Microeconomic Theory I
Midterm October 2014

Marcin Pęski

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Each question has the same value. You need to provide arguments for each answer. If you cannot solve one part of the problem, don't give up and try to solve the next one. If the question explicitly asks you to prove a result from the class, you must carefully describe the proof. Otherwise, you may use any result from the class given that you clearly state the assumptions, thesis and verify that the assumptions hold in your application. You have 110 minutes. Good luck!

1. (30 points) In the economy, there are two goods and two firms $i = 1, 2$ with production sets

$$\begin{aligned} Y_1 &= [0,2] \times [0,3] \subseteq R^2, \\ Y_2 &= \{(y_1, y_2) : y_1, y_2 \geq 0 \text{ and } y_1 + y_2 \leq 2\}. \end{aligned}$$

(a) (5 points) State the profit maximization problem of each firm.

(b) (20 points) Let $\pi_i(p)$ be the profit function of each firm. Show that there exists a set $Y^* \subseteq R^2$ such that if $\pi^*(p)$ is the profit function of a firm with technology Y^*, then for each p,

$$\pi^*(p) = \pi_1(p) + \pi_2(p).$$

(c) (5 points) Does production vector $y = (4, 5)$ belong to set Y^*? What about $y' = (0, 5)$?

2. (30 points) Let $(X, \mathcal{B}, C(.))$ be a choice structure such that $|X| \geq 3$. We assume that the family of choice situations $\mathcal{B}$ consists of all 3-element subsets of X (and it does not contain any other subsets). Define a binary relation $\lhd$on X: For any x, y,

$x \lhd y$ if and only if there exists $B \in \mathcal{B}$ such that $\{x, y\} \subseteq B$and $y \in C(B)$.

   (a) (5 points) State the Weak Axiom of Revealed Preferences.

(b) (10 points) From now on, suppose that the choice correspondence satisfies WARP. Show that the relation $\lhd$ is transitive. (Start with explaining what is "transitive".)

(c) (5 points) Explain that relation $\lhd$ rationalizes the choices.

(d) (10 points) Is relation $\lhd$ complete? If so, prove it. If not, provide a counterexample.

3. (30 points) Answer the following questions.

   (a) (5 points) Let $X = R^N$ for $N \geq 2$. Describe a partial order on X for which X is a lattice.

(b) (5 points) State the definition of a supermodular function $f : X \to R$.

(c) (10 points) Suppose that $N = 2$ and suppose that $h(x_1)$ and $g(x_2)$ are increasing functions. Let $f(x_1, x_2) = h(x_1) g(x_2)$. Is f supermodular? Either prove that f is supermodular, or provide a counterexample.

(d) (10 points) Suppose that $N = 3$ and suppose that $h(x_1)$, $g(x_2)$, and $k(x_3)$ are increasing functions. Let $f(x_1, x_2, x_3) = h(x_1)\, g(x_2)\, k(x_3)$. Is f supermodular? Either prove that f is supermodular, or provide a counterexample.

4. (30 points) Let S be a finite state space, Z be a finite set of prizes, and let X be the space of Anscombe-Aumann acts $f : S \to \Delta Z$.
For each act f, for any two states s_0, s_1, define an act f^{s_0,s_1} obtained from f by exchanging the lotteries in states s_0 and s_1:

$$f^{s_0,s_1}(s) = \begin{cases} f(s_1), & \text{if } s = s_0, \\ f(s_0), & \text{if } s = s_1, \\ f(s), & \text{if } s \neq s_0, s_1. \end{cases}$$

Assume that an agent has continuous preference relation $\preceq$ over AA acts that satisfies the Independence axiom. Additionally, the preference relation satisfies the **State Invariance Axiom:** For each act f, any two states s_0, s_1, the agent is indifferent between acts f and f^{s_0,s_1}:

$$f \sim f^{s_0,s_1}.$$

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(a) (7 points) Explain that the preferences have State-Dependent Expected Utility (SDEU) representation. (You don't need to prove anything, just refer to appropriate theorem from the class. Carefully state the assumptions of the theorem.) Carefully describe the form of the representation.

(b) (8 points) Let functions $u_s : Z \to R$ be as in the SDEU representation. Show that State Invariance implies that for each state s_0 and s_1, there exists a constant c^{s_0,s_1} such that for each z,

$$u_{s_0}(z) - u_{s_1}(z) = c^{s_0,s_1}.$$

In particular, the difference between the utilities of a prize z in any two states does not depend on z.

(c) (8 points) Conclude that the preferences have a State-Independent Representation in which the decision maker acts as if she assigns equal probability to each state.

(d) (7 points) Is the representation unique? Carefully explain using any result from the class.