Microeconomic Theory I

Midterm

November 3, 2016

Name: .....

Student number: .....

Q1	Points	Q2	Points	Q3	Points	Q4	Points
1a		2a		3a		4a	
1b		2b		3b		4b	
1c		2c				4c	
		2d				4d	

Each question has the same value. You need to provide arguments for each answer. If you cannot solve one part of the problem, don't give up and try to solve the next one. If the question explicitly asks you to prove a result from the class, you must carefully describe the proof. Otherwise, you may use any result from the class given that you clearly state the assumptions, thesis and verify that the assumptions hold in your application. You have 110 minutes. Good luck!

1. Let $X = R^2$ and let $\preceq$ be a rational preference relation on X.

   (a) Explain what it means that preferences $\preceq$ are monotonic and/or continuous.

(b) Give an example of preferences $\preceq$ that are monotonic but not continuous. Justify all your claims.

(c) Give an example of preferences $\preceq$ that do not have a utility representation. Justify all your claims. (You can use the same example as in case (a).)

2. Suppose that the space state $S = \{s, t\}$ and the space of prizes $Z = \{0, 1\}$ consist of two elements. We assume that $\preceq$ is a rational and preference relation over AA acts (i.e., the Axiom 1 of the expected utility theory holds).

   (a) Use a well-labeled diagram to graphically represent all AA acts.

(b) Suppose that $\preceq$ satisfy the Independence Axiom, and that the preferences are non-trivial (i.e., there are acts f and f' such that $f \prec f'$). Explain that the indifference curve are parallel lines.

(c) Give an example of preferences that have the State-Dependent Expected Utility (SDEU) representation, but not the State-Independent Expected Utility (SIEU) representations. Illustrate your answer on a diagram.

(d) For $x_1, x_2 \in \Delta Z$, let (x_s, x_t) be an act that gives prizes x_i in state i. Suppose that $\preceq$ satisfy the Continuity and the Independence axiom and that $(1,1) \prec (0,1) \prec (0,0)$. Show that $\preceq$ have the S**I**EU representation.

3. Consider an investor with the utility over final consumption given by function $u : \mathbb{R} \to \mathbb{R}$. We assume that u is twice continuously differentiable, increasing and concave and that

$$u'(x) + xu''(x) > 0 \text{ for each } x.$$

The investor chooses how much $a \in [0, w]$ of his wealth w to invest into the risky asset. The risky asset brings an uncertain return z. The safe asset yields a constant return r where $r > 0$.

(a) Let

$$v(a, z) = u(a(1 + z) + (w - a)(1 + r))$$

be the utility of the investor who invests a into the risky asset and the rate of return on the risky asset is z. Show that function v has increasing differences.

(b) Let $F_\gamma(z) = 1 - e^{-\gamma z}$ be a cumulative distribution function. Let a_γ be the optimal investment level if the uncertain return is distributed according to F_γ and the investor maximizes the expected utility. Show that a_γ is increasing in γ. Carefully state (and verify all the assumptions of) any result that you need to justify your claim.

4. Answer the following questions:

   (a) State the Laws of (Uncompensated) Demand.

(b) Use the Slustky equation to discuss conditions which ensure that the Law of Demand holds.

(c) Explain that homothetic preferences satisfy the Law of Demand.

(d) Explain that quasi-linear preferences satisfy the Law of Demand.