UNIVERSITY OF TORONTO

Faculty of Arts and Science

April 2014 Final exam

Advanced Economic Theory, ECO326H1S

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Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.)

The questions with * are more difficult and you should attempt to solve them if you have enough time.

You have 120 minutes.

Good luck!

Total pages (including the title page): 6

Total marks: 100

(1) (40 points) A firm and a labor union bargain over wages. There are N rounds of bargaining. Each round of the bargaining has the same structure: First, one of the sides proposes a wage $w \in [0, \pi]$, where $\pi > 0$ are the total revenues, and the other side accepts or rejects the offer. If the offer is accepted in round $t = 1, ..., N$, then the game ends with the labor union's payoff equal to $(N - t + 1)\, w$ (i.e., wages for all the remaining rounds) and the firm's payoff is equal to $(N + 1 - t)\,(\pi - w)$. If the offer is rejected, then the firm and the labor union play a strike-lockout game

Firm\labor union	Work	Strike
Work	$\frac{1}{2}\pi, \frac{1}{2}\pi$	$0, 0$
Lockout	0,0	$0, 0$

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In other words, both firm and the labor union decide whether to work under temporary agreement for one period or not. After the strike-lockout game is resolved, the next round of the bargaining commences.

In this exercise, "equilibrium" means "pure strategy subgame perfect equilibrium".

(a) Suppose that $N = 1$ and the first (and the only) proposal is made by the union. Explain that there exists an equilibrium of the bargaining game in which if the offer is rejected, neither the labor union nor the firm works. Carefully describe the strategies of the players. What is the payoff of the firm?

Strike-Lockout is an equilibrium in the subgame following the rejection. The equilibrium strategies:

$$\text{Period } 1: \begin{cases} w = \pi, \end{cases} \quad \begin{cases} A, & \text{if } w \leq \pi \\ R, & \text{otherwise} \end{cases}$$

$$\text{Period } 2: (S, L)$$

$$\text{Payoffs: } (\pi, 0)$$

(b) Suppose that $N = 1$ and the first (and the only) proposal is made by the union. Describe all equilibria of the game. What is the best possible payoff that the firm can get in an equilibrium? What is the worst? What is the best possible payoff that the worker can get in equilibrium?

One equilibrium is described above. In general, there exists an equilibrium for each $w^* \in \left[\frac{1}{2}\pi, \pi\right]$:

$$\text{Period } 1 : \begin{cases} w = w^*, & \begin{cases} A, & \text{if } w \leq w^* \\ R, & \text{otherwise} \end{cases} \end{cases}$$

$$\text{Period } 2 : \begin{cases} (W, W) & \text{if } w > w^* \\ (S, L) & \text{otherwise} \end{cases}$$

$$\text{Payoffs: } (w, \pi - w)$$

The best possible payoff of the firm is equal to $\frac{1}{2}\pi$. The worst possible is equal to 0. The best possible payoff to the firm is equal to

(c) Suppose that $N = 2$, the first offer is made by the firm and the second one by the union. Describe all equilibria. What is the best possible payoff that the firm can get in an equilibrium? What is the best possible payoff that the worker can get in equilibrium?

There are many equilibria. We describe four equilibria with the extreme payoffs for all the parites. The equilibria vary depending on the outcome of the strike-lockout game following rejection in each period:

- If in both periods, (W, W) follows rejected offers, the continuation payoff after the offer is rejected in the first period is equal to

$$\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)_{\text{in the strike game in the first period}} + \left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)_{\text{in the second period bargaining game}} = (\pi,$$

  (The first payoff is the union.) If the offer w is accepted, the labor and the firm get $(2w, 2(\pi - w))$. The offer is going to be accepted if $w \geq \frac{1}{2}\pi$, and the firm will offer $w^* = \frac{1}{2}\pi$. The payoff is (π, π).

- (W, W) in the first period after rejection, and (S, L) after rejection in the second period. Payoffs

$$\left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)_{\text{in the strike game in the first period}} + (\pi, 0)_{\text{in the second period bargaining game}} = \left(\frac{3}{2}\pi, \frac{1}{2}\pi\right)$$

  (The first payoff is the union.) If the offer w is accepted, the labor and the firm get $(2w, 2(\pi - w))$. The offer is going to be accepted if $w \geq \frac{3}{4}\pi$, and the firm will offer $w^* = \frac{3}{4}\pi$. The payoff is $\left(\frac{3}{2}\pi, \frac{1}{2}\pi\right)$.
- (S, L) in the first period after rejection, and (W, W) after rejection in the second periodBad equilibrium in the first period, followed by good equilibrium: Payoffs

$$(0, 0)_{\text{in the strike game in the first period}} + \left(\frac{1}{2}\pi, \frac{1}{2}\pi\right)_{\text{in the second period bargaining game}} = \left(\frac{1}{2}\pi, 0\right)$$

  (The first payoff is the union.) The offer is going to be accepted if $w \geq \frac{1}{4}\pi$, and the firm will offer $w^* = \frac{1}{4}\pi$. The payoff is $\left(\frac{1}{2}\pi, \frac{3}{2}\pi\right)$.
- (S, L) after rejections in both periods: Payoffs

$$(0, 0)_{\text{in the strike game in the first period}} + (\pi, 0)_{\text{in the second period bargaining game}} = (\pi, 0).$$

  (The first payoff is the union.) The offer is going to be accepted if $w \geq \frac{1}{2}\pi$, and the firm will offer $w^* = \frac{1}{2}\pi$. The payoff is (π, π).

The best equilibrium payoff for the firm is $\frac{3}{2}\pi$ and this is also the best equilibrium payoff for the worker.

(d) *Find the best equilibrium payoff of each player in the alternating offer game for general N.

(2) (25 points) Two players play infinitely repeated Prisoner's Dilemma with payoffs

Pl. 1\Pl. 2	C	D
C	x,x	0,y
D	y,0	1,1

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We assume that $y > x > 1$. The players discount future with discount factor $\delta < 1$.

Define a strategy σ_{TT} "Tit for Tat": Player i begins with C in period 1, and then in each period $t > 1$, player i repeats the action of player $-i$ in period $t-1$. For example, if player $-i$ plays C, D, C, C, D in periods $t = 1, ..., 5$, then Tit for Tat strategy calls for C, C, D, C, C.

(a) Suppose that both players are using Tit for Tat. Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

The outcome is $(CC, CC, CC, ...)$. The payoff of each player is equal to $\frac{x}{1-\delta}$.

(b) Suppose that player 1 plays Tit for Tat and player 2 always plays D. Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

The outcome is $(CD, DD, DD, ...)$. The payoff of player 1 is equal to $\frac{\delta}{1-\delta}$. The payoff of player 2 is $y + \frac{\delta}{1-\delta}$.

(c) Suppose that player 1 plays Tit for Tat and player 2 uses a strategy that looks like Tit for Tat with one difference: player 2 begins with action D in the first period and then continues with Tit for Tat in all subsequent periods (i.e., he repeats the action of player 1 from the previous period). Describe the outcome of the game (i.e., the history of actions taken in all periods). What are the players payoff?

The outcome is $(CD, DC, CD, DC, ...)$. The payoff of player 1 is

$$0 + \delta y + \delta^2 0 + \delta^3 y + ... = \delta y \left(1 + \delta^2 + ...\right) = \delta y \frac{1}{1-\delta^2}.$$

The payoff of player 2 is

$$y \frac{1}{1-\delta^2}.$$

(d) Suppose that $y = 5$ and $x = 2$. Is a profile Tit for Tat strategies an equilibrium? Does this claim depend on the discount factor? (Hint: It may help to recall that $a^2 - b^2 = (a-b)(a+b)$ for any $a, b \in R$.)

A simple proof: Player 2 compares A. the payoff $\frac{2}{1-\delta}$ from playing Tit for Tat and B. payoff $\frac{5}{1-\delta^2}$ from playing the strategy described in point (c). Because

$$\frac{5/\left(1-\delta^2\right)}{2/\left(1-\delta\right)} = \frac{5}{2}\frac{1-\delta}{1-\delta^2} = \frac{5}{2}\frac{1}{1+\delta} = \frac{5}{2\left(1+\delta\right)},$$

where we used the fact that $1-\delta^2 = (1-\delta)(1+\delta)$. Because $\delta < 1$, $5 > 2(1+\delta)$, and the payoff from B is higher than the payoff from A. Alternatively, notice that in case A, player 2 receives payoff x in each period. In case B, player 2 alternates between payoffs y in odd periods and 0 in even periods. Because

$$y + \delta 0 = 5 > 2(1+\delta) = x + \delta x,$$

the payoff from the first periods is higher in B than in A. Because similar comparison holds for any other group of subsequent periods, starting with an odd periods, the payoff in B must be higher than the payoff in A.

(e) * Suppose that $y = 3$ and $x = 2$. Show that Tit for Tat is an equilibrium strategy for sufficiently high $\delta < 1$.

(3) (35 points) Allan is an engineer who is working on a patent that, as he claims, is going to be revolutionary. He needs financing to fully develop the patent. Beth is a venture capitalist who specializes in financing risky projects. Being experienced investor, Beth does not trust Allan's claims about the quality of his discovery. Beth believes that the true value of the project is either high and equal to θ_H with probability $p \in (0, 1)$, or the value is low and equal to θ_L with probability $1-p$. We assume that $0 \leq \theta_L < \theta_H$. Allan knows the true value of the project θ (or, in our terminology, θ is Allan's type). Allan decide how much effort e_A to put into the development of the project. Beth decides how much money $e_B \geq 0$ to invest. Allan's payoff is

$$(\theta + e_B)\, e_A - e_A^2.$$

Beth's payoff is equal to

$$(\theta + e_A)\, e_B - e_B^2.$$

(a) Compute the best response of Allan's types given Beth strategy.

Allan's best response maximizes his payoffs. FOCs

$$\theta + e_B - 2e_A = 0,$$

which implies that

$$e_A(\theta) = \frac{1}{2}(\theta + e_B).$$

(b) Compute Beth's best response given Allan's strategy.

Beth's best response maximizes her expected payoffs. FOCs

$$E\theta + Ee_A - 2e_B = 0,$$

where $E\theta = p\theta_H + (1-p)\,\theta_L$, and $Ee_A = pe_{A,H} + (1-p)\,e_{A,L}$. This implies that

$$e_B = \frac{1}{2}(E\theta + Ee_A).$$

(c) Find the Bayesian Nash equilibrium of the game. Compute the expected payoffs of Beth and each of the Allan's types.

Notice that

$$Ee_A = \frac{1}{2}E\theta + \frac{1}{2}e_B.$$

Together with the Beth best response, we get

$$e_B = \frac{1}{2}E\theta + \frac{1}{2}\left(\frac{1}{2}E\theta + \frac{1}{2}e_B\right).$$

Solution

$$e_B^* = E\theta$$

amd

$$e_A^*(\theta) = \frac{1}{2}\theta + \frac{1}{2}E\theta.$$

In particular, the expected Allan's effort it equal to

$$Ee_A^* = E\theta.$$

action of Allan

Beth's expected payoffs are equal to

$$(E\theta + Ee_A^*)\, e_B^* - (e_B^*)^2 = (E\theta + E\theta)\, E\theta - (E\theta)^2 = (E\theta)^2.$$

Allan's type θ payoffs are equal to

$$(\theta + e_B^* - e_A^*(\theta))\, e_A^*(\theta) = \left(E\theta + \frac{1}{2}\theta - \frac{1}{2}E\theta\right)\left(\frac{1}{2}\theta + \frac{1}{2}E\theta\right).$$

(d) If Beth were able to obtain full access to Allan's documentation, she would be able to fully evaluate the project and learn the true value θ. Suppose that she gets such access. What is the new Bayesian Nash equilibrium? Compute all the payoffs. What is Beth expected payoff before she examins Allan's project?

If Beth knows θ, then the game becomes a game with perfect information. We can solve the game using standard methods: For each θ, there is a unique nash equilibrium with

$$e_A^{**}(\theta) = e_B^{**} = \theta.$$

Allans and Beth's payoffs are equal to

$$\theta^2.$$

(e) Is it in Beth interest to obtain more information about the project? To answer this question, compare the expected Beth's payoff that you found in point (c) with the expected payoff that Beth hopes to receive once she learns about the true quality of the project (i.e., the expectation over Beth's payoffs that you computed in point (d)). For simplicity, in this and subsequent question, you can assume that $\theta_L = 0, \theta_H = 2, p = \frac{1}{2}$.

Beth expected payoff without information is $(p\theta_H + (1-p)\,\theta_L)^2$. Beth expected payoff with the information is $p\theta_H^2 + (1-p)\,\theta_L^2$. The latter number is always bigger than the former one. (This is a mathematical fact. You can verify that the relationship is satisfied given the example payoffs).

(f) Is the in Allan's interest to grant such an information? What would happen, in your opinion, if Beth asked Allan for the information? Would he give it to her? How does your answer depend on Allan's type? Could Beth learn anything from observing Allan's reaction to Beth request?

Total pages: 6

Total marks: 100