Games with Incomplete Information

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Plan

Games with incomplete information

Introduction

- So far we talked about extending the basic model of a game to analyze actions that take place in time
- Today, we start with another extension: incomplete information.
- Players may not know about something that is important for their payoff, or
  - something that is important about payoffs of their opponent, or
  - something that is important to understand their beliefs, or
  - something about opponents beliefs about their own beliefs, etc.

Games with incomplete information

Battle of Sexes

- Example. Incomplete information in Battle of Sexes. Suppose that He is uncertain whether She wants to meet Him or avoid Him.
- Two states of the world: If She wants to meet Him, the payoffs are

He, She "meet"	O	S
O	3,5	0,0
S	0,0	5,3

If not, the payoffs are

He, She "avoid"	O	S
O	3,0	0,3
S	0,5	5,0.

- Notice that He only cares about meeting Her, not whether She wants to meet Him or not.

Games with incomplete information

Battle of Sexes: Information

- She knows her feelings.
  - We say that she knows the state of the world, or
  - Her type (her information) is to believe with probability 1 what is the true state of the world.
  - She has two possible types, "meet" and "avoid",
  - Here, type= state of the world. Not always the same.
- He does not know her feelings.
  - He has beliefs - 1/2 probability of state "meet" and 1/2 probability of state "avoid".

- His strategy $a_{He} \in \{S, O\}$.
  Or mixed strategy $\sigma_{He} \in \Delta\{S, O\}$
- Her choice depends on her feelings. We say that she has information about type and her choice depends on her information.
  - her strategy is $\sigma_{She} : \{meet, avoid\} \to \{S, O\}$,
    - $\sigma_{She}(meet)$ - her choice if she is a type "meet",
    - $\sigma_{She}(avoid)$ - her choice if she is a type "avoid",
  - Her strategy must be a complete contingency plan that says what she does in each of the cases.
  - This is important for Him - He needs to have beliefs what she does in order to figure out what she is best responding.

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Battle of Sexes: Strategies

- A candidate profile of strategies:
  - $\sigma_{He} = S$: He chooses S.
  - $\sigma_{She}(meet) = S, \sigma_{She}(avoid) = O$: She chooses S if she wants to meet and O if avoid.
- His payoffs against σ_{She}:
  - His expected payoff from $\sigma_{He} = S$ is

    $$\frac{1}{2}(5) + \frac{1}{2}(0) = 2.5.$$

    (Because he meets her with probability $\frac{1}{2}$ at S and he misses her with probability $\frac{1}{2}$.)
    His expected payoff from O is

    $$\frac{1}{2}(0) + \frac{1}{2}(3) = 1.5.$$

    (Because he meets her with probability $\frac{1}{2}$ at O and he misses her with probability $\frac{1}{2}$.)
  - Hence, $\sigma_{He} = S$ is a best response against σ_{She}.

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Battle of Sexes: Strategies

- Her "meet" type payoffs against σ_{He}:
  - from $\sigma_{She}(meet) = S$ is 3,
  - from O is 0,
  - hence, $\sigma_{She}(meet) = S$ is a best response.
- Her "avoid" type payoffs against σ_{He}:
  - from $\sigma_{She}(avoid) = O$ is 5,
  - from S is 0,
  - hence, $\sigma_{She}(avoid) = O$ is a best response.

Games with incomplete information

Battle of Sexes: Pure strategy equilibria

- All players (including, all her types) are best responding
- $\sigma_{He} = S$ and $\sigma_{She}(meet) = S, \sigma_{She}(avoid) = O$ is an Equilibrium!
- There is no other pure strategy Nash equilibrium.
  - Suppose that He chooses O in equilibrium.
  - Then Her best response is O if she wants to meet him, and S if she wants to avoid him.
  - His payoff from O is $\frac{1}{2}3 + \frac{1}{2}0 = 1.5$.
  - His payoff from S is $\frac{1}{2}0 + \frac{1}{2}5 = 2.5$, which means that O is not the best response.

Games with incomplete information

Battle of Sexes: Mixed strategy equilibria

- If He is not randomizing, then both of Her types have pure best responses (see above)
  - we already showed that only one of them leads to an equilibrium.
- Also, if both of her types play a pure strategy, His best response is also pure,
  - check!
- Hence, in a mixed strategy equilibrium, it must be that He, and at least one of Her types are randomizing.
- Two cases:
  - He and "meet" type,
  - He and "avoid" type.

Games with incomplete information

Battle of Sexes: Mixed strategy equilibria

- Suppose that He and Her"meet" type are randomizing.
  - let $\alpha = \sigma_{He}(S)$ be His probability to play S,
  - let $\beta = \sigma_{She}(S; meet)$ be Her "meet" type probability of going to S.
- Her "meet" type indifference condition:

$$\underbrace{\alpha 3 + (1-\alpha)\,0}_{\text{payoff from } S} = \underbrace{\alpha 0 + (1-\alpha)\,5}_{\text{payoff from } O}.$$

Solving the equation leads to

$$\alpha = \frac{5}{8}.$$

- Not surprisingly, Her indifference condition determines His mixing probabilities.

- What is the best response of Her “avoid” type?
  - payoff from S is
    $$\alpha 0 + (1-\alpha)\,3 = \frac{9}{8},$$
  - payoff from O is
    $$\alpha 5 + (1-\alpha)\,0 = \frac{25}{8}.$$
- Hence, Her “avoid” type best response is to play O.

- His indifference condition
  - his payoff from S is

$$\frac{1}{2}\underbrace{(\beta 5+(1-\beta)\,0)}_{\text{payoff facing "meet" type}}+\frac{1}{2}\underbrace{0}_{\text{payoff ffacing "avoid" type}}=\frac{5}{2}\beta,$$

  - his payoff from O is

$$\frac{1}{2}\underbrace{(\beta 0+(1-\beta)\,3)}_{\text{payoff facing "meet" type}}+\frac{1}{2}\underbrace{3}_{\text{payoff ffacing "avoid" type}}=\frac{3}{2}(2-\beta)\,.$$

  - indifference condition

$$\frac{5}{2}\beta=\frac{3}{2}(1-\beta)\,,$$

or $\beta=\frac{3}{8}$.

Games with incomplete information

Battle of Sexes: Mixed strategy equilibria

- We found an equilibrium
  - $\sigma_{He} = S^{\alpha} O^{1-\alpha} = S^{\frac{5}{8}} O^{\frac{3}{8}}$,
  - $\sigma_{She}(meet) = S^{\beta} O^{1-\beta} = S^{\frac{3}{8}} O^{\frac{5}{8}}$,
  - $\sigma_{She}(avoid) = O$.
- There is another equilibrium when He and She "avoit" type are randomizing.
  - find it yourself!

- We can consider more complicated information structure.

Example

Suppose that She tells her friend about her attitude towards Him, and next thing, she sees him walking the corridor. Did he hear? Or not?
She thinks he heard with probability p. If he heard, he knows her type. If the didn't, he is in the same situation as previously.

Games with incomplete information

Battle of Sexes: Beliefs about beliefs

- She still has two types “meet” and “avoid”.
- He has three types “ didn’t hear”, “heard meet”, “heard avoid”.
  - (we need three, instead of two, types for him to fully describe his beliefs about her types. See next slide.)
- To describe the situation formally, we need to specify beliefs
  - each type of each player assigns probabilities to the opponent types,
  - the probabilities may be different across types.

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Battle of Sexes: Mixed strategy equilibria

- His beliefs (each cell describes the probability that the row type assigns to the column type)

His types\Her types	meet	avoid
heard M	1	0
heard A	0	1
didn't hear	1/2	1/2

Games with incomplete information

Battle of Sexes: Mixed strategy equilibria

- Her beliefs:

Her types\His types	hear M	heard A	didn't hear
meet	p	0	1-p
avoid	0	p	1-p

Plan

Games with incomplete information

Model

- Next, we present a general model of a game with incomplete information.
- The model has two components:
  - game (players, actions, payoffs),
  - incomplete information (types, beliefs)
- Incomplete information
  - state of the world ω: complete description of everything there is to learn about the world
  - types of player i, $t_i \in T_i$: describes player i's information
    - the true type of player i is a private information of player i,
  - beliefs of type t_i about the state of the world and opponent types.

Formal model (in book, a slightly different model, more general than we need):

- players $i = 1, ..., I$,
- actions $a_i \in A_i$,
- state of the world $\omega \in \Omega$
- types $t_i \in T_i$ for each player. Types carry information about the state of the world and types of other players.
  - type profile $t = (t_1, ..., t_N) = (t_i, t_{-i})$,
  - belief function $\pi(\omega, t_{-i}|t_i) \geq 0$ - the probability of t_{-i} as believed by player i type t_i,
- Payoffs $u_i(a_i, a_{-i}, t_i, t_{-i}, \omega)$, depend on the state, and possibly, on all types.

Games with incomplete information

Model: strategies and payoffs

- Pure strategies $\sigma_i : T_i \to A_i$,
  - $\sigma_i(t_i)$ - action of player i type t_i.
- More generally, $\sigma(t_i)$ is a (possibly, mixed) action of player i.
- For any strategy of player i, (*interim*) expected payoff of player i type t_i from action a_i if the opponents are using strategy σ_{-i} is

$$U_i(a_i, \sigma_{-i}; t_i) = \sum_{\omega, t_{-i}} u_i(a_i, \sigma_{-i}(t_{-i}), \omega, t_i, t_{-i}; t_i)\, \pi(\omega, t_{-i} | t_i).$$

Games with incomplete information

Model: solutions

- Action a_i is a *best response* for type t_i against strategy σ_{-i} if for any other action a'_i,

$$U_i\left(a_i, \sigma_{-i}; t_i\right) \geq U_i\left(a'_i, \sigma_{-i}; t_i\right).$$

- Action a_i is *(strictly) dominated* for type t_i if there exists an action a'_i such that for all strategies of the other players σ_{-i},

$$U_i\left(a_i, \sigma_{-i}; t_i\right) < U_i\left(a'_i, \sigma_{-i}; t_i\right).$$

- A *Bayesian Nash equilibrium* is a profile of strategies $\sigma = (\sigma_1, ..., \sigma_I)$ such that a (possibly, mixed) action of each player and type is a best response given the strategies of the other players and their types.

Plan

Example

Two firms, 1,2.

- Each firm has a high cost c^H with probabiltiy p and low cost $c^L < c^H$ with prob. $1 - p$.
- The firms know their own costs, but not the costs of the other firm.
- each firm chooses quantity $q_i \geq 0$,
- payoffs depend only on own cost (and both quantities):

$$\pi_i\left(q_i, q_{-i}, c_i\right) = q_i\left(a - c_i - \left(q_i + q_{-i}\right)\right).$$

Cournot duopoly

Model

- Strategy (one quantity per type): $q_i = \left(q_i^H, q_i^L\right)$,
- Expected payoffs from quantity q_i, if i's costs are c_i and the opponent plays $q_{-i} = \left(q_{-i}^H, q_{-i}^L\right)$

$$\begin{aligned} & pq_i\left(a - c_i - q_i - q_{-i}^H\right) + (1-p)\left(a - c - q_i - q_{-i}^L\right) q_i \\ = \;& q_i\left(a - c_i - q_i - \left[pq_{-i}^H + (1-p)\, q_{-i}^L\right]\right) \\ = \;& q_i\left(a - c_i - q_i - Eq_{-i}\right), \end{aligned}$$

where we denote the expected quantity of $-i$ as

$$Eq_{-i} = pq_{-i}^H + (1-p)\, q_{-i}^L.$$

- Also, define expected cost as

$$Ec = pc^H + (1-p)\, c^L,$$

Cournot duopoly

Best responses

- Best response of player i with cost c_i against strategy $q_{-i} = \left(q_{-i}^H, q_{-i}^L\right)$ maximizes $q_i \left(a - c - q_i - Eq_{-i}\right)$.
  - standard analysis shows that
$$q_i^{BR}\left(q_{-i}; c_i\right) = \frac{1}{2}\max\left(0, a - c - Eq_{-i}\right).$$
  - for simplicity, we only look at the equilibrium, where both types of both players have strictly positive quantity.
  - Thus, the best response is
$$q_i^{BR}\left(q_{-i}; c\right) = \frac{a - c - Eq_{-i}}{2}.$$
- The expected best response quantity is
$$\begin{aligned} Eq_i^{BR} &= p\frac{a - c_i^H - Eq_{-i}}{2} + (1-p)\frac{a - c_i^L - Eq_{-i}}{2} \\ &= \frac{a - Ec - Eq_{-i}}{2}, \end{aligned}$$
where $Ec_i = pc^H + (1-p)\,c^L$ is the expected cost of player i.

Cournot duopoly

Equilibrium

- In equilibrium,

$$Eq_i = \frac{a - Ec_i - Eq_{-i}}{2} \text{ and } Eq_{-i} = \frac{a - Ec_{-i} - Eq_i}{2},$$

hence

$$Eq_i = \frac{a - Ec_i - Eq_{-i}}{2} = \frac{a - Ec_i - \frac{\alpha - Ec_{-i} - Eq_i}{2}}{2},$$

which implies that

$$Eq_i = \frac{a - 2Ec_i + Ec_{-i}}{3}.$$

- Because the distribution of costs is symmetric $Ec_i = Ec_{-i} = Ec$, it follows that

$$Eq_i = Eq_{-i} = \frac{a - Ec}{3}.$$

Cournot duopoly
Equilibrium

- The above describes only the equilibrium *expected* quantities.
- The equilibrium quantities for each type:

$$
\begin{aligned}
q_i^*(c_i) &= \frac{a - c_i - Eq_{-i}}{2} = \frac{a - c_i - \frac{a - Ec}{3}}{2} \\
&= \frac{1}{3}a - \frac{1}{2}c_i + \frac{1}{6}Ec.
\end{aligned}
$$

  In particular, higher cost firm produces less than the lowest cost firm!
- we need to be careful to check that $q_i^*(c) > 0$ for each c.
- this leads to conditions under which we have equilibrium with positive quantities.

Plan

Oil field auction

Model

Example

Two firms explore a possibility to bid for drilling rights.

- State of the world $\omega = 1, 2, ..., 1000$ of millions of barrels of oil in the ground.
  - unknown to each of the firms,
  - equal probability for each state,
- firm 1 observes whether the amount of oil is
  - low: $\omega < 500$, or
  - high $\omega \geq 500$,
- firm 2 observes whether the amount of oil is
  - not high $\omega < 750$ with probability 0.75,
  - very high $\omega \geq 750$ with probability 0.25.
- Information is not symmetric!

Oil field auction

Model: beliefs

- Beliefs of firm 1

1's types\2's types	not high	very high
low	$1, E_{L,NH}\omega = 250$	0
high	$\frac{1}{2}, E_{H,NH}\omega = 625$	$\frac{1}{2}, E_{H,VH}\omega = 875$

- Each cell contains the probability of the opponent type and the expected (average) amount of the oil conditionally on the two types
  - for example, if player 1 received High observation, and player 2 received Not High, then the oil must be between 500 and 750,
  - because the distribution of oil is uniform, the average amount of oil given H and NH is

$$E_{H,NH}\omega = 625.$$

Oil field auction

Model: beliefs

- Firm 2 beliefs:

2's types\1's types	high	low
not high	$\frac{1}{3}, E_{NH,H}\omega = 625$	$\frac{2}{3}, E_{H,VH}\omega = 250$
very high	$1, E_{H,VH}\omega = 875$	0

- for example, given Not High, the amount of oil is ≤ 750. Hence, the probability that type Not High assigns to type High ($\omega \geq 500$) is equal to $\frac{250}{750} = \frac{1}{3}$.

Oil field auction

Model: Actions

- Actions bid or not bid.
  If not bid, payoff $= 0$.
- If bid, the payoff is equal to (the value of the field minus the price) multiplied by the probability of winning the auction:

$$(\omega - \text{Price})\,\text{Prob}_{win}.$$

- The probability of winning and price depend on how many firms bid:
  - if only one firm bids, $\text{Prob}_{win} = 1, \text{Price} = 275$,
  - if two firms bid, $\text{Prob}_{win} = \frac{1}{2}, \text{Price} = 550$.

Oil field auction

Dominated actions

- We solve for equilibrium by, first, eliminating dominated strategies.
- Bidding is a dominated strategy for player 1 Low type.
  - notice that the expected value of the field for such a player is $E_{L,NH}\omega = 250$ (Low type knows that the opponent is Not High for sure),
  - expected payoff from bid is either
    - $E\omega - \text{Price} = 250 - 275 < 0$ if the other player does not bid, or
    - $\frac{1}{2}(250 - 550) < 0$ if the other player bids.
  - expected payoff from not bid is 0.
  - Hence, no matter what the other player does, not bid is better.

Oil field auction

Dominated actions

- Bidding is a dominant strategy for Very High player 2.
  - expected value of oil is $E_{VH,H}\omega = 875$
  - expected payoff from bid is either
    - $875 - 275 > 0$ if the other player does not bid, or
    - $\frac{1}{2}(875 - 550) > 0$ if the other player bids.
  - expected payoff from not bid is 0.
- A similar argument shows that bidding is a dominant strategy for High type of player 1.
  - recall that the lowest possible conditional expected value of the oil field is $625 > 550$.

Oil field auction

Equilibrium

- Neither bidding nor not bidding are not dominated for Not High type of player 2
  - If none of the types of player 1 bid, payoff from bidding for Not High type is

$$\frac{2}{3}E_{NH,L}\omega + \frac{1}{3}E_{NH,H}\omega - 275 = 375 - 275 > 0.$$

  - If both types of player 1 bid, payoff from bidding for Not High type is

$$\frac{1}{2}\left(\frac{2}{3}E_{NH,L}\omega + \frac{1}{3}E_{NH,H}\omega - 550\right) = \frac{1}{2}(375 - 550) < 0.$$

Oil field auction

Equilibrium

- But not bidding is a best response if Low type of player 1 does not bid, and High type bids.
  - payoff from bidding is

$$
\begin{aligned}
& \frac{2}{3}\left(E_{NH,L}\omega - 275\right) + \frac{1}{3}\frac{1}{2}\left(E_{NH,H}\omega - 550\right) \\
= & \frac{2}{3}\left(250 - 275\right) + \frac{1}{3}\frac{1}{2}\left(625 - 550\right) \\
= & -\frac{2}{3}25 + \frac{1}{6}75 < 0,
\end{aligned}
$$

    hence lower than payoff from not bidding, which is 0.

Oil field auction

Equilibrium

- Player 1
  - Low: not bid
  - High: bid
- Player 2:
  - Not High: not bid
  - Very High: bid.

Plan

Roommate problem

Model

Example

Two roommates. The apartment is dirty. It costs $c > 0$ of effort to clean the apartment . Will anyone of them do it?

- Actions: Clean or go to Movies.
- Each roommate has two types $v_H > v_L = 0$, where the probability of the high type is $p \in (0,1)$.
  The type is equal to the value of clean apartment.
  We assume that
  $$v_H > c \text{ and } v_H (1-p) < c.$$
- The payoffs of type v,
  $$u_i(a_i, a_{-i}, v) = \begin{cases} v - c, & \text{if } a_i = C, \\ v & \text{if } a_i = M, a_{-i} = C, \\ 0 & \text{if } a_i = M, a_{-i} = M. \end{cases}$$

Roommate problem

Model

- We will show that there is no efficient cleaning in equilibrium (σ_1, σ_2).
- Notice first that the low types never clean in equilibrium, $\sigma_i\,(Clean; low) = 0$.
- Let $\alpha_i = \sigma_i\,(Clean; high)$ be the probability that the high type of player i cleans.
  - The payoff of high type of player i from cleaning is

$$v_H - c$$

  - The expected payoff if she does not clean is

$$(1-p)\,0v_H + p\alpha_{-i}v_H = p\alpha_{-i}v_H.$$

Best response correspondence

$$BR\,(\alpha_{-i}; high) = \begin{cases} Clean, & \text{if } v_H - c \geq p\alpha_{-i}v_H, \\ Not & \text{if } v_H - c \leq p\alpha_{-i}v_H. \end{cases}$$

Roommate problem

Pure strategy equilibria

- Suppose that $\alpha_1 = 0$, i.e., player 1 never cleans.
  - payoff from cleaning for player 2 type high is equal to $v_H - c$,
  - payoff from not cleaning is 0,
  - hence, player 2 type high cleans, $\alpha_2 = 1$.
- Suppose that $\alpha_2 = 1$, i.e., player 2 high type cleans.
  - payoff from cleaning for player 1 type high is equal to $v_H - c$,
  - payoff from not cleaning is $p1v_H = pv_H$,
  - because we assumed that $v_H - c < pv_H$, player 1 high type does not want to clean.
- Thus, $\alpha_1 = 0, \alpha_2 = 1$, i.e., player 1 never cleans, and player 2 cleans when high, is an equilibrium.
- There is also a pure strategy equilibrium, where player 2 never cleans and player 1 cleans only if high.

Roommate problem

Mixed strategy equilibria

- Suppose that player i type high is indifferent between cleaning and not cleaning. Then,

$$v - c = p\alpha_{-i}c,$$

or

$$\alpha_{-i} = \frac{v - c}{pc}.$$

Thus, player $-i$ must be also indifferent, and both of them clean.

Roommate problem

Efficiency

- In any equilibrium there is always a possibility that at least one roommate has a high value from cleaning, and would like to clean it, but the room is not cleaned.
- Hence, the social outcome is not always efficient.

Plan

Bargaining with incomplete information

Labor union bargaining

- Next, we consider a model of bargaining between a labor union and a firm under incomplete information.
- Labor union bargains with a firm through an ultimatum game
  - labor makes an offer, firm accepts or rejects,
  - there is uncertainty about profits of the firm, and how much labor union can demand before it bankrupts the firm.
- We consider few versions of the game. The goal is to notice that, in strategic situations, more information may be bad for players.

Bargaining with incomplete information

Labor union bargaining

- State of the world - firm's profits π.
  - Cdf $F(.)$. It means that $F(x) = \text{Prob}(\pi \leq x)$.
  - Pdf $f(x) = \frac{dF(x)}{dx}$.
  - Uniform cdf: $F_{\text{uniform}}(x) = \begin{cases} 0, & x \leq 0 \\ x, & x \in [0,1] \\ 1, & x \geq 1. \end{cases}$
- Two stages:
  - labor union demands wage w.
  - the firm decides whether to
    - accept, with payoffs w for the union and $\pi - w$ for the firm,
    - or not, with payoffs 0 for the union and 0 for the firm.

Bargaining with incomplete information

Labor union bargaining: Version Ia

- Firm knows the profits π, labor union does not
  - labor union has beliefs $F_{\text{uniform}}(.)$.
- We find the SPE.
- Subgame w: Firm accepts if $w \leq \pi$, rejects otherwise.
- Subgame $\emptyset$:Labor union proposes w.
  - expected payoff:

$$\begin{aligned} & w\text{Prob}(\pi \geq w) + 0\text{Prob}(\pi < w) \\ = & w(1 - \text{Prob}(\pi < w)) \\ = & w(1 - F_{\text{uniform}}(w)) \\ = & w(1 - w), \end{aligned}$$

    as we can assume that $w \in (0, 1)$.
  - best response maximizes $w(1-w)$. FOCs leads to

$$w^* = \frac{1}{2}.$$

Bargaining with incomplete information

Labor union bargaining: Version Ia

- Equilibrium expected payoffs
  - labor union
$$u_{LU}^{(Ia)} = w^* (1 - w^*) = \frac{1}{4}.$$
  - firm with profits π:
$$\max (\pi - w^*, 0),$$
  - firm, expected payoffs from the point of view before it learns the profits:
$$\begin{aligned} u_F^{(Ia)} &= \int_0^1 \max (\pi - w^*, 0)\, f_{\text{uniform}}(\pi)\, d\pi \\ &= \int_{w^*}^1 (\pi - w^*)\, f_{\text{uniform}}(\pi)\, d\pi \\ &= \int_{\frac{1}{2}}^1 \left(\pi - \frac{1}{2}\right) 1 d\pi = \frac{1}{8}. \end{aligned}$$

(Check!)

Bargaining with incomplete information

Labor union bargaining: Version Ib

- Labor union knows the profits.
- In equilibrium, labor union demands all the profits, $w = \pi$, and the firm accepts it.
- The expected payoffs
  - labor union

$$u_{LU}^{Ib} = \int_0^1 \pi f_{\text{uniform}}(\pi)\, d\pi$$
$$= \int_0^1 \pi d\pi = \frac{1}{2},$$

  - firm

$$u_F^{Ib} = 0.$$

    Firm gets nothing.
- Notice that $u_{LU}^{Ib} > u_{LU}^{Ia}$. Labor union benefits from the extra information.

  Firm, clearly loses.
- Notice also that $u_{LU}^{Ib} + u_F^{Ib} > u_{LU}^{Ia} + u_F^{Ia}$, or that the sum of payoffs of two players is higher in *Ib* case.
  - this is because the behavior in Ia case is not efficient

Bargaining with incomplete information

Labor union bargaining: Version Ib

- The difference between cases Ia and Ib shows that more information can help a player.
- The question of more information in the bargaining has some economic motivation.
  Some countries (like Germany) have laws tat require that board of directors of a big company keeps a seat reserved for a representative of a labor union.
  Such representative is then privy to all the information available to the directors.
- One can whether laws like this are good or bad for the firm and the labor union.
  Our model can be used to illustrate why this question matters and some of the important issues (bargaining).
- Next, we present version II to show that more information can be actually worse.

Bargaining with incomplete information

Labor union bargaining: Version II

- Suppose that before the bargaining happens, before even profits are drawn, firm decides whether to build a factory or not.
  - Factory is costly $c > 0$. Assume that $c < \frac{1}{8}$.
  - If factory is not built, everybody gets payoff 0 and the game ends.
  - If factory is built, profits are drawn from F_{uniform}, and the bargaining commences.
- Will the firm build the factory?

Bargaining with incomplete information

Labor union bargaining: Version IIa

- Consider IIa model: firm will learn π, but labor union not.
- If firm builds the factory, payoffs
  - labor union u_{LU}^{Ia},
  - firm
$$u_F^{Ia} - c > 0.$$
- If firm does not build the factory, payoffs 0 to everybody.
- In equilibrium, firm builds the factory and the payoffs are

$$u_{LU}^{IIa} = u_{LU}^{Ia} = \frac{1}{4},$$
$$u_F^{IIa} = u_F^{Ia} - c = \frac{1}{8} - c.$$

Bargaining with incomplete information

Labor union bargaining: Version IIb

- Consider IIb model: firm and labor union will learn π.
- If firm builds the factory, payoffs
  - labor union u_{LU}^{Ib},
  - firm
  $$u_F^{Ib} - c = -c < 0.$$
- In equilibrium, firm does not build the factory and the payoffs are
$$u_{LU}^{IIb} = 0,$$
$$u_F^{IIb} = 0.$$
- Labor union is better under *IIa*-less information then under *IIb*-more information!