UNIVERSITY OF TORONTO

Faculty of Arts and Science

December 2016 Final exam

Advanced Economic Theory, ECO326H1F

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Duration - 120 minutes

No Aids Allowed

There are three questions with total worth of 100 points. Read the questions carefully. You must give a supporting argument and an answer in words to get full credit. If you don't know the answer to any of the parts, try to solve the next one. You do not need to compute the exact values of algebraic formulas (for example, it is OK to say that $x = 2 * \frac{1}{5} * \sqrt{100}$ instead of $x = 4$.) Questions denoted with (*) are for extra credit; they are more difficult, and you should attempt them only if you have extra time.

You have 120 minutes.

Good luck!

Total pages (including the title page): 6

Total marks: 100

(1) (25 points) In this question, you will analyze a 3-person version of the n-stage alternating offer bargaining game. In the first stage of the n-stage game, player 1 makes an offer (x_1, x_2, x_3) where $x_i \geq 0$ and $x_1 + x_2 + x_3 = 1$. Subsequently, player 2 first and player 3 next decide whether to accept or reject the offer. If the offer is accepted by both players, the game ends and each player i gets x_i. Otherwise, if the offer is rejected by one of the players, the game moves to the next stage. In the second stage, player 2 makes an offer and the players 3 and 1 (in that order) accept or reject. In stage 3 (if $n \geq 3$), player 3 makes the offer and players 1 and 2 (in that order) decide whether to accept or reject. And so on in the subsequent stages. The players discount future with factor $\delta < 1$.

(a) Suppose that $n = 1$, i.e., there is only the first stage. Find the SPE behavior and payoffs.

(b) Suppose that $n = 2$. Find the SPE payoffs.

(c) Suppose that $n = 3$. Find the SPE payoffs.

(d) Suppose that $n = 4$. Find the SPE payoffs.

(e) Let $(\pi_1^n, \pi_2^n, \pi_3^n)$ be the expected SPE payoffs in the n-stage game. Describe a recursive formula for $(\pi_1^n, \pi_2^n, \pi_3^n)$ as a function of the $(n-1)$-stage game, $\left(\pi_1^{n-1}, \pi_2^{n-1}, \pi_3^{n-1}\right)$. Carefully explain your answer.

(2) (40) There are two players involved in a multi-period interaction. In the first $N-1$ periods, the two players play the favor exchanging game: each player chooses effort level $e_i \geq 0$ and receives payoff equal to

$$\frac{3}{2}e_{-i} - e_i.$$

The interpretation is that e_i is the cost of i's effort, and $\frac{3}{2}e_{-i}$ is the benefit obtained from the effort of the other player. The game played in the last period is described below and it varies across different parts of this question. The payoffs in the multi-period interaction are equal to the sum of payoffs obtained in each period (i.e., the players do not discount future).

(a) (For this part of the question only, assume that the favor exchange game is played once and nothing follows it.) Find a static Nash equilibrium(-a) of the favor exchange game.

(b) From now on, we consider the N-period interaction described as above. Suppose that in the last (i.e, the Nth) period, the players play the same favor exchange game. Find the SPE(s) of the multi-period interaction. Is the SPE unique?

(c) Suppose that in the last period, the players play the coordinated investment game:

	I	N
I	1, 1	-2,0
N	0,-2	0, 0

Assume that $N = 2$ (i.e., one iteration of the favor exchange is followed by the investment game). Describe an SPE in which both players choose the same effort level $e^1 = e_i^1$ for $i = 1, 2$ in the first period. What is the largest value of e^1 that is consistent with SPE? What are the payoffs in such an equilibrium.

(d) Suppose that $N = 3$ and the last period game is the coordinated investment (i.e., two iterations of the favor exchange are followed by the investment game). Describe an SPE in which both players choose the same effort level $e^1 = e_i^1$ for $i = 1, 2$ in the first period. What is the largest value of e^1 that is consistent with SPE?

(e) Suppose that $N = 2$. Design a last period game such that in the 2-period interaction, (i) there is a unique SPE, and no effort is exerted in the first period in this SPE, (ii) there is a Nash equilibrium, in which the players exert strictly positive effort in the first period.

(3) (35) Two investors, Elo and Fran, live and work in Raxiland. Elo builds mines and Fran runs trains. They both simultaneously choose whether to Invest or Not and they receive payoffs

	I	N
I	π, π	-5,0
N	0,-5	0, 0

The profits from the successful investment π are unknown and they depend on the conditions of the economy. With equal probability $\frac{1}{3}$, they are equal to $-10, 1, 10$, which corresponds to horrible, so-so, and great conditions. In order to learn more about the state of the economy, Elo reads The Raxiland Times and learns with probability $\frac{1}{3}$ that the economy is good and with probability $\frac{2}{3}$ that the economy is either so-so or horrible. (Unfortunately, The Times analyst cannot help its readers to differentiate between so-so and horrible conditions.) Fran reads The Raxiland Gazette and she learns with probability $\frac{2}{3}$ that the economy is either great or so-so and with probability $\frac{1}{3}$ she learns that the economy is OK. (The Gazette cannot differentiate between so-so and great conditions.)The beliefs of Elo can be represented in the Table:

Elo's type\Fran's Type	Horrible	OK
Great	$p = 0$,	$p = 1$,$\pi = 10$
Not good	$p = \frac{1}{2}$,$\pi = -10$	$p = \frac{1}{2}$,$\pi = 1$.

For example, if Elo reads that the economy is Not good, he assigns equal probability $\frac{1}{2}$ that it is horrible (in which case Fran learns that it is Horrible and $\pi = -10$) and the same probability that it is so-so (in which case, Fran learns that the economy is OK, and $\pi = 1$). Fran's beliefs can be represented as follows

Elo's type\Fran's Type	Great	Not good
Horrible	$p = 0$,	$p = 1$,$\pi = -10$
OK	$p = \frac{1}{2}$,$\pi = 10$	$p = \frac{1}{2}$,$\pi = 1$.

For example, if Fran reads that the economy is Horrible, she knows for sure that $\pi = -10$ and that Elo must learn that the economy is Not good.

(a) Suppose that Fran uses a strategy that makes her always Invest. What is the Elo's best response?

(b) Explain carefully that (some) of Fran's types has a strictly dominant action.

(c) Does any type of Elo has a dominant action?

(d) Describe (pure-strategy) Bayesian Nash equilibrium(-ia). What are the expected payoffs of the players?

(e) Fran considers changing newspaper subscription and start reading The Times. In such a case, she would obtain the same information as Elo. What would be the new equilibrium? If the equilibrium is not unique, describe the equilibrium that gives the players the highest payoffs. Would Fran be better off? Would Elo be better off?

Total pages: 6

Total marks: 100