UNIVERSITY OF TORONTO

Faculty of Arts and Science

December 2013 Final exam

Advanced Economic Theory, ECO326H1F

Instructor: Marcin Pęski

Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.) You have 120 minutes.

Total pages (including the title page): 4

Total marks: 100

Good luck!

(1) (35 points) In the Rubinstein bargaining model, there are two players who are trying to divide a pie of size 1. In each period $t \leq T$, one player offers a division of the pie $(x, 1-x)$, and the other player accepts (in which case the game ends with payoffs $\delta^t x$ for player 1 and $\delta^t (1-x)$ for player 2) or rejects, in which case either, if $t < T$, the game moves to the next period $t+1$, or, if $t = T$ the game ends with payoffs 0 for both players. In the model that you learned in the class, the players alternate between making offers. Here, we consider a modification of the Rubinstein bargaining model in which player 1 has an opportunity to make an offer twice as often as player 2.

(a) Find the subgame perfect equilibrium strategies and payoffs in the game with 1 period only ($T = 1$), in which player 1 makes the only offer.

(b) Find the subgame perfect equilibrium payoffs in the game with $T = 2$ periods , in which player 1 makes the offer in the first and player 2 makes the offer in the second period.

(c) Find the subgame perfect equilibrium payoffs in the game with $T = 3$ periods , in which player 1 makes the offer in the first period and in the second period, and player 2 makes an offer in the last period.

(d) Find the subgame perfect equilibrium payoffs in the game with $T = 4$ periods , in which player 2 makes the offer in periods $t = 1$ and $t = 4$ and player 1 makes an offer in periods $t = 2$ and $t = 3$.

(e) (Extra credit) Find the equilibrium payoffs when $T = 3k$ and $k \to \infty$ with player 1 making offers in periods $t = 1, 2, 4, 5, 7, 8, ..., 3k-2, 3k-1$ and player 2 making offers in periods $t = 3, 6, 9, ..., 3k$.

(2) (30 points) Two entrepreneurs simultaneously make investment decisions. Andy chooses the level of investment equal to $e_1 \geq 0$, $e_1 \in R$. His payoff is equal to

$$e_1 (1 + \theta e_2) - e_1^2.$$

Here, $\theta = 6$ is a demand parameter. Beth chooses between two investment levels $e_2 = 0$ or $e_2 = 1$. (In particular, it is not possible to choose for her any intermediate level of effort). Her payoffs are equal to

$$\begin{cases} e_1 - c, & \text{if } e_2 = 1, \\ 0, & \text{if } e_2 = 0. \end{cases}$$

Here $c = 3$ is a cost parameter.

(a) Show that there exists a (Nash) equilibrium in which Beth invests.

(b) Suppose that Beth has two possible cost types, $c_L = \frac{7}{3} < c_H = \frac{11}{3}$. Andy does not know the cost type of Beth and he assigns probability $p = \frac{1}{2}$ to the low cost type, so that $pc_L + (1 - p) c_H = c = 3$. Beth knows her cost type. Show that Beth does not invest at all in any Bayesian Nash equilibrium of the game with incomplete information.

(c) You can interpret (a) as a game with incomplete information in which neither Beth nor Andy knows Beth's cost type and both of the players have the same beliefs p. Compare the answers to (a) and (b). Does providing Beth with more information about her cost type improves her equilibrium payoffs?

(d) Finally, suppose that both Beth and Andy know Beth cost type. Show that there exists an equilibrium in which Beth makes some investment.

(3) (35 points) Consider a two-player two-period game in which in the first period, the players play the Prisoner's Dilemma

Pl. 1\Pl. 2	Cooperate	Defect
Cooperate	9,9	0,10
Defect	10,0	5,5

,

and in the second period, the players play the following game

Pl. 1\Pl. 2	Normal	Crazy
Normal	0,0	-5,-10
Crazy	-10,-5	-10,-10

,

(a) Show that in any Subgame Perfect Equilibrium (SPE), no player Cooperates in the first period. Carefully explain the equilibrium strategies.

(b) Explain that the two-period game has a Nash equilibrium (not necessarily SPE) in which the players Cooperate in the first period. Carefully explain the equilibrium strategies and show that they indeed form an equilibrium.

(c) Suppose that instead of the game described above, the players play the Prisoner's Dilemma in the first period followed by the Battle of Sexes (with payoffs given in the table below)

Pl. 1\Pl. 2	Stadium	Opera
Stadium	5,3	0,0
Opera	0,0	3,5

.

in the second period. Show that there exists an SPE in which both players Cooperate in the first period. Carefully explain the equilibrium strategies and show that they indeed form an equilibrium.

Total pages: 4

Total marks: 100