Bargaining in Stationary Networks
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1 Motivation
   • The Basic Problem
   • Example

2 Benchmark Model
   • Framework
   • Main results

3 Extension

4 Conclusion
The Basic Problem

Explore the influence of the network structure on market outcomes
- How does an agent’s position determine his bargaining power?
- Who trades with whom and on what terms?
- When are prices uniform in the network?
Figure 1. The position with the largest number of connections is not necessarily the strongest.
Noncooperative model of decentralized infinite horizon bilateral bargaining in networks:

- **N**: the set of n players
- A network is an undirected graph $H = (V, E)$
- Vertices and edges: $V \subset N$, $E \subset \{(i, j) \mid i \neq j \in V\}$
- Player i has an H link to j if $ij \in E$
- A network $H' = (V', E')$ is a subnetwork of H if $V' \subset V$ and $E' \subset E$
- A network $H' = (V', E')$ is the subnetwork of H induced by $V'$ if $E' = E \cap (V' \times V')$
$(p_{ij} > 0)_{ij \in G}$ is the matching technology for network $G$.

A link $ij$ is interpreted as the ability of players $i$ and $j$ to jointly generate a unit surplus.

Each period $t = 0, 1, \ldots$ a link $ij$ is selected with probability $p_{ij}$ and one of the players is randomly chosen to make an offer.

If the responder accepts the offer, the players exit the game with the shares agreed upon and two new players would take the same positions in the next period.

If the responder rejects, they remain in the game for the next period.

All players share a discount factor $\delta \in (0, 1)$.

The game is denoted $\Gamma^{\delta}$. 
There exists a sequence $i_0, i_1, \ldots, i_\tau, \ldots$ of players of type $i \in N$.

All players have common knowledge of the game, including the network structure, the matching technology and the events preceding any of their decision nodes in the game.

$H(i_\tau)$ is the set of complete histories, or subgames, where $i_\tau$ is player of type i present in the game.

A strategy profile is stationary if each player’s strategy at any time $t$ depends exclusively on his position in the network and the play of the game in period $t$. 
For every $\delta \in (0,1)$, there exists a payoff vector $(v_i^{*\delta})_{i \in N}$ such that for every subgame perfect equilibrium of $\Gamma^\delta$ the expected payoff of player $i_\tau$ in any $H(i_\tau)$ subgame is uniquely given by $v_i^{*\delta}$ for all $i \in N$, $\tau \geq 0$. For every equilibrium of $\Gamma^\delta$, in any subgame where $i_\tau$ is selected to make an offer to $j_{\tau'}$, the following statement hold with probability one:

1. If $\delta \left(v_i^{*\delta} + v_j^{*\delta}\right) < 1$ then $i_\tau$ offers $\delta v_j^{*\delta}$ and $j_{\tau'}$ accepts;
2. If $\delta \left(v_i^{*\delta} + v_j^{*\delta}\right) > 1$ then $i_\tau$ makes an offer that $j_{\tau'}$ rejects.
(v_i^*δ)_{i \in N} is the equilibrium payoff vector at δ. The equilibrium agreement network at δ, denoted G^*δ, is defined as the subnetwork of G with the link included if and only if \( \delta \left( v_i^*δ + v_j^*δ \right) < 1. \) They show that the condition \( \delta \left( v_i^*δ + v_j^*δ \right) \neq 1, \forall ij \in G \) holds for all but a finite set of δ.
Main Results

Theorem

There exists $\delta \in (0,1)$ and a subnetwork $G^*$ of $G$ such that the equilibrium agreement network $G^*\delta$ is equal to $G^*$ for all $\delta \geq \delta$. The equilibrium payoff vector $v_i^{*\delta}$ converges to a vector $v^*$ as $\delta$ goes to 1. The rate of convergence of $v_i^{*\delta}$ to $v^*$ is $O(1 - \delta)$. 
Main Results

$G^*$ is the limit equilibrium agreement network and $v^*$ is the limit equilibrium payoff vector.

- If $ij \in G$, then $v_i^* + v_j^* \geq 1$. If $ij \in G^*$, then $v_i^* + v_j^* = 1$
- Every player has at least one link in $G^*$ under the assumption that every player has at least one link in $G$
For network $H$ and subset vertices $M$,
$$L^H(M) = \{j \mid ij \in H, i \in M\}$$

A set $M$ is $H$-independent if there exists no $H$-link between two players in $M$.

A nonempty set of players is mutually estranged if it is $G^*$-independent.

The set of partners for a mutually estranged set $M$ is $L^{G^*}(M)$.

The shortage ratio of $M$ is $|L| / |M|$. 
Bounds for Limit Equilibrium Payoffs

**Theorem**

For every mutually estranged set $M$ with partner set $L$, the following bounds on limit equilibrium payoffs hold:

$$\min_{i \in M} v_i^* \leq \frac{|L|}{|L| + |M|}$$

$$\max_{j \in L} v_j^* \geq \frac{|M|}{|L| + |M|}$$
Limit Equilibrium Payoff Computation

**Definition**

Define the sequence \((r_s, x_s, M_s, L_s, N_s, G_s)\)_s recursively as follows:
Let \(N_1 = N\) and \(G_1 = G\). For \(s \geq 1\), if \(N_s = \emptyset\) then stop. Otherwise, let \(r_s = \min_{M \subset N_s, M \subset I(G)} \frac{|L^{G_s}(M)|}{|M|}\). If \(r_s \geq 1\) then stop. Else, set \(x_s = \frac{r_s}{(1+r_s)}\). Let \(M_s\) be the union of all minimizers \(M\). Denote \(L_s = L^{G_s}(M_s)\). Let \(N_{s+1} = N_s \setminus (M_s \cup L_s)\) and \(G_{s+1}\) be the subnetwork of \(G\) induced by the players in \(N_{s+1}\). Denote by \(\bar{s}\) the finite step at which the algorithm ends.
Theorem

Let \((r_s, x_s, M_s, L_s, N_s, G_s)\) for \(s = 1, 2, \ldots, \bar{s}\) be the outcome of the algorithm. The limit equilibrium payoffs for \(\Gamma^\delta\) as \(\delta \to 1\) are given by

\[
\nu_i^* = x_s, \forall i \in M_s, \forall s < \bar{s}
\]

\[
\nu_j^* = 1 - x_s, \forall j \in L_s, \forall s < \bar{s}
\]

\[
\nu_k^* = \frac{1}{2}, \forall k \in N_{\bar{s}}
\]
Example
Example

A network is depicted with nodes 1, 2, 3, 4, and 5. The connections and their associated values are as follows:
- Node 1 connects to nodes 3 and 4 with values 2/3 and 1/3, respectively.
- Node 2 connects to node 5 with a value of 1/2.
- Node 3 connects to nodes 3 and 4 with values 1/3 and 1/3, respectively.
- Node 5 connects to node 5 with a value of 1/2.
Submarkets endogenously emerge in equilibrium
The limit equilibrium prices are uniform within every submarket
Each agent self-selects into the most favorable submarket to which he has access
Equitable Networks

- A network is equitable if the limit equilibrium payoffs of all players are identical.
- The common limit payoffs must be equal to $1/2$.
- A network $G$ is equitable if and only if $|L^G(M)| \geq |M|$ for every $G$-independent set $M$. 
Extension

- Buyer and seller networks
- Steady state with a continuum of players
The model is well behaved in that equilibria are essentially unique and converge as players become patient.

Main result: characterization of the limit equilibrium payoffs.