LAST (Family) NAME: ______________________________

FIRST (Given) NAME: ______________________________

STUDENT NUMBER: ______________________________

UNIVERSITY OF TORONTO

Faculty of Arts and Science

DECEMBER 2019 EXAMINATIONS

ECO2020 part II, General Equilibrium

Instructor: Marcin Pęski

Duration - 120 minutes

No Aids Allowed

Exam Reminders:

- Fill out your name and student number on the top of this page.
- Do not begin writing the actual exam until the announcements have ended and the Exam Facilitator has started the exam.
- Your answer must fit into the space provided. Use a PENCIL. Ink can't be erased if you change your mind.
- Two last pages contain the space for the rough-work. If you must use the rough-work pages for an answer, indicate this clearly by writing "Continued on last page".
- There are four questions with total worth of 100 points. Each question has the same value.
- You need to provide arguments for each answer.
- If you cannot solve one part of the problem, don't give up and try to solve the next one.
- If the question explicitly asks you to prove a result from the class, you must carefully describe the proof. Otherwise, you may use any result from the class given that you clearly state the assumptions, thesis and verify that the assumptions hold in your application.
- An advice: It is always a good idea to start each "proof" answer with writing precisely what is that you want to show. It will help you to make it precise when thinking about solution. It will also allow me to give you a tiny bit of partial credit if the rest of the answer turns out to be wrong.

Good luck!

Students must hand in all examination materials at the end.

1. (25) Consider the following social choice problem. A society collectively chooses an action $a \in A$ and a system of transfers τ_i to each individual. The transfers have to satisfy the feasibility condition $\sum \tau_i \leq 0$. An allocation is a pair (a, τ) of $a \in A$ and feasible transfers. Agent i's utility from the allocation is

$$u_i(a) + k_i \tau_i,$$

where $k_i > 0$ is some agent-specific constant.

   (a) (5) State a definition of a Pareto-optimal allocation appropriate for this environment.

(a) (20) Show (using the definition of Pareto-optimality) that if an allocation (a, τ) is Pareto-optimal, then $a \in \arg\max \sum_i \frac{1}{k_i} u_i(a)$.

2. (25) A farmland is divided by the Deep River flowing east-west. The Deep River is crossed by the Aurora Highway going north-south. In order to irrigate farms on the northern side of the Deep River, a ditch needs to be built. The farms lie along the Highway, starting from farm 1 and ending with farm I and a ditch to farm $i \leq I$ will necessarily irrigate all farms $j \leq i$. The cost of running the ditch to farm i is equal to $c_i > 0$ and it is increasing in i. The value of a coalition of farmers S is defined as the negative of the total cost of running a ditch through all farms in S, or

$$v(S) = -\max_{i \in S} c_i.$$

From now on, we consider the irrigation problem (I, v) as the TU-cooperative game with characteristic function v.

(a) (5) State the definition of a convex game.

(b) (10) Show that the above defined game is convex.

(c) (10) Suppose that the local authority proposes to build a ditch to the last farm I. To pay for it, the authority proposes to charge taxes

$$t_1 = c_1,$$
$$t_2 = c_2 - c_1,$$
$$t_3 = c_3 - c_2,$$
$$\dots$$
$$t_I = c_I - c_{I-1}.$$

Let $x_i = -t_i$ be the utility vectors associated with the above taxes. Is x in the core of the game? Justify your answer without relying on any results from the class.

(d) (5) Is there any other core allocation?

3. (25) Consider an exchange economy, with two consumers $i = A, B$ who trade on two markets $m = S, T$. There are 8 goods $1, 2..., 8$; goods $1...., 4$ are traded on market S and goods $5, ..., 8$, are traded on market T. The consumers have identical preferences

$$u(x_{i1}, ..., x_{i8}) = (x_{i1} \cdot x_{i2} \cdot ... \cdot x_{i8})^{1/8}.$$

Suppose that initial endowments of the two goods are equal to

$$\omega_A = (2, 4, 6, 8, 10, 8, 3, 8),$$
$$\omega_B = (3, 6, 9, 12, 10, 8, 3, 8).$$

(a) (5) Find an allocation in a Radner equilibrium. (Hint: I am not asking for the equilibrium prices.)

(b) (10) Explain that the allocation that you found in previous question is not Pareto-optimal. Give an example of a Pareto-improving allocation.

(c) (10) Assume again that goods are traded only on their own respective markets. However, suppose that there is another market F, on which a single financial asset can be traded. The asset can be traded in both positive and negative amounts. Each unit of a financial asset r delivers r_S units of currency in market S and $-r_T$ in market T, where $r_S, r_T > 0$. The currency later can be exchanged for goods. Find a Radner equilibrium of the economy in which the asset plays non-trivial role. Show that the equilibrium allocation Pareto improves on the allocation that you found in part 1. (Your answer may depend on a numerical calculation. In such a case, if you don't have a calculator and cannot do it by hand - don't worry about it, just clearly describe the objective as an outcome of some calculation.)

Extra space for question part (c) of Q3.

4. (25) Consider an exchange economy with I consumers, and L goods. The initial endowments are $\omega_i \in \mathbb{R}^L, \omega_{il} \gg 0$ for each i and l, and for each i, the utilitiy function u_i of agent i is strongly monotone.

   (a) (5) State the definition of a Walrasian equilibrium for this economy

(b) (5) State the definition of the core.

(c) (15) Prove (using only the definitions) that any Walrasian equilibrium allocation is in the core.

SPACE FOR ROUGH WORK ONLY. WORK HERE WILL NOT BE GRADED

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