{\alpha}{4} \le \beta \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$. Q.E.D.

For the overlap between SSP1 and SSP4 ($\frac{\alpha}{2}(1-\frac{1}{r}) \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$ and $2 \le r \le \frac{4-3\alpha}{2(1-\alpha)}$), we have the following results.

Lemma EC.2. *There always exists* $\beta_1(r)$ *such that SSP1 dominates SSP4 when* $\frac{\alpha}{2}(1-\frac{1}{r}) \le \frac{c}{\gamma} \le \beta_1(r)$*; otherwise when* $\beta_1(r) \le \frac{c}{\gamma} \le \frac{\alpha(2-\alpha)}{2(4-3\alpha)}$*, SSP4 dominates SSP1.*

It is possible that $\beta_1(r)$ takes the value of $\frac{\alpha(2-\alpha)}{2(4-3\alpha)}$, then SSP1 always dominates SSP4. The proof is similar that for Lemma EC.1 and thus omitted. The only difference is that the threshold β_1 depends on r. Q.E.D.

Proof of Proposition 3. Recall that $\underline{v} = p_1$, if $p_1 \le \frac{p_2}{\alpha}$. Suppose $(1-\underline{v})r \le 1$. The remaining riders at the beginning of Period 2 are those with a valuation $v \le \underline{v}$, and thus no transaction incurs in Period 2. It is easy to see the optimal solution is $p_1^* = \frac{1}{2}$ with $\pi_1^* = \frac{1}{2}$ if $r \le 2$; otherwise, $p_1^* = 1 - \frac{1}{r}$ with $\pi_1^* = 1 - \frac{1}{r}$. We next study the case $(1-\underline{v})r \ge 1$, where we have $\rho_1^r = \frac{1}{(1-\underline{v})r}$ and $y = 0$. Since we have $\frac{p_2}{\alpha} \le \underline{v}$ in Solutions 3 and 4 of Period 2, it suffices to consider Solutions 1 and 2.

Suppose Solution 1 is the optimal solution in Period 2. Since $\rho_2^d = 1$, by $\gamma p_2^* \rho_2^d = c$, we have $p_2^* = \frac{c}{\gamma}$. Moreover, since $p_2^* = [1 - \frac{d+y}{(1-\rho_1^r)r}]\alpha$, it follows that $d + y = (1 - \frac{c}{\alpha\gamma})(1 - \rho_1^r)r = (1 - \frac{c}{\alpha\gamma})(r - \frac{1}{1-\underline{v}})$, and thus $\pi_2^* = (1-\gamma)\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})(r - \frac{1}{1-\underline{v}})$. Then the total profit over two periods is given by $\pi = (1-\gamma)[p_1 + \frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})(r - \frac{1}{1-\underline{v}})]$. Let $\frac{d\pi}{dp_1} = 0$, we have $p_1 = 1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}$. In addition, recall that $(1-\underline{v})r \geq 1$, i.e., $p_1 \leq 1 - \frac{1}{r}$. Then $p_1^* = \min\{1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}, 1 - \frac{1}{r}\}$.

Specifically, if $1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})} \leq 1 - \frac{1}{r}$, i.e., $r \geq \frac{2}{\sqrt{\alpha}}$ and $\frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}} \leq \frac{c}{\gamma} \leq \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$, then $p_1^* = \underline{v} = 1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}$ and $\pi^* = (1-\gamma)[1 - 2\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})} + \frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})r]$. In addition, we have $d = (1 - \frac{p_2}{\alpha})(1 - \rho_1^r)r = (1 - \frac{c}{\alpha\gamma})(r - \frac{1}{\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}})$. To guarantee $(1-\underline{v})(1-\rho_1^r)r \geq d + y$, and $\frac{1}{2}(1-\rho_1^r)r \geq d + y$, we need the condition $\frac{c}{\gamma} \geq \frac{\alpha}{1+\alpha}$ and $\frac{c}{\gamma} \geq \frac{\alpha}{2}$, respectively. By $\frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}} \leq \frac{\alpha}{2} \leq \frac{\alpha}{1+\alpha} \leq \frac{c}{\gamma} \leq \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}} \leq \alpha$, the equilibrium with $p_1^* = 1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}$ exists when $\frac{\alpha}{1+\alpha} \leq \frac{c}{\gamma} \leq \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$ and $r \geq \frac{2}{\sqrt{\alpha}}$. If $1 - \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})} \geq 1 - \frac{1}{r}$, i.e., $\frac{c}{\gamma} \leq \frac{\alpha}{2} - \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$ or $\frac{c}{\gamma} \geq \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$ or $r < \frac{2}{\sqrt{\alpha}}$, then $p_1^* = \underline{v} = 1 - \frac{1}{r}$ and $\pi^* = 1 - \frac{1}{r}$. Note that it follows that $(1-\underline{v})r = 1$, which implies $d + y = 0$ and no transaction Period 2.

Suppose Solution 2 is the optimal solution in Period 2. Since $\rho_1^r = \frac{1}{(1-\underline{v})r}$, $\pi_2^* = \frac{1-\gamma}{4}(r - \frac{1}{1-\underline{v}})\alpha$. Then the platform's profit over two periods is given by $\pi = (1-\gamma)[p_1 + \frac{\alpha}{4}(r - \frac{1}{1-\underline{v}})]$. Let $\frac{d\pi}{dp_1} = 0$, we have $p_1 = 1 - \frac{\sqrt{\alpha}}{2}$. In addition, recall that $(1-\underline{v})r \geq 1$, i.e., $p_1 \leq 1 - \frac{1}{r}$. Then $p_1^* = \min\{1 - \frac{\sqrt{\alpha}}{2}, 1 - \frac{1}{r}\}$.

Specifically, if $1 - \frac{\sqrt{\alpha}}{2} \leq 1 - \frac{1}{r}$, i.e., $r \geq \frac{2}{\sqrt{\alpha}}$, it follows that $p_1^* = 1 - \frac{\sqrt{\alpha}}{2}$. However, since we have $p_1 \leq \frac{p_2}{\alpha}$ in this case, there is no equilibrium with $p_1 \leq \frac{p_2}{\alpha}$. If $1 - \frac{\sqrt{\alpha}}{2} \geq 1 - \frac{1}{r}$, i.e., $r \leq \frac{2}{\sqrt{\alpha}}$, we have $p_1^* = \underline{v} = 1 - \frac{1}{r}$. Again, no transaction takes place in the second period.

When $p_1 \leq \frac{p_2}{\alpha}$, riders' total surplus and total transaction volume are given by $S = \frac{1-p_1}{2} + (1 - \frac{p_2}{\alpha})(1-\rho_1^r)r \times \frac{\alpha - p_2}{2} = \frac{1}{2}\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})} + \frac{1}{2\alpha}(\alpha - \frac{c}{\gamma})^2(r - \frac{1}{\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}})$ and $V = 1 + (1 - \frac{p_2}{\alpha})(1-\rho_1^r)r = 1 + (1 - \frac{p_2}{\alpha})r - \frac{1 - p_2/\alpha}{1-p_1}$, where $(1 - \frac{p_2}{\alpha})(1-\rho_1^r)r$ and $\frac{\alpha - p_2}{2}$ are the number and average surplus of the riders in Period 2, respectively. Drivers' surplus is given by $S_D = \gamma(p_1 V_1 + p_2 V_2) - dc = \gamma(p_1 + (1 - \frac{p_2}{\alpha})(1-\rho_1^r)r \cdot p_2) - dc$, where V_i is transaction volume in period i and d is the number of new drivers arrived in Period 2. Then the total social welfare is given by $W = S + S_D + \pi$, where S is the riders' surplus and $\pi = (1-\gamma)(p_1 V_1 + p_2 V_2)$ is the platform's profit. Q.E.D.

Proof of Proposition 4. PSP equilibrium has overlap regions with GSP2 and SSP4 separately. First, we compare PSP and GSP2. The overlap regions between the two equilibria can be expressed as $\alpha(1 - \frac{1}{r}) \leq \frac{c}{\gamma} \leq \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$ and $r \geq \frac{2}{\sqrt{\alpha}}$. It is easy to see the profit gap between PSP and GSP2 is given by $\Delta\pi = (1-\gamma)(\frac{1}{r} - 2\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})} + \frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})r) = (1-\gamma)\frac{1}{r}[\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}r - 1]^2 \geq 0$. The volume gap between PSP and GSP2 is given by $\Delta V = \frac{1}{r}[(1 - \frac{c}{\alpha\gamma})r^2 + \sqrt{\frac{\gamma}{c} - \frac{1}{\alpha}}r + 1] \geq 0$.

Second, we compare Equilibria PSP and SSP4. The overlap regions are given by $\frac{\alpha}{1+\alpha} \leq \frac{c}{\gamma} \leq \alpha(1 - \frac{1}{r})$ and $r \geq \frac{2}{\sqrt{\alpha}}$. It can be easily checked that the gap of the profits (SSP4-PSP) is $(1 - \frac{1}{r})(1 -$

$\alpha) - (1 - 2\sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})})$. Then, when $r \leq \frac{1-\alpha}{\alpha - 2\sqrt{\frac{c}{\gamma}(1-\frac{c}{\gamma})}}$, the profit under PSP is larger than that under SSP4; otherwise the profit under SSP4 is larger.

It can be easily checked that the there are more new drivers attracted (d) in PSP than that in SSP4, if $\frac{c}{\gamma} \geq \frac{\alpha}{1+\alpha}$, which is also part of the conditions for PSP. Therefore, more drivers are attracted to the current region in PSP than in SSP4. Since in PSP and SSP4, no drivers are left to Period 2 and $\rho_d = 1$, then the total transaction volume can be expressed as $1 + d$, and thus the matching volume in PSP is larger.

We next show $|p_1^* - p_2^*|$ in SSP4 is larger than that PSP, i.e., $(1 - \frac{1}{r})(1 - \alpha) \geq \frac{c}{\gamma} - 1 + \sqrt{\frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})}$. Recall that in the conditions of SSP4, we have $1 - \frac{1}{r} \geq \frac{c}{\alpha\gamma}$. Therefore, it suffices to show for for $\frac{c}{\gamma} \geq \frac{\alpha}{1+\alpha}$, $[(1-\alpha)\frac{c}{\alpha\gamma} - \frac{c}{\gamma} + 1]^2 \geq \frac{c}{\gamma}(1 - \frac{c}{\alpha\gamma})$, which can be further simplified as $g(\frac{c}{\gamma}) \equiv (4\alpha^2 - 3\alpha + 1)(\frac{c}{\gamma})^2 + (2 - 5\alpha)\alpha\frac{c}{\gamma} + \alpha^2 \geq 0$. One can verify that discriminant for the quadratic function $g(\frac{c}{\gamma}) = 0$, $\delta = [(2-5\alpha)\alpha]^2 - 4\alpha^2(4\alpha^2 - 3\alpha + 1) \geq 0$ when $\alpha \leq \frac{8}{9}$. Therefore, we next show $g(\frac{c}{\gamma}) \geq 0$ for $\frac{c}{\gamma} \geq \frac{\alpha}{1+\alpha}$ when $\alpha \geq \frac{8}{9}$. It can be checked that $g(\frac{c}{\gamma})$ is increasing for $\frac{c}{\gamma} \geq \frac{\alpha}{1+\alpha}$ when $\alpha \geq \frac{8}{9}$. Moreover, $g(\frac{c}{\gamma} = \frac{\alpha}{1+\alpha}) = \frac{\alpha^2}{(1+\alpha)^2}(4 - 4\alpha) \geq 0$. This completes the proof. Q.E.D.

Proof of Proposition 5. When r is relatively small, $r \leq \frac{2}{\sqrt{\alpha}}$, the boundary line between GSP and SSP is $\frac{c}{\gamma} = \frac{\alpha}{2}$ for $r \leq 2$ and $\frac{c}{\gamma} = \alpha(1 - \frac{1}{r})$ for $r \geq 2$, both of which are increasing in α. It follows that the boundary line between GSP and SSP is raised. For $r \geq \frac{2}{\sqrt{\alpha}}$, the boundary line between PSP and GSP is $\frac{c}{\gamma} = \frac{\alpha}{2} + \sqrt{\frac{\alpha^2}{4} - \frac{\alpha}{r^2}}$, which is increasing α; and thus the boundary line between GSP and PSP is raised. Similarly, it can be checked that the boundary line between PSP and SSP $r = \frac{1-\alpha}{\alpha - 2\sqrt{\frac{c}{\gamma}(1-\frac{c}{\gamma})}}$ goes down. Q.E.D.

Proof of Proposition 6. It is easy to check that the GSP equilibria remain the same as previous. We next study the SSP and PSP equilibria. Specifically, for SSP1 and SSP2, most outcomes remain the same, including p_1, π, S $\underline{v}$, p_2. The only differences are d, ρ_2^d and the conditions. d is the solution to $\gamma\frac{1}{2}\underline{v}\alpha\frac{\frac{1}{2}\underline{v}r}{d+y} = c_0 + c_1 f(d)$, and $\rho_2^d = \frac{1}{2}\underline{v}r/(d+y)$, where y is the same as before, i.e., $y = 1 - (1 - \underline{v})r$. In addition, the conditions for SSP1 and SSP2, $d + y \geq \frac{1}{2}[(1-\underline{v})(1-\rho_1^r) + \underline{v}]r$ can be expressed as $d \geq \frac{r-3}{2} + (1 - \underline{v})r$.

For SSP3 and SSP4, since Solution 4 is the optimal solution in Period 2, we still have 1) $\rho_2^d = 1$, and $p_2^* = \frac{c_0 + c_1 f(d)}{\gamma}$; and 2) $d + y = (\underline{v} - \frac{p_2^*}{\alpha})r$ and $p_2^* = (1 - \frac{d+1}{r})\alpha$. Then it follows that d is the solution to $\frac{c_0 + c_1 f(d)}{\gamma} = (1 - \frac{d+1}{r})\alpha$. Plugging d into $p_2^* = (1 - \frac{d+1}{r})\alpha$, we can obtain p_2^*. The rest proof resembles that of Proposition 2. We have $p_1^* = \max\{\frac{1-\alpha}{2} + p_2^*, (1 - \frac{1}{r})(1 - \alpha) + p_2^*\}$. The conditions for SSP3 are given by $r \leq 2$ and $\frac{\alpha}{4} \leq p_2^* \leq \frac{\alpha}{2}$. The conditions for SSP4 are given by $r \geq 2$ and $\frac{\alpha}{2}(1 - \frac{1}{r}) \leq p_2^* \leq \alpha(1 - \frac{1}{r})$. In both equilibria, the matching volume is given by $V = (1 - \frac{p_2^*}{\alpha})r$.

For PSP, similar to the proof in Proposition 3, it suffices to consider Solutions 1 as the optimal solution in Period2. Since $\rho_2^d = 1$, by $\gamma p_2^* \rho_2^d = c_0 + c_1 f(d)$, we have $p_2^* = \frac{c_0 + c_1 f(d)}{\gamma}$. Moreover, since $p_2^* = [1 - \frac{d+y}{(1-\rho_1^r)r}]\alpha$ in Solution 1, it follows that d is the solution to $\frac{c_0+c_1 f(d)}{\gamma} = [1 - \frac{d+y}{(1-\rho_1^r)r}]\alpha$. Note that the solution is in terms of $\underline{v}$. Plugging d into $p_2^* = \frac{c_0+c_1 f(d)}{\gamma}$, one can easily obtain p_2^* (also in terms of $\underline{v}$). Recall that $\pi = (1-\gamma)(\underline{v} + dp_2^*)$ and $p_1 = \underline{v}$ in PSP. Let p_1^{PSP} be the real solution to $\frac{d\pi}{d\underline{v}} = 0$ such that p_1^{PSP} maximizes π and $0 < p_1^{PSP} < 1$. By $(1-\underline{v})r \geq 1$, we need the condition $p_1^{PSP} \leq 1 - \frac{1}{r}$. In addition, as Solution 1 is the optimal solution, we also need the conditions $(1-\underline{v})(1-\rho_1^r)r > d+y$ and $0.5(1-\rho_1^r)r > d+y$. Q.E.D.

Proof of Proposition 7. Since the firm is myopic to maximize the revenue in period 1 when setting p_1, it is easy to see that $p_1^* = 1 - \frac{1}{r}$. When setting p_2, Lemma 3 still holds with a myopic firm. In addition, by $p_1^* = 1 - \frac{1}{r}$, we have $(1-\underline{v})r \leq 1$. Thus, we only need to consider the scenario $\underline{v} \geq p_1 \geq \frac{p_2}{\alpha}$, i.e., only Solutions 3 and 4 in Lemma 3 can arise. Suppose Solution 3 is the optimal solution in Period 2. It follows that $\rho_1^r = 1$, $\rho_2^d = \frac{\underline{v}r}{2(d+y)}$, $\pi_2^* = \frac{1}{4}(1-\gamma)\alpha\underline{v}^2 r$ and $p_2^* = \frac{1}{2}\underline{v}\alpha$. By $\underline{v} = \frac{p_1 - p_2}{1-\alpha}$, we have $\underline{v} = (1-1/r)/(1-\alpha/2)$ and $p_2^* = \frac{\alpha}{2-\alpha}(1-\frac{1}{r})$. By $\gamma p_2^* \frac{(\underline{v} - p_2^*/\alpha)r}{d+y} = c$, we have $d + y = \frac{\alpha\gamma r}{4c}(\underline{v})^2$. To guarantee $d + y \geq \frac{1}{2}[(1-\underline{v})(1-\rho_r^r) + \underline{v}]r$ and $\underline{v} \leq 1$, we need the conditions $\frac{c}{\gamma} \leq \frac{\alpha}{2-\alpha}(1-\frac{1}{r})$ and $\alpha \leq \frac{2}{r}$. Suppose Solution 4 is the optimal solution in Period 2. Since $\rho_2^d = 1$, we always have $p_2^* = \frac{c}{\gamma}$. It follows that $\underline{v} = \frac{p_1-p_2}{1-\alpha} = (1-1/r-c/\gamma)/(1-\alpha)$ and $d+y = (\underline{v} - \frac{c}{\alpha\gamma})r$. To guarantee $d+y \leq \frac{1}{2}[(1-\underline{v})(1-\rho_r^r)+\underline{v}]r$, we need the condition $\frac{c}{\gamma} \geq \frac{\alpha}{2-\alpha}(1-\frac{1}{r})$. In addition, by $p_1 \geq \frac{p_2}{\alpha}$ and $\underline{v} \leq 1$, we have the conditions $\frac{c}{\gamma} \leq \alpha(1-\frac{1}{r})$ and $\alpha \leq \frac{1}{r} + \frac{c}{\gamma}$. For the above two scenarios, the firm's total revenue over two periods is expressed as $\pi = (1-\gamma)[p_1(1-\underline{v})r + p_2(\underline{v} - \frac{p_2}{\alpha})r]$, and the riders' surplus is given by $S = (\frac{1+\underline{v}}{2} - p_1)(1-\underline{v})r + (\frac{\alpha\underline{v}-p_2}{2})(\underline{v} - \frac{p_2}{\alpha})r$. As the drivers' surplus is $S_D = \gamma[p_1(1-\underline{v})r + p_2(\underline{v} - \frac{p_2}{\alpha})r] - dc$, the total social welfare is given by $W = S_D + S + \pi = \frac{1+\underline{v}}{2}(1-\underline{v})r + \frac{\alpha\underline{v}+p_2}{2}(\underline{v} - \frac{p_2}{\alpha})r - dc$.

It is also possible that $\underline{v} \geq 1$, then no transaction takes place in period 1, i.e., when 1) $\frac{c}{\gamma} \leq \frac{\alpha}{2-\alpha}(1-\frac{1}{r})$ and $\alpha \geq \frac{2}{r}$; or 2) $\frac{\alpha}{2-\alpha}(1-\frac{1}{r}) \leq \frac{c}{\gamma} \leq \alpha(1-\frac{1}{r})$. In these scenarios, at the beginning of period 2, the firm sets the price p_2 to maximize its revenue $\min\{(1-\frac{p_2}{\alpha}r, 1+d)\}p_2$. It is easy to check the optimal solution is $p_2^* = \frac{\alpha}{2}$ with $d = \frac{\alpha\gamma r}{4c} - 1$ when $\frac{c}{\gamma} \leq \frac{\alpha}{2}$ and $p_2^* = \frac{c}{\gamma}$ with $d = (1-\frac{c}{\alpha\gamma})r - 1$ when $\frac{c}{\gamma} \geq \frac{\alpha}{2}$. In these scenarios, the transaction volume, platform profit, rider surplus, driver surplus and social welfare are given by $V = (1-\frac{p_2}{\alpha})r$, $\pi = (1-\gamma)(1-\frac{p_2}{\alpha})rp_2$, $S = (1-\frac{p_2}{\alpha})r\frac{\alpha-p_2}{2}$, $S_D = \gamma(1-\frac{p_2}{\alpha})rp_2 - dc$ and $W = \frac{\alpha+p_2}{2}(1-\frac{p_2}{\alpha})r - dc$. Q.E.D.

Proof of Proposition 8. The proof of this lemma resembles that of Propositions 2 and 3. Q.E.D.

Proof of Proposition 9. Note that GSP has overlaps with SSP4 in which $(1-\underline{v})\theta r+(1-p_1^*)(1-\theta)r=1$. Therefore, the profit in SSP4 can be expressed as $\pi^{SSP4}=(1-\gamma)[p_1^*+(\underline{v}-\frac{c}{\alpha\gamma})\theta r\frac{c}{\gamma}]=(1-\gamma)\frac{(1-\frac{1}{r})(1-\alpha)+\theta\frac{c}{\gamma}(1-\frac{c}{\alpha\gamma})r}{1-\alpha+\alpha\theta}$. It can be checked that 1)when $r\geq 3-\alpha(1-\theta)$, π^{SSP4} is increasing in θ; and when $r\geq 3-\alpha(1-\theta)$, π^{SSP4} is decreasing in θ for $\frac{1-\frac{1}{r}}{2-\alpha+\alpha\theta}\leq\frac{c}{\alpha\gamma}\leq\frac{1}{r}$, and increasing in θ for $\frac{1}{r}\leq\frac{c}{\alpha\gamma}\leq 1-\frac{1}{r}$. Q.E.D.

Proof of Lemma 4. The proof of this lemma resembles that of Lemma 3. Q.E.D.

Proof of Proposition 10. The proof of this proposition resembles that of Proposition 2. Q.E.D.

Proof of Proposition 11. We consider the scenario where SSP2 and MSP4 are the outcomes of SSP and MSP, respectively. Note that the conditions for SSP2 is a subset of those for MSP4. Therefore, the conditions for this scenario are the same as those for SSP2, i.e., $r\geq\frac{4-3\alpha}{2(1-\alpha)}$ and $\frac{c}{\gamma}\leq\frac{\alpha}{2}(1-\frac{1}{r})$. The first period prices p_1^* in MSP4 and SSP2 are $1-\frac{1}{r}$ and $(1-\frac{1}{r})(1-\frac{\alpha}{2})$, respectively. The second period prices p_2^* in MSP4 and SSP2 are $\frac{\alpha}{2}$ and $\frac{\alpha}{2}(1-\frac{1}{r})$, respectively. In MSP4, the social welfare is given by $W_{MSP4}=\frac{\alpha+p_2}{2}(1-\frac{p_2}{\alpha})r-dc=\alpha r(\frac{3}{8}-\frac{\gamma}{4})+c$. The social welfare in SSP2 is given by $W_{SSP2}=1-\frac{1}{2r}+(\frac{3}{8}-\frac{\gamma}{4})\alpha r(1-\frac{1}{r})^2$. Hence, the social welfare gap can be expressed as $\Delta=W_{SSP2}-W_{MSP4}=[1-(\frac{3}{4}-\frac{\gamma}{2})\alpha](1-\frac{1}{2r})-c$. Recall that the conditions for SSP2 are $\frac{c}{\gamma}\leq\frac{\alpha}{2}(1-\frac{1}{r})$ and $r\geq\frac{4-3\alpha}{2(1-\alpha)}\geq 2$. Therefore, $\Delta\geq[1-(\frac{3}{4}-\frac{\gamma}{2})\alpha](1-\frac{1}{2r})-\frac{\alpha\gamma}{2}(1-\frac{1}{r})=(1-\frac{3\alpha}{4})(1-\frac{2}{r})+\frac{\alpha\gamma}{4r}\geq 0$. Q.E.D.