UNIVERSITY OF TORONTO

Faculty of Arts and Science

DECEMBER 2011 EXAMINATIONS

Advanced Economic Theory, ECO326F1H

Duration - 2 hours

No Aids Allowed

There are three questions. Each question has 20 points. Read questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You have 120 minutes.

(1) There are two periods. In the first period, two players play game

Player 1\ Player 2	Fight	Accommodate
Fight	-1,-1	-1,0
Accommodate	0,-1	1,1

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In the second period, they play game

Player 1\ Player 2	Nice	Mean
Nice	x,x	-1,-1
Mean	-1,-1	0,0

,

where $x > 0$. Their total payoff is equal to the sum of the payoffs received in each period.

(a) Find a subgame perfect equilibrium in which both players play Accommodate in the first period. (Carefully describe the strategies of both players.) Is such a subgame perfect equilibrium unique?

Consider strategies σ_i in which player $i = 1, 2$ plays Accommodate in the first period, and plays Nice in the second period, after each period-1 history.

- In any subgame $h \in \{AA, AF, FA, FF\}$, the payoff from playing Nice against Nice is x and the payoff from playing Mean against Nice is -1. Because $x > -1$, (Nice, Nice) is an equilibrium behavior in any period-2 subgame.
- Given the continuation behavior, the payoff from playing Accommodate against Acommodate is 1(+x) and the payoff from playing Fight is -1(+x). Because $1 > -1$, and because the continuation behavior does not depend on period-1 history, Accommodate is a best response to Accommodate in the first period.

(b) For which $x > 0$, does there exist a subgame perfect equilibrium in which both players play Fight in the first period? Explain.

We are going to show that if $x \geq 1$, then there is an subgame perfect equilibrium, in which both players Figth in the first period. Consider strategy profile in which both players Fight in the first period. In the second period, both players are Nice after history FF, and they are Mean after any other history.

- In any subgame $h \in \{AA, AF, FA, FF\}$, both (Nice,Nice) and (Mean,Mean) are equilibrium profiles in the subgame.
- Given the continuation behavior, the payoff from playing Fight against Fight is $-1 + x$ and Acommodate against Fight is $0 + 0$. If $x \geq 1$, then Fight is best response.

Next, suppose that $x < 1$. Notice that both (Nice,Nice) and (Mean,Mean) are equilibrium profiles in the subgame, and the only subgame perfect possible payoffs in the continuation game are x (from (Nice,Nice) equilibrium) or 0 (from (mean, Mean) equilibrium). Thus, the payoff from playing from playing Fight against Fight is not higher than $-1 + x$ and from playing Acommodate against Fight is not smaller than $0 + 0$. If $x < 1$, then the latter is higher than the former. It Fight, Fight cannot be a part of subgame perfect equilibrium profile.

(2) There are two taxpayers. Each of the taxpayers has low income with probability p and has high income with probability $1-p$. The level of income is unobservable neither by the Government nor the other taxpayer. Each of the taxpayers reports either low or high income. The Government collects tax t from each person who reported high income. Additionally, the Government can audit exactly one taxpayer. If both taxpayers report high income, then the Government does not audit anybody. If there is only one taxpayer with high income, the Government audits the taxpayer who reports low income. If both taxpayers report low income, the Government audits each of the taxpayers with equal probability $\frac{1}{2}$. If the taxpayer is audited and he is caught lying, he pays fine $F > t$. If the taxpayer is audited and his report is correct, nothing happens.

For example, if taxpayer 2 always reports low income (no matter what his true type is), then the expected payment of high income type of taxpayer 1 from reporting truthfully is equal to her tax, t, and the expected payment from lying and reporting low income is equal to the probability of being audited times the fine, $\frac{1}{2}F$.

We model the taxpayer game as a Bayesian game with two players - taxpayers, and where each taxpayer has one of two types $\{h, l\}$ and chooses one of two actions H or L. Each taxpayer minimizes the expected tax or fine payment.

(a) Suppose that taxpayer 2 always reports his income truthfully (i.e., he reports H, if he is type h, and he reports L, if he is type l). Compute the expected payment of type h of taxpayer 1 from reporting his income truthfully. Compute the expected payoff of type h of taxpayer 1 from always reporting low income. (Hint: notice that the probability that taxpayer 1 is audited depends on the type of taxpayer 2.)

If the taxpayer reports low income, her probability of being audited is $\frac{1}{2}$ if the other player's type is h (which happens with probability $1-p$) and 1 otherwise (which happens with probability p). If type h is audited, he gets to pay fine F. Thus, the payoff of the high type from reporting low income is $-\left(\frac{1}{2}F\,(1-p) + pF\right)$.

(b) Suppose that $t < \frac{1}{2}F$. Show that in the unique Bayesian Nash equilibrium, all players report their income truthfully.

Notice first that the low type always reports low income. Suppose that σ_{-i} is the probabilty that the type h of player $-i$ reports high income truthfully. Then the payoff of type h of player from:

- reporting H is equal to $-t$,
- reporting L is equal to $-(1-p)\left(\sigma_{-i}F + (1-\sigma_{-i})\frac{1}{2}F\right) - p\frac{1}{2}F \leq -(1-p)\frac{1}{2}F - p\frac{1}{2}F = -\frac{1}{2}F$. (Notice that
$$\sigma_{-i}F + (1-\sigma_{-i})\frac{1}{2}F \geq \frac{1}{2}F.)$$

If $t < \frac{1}{2}F$, then reporting H leads to higher payoff for type h, no matter what the other taxpayer doing. Thus, reporting truthfully is the only equilibrium.

(c) Suppose that $\left[\frac{1}{2}p + (1-p)\right]F < t$. Find all Bayesian Nash equilibria. Is the equilibrium unique?

The low type always reports low income. If $\left[\frac{1}{2}(1-p) + p\right]F < t$, we check in the same way as above that no matter what is the other taxpayer doing, the payment from L is lower than from H. Thus, always reporting low income is the only equilibrium.

(d) Finally, suppose that
$$\frac{1}{2}F < t < \left[\frac{1}{2}p + (1-p)\,p\right]F.$$
Find all pure strategy Bayesian Nash equilibria. Is the equilibrium unique?

There are two pure strategy equilibria: one described in (b) and the other described in (c).

(3) A firm and a worker bargain about the split of the firm's revenue R. The bargaining process takes form of alternating offers made by the worker or the firm. At each stage, one party makes an offer of splitting revenue $(w, R-w)$,

where w goes to the worker and $R-w$ goes to the firm. If the offer is rejected, the other party gets to make an offer in the next period. If the offer is accepted, the game ends and players receive their share of the revenue. In each period in which the offer is rejected, the firm pays $c > 0$ for the cost of up-keeping the machines and the worker receives picketing wage $b > 0$ from the labor union fund. Thus, if the offer $(w, R-w)$ is accepted in period t, the firm's payoff is equal to

$$R - (t-1)\,c,$$

and the worker's payoff is equal to

$$(t-1)\,b + w.$$

If the players do not reach an agreement in the first T periods, the game ends with payoff $-Tc$ to the firm and payoff Tb to the worker.
We assume that $b < c$ and that the revenue R is perfectly divisible.

(a) Suppose that $T = 1$ and the firm makes the first and only offer. Find the unique subgame perfect equilibrium payoffs. (Hint: remember that the worker receives his benefit b if and only if he rejects the firm's offer.)

The firm offers $(b, R-b)$ and the offer is accepted.

(b) Suppose that $T = 2$ and the worker makes the first offer (which, if rejected, is followed by the firm's offer). Find the unique subgame perfect equilibrium payoffs. (Hint: use part (a).)

If the worker's offer is rejected, the payoffs are$(2b, R-b-c)$. If the worker wants its offer to be accepted, it must propose $(b+c, R-b-c)$.Because $c+b > 2b$, the worker prefers to propose an offer that will be accepted.

(c) Suppose that $T = 3$ and the firm makes the first offer. Find the subgame perfect equilibrium payoffs.

If the firm's offer is rejected, the payoffs are$(2b+c, R-b-2c)$. If the firm wants its offer to be accepted, it must propose $(2b+c, R-2b-c)$. The firm prefers its offer to be accepted.

(d) Suppose that $T = 2n + 1$ for some n and that the firm makes the first offer. Find the subgame perfect equilibrium payoffs.

If the firm's offer is rejected, the payoffs are $(b + n(b + c), R - n(b + c) - c)$.
If the firm wants its offer to be accepted, it must propose $(b + n(b + c), R - b - n(b + c))$.
The firm prefers its offer to be accepted.