UNIVERSITY OF TORONTO

Faculty of Arts and Science

December 2015 Final exam

Advanced Economic Theory, ECO326H1F

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Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.) Questions denoted with (*) are for extra credit; they are more difficult, and you should attempt them only if you have extra time.

You have 120 minutes.

Good luck!

Total pages (including the title page): 5

Total marks: 100

(1) (40 points) We consider a version of the alternating offer bargaining model. There are two players and $T \geq 1$ rounds of bargaining. In each round t, the size of the pie is equal to $\pi_t > 0$ and it may differ across periods. In round t, one player (player 1 if t is odd and player 2 if t is even) proposes to divide the pie so that the player who makes an offer gets x and the other player gets $\pi_t - x$. We allow for offers $x > \pi_t$ or $x < 0$. The other player either accepts or rejects the offer. If the offer is accepted, the game ends and the players receive payoffs equal to their share of the pie. If the offer is rejected in round $t < T$, the game moves to the next round. If the offer is rejected in the last round, both players receive payoff 0.

Comment: This is a more general version of the game that we considered in the class (in the game in class, we assumed $\pi_t = \delta^{t-1}$, where $\delta < 1$ was a discount factor). The more general version allows us to see how does bargaining depends on how the size of the pie evolves in time.

(a) Suppose that $T = 1$. Let π_1 be the size of the pie in period 1. Find the subgame perfect equilibrium (SPE) strategies and payoffs.

(b) Suppose that $T = 2$ and $\pi_1 > \pi_2$. Find the SPE strategies and payoffs.

(c) Suppose that $T = 3$ and $\pi_1 > \pi_2 > \pi_3$. Find the SPE strategies and payoffs.

(d) Suppose that that $\pi_1 < \pi_2 < ... < \pi_T$, or, in other words, the size of the pie grows in each period. Describe the SPE behavior. (For partial credit, assume that $T = 2$).

(e) Finally, suppose that $T = 3$ and $\pi_1 = 1, \pi_2 = 3, \pi_3 = 2$. When is the agreement (i.e., an offer being accepted) going to be reached in the SPE?

(2) (30) There are two firms $i = 1, 2$ that interact with each other through $T \geq 1$ periods.

(a) Suppose that $T = 1$ and that the two firms play Cournot duopoly, i.e., they choose quantities $q_i \geq 0$ and receive payoffs equal to $\pi_i (q_i, q_{-i}) = q_i (\alpha - c - (q_i + q_{-i}))$. Explain that a symmetric profile (q^*, q^*), where $q^* = \frac{1}{4} (\alpha - c)$, maximizes their joint payoffs. Is (q^*, q^*) a Nash equilibrium profile? Why or why not?

(b) Suppose that $T = 2$ and that the two firms play Cournot duopoly in each period. Does there exist an SPE in which the two firms play q^* in the first period? Find such an SPE or prove that it does not exist.

(c) Will your answer to the previous question change if $T > 2$ and the two firms play Cournot duopoly in each period?

(d) Suppose now that $T = 2$, the firms play Cournot duopoly in the first period, and they play an investment game in the second period. The payoffs in the investment game are presented below. For what values of x does there exist an SPE such that the firms choose q^* in the first period?

	Invest	Don"t Invest
Invest	x, x	$-2, 0$
Don't Invest	$0, -2$	$0, 0$

(e) Suppose that the firms play an infinitely repeated Cournot duopoly. They discount future payoffs with discount factor $\delta < 1$. Show that if δ is high enough, then there exist a Nash equilibrium, in which the two firms play q^* in each period.

(3) (30) Ann and Bob have boarded a train at the same station. They discovered that there was only one empty seat left. Each of them must choose whether to try to Take the seat or whether to Pass. Each of them can have one of two types: polite or impolite. Each individual knows his or her own type. They also know that the probability that the other individual $i = A, B$ has a polite type is equal to π_i.

If individual i is polite, and she faces a polite individual $-i$, individual i gets payoff 5 if he or she plays P and she gets 0 if he or she plays T.

In all other cases (i.e., if i is impolite or i is polite and $-i$ is impolite), i gets payoffs described in the following table:

Action of i \ Action of $-i$	T	P
T	2	10
P	4	0

For example, if Ann is polite and Bob is impolite and she chooses T and he chooses P, she gets payoff 10, and he gets 4. If, instead, Bob were to be polite, she would get only 0, and he would get 5.

(a) Suppose that Ann plays the following strategy: T if she is impolite, and P if she is polite. Explain that Bob's polite type expected payoff from playing T is equal to $2 - 2\pi_A$.

(b) What is Bob's best response strategy to Ann's strategy from the previous question? How does your answer depend on the probability that Ann is polite π_A? How does it depend on the probability that Bob is impolite?

(c) Find the conditions on the beliefs π_i such that there exists a Bayesian Nash equilibrium, in which all impolite types play T and all polite types play P.

(d) Find the conditions on the beliefs such that there exists a Bayesian Nash equilibrium, in which both types of Ann always play T, and both types of Bob play P.

(e) Does polite Bob have a dominant strategy? How does your answer depend on π_A and/or π_B?

Total pages: 5

Total marks: 100