Microeconomic Theory I
Midterm October 2017

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Each question has the same value. You need to provide arguments for each answer. If you cannot solve one part of the problem, don't give up and try to solve the next one. If the question explicitly asks you to prove a result from the class, you must carefully describe the proof. Otherwise, you may use any result from the class given that you clearly state the assumptions, thesis and verify that the assumptions hold in your application. You have 110 minutes.

An advice: It is always a good idea to start each "proof" answer with writing precisely what is that you want to show. It will help you to make it precise when thinking about solution. It will also allow me to give you a tiny bit of partial credit if the rest of the answer turns out to be wrong.

Good luck!

1. *Quasi-linear and homothetic preferences.* Consider a consumer with Walrasian demand $x\,(p,w) \in \mathbb{R}^L$. Let

$$D_p x = \begin{bmatrix} \frac{\partial x_1}{\partial p_1} & \frac{\partial x_1}{\partial p_2} & & \frac{\partial x_1}{\partial p_L} \\ \frac{\partial x_2}{\partial p_1} & \frac{\partial x_2}{\partial p_2} & & \frac{\partial x_2}{\partial p_L} \\ & & & \\ & & & \\ \frac{\partial x_L}{\partial p_1} & \frac{\partial x_L}{\partial p_2} & & \frac{\partial x_L}{\partial p_L} \end{bmatrix}$$

be the matrix of the derivatives of x.

(a) State the Slutsky equation and briefly (no more than 1 sentence) describe each term.

(b) Show that if the consumer's preferences are quasi-linear in good L, the restriction of the matrix $D_p x$ to the first $L-1$ terms is symmetric and positive semi-definite.

(c) Show that if the consumer's preferences are homothetic, the matrix $D_p x$ is symmetric and positive semi-definite.

2. *Non-linear budget sets.* In some countries, anti-poverty government programs provide for distribution of certain amounts $f_i \geq 0$ of basic goods (food, fuel, etc.) free to all individuals. If an individual wants to consumer more than the free amount, he or she can purchase the good on the market for price p_i. We can model the consumer's choice as a non-linear budget set:

$$B_i(p, w, f) = \left\{ x \in (\mathbb{R}_+)^L : \sum_i p_i \max(0, x_i - f_i) \leq w \right\}.$$

Here, L is the number of goods and $f = (f_1, ..., f_L) \in (\mathbb{R}_+)^L$ is the "free" bundle.

(a) Suppose that $L = 2$, $f_1, f_2 > 0$, $p_1 = p_2$ and $w > 0$. Describe the budget set graphically on a diagram.

(b) From now on, assume that the consumer has a strictly monotone, strictly convex utility. Define the indirect utility in the standard way

$$v(p, w, f) = \max_{x \in B(p,w,f)} u(x).$$

Show that $v(., w, f)$ is quasi-convex.

(c) Define the expenditure function as

$$e(p,u,f) := \min_x \sum_i p_i \max(0, x_i - f_i) \;\text{ st. }\; u(x) \geq u.$$

Show that $e(.,u,f)$ is concave.

3. *Investment choice.* Consider a consumer who lives for two periods. The consumer has fixed wealth w, no other source of income, and decides how much of his wealth to invest. The consumer maximizes the lifetime utility

$$\max_{s} u_1 (w - s) + u_2 (\gamma s)$$

where s is the choice level of savings and $\gamma > 0$ is the return rate on the investment.

   (a) Suppose that the second period utility u_2 is twice continuously differentiable, increasing and concave, and the Arrow-Pratt *relative* measure of risk aversion is strictly smaller than 1.

   $$-\frac{x (u_2'' (x))}{u_2' (x)} \leq 1.$$

   Does function

   $$u_2 (\gamma s)$$

   have increasing differences in γ and s.

(b) Conclude that the optimal level of savings changes monotonically with the return rate in the strong set order sense. (For full credit, do not rely on the results from the class.)

(c) What is the comparative statics of the optimal savings if the relative measure of risk aversion is strictly larger than 1?

4. *Prize State Independence.* Consider an agent with preferences $\preceq$ over Anscombe-Aumann acts $f : S \to \Delta Z$, where S is the state space, and Z is the space of prizes. For any act f, any state s, and any prize z, let

$$f_s z = (f_1, f_2, ..., f_{s-1}, z, f_{s+1}, ..., f_n)$$

be an act obtained from f by replacing the state s lottery by prize z. Say that $\preceq$ satisfy **Prize State Independence** (PSI) if for any act f, any two states s, s', and any prizes z, z', we have

$$f_s z \preceq f_s z' \Longleftrightarrow f_{s'} z \preceq f_{s'} z'.$$

(a) Explain the difference between the PSI and the State Independence (SI) axiom from the class.

(b) Describe a concrete example of State-Dependent Expected Utility (SDEU) preferences that violate PSI.

(c) Show that if $|Z| = 2$ (i.e., there are only two prizes), then the any SDEU preferences that satisfy PSI have State-Independent Expected Utility (SIEU) representation.

(d) Does the above observation remain true if $|Z| > 2$? Prove it or disprove it using a counterexample.