UNIVERSITY OF TORONTO

Faculty of Arts and Science

April 2014 Final exam

Advanced Economic Theory, ECO326H1S

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Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.)

The questions with * are more difficult and you should attempt to solve them if you have enough time.

You have 120 minutes.

Good luck!

Total pages (including the title page): 6

Total marks: 100

(1) (40 points) A firm and a labor union bargain over wages. There are N rounds of bargaining. Each round of the bargaining has the same structure: First, one of the sides proposes a wage $w \in [0, \pi]$, where $\pi > 0$ are the total revenues, and the other side accepts or rejects the offer. If the offer is accepted in round $t = 1, ..., N$, then the game ends with the labor union's payoff equal to $(N - t + 1)\, w$ (i.e., wages for all the remaining rounds) and the firm's payoff is equal to $(N + 1 - t)\,(\pi - w)$. If the offer is rejected, then the firm and the labor union play a strike-lockout game

Firm\labor union	Work	Strike
Work	$\frac{1}{2}\pi, \frac{1}{2}\pi$	$0, 0$
Lockout	0,0	$0, 0$

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In other words, both firm and the labor union decide whether to work under temporary agreement for one period or not. After the strike-lockout game is resolved, the next round of the bargaining commences.

In this exercise, "equilibrium" means "pure strategy subgame perfect equilibrium".

(a) Suppose that $N = 1$ and the first (and the only) proposal is made by the union. Explain that there exists an equilibrium of the bargaining game in which if the offer is rejected, neither the labor union nor the firm works. Carefully describe the strategies of the players. What is the payoff of the firm?

(b) Suppose that $N = 1$ and the first (and the only) proposal is made by the union. Describe all equilibria of the game. What is the best possible payoff that the firm can get in an equilibrium? What is the worst? What is the best possible payoff that the worker can get in equilibrium?

(c) Suppose that $N = 2$, the first offer is made by the firm and the second one by the union. Describe all equilibria. What is the best possible payoff that the firm can get in an equilibrium? What is the best possible payoff that the worker can get in equilibrium?

(d) *Find the best equilibrium payoff of each player in the alternating offer game for general N.

(2) (25 points) Two players play infinitely repeated Prisoner's Dilemma with payoffs

Pl. 1\Pl. 2	C	D
C	x,x	0,y
D	y,0	1,1

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We assume that $y > x > 1$. The players discount future with discount factor $\delta < 1$.

Define a strategy σ_{TT} "Tit for Tat": Player i begins with C in period 1, and then in each period $t > 1$, player i repeats the action of player $-i$ in period $t-1$. For example, if player $-i$ plays C, D, C, C, D in periods $t = 1, ..., 5$, then Tit for Tat strategy calls for C, C, D, C, C.

(a) Suppose that both players are using Tit for Tat. Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

(b) Suppose that player 1 plays Tit for Tat and player 2 always plays D. Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

(c) Suppose that player 1 plays Tit for Tat and player 2 uses a strategy that looks like Tit for Tat with one difference: player 2 begins with action D in the first period and then continues with Tit for Tat in all subsequent periods (i.e., he repeats the action of player 1 from the previous period). Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

(d) Suppose that $y = 5$ and $x = 2$. Is a profile Tit for Tat strategies an equilibrium? Does this claim depend on the discount factor? (Hint: It may help to recall that $a^2 - b^2 = (a - b)(a + b)$ for any $a, b \in R$.)

(e) * Suppose that $y = 3$ and $x = 2$. Show that Tit for Tat is an equilibrium strategy for sufficiently high $\delta < 1$.

(3) (35 points) Allan is an engineer who is working on a patent that, as he claims, is going to be revolutionary. He needs financing to fully develop the patent. Beth is a venture capitalist who specializes in financing risky projects. Being experienced investor, Beth does not trust Allan's claims about the quality of his discovery. Beth believes that the true value of the project is either high and equal to θ_H with probability $p \in (0,1)$, or the value is low and equal to θ_L with probability $1-p$. We assume that $0 \leq \theta_L < \theta_H$. Allan knows the true value of the project θ (or, in our terminology, θ is Allan's type). Allan decide how much effort e_A to put into the development of the project. Beth decides how much money $e_B \geq 0$ to invest. Allan's payoff is

$$(\theta + e_B)\, e_A - e_A^2.$$

Beth's payoff is equal to

$$(\theta + e_A)\, e_B - e_B^2.$$

(a) Compute the best response of Allan's types given Beth strategy.

(b) Compute Beth's best response given Allan's strategy.

(c) Find the Bayesian Nash equilibrium of the game. Compute the expected payoffs of Beth and each of the Allan's types.

(d) If Beth were able to obtain full access to Allan's documentation, she would be able to fully evaluate the project and learn the true value θ. Suppose that she gets such access. What is the new Bayesian Nash equilibrium? Compute all the payoffs. What is Beth expected payoff before she examins Allan's project?

(e) Is it in Beth interest to obtain more information about the project? To answer this question, compare the expected Beth's payoff that you found in point (c) with the expected payoff that Beth hopes to receive once she learns about the true quality of the project (i.e., the expectation over Beth's payoffs that you computed in point (d)). For simplicity, in this and subsequent question, you can assume that $\theta_L = 0, \theta_H = 2, p = \frac{1}{2}$.

(f) Is the in Allan's interest to grant such an information? What would happen, in your opinion, if Beth asked Allan for the information? Would

he give it to her? How does your answer depend on Allan's type? Could Beth learn anything from observing Allan's reaction to Beth request?

Total pages: 6

Total marks: 100