MICROECONOMIC THEORY I
MIDTERM

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Each question has the same value. You have 120 minutes. Good luck!

(1) Answer the following questions.

   (a) Write down the consumer's expenditure minimization problem and show that the expenditure function $e(p, u)$ is concave in prices.

   See the lecture.

(b) Explain the identity

$$x\left(p, e\left(p, u\right)\right) = h\left(p, u\right),$$

where $x\left(p, w\right)$ is Walrasian demand, and $h\left(p, u\right)$ is Hicksian demand. Use the above identity to derive the Slutsky matrix equation.

See the lecture.

(2) A competitive firm employs two factors of production: labor l, and management m. We interpret m as the measure of the quality of the managers employed by the firm. The cost of m units of managerial quality is equal to $F(m)$. The profits are equal to

$$A(m) f(l) - wl - F(m),$$

where w is the price of unit of labor. Function A measures the impact of the quality management on the levels of production. We assume that both A and f are increasing.

(a) Define function g

$$g(l, m; \theta) = A(m) f(l) + \theta l - F(m).$$

Show that $g(.,.;\theta)$ is supermodular for each θ. Carefully explain the notion of lattice and order appropriate in this setting.

We use the standard two-dimensional notion of lattice on $\left\{(l, m) \in R_+^2\right\}$. In such a case, supermodularity of the production function is equivalent to showing that

$$g(\max(l, l'), \max(m, m'); \theta) + g(\min(l, l'), \min(m, m'); \theta)$$
$$\geq g(l, m; \theta) + g(l', m'; \theta).$$

There are four cases to check. For example, when $l < l'$ and $m < m'$, the difference

$$g(l', m'; \theta) - g(l, m'; \theta) - [g(l', m; \theta) - g(l, m; \theta)]$$

is equal to

$$= (A(m') - A(m'))(f(l') - f(l)),$$

which is positive due to the monotonicity of functions A and f.

cont.

(b) Show that family $g(.,.;\theta)$ is ordered by increasing differences.

For each $l < l'$ and $m < m'$, the difference

$$g(l', m'; \theta') - g(l, m; \theta) = \theta (l' - l)$$

is increasing in θ.

(c) Conclude that the optimal managerial quality decreases with the cost of labor. State the appropriate Theorem (carefully list all the assumptions).

We use the multi-dimensional version of the comparative statics result.

Theorem 1. *Suppose that* $(X, \leq_X)$ *is a lattice,* $(T, \leq_T)$ *is a partially ordered set, and* $f : X \times T \to R$ *is a function with increasing differences and such that* $f(., t)$ *is supermodular for each* t*. Then, for each* $t \leq t'$*,* $x^*(t)$ *is a lattice (i.e., for each* $x, x' \in x^*(t)$ *,* $x \wedge x' \in x^*(t)$ *and* $x \vee x' \in x^*(t)$ *, and*

$$x^*(t) \leq_{Strong} x^*(t').$$

The Theorem implies that $(l^*(\theta), m^*(\theta))$ increases in the strong s et order in θ. Thus, $m^*(w)$ decreases with w.

(3) Consider a decision problem under uncertainty with two states of the world, $S = \{s_1, s_2\}$, and two prizes, $Z = \{z_1, z_2\}$. Let $\mathcal{F}$ be the space of Anscombe-Aumann acts $f : S \to \Delta Z$.

  (a) State the Independence Axiom.

  See the notes.

(b) Suppose that preferences $\preceq$ over acts are represented by function

$$U(f) = f_{s_1}(z_1)\, f_{s_2}(z_1).$$

Show that the preferences fail the independence axiom.

Take $f = (1, 0)$, $g = (0, 1)$, and $h = (0, 1)$. Then,

$$U(f) = 0 \leq 0 = U(g)$$

but

$$U\left(\frac{1}{2}f + \frac{1}{2}g\right) = \frac{1}{4} > 0 = U(g) = U\left(\frac{1}{2}g + \frac{1}{2}g\right).$$

(c) In order to analyze the preferences that exhibit Ellsberg type of behavior, Gilboa and Shmeidler introduced the following axiom:

Uncertainty Aversion: For any two acts f, g, if $f \sim g$, then for each $\alpha \in (0,1)$, $f \preceq \alpha f + (1-\alpha) g$.

Show that the above preferences satisfy the Uncertainty Aversion. (Hint: You may find the following inequality useful

$$x + \frac{1}{x} \geq 2 \text{ for each } x > 0.)$$

Suppose that $f \sim g$. This means that there exists U such that

$$f_s(z_1) = \frac{U}{f_{s_2}(z_1)},$$
$$g_s(z_1) = \frac{U}{g_{s_2}(z_1)}.$$

Then,

$$\begin{aligned} U(\alpha f + (1-\alpha) g) &= (\alpha f_{s_1}(z_1) + (1-\alpha) g_{s_1}(z_1))(\alpha f_{s_2}(z_1) + (1-\alpha) g_{s_2}(z_1)) \\ &= \left(\alpha^2 + \left(1-\alpha^2\right)\right) U + \alpha(1-\alpha) U \left(\frac{g_{s_1}(z_1)}{f_{s_2}(z_1)} + \frac{g_{s_2}(z_1)}{f_{s_1}(z_1)}\right) \\ &\geq \left(\alpha^2 + \left(1-\alpha^2\right) + 2\alpha(1-\alpha)\right) U = U. \end{aligned}$$

(4) A decision maker has strictly concave Bernoulli utility function $u(c)$ over monetary consumption c. Her current consumption level is equal to x. She considers an investment in a start-up company. If she buys a single share of the start-up, the consumption after investment is equal to $x+Y$, where Y is a random variable. The decision maker maximizes her expected utility.

(a) Let
$$E_Y u(x+nY)$$
be the expected utility from buying n shares of the start-up. Argue that the above function is concave in the number of shares $n \geq 0$.

Define
$$f(n) = E_Y u(x+nY)$$
For each x and any n, n' and $\beta \in (0,1)$,
$$u(x + (\beta n + (1-\beta) n') Y) > \beta u(x+ny) + (1-\beta) u(x+n'y)$$
by the strict concavity of U. But this implies that $f(.,)$ is strictly concave in n. In turn, the concavity of $f(.)$ implies that

(b) Suppose that the decision maker would prefer not to buy anything than to buy a single share,

$$E_Y u(x+Y) < u(x).$$

Show that the consumer would rather not buy anything than to buy $n > 0$ shares,

$$E_Y u(x+nY) < u(x).$$

The concavity of $E_Y u(x+nY)$ implies that for each $n \geq 1$,

$$\frac{1}{n} E_Y u(x+nY) + \frac{n-1}{n} E_Y u(x) \leq E_Y u\left(x + \left(\frac{1}{n} n + \frac{n-1}{n} 0\right) Y\right) = E_Y u(x+Y).$$

If

$$E_Y u(x+Y) < u(x),$$

then

$$\begin{aligned}\frac{1}{n} E_Y u(x+nY) &\leq \frac{1}{n} E_Y u(x+Y) + \frac{n-1}{n} E_Y u(x+Y) - \frac{n-1}{n} E_Y u(x) \\ &< \frac{1}{n} E_Y u(x+Y).\end{aligned}$$

The result is not true for $n \leq 1$.

(c) Use an example to show that the claim from (a) does not hold if the consumer is risk-loving.

I made a mistake when I wrote the question. I wanted you to counter the claim from part (b). The question as it stands now is trivial given the solution to part (a). (More precisely, the same argument as in the proof of part (a) shows that if the consumer has convex Bernoulli utility, then the expectation is convex in n.)

I gave credit both for the counterexample to part (a) and to part (b).